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Taming Risk

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ABSTRACT

We look to optimize the weighting of any portfolio of stocks, in the S&P 500, by maximizing the Sharpe Ratio: we limit the covariance (risk) of the portfolio while maintaining accelerated return rates using gradient-based non-linear optimization, followed by a downstream Markov-Chain Monte Carlo (MCMC) forecasting model. In development, we utilized 3 years of historical data from 2020-2023 and multiple randomly selected portfolios to compute the maximum likelihood estimate for the weight of each stock. We then compared the initial portfolios to the optimized portfolios' predicted Sharpe Ratio in 2025. We find that our model consistently outperforms the initial weights, both in long-term and quarterly gains.

1. BACKGROUND

Our research aims to create a model that obtains the optimal portfolio from randomly selected tickers. As such, we look to create a more profitable and less risky portfolio without looking at different stocks individually. To do this, we use the Sharpe Ratio (4), the most commonly used portfolio metric. Specifically, it looks at a portfolio's return-to-volatility ratio: the higher the Sharpe ratio, the greater the return for one unit of risk taken, meaning a greater asymmetry of returns. Every stock in our simulated portfolios is tracked with large-scale periodized quarterly return data, which is used to calculate the Sharpe Ratio accurately.

We allow short-selling in our portfolio construction, meaning weights can take on negative values, reflecting positions taken by betting on price declines in particular assets. Short selling involves selling stocks in anticipation of a price drop, then repurchasing them at a lower price; we profit by betting against those stocks. This flexibility expands the solution space and enables the model to exploit positive and negative return expectations across tickers. Additionally, it reflects a more realistic and comprehensive trading environment.

We utilized the scaled gradient descent minimization technique as defined by Amir Beck (2) by formulating the Sharpe Ratio with the covariance matrix of the tickers' quarterly returns and the weights of each ticker; we transmuted the ill-conditioned covariance matrices using diagonal scaling. Additionally, to ensure mathematical stability, we assumed a zero risk-free rate; we are merely comparing the metric internally, making pure financial accuracy less important than relative values. The theory behind our optimizer goes as follows:

$$\text{Objective: } f(w) = -\frac{w^\top \mu}{\sqrt{w^\top \Sigma w}} \tag{1}$$

Where:

- w = vector of portfolio weights (normalized to sum to 1)
- μ = vector of expected returns
- Σ = diagonally scaled covariance matrix of asset returns

This objective maximizes the return-to-volatility ratio, commonly used in practical scenarios where only relative performance matters.

The numerator is the portfolio return:

$$R_p = w^\top \mu$$

The denominator is the portfolio volatility:

$$\sigma_p = \sqrt{w^\top \Sigma w}$$

We compute the gradient of the negative Sharpe objective as:

$$\nabla f(w) = -\left(\frac{\mu \cdot \sigma_p - R_p \cdot \Sigma w/\sigma_p}{\sigma_p^2}\right) \tag{2}$$

This expression corresponds to:

$$\nabla f(w) = -\frac{\mu \cdot \sqrt{w^\top \Sigma w} - \frac{w^\top \mu}{\sqrt{w^\top \Sigma w}} \cdot \Sigma w}{w^\top \Sigma w}$$

The weights are updated using gradient descent:

$$w^{(t+1)} = w^{(t)} + \eta \cdot (-\nabla f(w^{(t)})) \tag{3}$$

After each step, the weights are renormalized:

$$w^{(t+1)} \leftarrow \frac{w^{(t+1)}}{\sum w^{(t+1)}} \tag{4}$$

At each iteration t, the following are computed and tracked:

- Objective Value:

$$f(w^{(t)}) = -\frac{(w^{(t)})^\top \mu}{\sqrt{(w^{(t)})^\top \Sigma w^{(t)}}}$$

- Portfolio Return:

$$R_p^{(t)} = (w^{(t)})^\top \mu$$

- Portfolio Variance:

$$\sigma_p^2 = (w^{(t)})^\top \Sigma w^{(t)}$$

- Weights History: The full weight vector $w^{(t)}$

The optimization terminates when either the gradient norm falls below a threshold ϵ:

$$\|\nabla f(w^{(t)})\| < \epsilon$$

or the maximum number of iterations is reached.

