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1 Assumptions

- Rope is vertical.
- Kelp is horizontal.
- Kelp is infinitesimally thin.
- Light comes straight down.
- Plants are evenly spaced with spacing Δz.
- Each plant absorbs a certain percentage of the light it receives, denoted by α.
- The lengths of kelp plants in the x direction are normally distributed with mean μ and standard deviation σ.

That is, the lengths are distributed as

$$p(l) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{l-\mu}{\sigma}\right)^2} \tag{1}$$

2 Analysis

Then for the kth plant from the top, at a horizontal position x, the expected number of plants shading this spot is

$$\begin{aligned} N(k,x) &= (k-1) \cdot p(l \geq x) \\ &= (k-1) \int_x^\infty p(l)\, dl \\ &= (k-1) \int_x^\infty \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{l-\mu}{\sigma}\right)^2} dl \\ &= \frac{k-1}{2} \left[1 - \operatorname{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma} \right) \right] \end{aligned} \tag{2}$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

The irradiance as a function of depth and length is given by

$$I(z,x) = I_0 (1-\alpha)^{N(k,x)} e^{-K_d z} \tag{3}$$

3 Plot

The following parameters are used in the plot below.

$$
\begin{aligned}
z_{max} &= 10 \\
x_{max} &= 10 \\
\alpha &= 0.1 \\
I_0 &= 5 \\
K_d &= 0.1 \\
\Delta z &= 1
\end{aligned}
$$

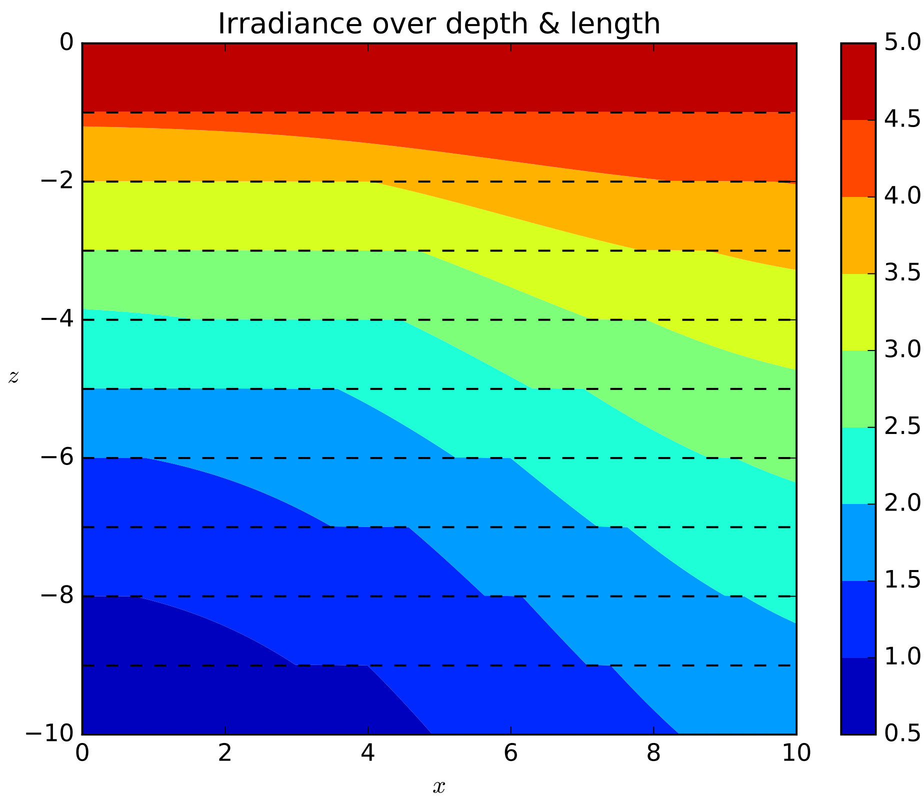

Figure 1: Irradiance as a function of space