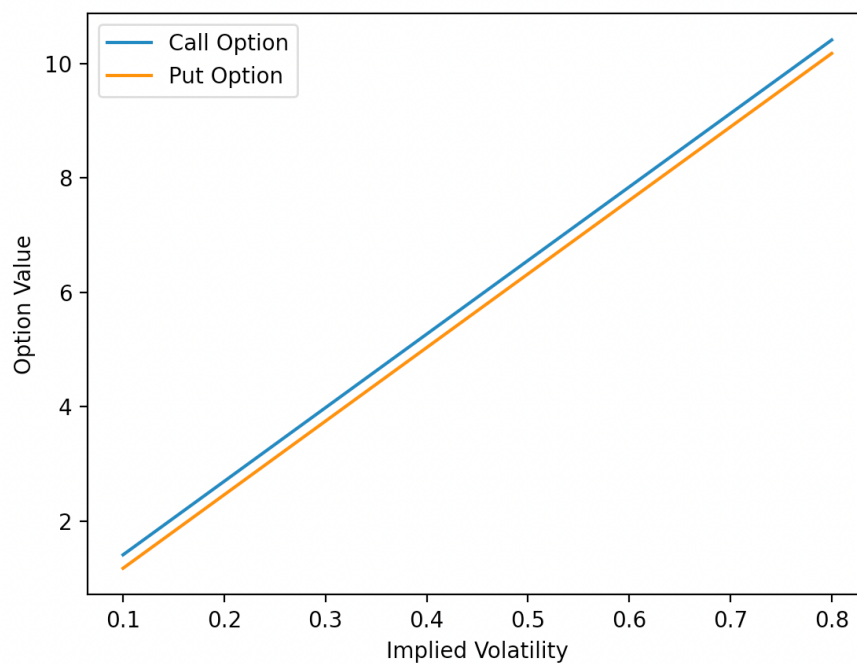


## Problem 1

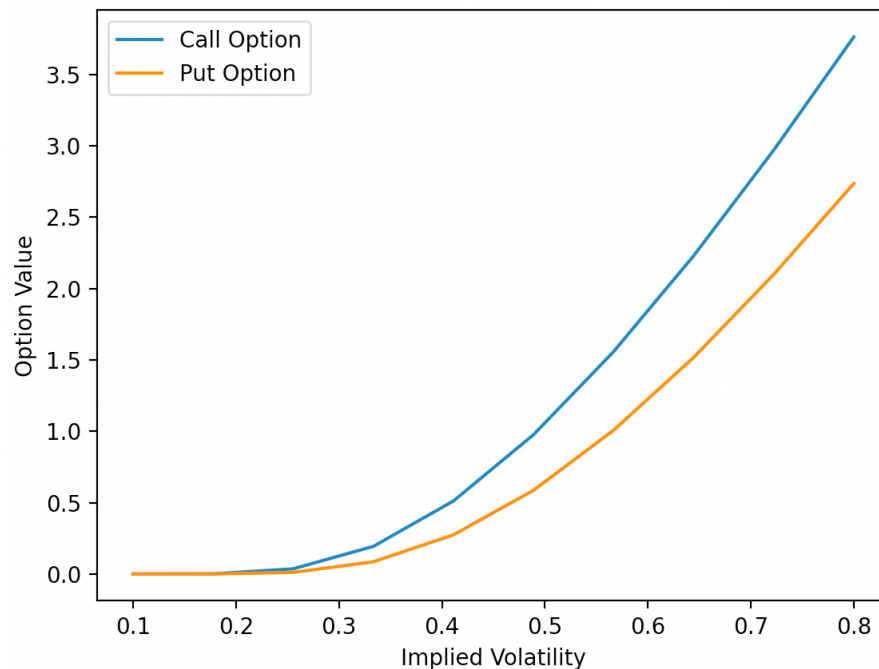
For this problem, we first use the module datetime to calculate the time to maturity from 3/3/2023 to 3/17/2023

**Time to maturity: 0.038356164383561646**

Then we define a function to calculate the value of options and input different level of implied volatility. Here is the plot of both options for different volatility with same strike.



Now we need to graph it with different strikes



The plotted graphs show the value of the call and put options at various implied volatility levels. As the implied volatility increases, the value of both call and put options increases. This is because higher implied volatility indicates a higher expected price movement in the underlying asset, making the options more valuable due to the increased potential for profit. Supply and demand can affect the implied volatility of options. When there's a higher demand for options, their prices increase, which in turn increases the implied volatility. This higher demand can be driven by various factors. On the other hand, when the demand for options is low, their prices decrease, leading to lower implied volatility.

