Final project - Highway

Oliver Bielke

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1 Problem

1.1 Problem description

Simulate a 1, 2 and 3 lane highway with overtaking/lane-changing (for 2 and 3 lanes) and cars with different maximum speeds. How does the road capacity (flow rate) compare for different numbers of lanes?

1.2 Problem interpretation

For this project, a simulation of a highway with varying amounts of lanes (1-3 lanes) will be done. Then, the overall traffic flow (sum of the cars speeds per length of road) will be calculated for the different amount of lanes. This will then give an understanding of how the number of lanes affect the flow of traffic.

2 Model

2.1 Approximations

2.1.1 Highway

The highway had a set length, where the end of the road was connected to the beginning of the road. So, the cars could run indefinitely, but they would loop around and encounter each other over and over. This simplification was made to have the simulation be smaller while the traffic still mimicked a long highway.

2.1.2 Car dimensions

The cars were all modeled to have the same given length, and the width of the car was not taken into account.

2.1.3 Car position

The speed of the car was first calculated (see Section 2.1.4) for each car and then the new position was calculated based on the speed (Euler-Cromer).

The cars were densely packed at the beginning of the simulation.

2.1.4 Car speed

The cars had different maximum speeds with a normal distribution around a given speed (could be seen as the speed limit of the highway) with a given standard deviation.

The car speed was calculated based on the distance from the car ahead, while not exceeding the maximum speed. They all followed the "three second" rule, so their speed never exceeded one where it would take less than 3 seconds to reach the car ahead (if that car stood still at its current position).

2.1.5 Car acceleration

All the cars had the same maximum acceleration, which they could not exceed.

2.2 The parameters

2.2.1 Highway

The length of the road was set to 100 m for most of the study.

2.2.2 Car dimension

The length of the cars was set to 2 m.

2.2.3 Car speed

The average max speed was set to 30 m/s (which is $\sim$ 100 km/h) with standard deviation of 2 m/s ($\sim$ 7 km/h). This seemed like a reasonable variation of a normal highway.

2.2.4 Car acceleration

The maximum acceleration was set to 4.3 m/s^2, which is the acceleration of a Saab 9-5 Hirsch.[1]

[1] `https://en.wikipedia.org/wiki/Orders_of_magnitude_(acceleration)`

2.3 Discretization

2.3.1 Integrator

Euler-Cromer was used as an integrator, see Section 2.1.3 and Section 5.1.1 for more information.

2.3.2 Time step

A time step of 0.1 s was used, see Section 5.1.2 for more information.

3 Simulation

3.1 Animation

The simulation was tested by animating the cars (see Figure 1). This was so that the behavior of the cars could be more easily studied and compared to a real highway.

