

Q1

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Answer:

By using repeated squaring, we can simply write n in binary. For example, let us say $n = 13$, thus, the binary form of 13 is $(1101)_2$, then we can write $13 = 2^3 + 2^2 + 2^0 = 8 + 4 + 1$. Clearly, the representation of n is $2^{k_1} + 2^{k_2} + \dots + 2^{k_m}$ where $k_1 > k_2 > \dots > k_m$ and $k_1 = \text{floor}(\log_2 n) = \lfloor \log_2 n \rfloor$. Hence, M^n can be represented as $M^{2^{k_1}} \cdot M^{2^{k_2}} \dots M^{2^{k_m}}$ and this algorithm does only $O(\log n)$ multiplications.