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Answer:

Clearly, x is a sequence of length $5 + 1 - 3 = 3$, then x can be written as $\langle a, b, c \rangle$.

We transform the x and $\langle 1, 1, -1 \rangle$ into corresponding polynomial

$$P_A(x) = a + bx + cx^2$$

$$P_B(x) = 1 + x - x^2$$

We multiply P_A and P_B

$$\begin{aligned} P_A(x) \cdot P_B(x) &= \sum_{j=0}^4 \left(\sum_{i=0}^j A_i \cdot B_{j-i} \right) x^j \\ &= a + (a + b)x + (c + b - a)x^2 + (c - b)x^3 - cx^4 \end{aligned}$$

The corresponding polynomial of sequence $\langle 1, 0, -1, 2, -1 \rangle$ is

$$P_c(x) = 1 - x^2 + 2x^3 - x^4$$

Since $P_c(x) = P_A(x) \cdot P_B(x)$ then we got,

$$a = 1$$

$$a + b = 0$$

$$c + b - a = -1$$

$$c - b = 2$$

$$c = 1$$

Hence,

$$a = 1$$

$$b = -1$$

$$c = 1$$

Finally, the sequence x will be $\langle 1, -1, 1 \rangle$.