This research is especially timely given today's exacerbated market volatility and the faltering of traditional investment strategies. As risk-aware investing becomes increasingly important, optimizing portfolios based on the Sharpe Ratio offers a robust, data-driven way to manage uncertainty. Moreover, we leverage open-source tools, empowering laymen to make risk-adjusted decisions.

Our research is critical in creating a model that can successfully capture financial trends regardless of the input portfolio. Its efficacy in maximizing the return-to-volatility ratio applies to all investors looking to capitalize on mathematically sound portfolio optimization.

2. DATA

To test our model's true efficacy, we focused our attention on data from 2020-2023, a period marked by meteoric recession and bearish gains, which put extreme pressure on our calculations and assumptions. Thus, by nature, our data is extremely volatile, with vastly differing, ill-conditioned values.

Our data were obtained using yfinance (1). This Python library fetches financial data from Yahoo Finance for any known ticker in the system: it is updated daily with up-to-date financial data. The data were filtered to contain only quarterly financial data, giving each asset a representative, holistic overview. We dropped tickers with excessive NaN values to clean the data and performed a consistent backfilling procedure for individual NaN data points. Our filtration ensured numerical accuracy in computation in all of the critical components of the Sharpe Ratio, ensuring we yielded accurate results. We also utilized a publicly sourced table of all S&P 500 companies to randomly select assets for our portfolio.

Potential sources of bias in the data include the missing information from the returns not captured in the quarterly return; inside each quarter, there could have been a potential crash and resurgence that our model never analyzed. Moreover, the backfilling procedure in our data scraping could lead to an intrinsic bias by forcefully filling in spotty, unavailable data with unrepresentative values from the last quarter. Although it is of note, this practice is commonplace when utilizing financial data and is not unique to our study.