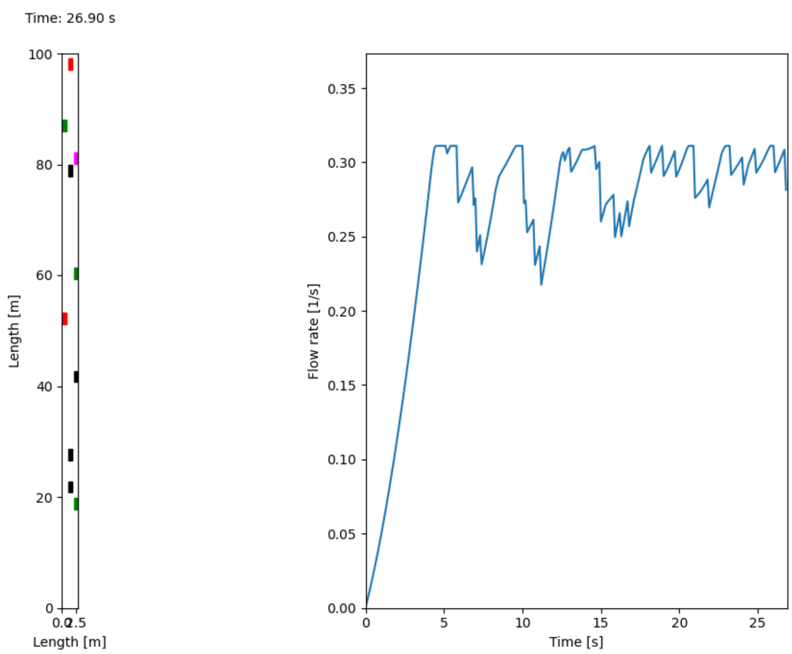


Figure 1: Example of the highway animation

3.2 Lane change

For each time step in the simulation, the lane changes are the first things that happen. This is so that the speeds and new positions are then calculated with the correct cars in front of them (as to avoid collisions).

3.2.1 Left lane change

To simulate normal driving behavior, it was decided that a car should want to change lane to the left if it can drive faster in that lane. This happens if there is a car in front in the current lane that makes the car slow down and if it were to change to the lane to the left, it would be able to speed up. Otherwise it stays in the same lane. Notice that the car does not want to overtake to the right, which is not allowed on a Swedish highway.

3.2.2 Right lane change

The cars want to change lane to the right as long as it does not affect their speed negatively. This is similar to normal driving behavior as the driver should be in the right-most lane, unless they are driving faster than the cars in that lane.

3.3 Speed change

The speed is updated by taking the minimum of the cars maximum speed, the speed reached with maximum acceleration, and the fastest speed to still follow the "3 second rule" (see Section 2.1.4).

$$v(t + \Delta t) = min(v_{max}, v(t) + a_{max} \cdot \Delta t, \frac{d_{car\_in\_front}}{3 \text{ s}})$$

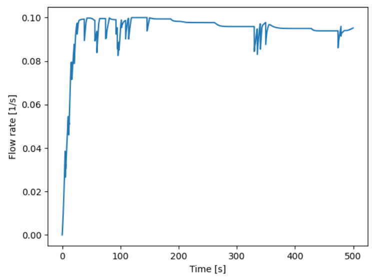
3.4 Position change

The position is simply calculated by adding the new calculated speed times the time step to the old position.

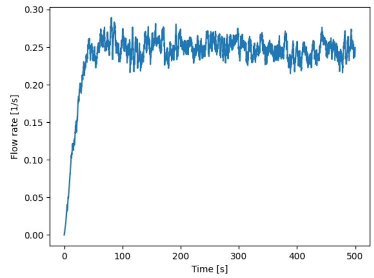
$$x(t + \Delta t) = x(t) + v(t + \Delta t) \cdot \Delta t$$

3.5 Flow rate

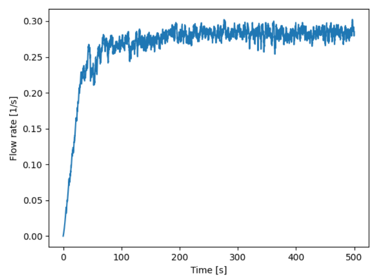
The flow rate was calculated as the sum of all the cars' speeds, divided by the total road length (road length times number of lanes). In Figure 2, we can see some examples of the flow rate for different simulations.



(a) 10 cars



(b) 40 cars

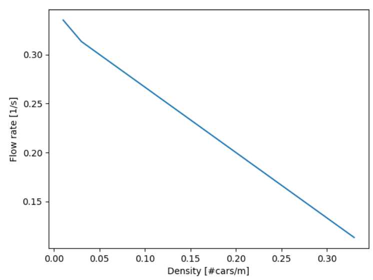


(c) 100 cars

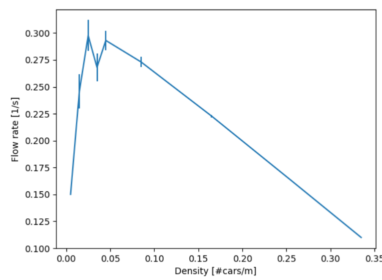
Figure 2: Flow rate vs time for different number of cars (number of lanes = 3, road length = 1 km, time step = 0.1 s)

