Problem 1

From Note:

1. Classical Brownian Motion

$$P_t = P_{t-1} + r_t$$

2. Arithmetic Return System

$$P_t = P_{t-1}\left(1 + r_t\right)$$

3. Log Return or Geometric Brownian Motion

$$P_t = P_{t-1} e^{r_t}$$

Here is the calculation of the three types of price returns

Classical brownian Motion:

$$P_t = P_{t-1} + r_t$$

$$E(P_t) = E(P_{t-1}) + E(r_t)$$

$$E(P_t) = P_{t-1}$$

$$sd(P_t) = sd(P_{t-1} + r_t)$$

$$sd(P_t) = sd(P_{t-1}) + sd(r_t)$$

$$sd(P_t) = \sigma$$

Arithmetic Return System

$$P_t = P_{t-1} e^{r_t}$$

$$E(\ln(P_t)) = E(\ln(P_{t-1} e^{r_t}))$$

$$E(\ln(P_t)) = E(\ln(P_{t-1})) + E(r_t)$$

$$E(\ln(P_t)) = \ln(P_{t-1})$$

$$sd(\ln(P_t)) = sd(\ln(P_{t-1} e^{r_t}))$$

$$sd(\ln(P_t)) = sd(\ln(P_{t-1})) + sd(r_t)$$

$$sd(\ln(P_t)) = \sigma$$

Log Return or Geometric brownian Motion

$$P_t = P_{t-1} e^{r_t}$$

$$E(\ln(P_t)) = E(\ln(P_{t-1} e^{r_t}))$$

$$E(\ln(P_t)) = E(\ln(P_{t-1})) + E(r_t)$$

$$E(\ln(P_t)) = \ln(P_{t-1})$$

$$sd(\ln(P_t)) = sd(\ln(P_{t-1} e^{r_t}))$$

$$sd(\ln(P_t)) = sd(\ln(P_{t-1})) + sd(r_t)$$

$$sd(\ln(P_t)) = \sigma$$

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For Classical Brownian Motion, the mean value is 50.00582620178858, the standard deviation is 0.5016530460284763
For Arithmetic Return System, the mean value is 50.29131008942892, the standard deviation is 25.08265230142381
For Log Return or Geometric Brownian Motion 3.9178492072167237, the standard deviation is 0.5016530460284763
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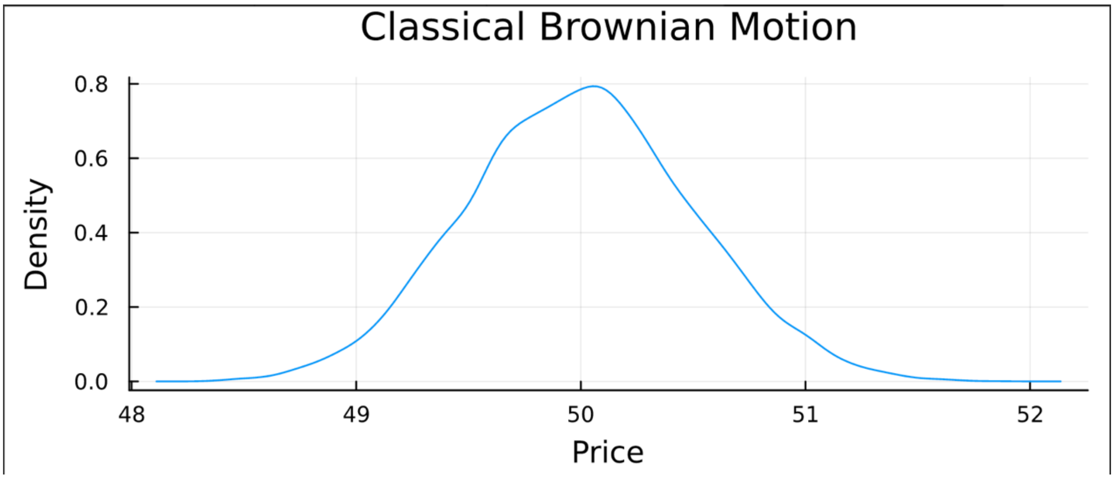

