Textbook Exercises: Chapters 15, 16, 17, and 18

Due on the 22nd of May, 2025.

1. A publisher of business and economics statistics textbook has realised that there are three different approaches to teaching introductory applied statistics:

   - use of computer software and almost no hand calculations;
   - traditional teaching of statistical concepts and limited hand calculations;
   - emphasis on mathematical proofs and development of a rigorous understanding.

   The publisher is now interested to know whether or not the market can be segmented on the basis of the teacher's background. To answer this question, he randomly sampled business and economics professors who taught introductory applied statistics. He asked them about their pedagogical approach and their highest academic degree.

	business	economics	STEM	others
computer software	55	10	6	11
traditional teaching	26	17	15	9
mathematical proofs	28	11	24	7

   (a) How many times is it as likely that a business or economics professor who teaches introductory applied statistics emphasises mathematical proofs, given that he has a STEM background, compared to those who has a different background (e.g., business or economics)? (6 marks)

   (b) Determine with 95% confidence whether or not a business or economics professor's pedagogical approach to introductory applied statistics is dependent on his highest academic degree. Perform a chi-square test. (12 marks)

   (c) Consider the following regression model:

$$\ln\left(\frac{\mathbb{P}\big[Y = y_k\big]}{\mathbb{P}\big[Y = y_3\big]}\right) = \alpha_k + \beta_{k,1}X_1 + \beta_{k,2}X_2 + \beta_{k,3}X_3$$

   for all $k \in \{1, 2\}$. X_1, X_2, $X_3 \in \{0, 1\}$ and $Y \in \{y_1, y_2, y_3\}$. X_1, X_2, and X_3 are dummy variables defined as follows:

   - $X_1 = 1$ if and only if the teacher's highest degree is in business;
   - $X_2 = 1$ if and only if the teacher's highest degree is in economics; and
   - $X_3 = 1$ if and only if the teacher's highest degree is not in business, economics, or STEM.

   y_1 denotes a software orientation, y_2 denotes the traditional approach, and y_3 denotes the mathematical approach. Answer the following questions accordingly:

i. Interpret all of the model parameters. (16 marks)

ii. Express the probability $\mathbb{P}[Y = y_3]$ in terms of Xs and the relevant model parameters. (6 marks)

iii. Derive the likelihood function for the model. Incorporate the tabulated information. (8 marks)

iv. Estimate the parameters using the maximum likelihood method. You can use computer software or programming tools to numerically approximate the solution. (8 marks)

2. Review Problem 1 in Homework 3. Recall that you fitted a linear equation for each commercial style; unsurprisingly, the result appeared to be style-dependent.

   (a) Construct a linear regression model accommodating different styles. Use dummy variables to allow the intercept and slope to be style-specific. (6 marks)

   (b) Estimate the parameters of the model that you constructed in Part (a). Use the ordinary least-square method. No software or programming tools. (6 marks)

   (c) Following Parts (a) and (b), test at a 5% significance level whether or not the slope for style A differs from that for C. You can use computer software or programming tools to calculate the standard error of the relevant parameter, but after that, you have to hand calculate the test statistic. (8 marks)

3. Review Problem 1 in Homework 5. Suppose that the student organisation disused the previous sample and surveyed another 50 economics professors. They found that on average, an economics professor had a normal salary of 65,292 US dollars. Assume the following regression model:

$$\text{Evaluation} = \alpha + \beta \cdot \text{Salary}.$$

Using the ordinary least-square method, making all the standard assumptions (e.g., independent and identically distributed errors), the student organisation is 95% confident that $2.5629 \leq \alpha \leq 5.5275$ and that $0.016397 \leq \beta \leq 0.060757$. Given these results, estimate the following alternative model:

$$\text{Salary} = \alpha' + \beta' \cdot \text{Evaluation}.$$

Specifically, estimate α' and β' each with 95% confidence. (24 marks)