Survey of Solution Techniques for Linear Systems from Finite Difference Methods in Numerical Radiative Transfer

Advanced Numerical Analysis II
Final Project

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May 11, 2017

1 Introduction

1.1 Kelp

1.2 Radiative Transfer

Let n be the number of spatial dimensions for the problem. (i.e., 2 or 3)
Let $x \in \mathcal{R}^n$.
Let Ω be the unit sphere in $\mathcal{R}^n$. Let $\omega \in \Omega$ be a unit vector in $\mathcal{R}^n$.
Let $L(x, \omega)$ denote *radiance* - the intensity of light at position x in the direction ω. Let $I(x)$ denote *irradiance* - the total intensity of light at position x. Let $a(x)$ and $b(x)$ denote the absorption and scattering coefficients respectively of the medium. Then, the Radiative Transfer Equation is

$$\omega \cdot \nabla_x L(x, \omega) = -(a(x) + b(x)) L(x, \omega) + \int_\Omega \beta(\omega \cdot \omega') L(x, \omega') \, d\omega' \tag{1}$$

2 Discretization

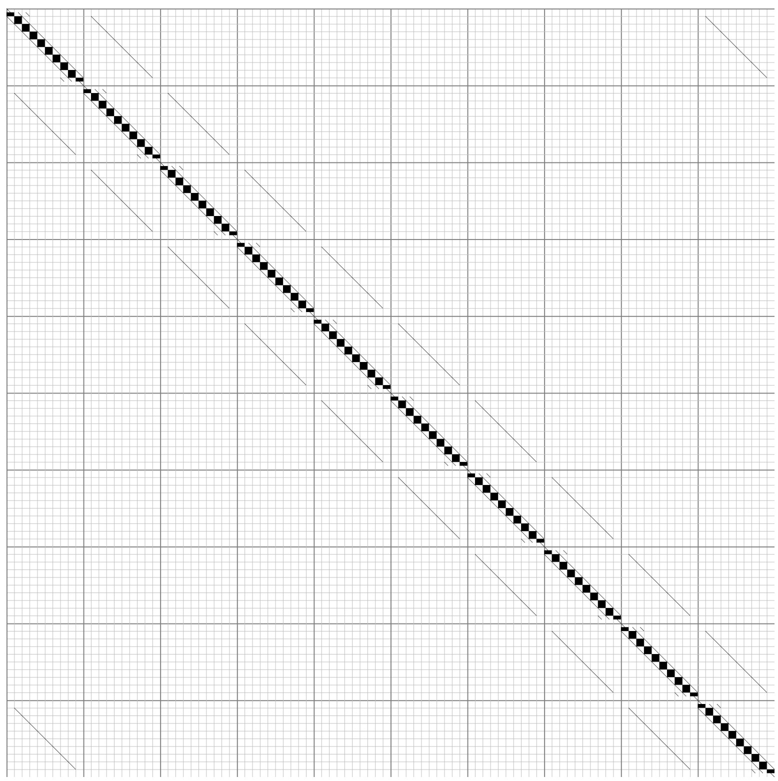

Figure 1: Sparsity plot: 10x10x16, ordering 012

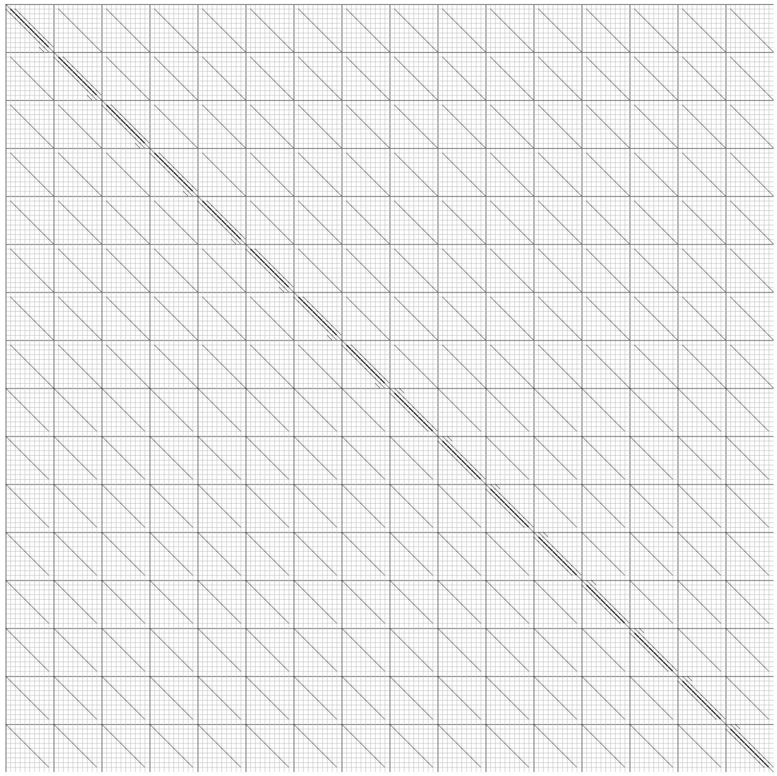

Figure 2: Sparsity plot: 10x10x16, ordering 210

2.1 Sparsity Plots

2.2 Matrix Properties

2.2.1 Diagonal Dominance

2.2.2 Spectral Radius

3 Direct Methods

3.1 Factorizations

3.2 Software Packages

4 Stationary Iterative Methods

4.1 Fixed-Point Iteration

4.2 Convergence and Preconditioning

5 Nonstationary Iterative Methods

5.1 Krylov Subspace Methods

5.2 Convergence and Preconditioning

6 Numerical Results

7 Conclusions