Problem 1

In this question, first we need to define functions to calculate first derivative, second derivatives, partial derivatives and greeks for GBSM. The values are shown below.

Using closed-form formula

	Call Greeks	**Put Greeks**
Delta	**0.5342650692693474**	**-0.46573493073065264**
Gamma	**0.04005712070020568**	**0.04005712070020568**
Vega	**19.71962666579851**	**19.71962666579851**
Theta	**-24.898522316969515**	**-18.786996965277233**
Rho	**7.583586080244792**	**-7.277010958127815**
Carry Rho	**7.966245676523029**	**-6.944415968299725**

Using finite derivative function

	Call	**Put**
Delta	**0.5340091223970944**	**-0.46551181431908617**
Gamma	**0.040037932080849714**	**0.040037960502559145**
Vega	**19.71017887198201**	**19.71017887198201**
Theta	**-24.898857359268334**	**-18.78733200548055**
Rho	**-0.3826595967950652**	**-0.3325949902830416**
Carry Rho	**7.96624576389604**	**-6.944415901223522**

Comparing those results, we can see that they are very similar, except Rho. Then we need to implement the binomial tree valuation for American options with and without discrete dividends.

American option value

Binomial tree value without dividend for call: 4.2698585632362684

Binomial tree value without dividend for put: 3.684138176821656

Binomial tree value with dividend for call: 4.112836095267345

Binomial tree value with dividend for put: 4.1105345298444895

GBSM

	Call (With Dividend)	**Put (With Dividend)**
Delta	**0.5307762251476333**	**-0.49309859110868715**
Gamma	**0.039547614557584154**	**0.03430520707665785**
Vega	**19.574632054868868**	**19.82401324750338**
Theta	**-24.86404674998166**	**-18.54000028767766**
Rho	**6.835035197376715**	**-7.20498855727314**
Sensitivity	**-0.115**	**0.512**

Here we find out that the sensitivity of the American call option is -0.115 and the sensitivity of the American put option is 0.512

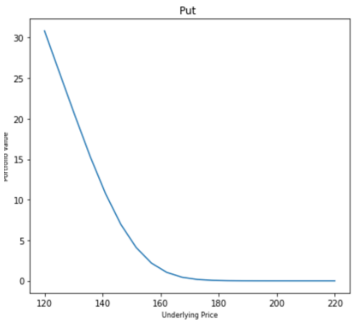
Problem 2

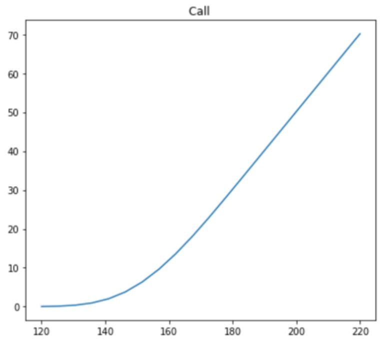
First we need to use the prepared function to turn the prices into the returns and make the mean value to be 0. Then I fit it with normal distribution and calculate the price 10 days ahead of AAPL, just like problem 3 in last week. Then we have the matrix below.

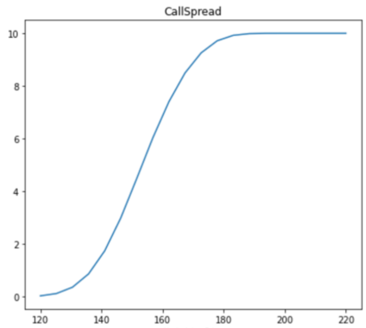
	Mean	**VaR**	**ES**
Call	6.67864522	6.679564564	6.876457
CallSpread	4.08644531	4.670913546	4.7653311
CoverCall	145.2465547	13.65464545	18.2362487
ProtectedPut	154.2654672	7.245642659	8.7648327
Put	6.76864845	4.472465221	4.7542624
PutSpread	3.89764355	2.746863114	2.8797589
Stock	149.378067	17.45753744	22.04687654
Straddle	13.1543546	1.342444311	1.087788674
SynLong	-0.1345534	19.35464765	24.6798679

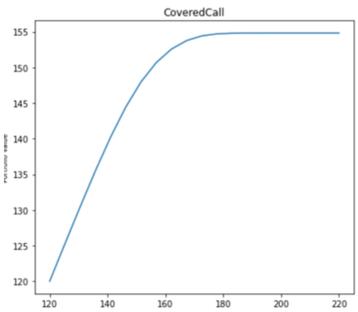
	Mean	**VaR**	**ES**
Call	0	9.35262476	11.13647113
CallSpread	0	5.475377756	6.8768675
CoverCall	0	10.537353	13.134513
ProtectedPut	0	12.7870760	15.47686798

