Oliver Collins Lattice Paths

With the 2x2 path, there are a total of 4 segments you MUST travel on, regardless of the way you move. There MUST also be two paths that must go south. Therefore, there are a total of:

$$\binom{4}{2}=6 \text{ total paths}$$

We can apply this same logic to a 20x20 grid. There are a total of 40 possible segments with half (20) that must travel south. Therefore:

$$\binom{40}{20} = 137,846,528,820 \text{ total paths}$$