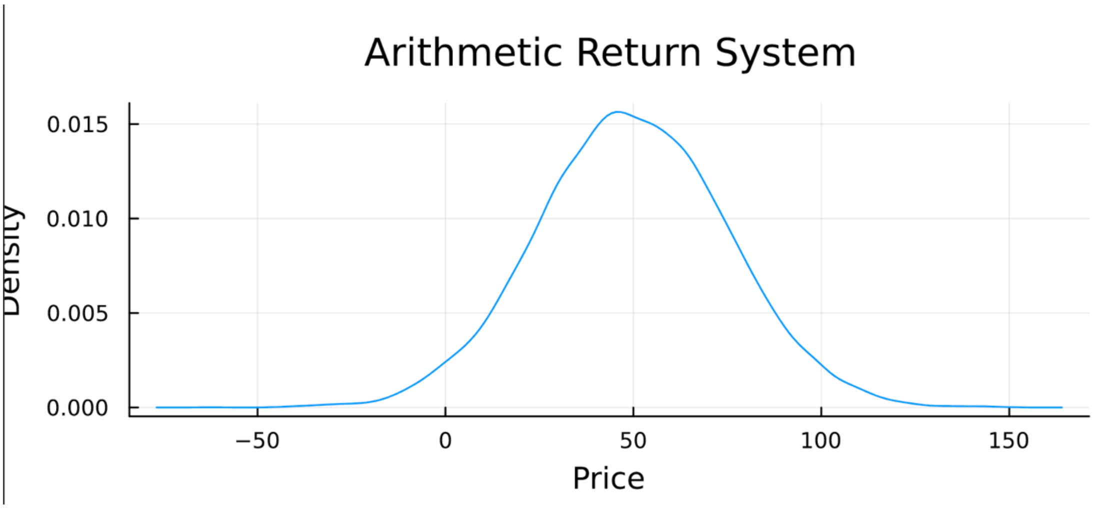

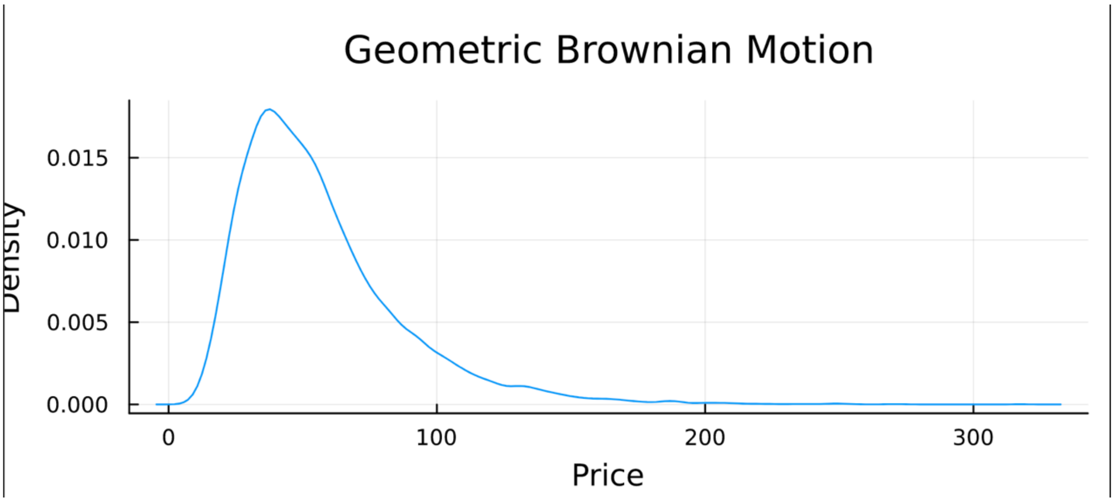

Geometric Brownian Motion
Density
0.015
0.010
0.005
0.000
0
100
200
300
Price

Problem 2

For this problem, I first modified the return_calculate() code which allow the user to choose whether to use discrete or log method for the calculation. Then I calculate the arithmetic returns for all prices. Following the instruction, I picked the stock META and calculated the mean value of it. I subtract each value by the mean of META so that the mean of META is 0, and I check it in my code. Here are the results of Var using different methods.

Normal distribution: 0.06560156967533284

Normal distribution with an Exponentially Weighted variance (λ = 0. 94) : 0.09184203076434297

MLE fitted T distribution: 0.0843785689437

Fitted AR(1) model: 0.06560235464639878

Historic Simulation: 0.05462007908237871

Result: Normal distribution with an Exponentially Weighted variance > MLE fitted T distribution > Fitted AR(1) model > Normal distribution > Historic Simulation

Problem 3

For this problem, I use an exponentially weighted covariance with lambda = 0.94, and here are the VaR of each portfolio

	Portfolio A	Portfolio B	Portfolio C	Total
Delta Normal VaR	$5670.7075	$4494.9984	$3786.9260	$13952.6319
Monte Carlo Method	$8032.5840	$6732.9929	$5638.2563	$20403.8332