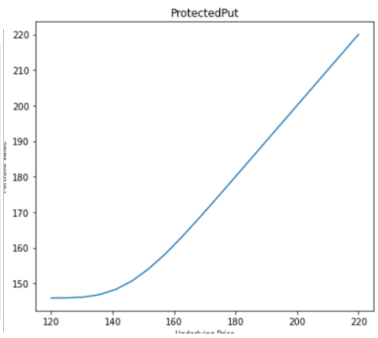
Put	0	8.15423626	10.1534315
PutSpread	0	4.78908709	6.8978975
Stock	0	17.6575381	22.457547
Straddle	0	1.24576587	1.8756755
SynLong	0	17.98764876	33.8764965

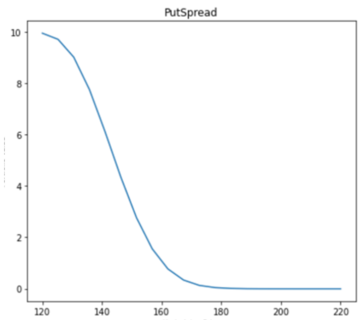


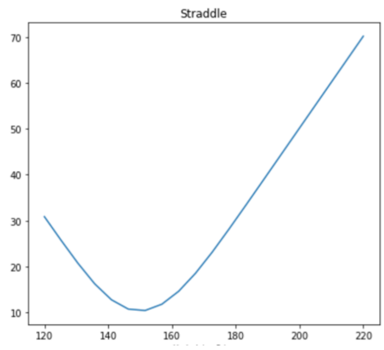


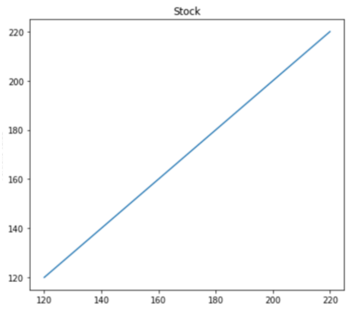


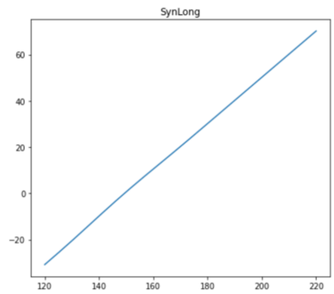












Looking at the graph above and compare it with problem 3 last week, we can see that the graph of Call, CallSpread, Stock, SynLong and CoveredCall are similar from the graph last week. But for Protected Put, Put, PutSpread and Straddle, the graph is different.

Problem 3

For this problem, we first need to read the files and put the data together, and we can use a prepared function to turn the prices into returns. After we have the return, we calculate the arithmetic expected return in the past 10 years. We find the geometric mean and covariance matrix of the stocks and from here, we try to find a maximized sharpe ratio.

```
AAPL      0.171146
META      0.762903
UNH      -0.037641
MA        0.271602
MSFT      0.192290
NVDA      0.950026
HD        0.282706
PFE       0.005805
AMZN      0.316488
BRK-B     0.146426
PG        0.145970
XOM       0.110124
TSLA      0.139654
JPM       0.407970
V         0.260227
DIS       0.462612
GOOGL     0.262884
JNJ      -0.038059
BAC       0.371124
CSCO      0.263864
dtype: float64
```

	AAPL	META	UNH	MA
AAPL	0.187678	0.312484	0.043679	0.127405
META	0.312484	1.579908	0.031448	0.243338
UNH	0.043679	0.031448	0.058855	0.039281
MA	0.127405	0.243338	0.039281	0.164553
MSFT	0.152881	0.322789	0.043127	0.127062
NVDA	0.430491	0.931842	0.091133	0.368959
HD	0.104033	0.233174	0.033209	0.096262
PFE	0.039731	0.083442	0.031809	0.044228
AMZN	0.203032	0.503301	0.046285	0.172149
BRK-B	0.077363	0.129622	0.031884	0.071805
PG	0.051068	0.069139	0.031573	0.046433
XOM	0.050553	0.042602	0.029309	0.045088

The maximized sharpe ratio is 1.5345634522, and the weights are

AAPL	0
META	4.26
UNH	0
MA	0
MSFT	0
NVDA	0
HD	0
PFE	0

AMZN	0
BRK-B	24.7
PG	2.4
XOM	7.65
TSLA	1.4
JPM	58.64
V	0
DIS	0
GOOGL	0
JNJ	0
BAC	0.94
CSCO	0