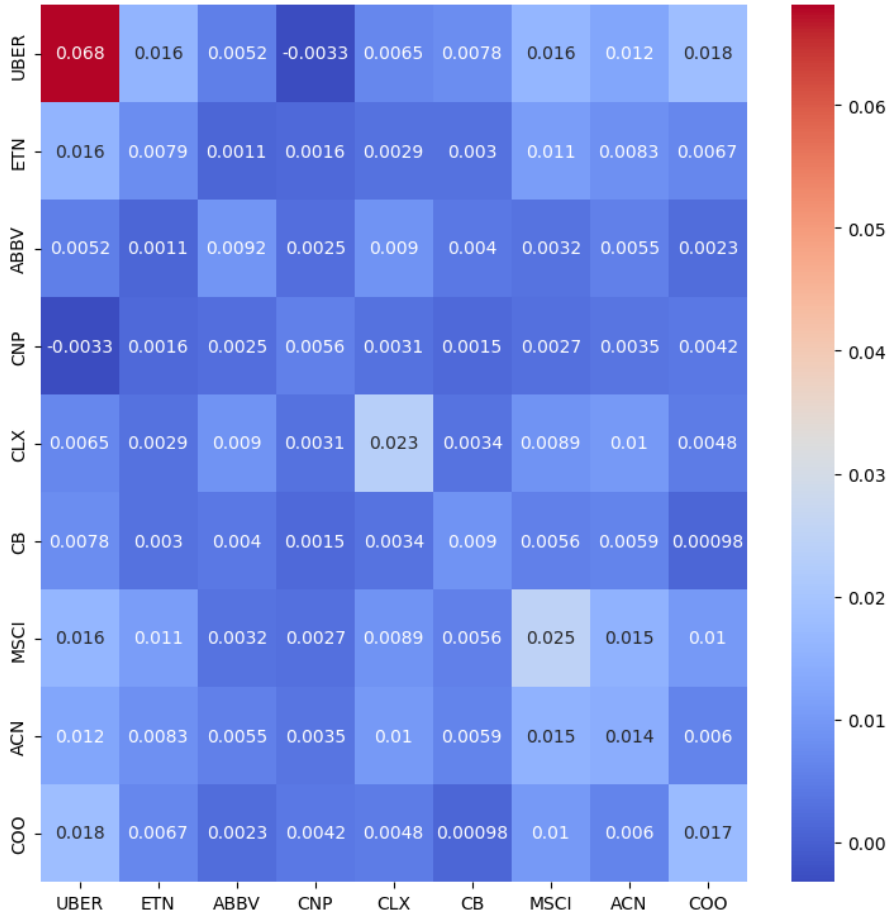

Figure 1. A heatmap demonstrating the covariance between quarterly returns for our randomly selected portfolio. The internal numbers represent the covariance between two assets, with their tickers labeled along the axis.

3. PROCEDURE

Initially, we conducted a string of preprocessing events. We used random selection to create an adaptable portfolio with various amounts of unique stocks. We then set up the optimization problem by calculating the covariance matrix, which quantified the dependencies between each selected asset's quarterly returns data; this can be seen in Fig. 1. This informed us how the stocks moved as a unit and the volatility involved in each ticker. Next, we determined the initial weights to be uniform; these values then needed to be subsequently normalized so the weights were all of valid proportions, adding up to one. This decision is mathematically valid, as our model looks to improve existing portfolios, so the starting point is arbitrary. Additionally, we computed the returns for each asset to measure the mean quarterly return. We are looking to compute the long-term optimized weights, making the mean quarterly return a representative value for the stock.

Secondly, we formulated the earlier defined optimization problem, using a string of linear algebra functions and a fixed constant step-size to automate the calculation via Python. This uncovered the portfolio weights yielding the greatest Sharpe Ratio. Additionally, it retrieves the weights, the expected returns, and the portfolio volatility as a function of the iterative steps. These additional metrics were important in determining our model's efficacy and each portfolio's accurate convergence tendencies. The productive difference between the initial weights and the optimized weights Sharpe Ratio for an arbitrary portfolio can be seen in Fig. 2. Furthermore, Figs. 3 and 4 visualize the convergence of our objective function and the weights. All of the figures in our paper are selected from one of the portfolios we analyzed randomly: this was selected because it was a mid-level performing, illustrative example.

Finally, we created two distributions of the Sharpe Ratios for each quarter from 2020 to 2023 using an adaptive rolling window: one for the optimized and one for the initial weights. From there, we utilize MCMC to fit a piecewise

