Q5

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Answer:

a. Using basic property of log function which is $\log a^b = b\log a$, then $g(n) = \log_2(n^{\log_2 n})^2 = \log_2(n^{2\log_2 n}) = 2\log_2 n * \log_2 n = 2\,(\log_2 n)^2$. Clearly, $g(n) = \theta(f(n))$.

b. In order to show $f(n) = O(g(n))$ we have to show $f(n) \le cg(n)$.

We take log on the both sides, then we have

$$\log n^{10} \le \log c + \log 2^{\sqrt[10]{n}}$$

$$10\log n \le \log c + \sqrt[10]{n}$$

We take $c = 1$, then we have

$$10\log n \le \sqrt[10]{n}$$

$$\frac{10\log n}{\sqrt[10]{n}} \le 1$$

We have to show

$$\lim_{n\to\infty} \frac{10\log n}{\sqrt[10]{n}} \le 1$$

We use L'Hopital's rule

$$\lim_{n\to\infty} \frac{10\log n}{\sqrt[10]{n}} = \lim_{n\to\infty} \frac{(10\log n)'}{(\sqrt[10]{n})'} = \lim_{n\to\infty} \frac{100}{n^{-0.9}\ln 2 \cdot n} = \lim_{n\to\infty} \frac{100}{n^{0.1}\ln 2} = 0 \le 1$$

Hence, we take $c = 1$ is enough to show $f(n) = O(g(n))$ for all sufficiently large n.

c. Clearly, we can see when $n = 2k\ (k \in N)$, $(-1)^n = 1$, then we have the equation $f(n) = n^2$, thus for any fixed constant $c > 0,\ n^2 > cn$.

When n = $n = 2k - 1\ (k \in N)$, $(-1)^n = -1$, then we have the equation $f(n) = n^0 = 1$, thus for any fixed constant $c > 0$, $1 \le cn$.

Hence, neither $f(n) = O(g(n))$ nor $f(n) = \Omega(g(n))$.