AUTOMATIC POLYPHONIC MUSIC TRANSCRIPTION

by

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Declaration

I hereby declare that this thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Oliver Ignetik

July 5th 2020

Contents

Abstract

Automatic Polyphonic Music Transcription

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This research paper explores the concept of Automatic Music Transcription (AMT). A literature review is conducted to provide a concise overview of the subject, including state of the art methods and how they can be used to better improve user satisfaction of current systems.

In particular, this paper explores the method known as Non-Negative Matrix Factorization as applied to time-frequency representations of audio signals. The primary concept that will be reviewed to aid with understanding this technique is the Short Time Fourier Transform.

A secondary avenue of exploration is machine learning algorithms and their application to AMT systems. A preliminary review is provided to readily prepare the reader for the related discussions and insights uncovered in this investigation.

Finally the design and methodology of a monophonic AMT system is presented. Thereafter, a discussion is presented on how higher level musical knowledge such as diatonic harmonies, time signatures and modulations can be incorporated into future models to improve their accuracy.

List of Figures

List of Tables

Chapter 1

Introduction

1.1 What is Automatic transcription ?

The nature of music signals, which often contain several sound sources that are highly correlated over both time and frequency, means that Automatic Music Transcription (AMT) is still considered an open problem in the literature. Usually an AMT system takes an audio waveform as input, computes a time-frequency representation and outputs pitches over time or ideally a typeset music score. Most approaches are designed to achieve an intermediate goal in AMT which does not actually resemble musical notation as shown in Figure 1.1.

The capability of transcribing music audio into music notation is a fascinating example of human intelligence. It involves analyzing complex auditory scenes, recognizing musical objects, forming musical structures and checking alternative hypotheses. AMT refers to the design of computational algorithms to convert acoustic music signals into some form of music notation. It is a challenging task and considered an unsolved problem in signal processing and artificial intelligence. This problem is particularly challenging in polyphonic music were even the most advanced systems are far behind meeting the accuracy of trained musicians. [1]

1.2 Key Challenges

Despite significant progress in AMT research, there exists no end-user application that can accurately and realiably transcribe music containing the range of instrument combinations and genres found in recorded music.

There are several factors that make AMT particularly challenging :

1. *Polyphonic mixtures* - inferring musical attributes from a signal containing multiple simultaneous sources with different pitch, loudness and sound quality is extremely difficult. Even the task of disentangling the harmonics of two coinciding pitches is not trivial. For consonant intervals, which are often seen in diatonic harmonies and form basic harmonic buidling blocks, the notes share many of the same harmonics making the seperation of voices even more difficult. [2]

2. *Synchronous sound sources* - musicians pay close attention to metrical structure and rhythmic synchronicity, which violates statistical independence between sources which is often used in Automatic Speech Recognition to facilitate seperation.

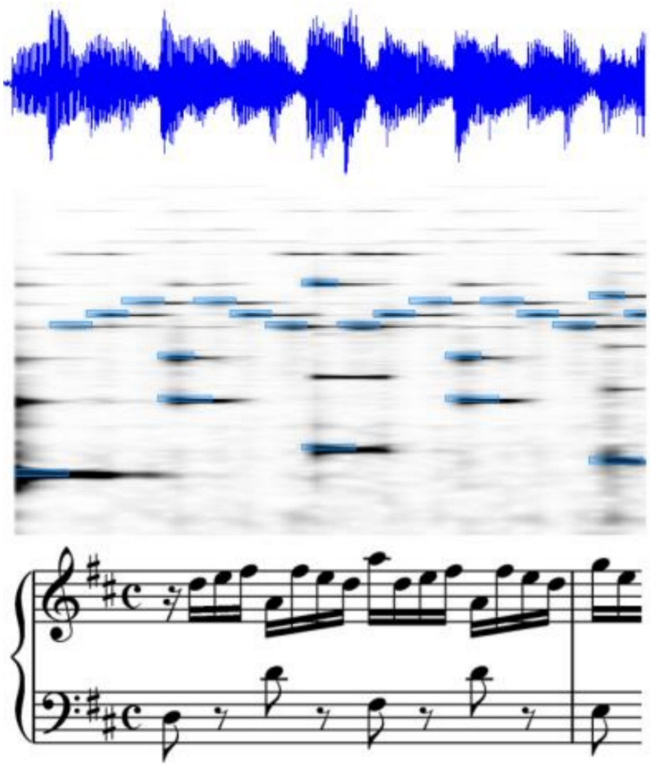

Figure 1.1: AMT process figure taken from NUS ISMIR 2019 tutorial [2]

3. *Lack of ground-truth transcriptions* - the annotation of polyphonic music is extremely time consuming and requires high expertise especially in symphonic pieces were there are many concurrent sound events. Even when there is sheet music available for a particular piece, they are difficult to align with an audio signal. Sheet music at best is considered as a weak label due to the fact subjective interpretation often plays a role. This is true even in the most prudent genres like classical music were musicians strive to pertain to the score as much as possible. [3]

1.3 Commercial Applications

A successful AMT system would enable a broad range of interactions between people and music, including automatic instrument tutoring, dictating improvised musical ideas and automatic music accompaniment, music content visualization and intelligent content-based editing, indexing and recommendation of music and analyzing jazz improvisations and other nonannotated music. Given the potential applications, the problem has attracted commerical interest and a number of AMT software exists. [4]

Commercially available applications include Melodyne (`http://www.celemony.com/en/melodyne`), Transcribe!(`https://www.seventhstring.com/xscribe/`) and AudioScore exist. In context-specific transcription scenarios these applications can reach multipitch detection accuracies of 90% or more. Even some open source academically produced applications can reach similar performance levels. [5] However, given complex ensemble pieces with multiple instruments the performance of such systems is still far behind that of a trained musician.