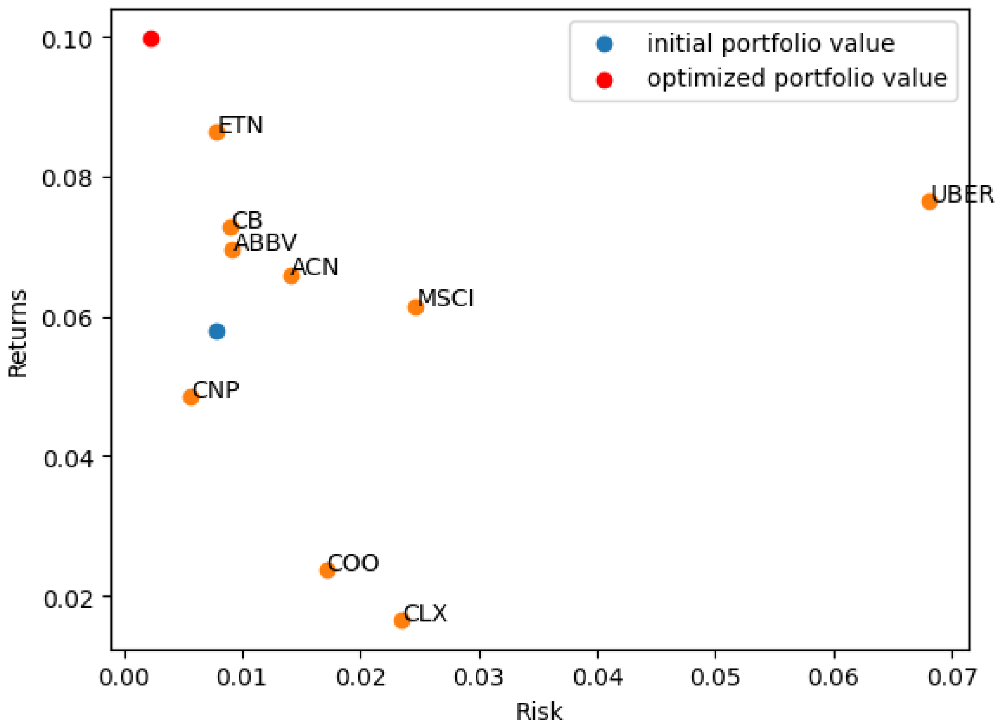

Figure 2. Visualization of the Risk vs. Return for the initial weights and the optimized weights, with the individual tickers' unweighted influence also plotted for further visualization.

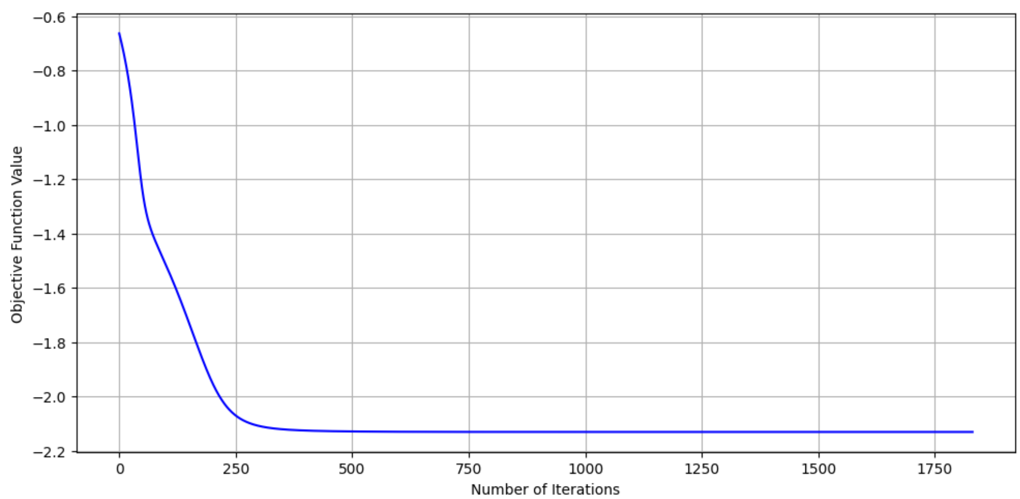

Figure 3. The objective Sharpe Ratio function as a product of the iteration count for the arbitrary portfolio. Note, the value is minimized as it is the negative Sharpe Ratio, so in practice is the same as maximizing the non-negative equivalent.

linear best-fit model–with two flexible breakpoints–to the data. We utilized narrow, uninformative priors to encourage less drastic, more representative slopes and change points. Our logic also induces noise into the data to better fit the sparsely populated, discrete Sharpe Ratios. Finally, we used predictive forecasting to estimate the portfolio's Sharpe Ratio at the start of 2025 with a 95% confidence interval derived from the Bayesian inference. These values were then compared to see if the initial optimization model consistently produced better Sharpe Ratios each quarter. Figs. 5 and 6 demonstrate the forecasting period and the initial data fitting found by the MCMC chains for each parameter.