4 Results

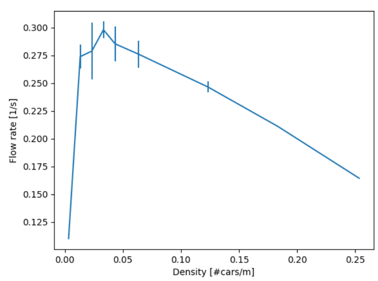
The fundamental diagrams for 1, 2 and 3 lanes were created, with estimated root mean square fluctuation for one simulation (per point) are seen in Figure 3 and with standard error estimate in Figure 4.



(a) 1 lane

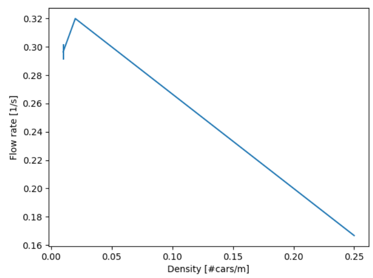


(b) 2 lanes

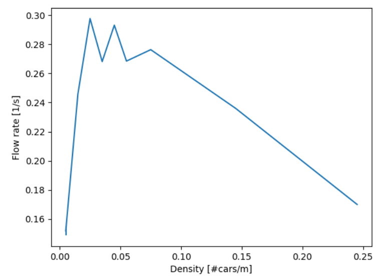


(c) 3 lanes

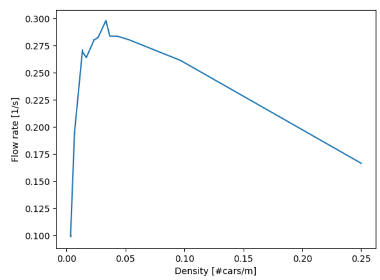
Figure 3: Fundamental diagram with estimated root mean square fluctuation as error bars (number of lanes = 3, road length = 100 m, time = 200 s)



(a) 1 lane



(b) 2 lanes



(c) 3 lanes

Figure 4: Fundamental diagram with standard error estimate as error bars (number of lanes = 3, road length = 100 m, time = 200 s, iterations = 5)

5 Discussion

5.1 Motivations

5.1.1 Integrator

In this project, Euler-Cromer was used as an integrator. This is when the updated speed is calculated with the current variables, and then the updated position is calculated with the updated speed.

This was chosen as the integrator because the cars are driven by humans. And the human reacts to the surroundings by changing the speed when breaking and accelerating. Then the position changes based on that. Any other more complicated integrator would not reflect the human factor of the simulation.

5.1.2 Time step

A time step of 0.1 s was chosen for the simulation. This was partly due to limitations of computer power and the efficiency of the code. But one argument for not having a lower time step would be that the human reaction time is not that fast, and having a smaller time step would give an unrealistic reaction time. The drawback is that the physics of the cars movements will be a bit worse.

5.2 Discussion of results

By observing the fundemental diagrams of Figure 4, we don't see as big of a difference as might be expected. It seems like for all three cases, they have a peak around 0.03-0.05 cars/m and they all decrease at around the same speed. This might be due to the fact that the simulation is quite small with only 100 m of road and cues build up very quickly. This is also supported by the very small standard error between the simulations (see Figure 4), which means that the simulations are very similar to one another, which could mean that there is a cue, which is not very dynamic. This is however contested by the root mean square fluctuation of one simulation (see Figure 3) which suggest that the simulation is actually more dynamic.
The short road length was chosen due to the increase in time taken for the simulation with each new car. So for a large road, many more cars would be needed for the density measured in this study.

6 Conclusion

There does not seem to be a major difference on the flow rate of traffic between the different amount of lanes on the highway. This could however be due to the relative small size of the simulation and a simulation of a larger road could give more accurate results.