

Textbook Exercises: Chapters 13, 14, and 16

Due on the 3rd of April, 2025.

Problem 1 involves a fraction of Chapter 16 that Professor Chin has yet to cover in the 20th March class, but that fraction has actually been introduced in Chapter 4, where you learnt descriptive statistics. Chapter 4 was covered in the previous semester already. Review Chapter 4, if needed.

1. In an experiment to determine how the length of a television commercial affects people's memory of the advertised product, 60 people were recruited. They watched a one-hour television programme in the middle of which, a commercial appeared; it advertised a fictitious brand of toothpaste, with which people were unfamiliar. The following table summarises the commercial's length (in seconds) and style (denoted by A, B, or C) together with the consequent memory test score (out of 30).

A		B		C	
score	length	score	length	score	length
24	52	20	40	10	44
11	28	16	36	16	60
4	16	18	52	21	60
23	48	15	44	10	24
14	36	15	44	8	40
7	32	17	52	13	32
9	24	28	60	15	36
22	56	11	20	8	40
13	20	15	52	2	24
13	48	18	52	8	36
24	60	16	16	7	28
10	44	19	32	10	20
26	56	15	60	8	20
12	40	22	48	10	16
14	44	14	16	4	20
24	56	16	40	11	56
14	32	25	36	9	24
8	32	17	56	6	16
15	28	19	28	11	24
8	48	21	48	5	28

Below we assume a linear model and the ordinary least-square method. These assumptions will be unspoken often in this course, for convenience.

- (a) Regress test scores on commercial lengths, regardless of styles.
- Specify the model equation. (5 marks)
 - Draw a scatter plot with the fitted line. (2 marks)
 - Calculate the coefficient of determination. (3 marks)
- (b) Repeat Part (a), but for each commercial style. (10 marks for each)
- (c) Compare the results of (a) and (b). Calculate the additional proportion of explained variation that is due to the partition of the sample. (10 marks)
- (d) Perform post-hoc pairwise comparisons of the residuals produced by the models for the three different commercial styles. Calculate Snedecor's F statistics to test for heteroscedasticity at a 20% familywise significance level. (15 marks)

2. Refer to the following table:

	A	B	C	D	E
skilled	25.9	22.2	25.3	25.5	24.4
	26.1	26.5	24.5	28.6	25.3
	24.3	24.8	22.4	27.2	27.9
	27.2	22.9	26.6	25.8	27.7
	21.7	24.1	25.9	26.5	28.1
	26.1	25.7	25.2	23.9	30.3
	24.1	25.6	26.1	28.8	24.2
	25.0	26.2	21.6	27.1	26.9
average	24.2	19.5	22.1	20.1	23.8
	21.2	19.0	19.9	25.8	23.7
	23.9	24.2	20.9	24.6	25.3
	23.4	25.9	21.7	19.8	27.1
	20.4	22.0	20.4	23.7	19.3
	24.7	21.7	18.0	22.5	22.2
	19.4	22.2	20.4	22.0	22.2
	25.9	22.0	25.4	23.2	24.9
unskilled	20.5	18.1	19.4	25.0	24.3
	20.2	19.8	21.3	19.8	20.6
	18.2	19.4	21.7	23.1	20.5
	18.0	21.1	20.6	20.7	22.6
	17.4	21.6	16.7	19.8	21.2
	20.8	17.0	21.5	21.9	21.6
	21.1	21.7	19.3	19.2	19.2
	23.0	18.2	19.1	20.3	21.7

The marketing department of a firm that manufactures office furniture has ascertained that there is a growing market for a specialised desk that houses the various parts of a computer system. The operations manager was recently summoned to put together a production plan. The marketing department has specified the materials from which the desk will be made, and the machines used to produce the parts. However, there are different methods that can be used, denoted by A, B, C, D, and E. Moreover, because of the complexity of the desk production, the operations manager realises that it is possible that different skill levels of the workers will yield different results. Accordingly, he organised an experiment where skilled, average, and unskilled workers were randomly sampled and assigned to five different groups, in each of which, they were asked to assemble the specialised desks using the specified method. The manager recorded the number of minutes that the task took each worker. The results were as tabulated on the previous page.

- (a) Perform a two-way analysis of variance without an interaction.
 - i. Calculate the proportion of unexplained variation. (7 marks)
 - ii. Determine with 95% confidence whether or not there is a main effect of the machine on the assembly speeds. (7 marks)
- (b) Repeat Part (a), but include an interaction. Additionally determine with 95% confidence whether or not there is an interaction between the machine and the skill level of the worker. (21 marks)