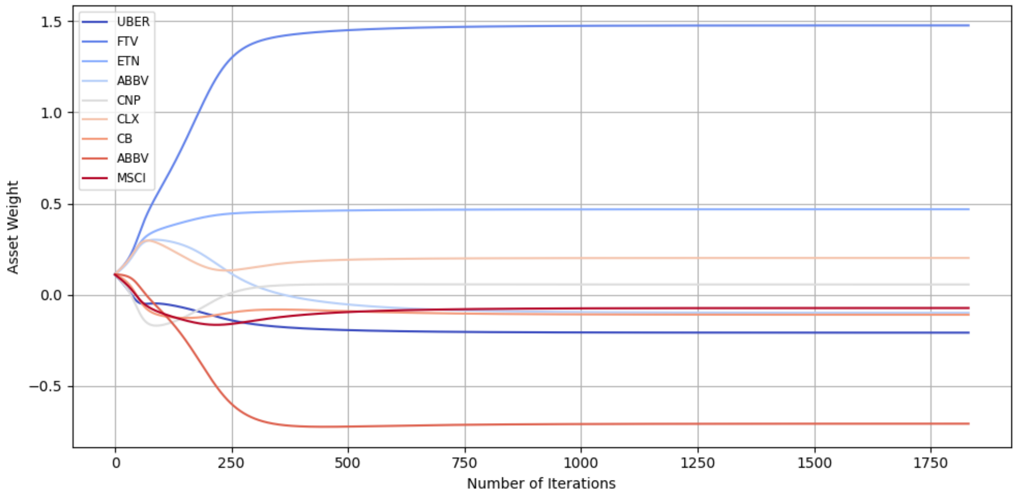

Figure 4. The asset weights visualized over the iteration count, demonstrating the vast difference between the initial and the optimized values. Additionally, it shows each of the parameters convergence.

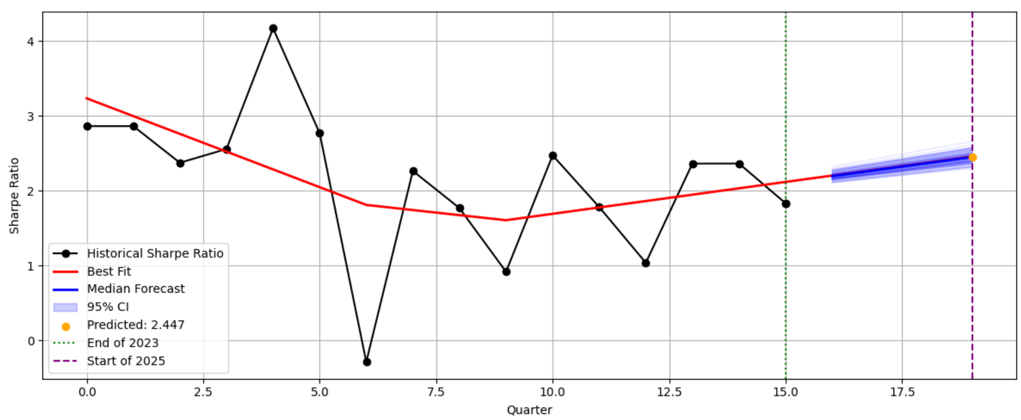

Figure 5. The fitted Sharpe Ratio data for our portfolio using optimized weights. The original Sharpe Ratios are underplotted, with the linear fits overplotted. The forecasting period along with the confidence interval are also shown.

