Q4

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Answer:

a. We let A be the sequence $< 1, \underbrace{0, 0, \cdots, 0}_{k}, 1 >$ we transform the sequence into corresponding polynomial $P_A(x) = 1 + x^{k+1}$ then we square it.

$$\left(P_A(x)\right)^2 = \sum_{j=0}^{2k+2} \left(\sum_{i=0}^{j} A_i \cdot A_{j-i} \right) x^j$$

$when\ j = 0; i = 0 \Rightarrow A_0 \cdot A_0 \cdot x^0 = 1$

$when\ j = k + 1; i = 0 \Rightarrow A_0 \cdot A_{k+1} \cdot x^{k+1} = x^{k+1}$

$when\ j = k + 1; i = k + 1 \Rightarrow A_{k+1} \cdot A_0 \cdot x^{k+1} = x^{k+1}$

$when\ j = 2k + 2; i = k + 1 \Rightarrow A_0 \cdot A_{k+1} \cdot x^{2k+2} = x^{2k+2}$

Hence, $\left(P_A(x)\right)^2 = 1 + 2 \cdot x^{k+1} + x^{2k+2}$ and the convolution of sequence A is

$$< 1, \underbrace{0, 0, \cdots, 0}_{k}, 2, \underbrace{0, 0, \cdots, 0}_{k}, 1 >$$

b. Since the sequence A is $< 1, \underbrace{0, 0, \cdots, 0}_{k}, 1 >$ and polynomial $P_A(x) = 1 + x^{k+1}$

$$\begin{aligned} DFT(A) &= < P_A(\omega_{k+2}^{0}), P_A(\omega_{k+2}^{1}), \cdots, P_A\left(\omega_{k+2}^{k+1}\right) > \\ &= < 1 + \omega_{k+2}^{0\cdot(k+1)}, 1 + \omega_{k+2}^{1\cdot(k+1)}, \cdots, 1 + \omega_{k+2}^{(k+1)^2} > \\ &= < 2, 1 + \omega_{k+2}^{k+1}, \cdots, 1 + \omega_{k+2}^{(k+1)^2} > \end{aligned}$$