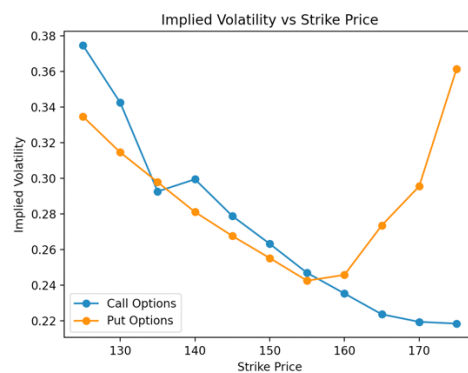
For example, if market participants expect significant price movements soon, they may demand more options to hedge their positions. This increased demand drives up option prices, which then results in higher implied volatility. Conversely, if market participants expect minimal price movements or have a general lack of interest in options, the demand and prices for options would be lower, causing the implied volatility to decrease.

## Problem 2

Using the function defined in problem 1, we can calculate the implied volatility for all the stocks.

|    | Stock | Expiration | Type | Strike | Last Price | Implied Volatility |
|----|-------|------------|------|--------|------------|--------------------|
| 0  | AAPL  | 4/21/2023  | Call | 125    | 27.300     | 0.374597           |
| 1  | AAPL  | 4/21/2023  | Call | 130    | 22.575     | 0.342351           |
| 2  | AAPL  | 4/21/2023  | Call | 135    | 17.750     | 0.292522           |
| 3  | AAPL  | 4/21/2023  | Call | 140    | 13.850     | 0.299358           |
| 4  | AAPL  | 4/21/2023  | Call | 145    | 9.975      | 0.278743           |
| 5  | AAPL  | 4/21/2023  | Call | 150    | 6.700      | 0.263141           |
| 6  | AAPL  | 4/21/2023  | Call | 155    | 4.050      | 0.246828           |
| 7  | AAPL  | 4/21/2023  | Call | 160    | 2.210      | 0.235242           |
| 8  | AAPL  | 4/21/2023  | Call | 165    | 1.035      | 0.223567           |
| 9  | AAPL  | 4/21/2023  | Call | 170    | 0.460      | 0.219339           |
| 10 | AAPL  | 4/21/2023  | Call | 175    | 0.195      | 0.218342           |
| 11 | AAPL  | 4/21/2023  | Put  | 125    | 0.405      | 0.334615           |
| 12 | AAPL  | 4/21/2023  | Put  | 130    | 0.665      | 0.314473           |
| 13 | AAPL  | 4/21/2023  | Put  | 135    | 1.120      | 0.297772           |
| 14 | AAPL  | 4/21/2023  | Put  | 140    | 1.840      | 0.280994           |
| 15 | AAPL  | 4/21/2023  | Put  | 145    | 3.010      | 0.267532           |
| 16 | AAPL  | 4/21/2023  | Put  | 150    | 4.750      | 0.255134           |
| 17 | AAPL  | 4/21/2023  | Put  | 155    | 7.150      | 0.242417           |
| 18 | AAPL  | 4/21/2023  | Put  | 160    | 10.575     | 0.245700           |
| 19 | AAPL  | 4/21/2023  | Put  | 165    | 14.925     | 0.273493           |
| 20 | AAPL  | 4/21/2023  | Put  | 170    | 19.425     | 0.295414           |
| 21 | AAPL  | 4/21/2023  | Put  | 175    | 24.625     | 0.361243           |

Then we can plot the graph of implied volatilities vs. strike prices



The shape of these graphs can provide insights into the market's expectations and potential risks. A common pattern seen in equity options is the "volatility smile" or "volatility smirk". In the case of a volatility smile, implied volatility is lower for at-the-money (ATM) options and higher for out-of-the-money (OTM) and in-the-money (ITM) options. This pattern suggests that market participants expect larger price movements in either direction. A volatility smirk, on the other hand, shows higher implied volatility for OTM put options compared to OTM call options, indicating that market participants anticipate higher risks or larger downward price movements. There are some related market dynamics, such as:

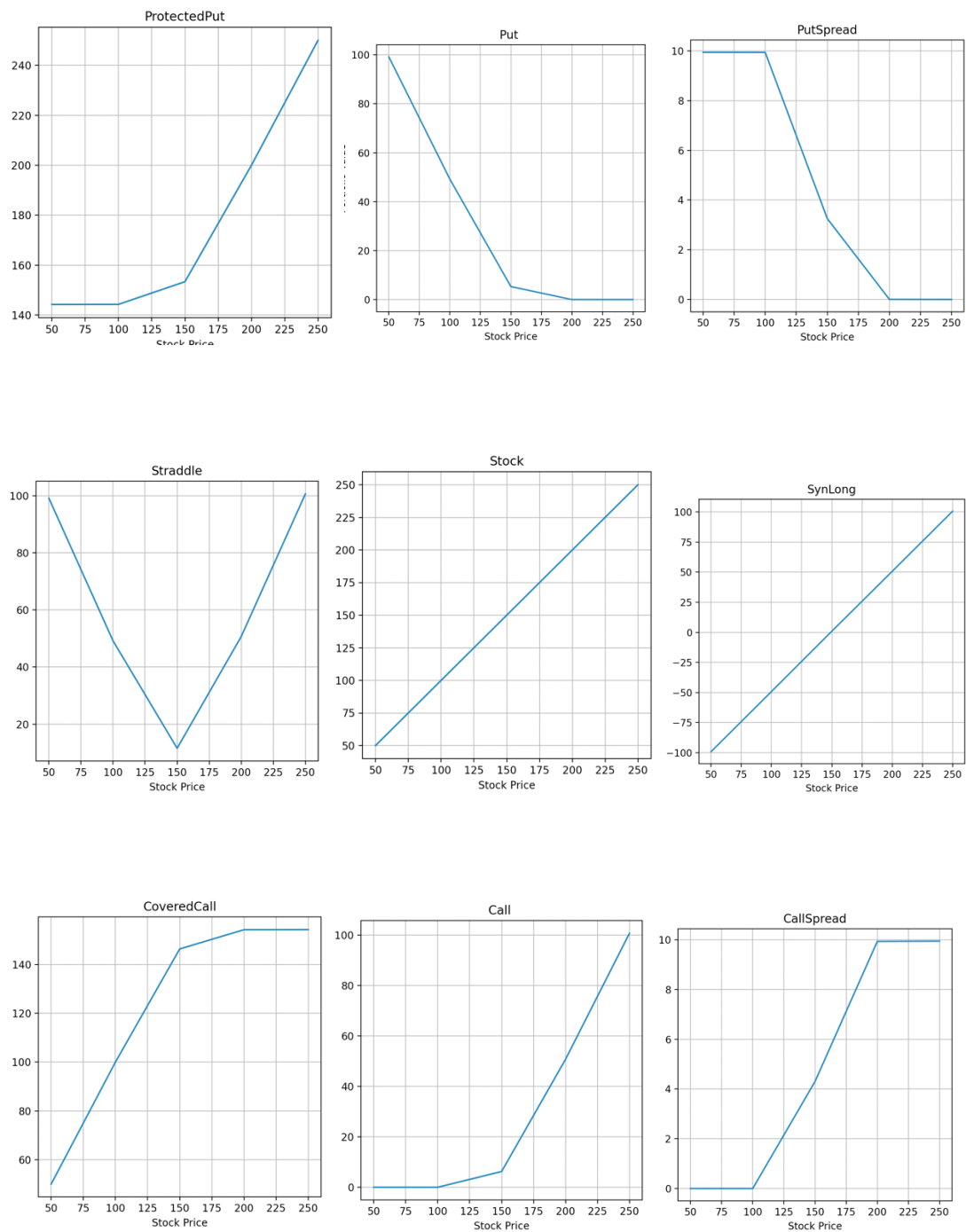
1. Market sentiment: The shape of the graph can be influenced by market participants' collective expectations of future price movements. For example, a volatility smirk might indicate that investors are more concerned about potential downside risks than upside potential.
2. Supply and demand: The demand for options can influence their prices and, consequently, their implied volatilities. Higher demand for OTM put options, for example, can lead to higher implied volatilities for these options.
3. Skewness and kurtosis of returns: If the underlying asset's return distribution is not symmetric or has fat tails, the implied volatility pattern may deviate from a flat line. In such cases, the volatility smile or smirk can emerge because of the market pricing in these

characteristics.

4. Market events and uncertainties: Upcoming events, such as earnings announcements or macroeconomic news, can influence the market's expectations of future price movements, leading to changes in the shape of the implied volatility graph.

We should keep in mind that the shape of the implied volatility graph can change over time as market dynamics and expectations evolve.

### Problem 3



For Call and Put options, it makes or loses money when the stock price goes up or down, they are just basic options. For CallSpread and PutSpread,

both of them have lower return compare to the normal options and the return is lower too. For CoveredCall and ProtectedPut, they put stocks and options together and it is very similar to a normal option, and they have mixed return rate. SynLong and Stock both have high risks and high return but SynLong has extremely high risk. Straddle always makes money as long as the price moves, so the risk is relatively low.

After we calculate the log returns of AAPL, demean the series so there is 0 mean and we fit an AR(1) model to AAPL returns. We simulate AAPL returns 10 days ahead and calculate the corresponding mean, Var, and ES for each portfolio, and the result is shown below.

|             | Mean      | VaR       | ES        |
|-------------|-----------|-----------|-----------|
| Call        | 8.815666  | 1.484726  | 0.940099  |
| CallSpread  | 4.812076  | 1.247209  | 0.813579  |
| CoveredCall | 145.36643 | 134.47055 | 130.88859 |
| ProtectedPu | 155.80697 | 146.64037 | 145.78211 |
| Put         | 6.341292  | 0.970105  | 0.605764  |
| PutSpread   | 3.335317  | 0.653875  | 0.415373  |
| Stock       | 151.41599 | 135.08739 | 131.24506 |
| Straddle    | 15.156958 | 12.590395 | 12.583541 |
| SynLong     | 2.474374  | -13.88346 | -17.75213 |