1 Introduction

1.1 Coordinate System

1.2 Radiative Transfer Equation

$$\vec{\omega} \cdot \nabla L(\vec{r}, \vec{\omega}) = -(a(\vec{r}) + b(\vec{r}) + b \int_{4\pi} \beta(\vec{\omega} \cdot \vec{\omega'}) L(\vec{r}, \vec{\omega'}) \, d\vec{\omega'}) \tag{1}$$

1.3 Boundary conditions

Downwelling light:

$$L(x, y, 0, \vec{\omega}) = f(\vec{\omega}) \tag{2}$$

Upwelling light:

$$L(x, y, M, \vec{\omega}) = 0 \tag{3}$$

1.4 All angles are coupled by scattering

2 Nondimensionalization

2.1 Assumptions

2.2 Scattering constant (same for kelp/water)

2.3 Table of variables

2.4 Rescale space, time

3 Limitations of Discrete Ordinates

3.1 Memory

3.2 CPU

3.3 GMRES unreliable

3.4 Also, need an initial guess for GMRES

4 Asymptotics

4.1 Solve each angular problem independently

4.2 Relatively computationally cheap

4.3 Much lower memory cost

4.4 Known number of operations

4.5 Low and high accuracy available

5 Mathematical Procedure

5.1 Substitute asymptotic series

$$L(\vec{r},\vec{\omega}) = L_0(\vec{r},\vec{\omega}) + bL_1(\vec{r},\vec{\omega}) + b^2 L_2(\vec{r},\vec{\omega}) + \cdots \tag{4}$$

$$\vec{\omega}\cdot\nabla\left[L_0(\vec{r},\vec{\omega}) + bL_1(\vec{r},\vec{\omega}) + b^2 L_2(\vec{r},\vec{\omega}) + \cdots\right] = -(a(\vec{r})+b(\vec{r})+b\int_{4\pi}\beta(\vec{\omega}\cdot\vec{\omega'})\left[L_0(\vec{r},\vec{\omega'}) + bL_1(\vec{r},\vec{\omega'}) + b^2 L_2(\vec{r},\vec{\omega'}) + \cdots\right]\,d\vec{\omega'}) \tag{5}$$

5.2 Group like powers of b

5.3 Boundary conditions

5.4 Rewrite as ODE along ray path

5.5 Solve ODE as 1st order linear via I.F.

6 Numerical Implementation

6.1 Discrete grid

6.2 Numerical integration

6.3 Storing pole values

6.4 Loop rolling

6.5 Scattering integral