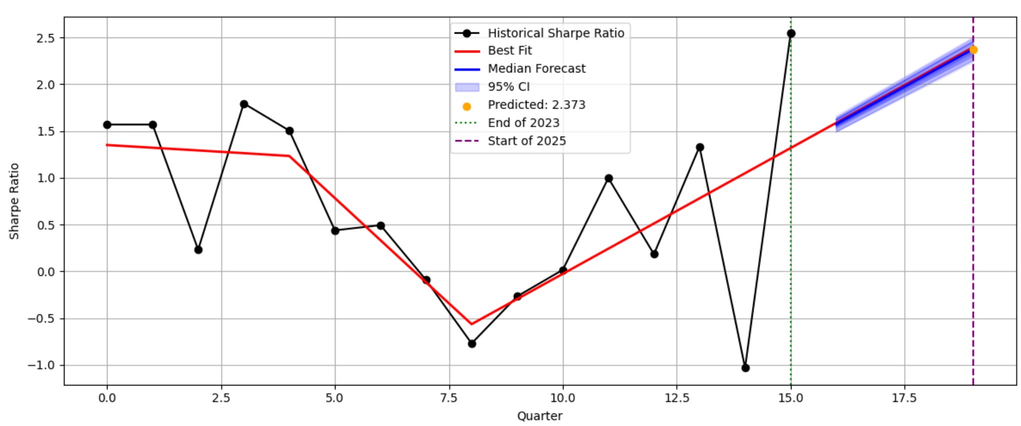

Figure 6. The fitted Sharpe Ratio data for our portfolio using the initial weights. The original Sharpe Ratios are underplotted, with the linear fits overplotted. The forecasting period along with the confidence interval are also shown.

4. ANALYSIS AND DISCUSSION

We looked at 10 different randomized portfolios, each selecting 11 different assets, and ran our model to determine the meaningful difference in our results. The following is the increase in ratio for each random portfolio we looked at:

Increase
1.310
3.384
1.016
1.473
0.295
1.138
0.403
0.837
0.949
0.685

Table 1. The increase in Sharpe Ratio for each randomized portfolio.

The optimizer provided a consistent and tangible increase in our long-term Sharpe Ratio, proving the model's efficacy. It is important to note that given the uniform nature of the initial weights, along with differing quarterly returns for each ticker, the level of increase is unimportant as long as it demonstrates growth; the purpose is not to create the most significant change in Sharpe Ratio but to obtain the best portfolio in every scenario. Fig. 2 and 3 show this increase effectively for our demonstrative portfolio, and the actual impact of our optimizer.

Furthermore, as exhibited by Fig.2, the optimizer yields higher performing Sharpe Ratios using the long-term data. Characteristically, the new weights return a heightened return and maintain relative stability for risk; logically, this relationship makes sense as the Sharpe Ratio is the ratio of these two values.

Additionally, in Fig. 3, we can see that the negative Sharpe Ratio is minimized until stability, demonstrating that the model actually obtains the maximizing weights for the Sharpe Ratio. This is further demonstrated by the convergence of the weights in Fig. 4, which are no longer exploring the parameter space.

Next, when analyzing our MCMC forecasting, due to the sparsity of quarterly return data, it cannot perfectly reflect what the values will be in 2025, as demonstrated by the example in Figs. 5 and 6. As such, we do not report the actual values but just the estimated general trend, and if we move towards a higher Sharpe Ratio in 2025 when inputting our long-term optimized weights. The results go as follows:

Trend	**Confidence Interval**
Greater	Not Overlapping
Greater	Not Overlapping
Greater	Not Overlapping
Greater	Overlapping
Greater	Not Overlapping
Greater	Not Overlapping
Greater	Overlapping
Less than	Overlapping
Greater	Not Overlapping
Greater	Overlapping

Table 2. Comparison of Sharpe Ratio increases across randomized portfolios, showing trend and confidence interval overlap for the optimized compared to the initial weights.

Again, we can see that the optimized weights perform better than the initial weights when looking at future values, another testament to the robustness of our initial optimization. The continued results demonstrate an improved Sharpe Ratio for short and long-term optimization.

5. CONCLUSIONS

In this study, we examined how the gradient descent method could adequately maximize the long-term Sharpe Ratio of a randomized portfolio. We analyzed quarterly data from one of the most economically unpredictable periods in financial history to test the robustness of our model. After analyzing multiple optimization runs, we see a consistent uptick in the Sharpe Ratio from the initial weights.

Furthermore, the forecasting period solidified that long-term benefits also translated into short-term profits. The consistently greater Sharpe Ratio in 2025 showed that our initial optimization went beyond minimizing long-term risk and created a better portfolio for the future.

For testing, we utilized historical data from 2020-2023 and forecasting through 2024, but the model can easily be used with current data or extended for longer-period optimization. Our research unveiled an operational portfolio optimizer, successfully guiding all tested portfolios to higher-profit regions. Using publicly sourced datasets, we have designed a model that anyone, regardless of resources, can operate to make calculated financial decisions.

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