1.4 Overview

1.4.1 Research Question

Can incorporating higher level musical knowledge into existing AMT systems improve the perceived quality of the output for human listeners ?

1.4.2 Project Scope

The scope of the thesis will focused on western music with its associated modes and scales. This paper will be restricted to approaches that analyze music produced by pitched instruments such as pianos and guitars. Outside of the scope of the paper will be methods for transcribing percussive instruments such as drums.

1.5 Thesis Synopsis

Chapter 2 serves as a review of important musical concepts and a literature review in music signal processing. Crucial Digital Signal Processing (DSP) techniques are relations are presented. Finally a number of state of the art methods and their characteristics are explored.

Chapter 3 provides details on the system architecture of the AMT systems used in this research project. Chapter 4 presents the crucial results of this research paper and discusses their implications. Chapter 5 discusses the outcomes of the thesis and explores directions for future research.

1.6 Resources

1. Core Python Libraries - see github repo for thesis.yaml file for all dependencies

    a) LibROSA - LibROSA is a python package for music and audio analysis. It provides the building blocks necessary to create music information retrieval systems.
    `https://librosa.github.io/librosa/`

    b) mir_eval - Python library for computing common heuristic accuracy scores for various music/audio information retrieval/signal processing tasks.
    `https://pypi.org/project/mir_eval/`

2. Datasets

    a) MAPS - A piano database for multipitch estimation and automatic transcription of music [6]

   b) MusicNet - A curated collection of labeled classical music [7]
   c) MAESTRO - MIDI and Audio Edited for Synchronous TRacks and Organization [8]

3. Work Environment
   a) Anaconda Package Manager
   b) Jupyter Lab NoteBooks

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Chapter 2

Background

2.1 Musical Concepts

2.1.1 Pitch and Harmony

The existence of sequences of sounds with well-defined fundamental periods is a very common feature in music. Most musical instruments such as pianos, guitars, flutes and trumpets are constrcuted to allow performers to produce sounds with easily controlled fundamental periods and associated harmonics. Such a signal is described as a harmonic series of sinusoids at multiples of the fundamental frequency and results in the perception of a pitch in the mind of the listener.

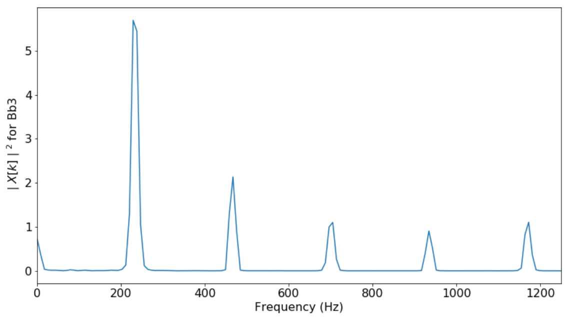

Figure 2.1: Spectrum of Bb3 with fundamental frequency 233.08 Hz

Although different cultures have developed different musical conventions, a common feature is the musical "scale", a set of discrete pitches that repeats every octave. In contemporary western music an "equal tempered scale" is used, which divides the octave into 12 steps on a logarithmic axis called semitones.[9]

$$P_n = P_a(\sqrt[12]{2})^{n-a} \tag{2.1}$$

Where :

P_n = Query pitch
P_a = Reference pitch

In musical theory, the spacing in between these steps are known as semitones and form musical intervals. Different combinations of notes that form intervals result in different harmonic structures or "colours" known as chords. Consonant intervals like a perfect fifth are made up of seven semitones and have a frequency ratio of $(2^{\frac{1}{12}})^7 = \frac{3}{2}$ sounding pleasant to the ear. They share many harmonics and ubiquitous in western music. This is partly the reason why transcription can be so difficult. The tritone is considered dissonant and has a intervallic frequency ratio of $(2^{\frac{1}{12}})^6 = \frac{45}{32}$. This interval sounds jarring to the ear and is associated with musical tension. Tritones provide a harmonic spine for the movement of groups of notes because they are so noticable to the listener.

Musical Instrument Digital Interface (MIDI) is one of the most important tools for musicians. It is a protocal that allows computers, musical instruments and other hardware to communicate. It encodes an audio signal into an multi-dimensional array which contains information about the pitch and onset/offset times of notes. Of particular note is the MIDI pitch which has the formula below :

$$d = 69 + 12\log_2(\frac{f_0}{12}) \tag{2.2}$$

Where :

f_0 = fundamental frequency
d = MIDI number

On a grand piano the lowest note A0 has a frequency of 27.50 Hz and midi number of 21. The highest note C8 has a frequency of 4186.0 Hz and midi number 108.

2.1.2 Tempo, Beat and Rhythm

The musical aspects of tempo, beat and rhythm play a fundamental role. The *beat* can be described as a sequence of perceived pulses that are regularly spaced in time and correspond to the pulse a human taps along when listening to the the music. [10]

The strength or stress of the musical pulse and how it varies determines the metrical signature of a piece of music. Notes are grouped in rhythmic units in each bar according to the time signature.

Figure 2.2: Excerpt from a piano arrangement for the tune Nearness of You with a common time signature of 4 quarter notes per bar

The term *tempo* refers to the rate of this pulse as is often denoted as *beats per minute* or *bpm*. Musical pulses typically coincide with note onsets or percussive events. In the context of AMT this tasks constitutes finding a *novelty curve* known as onset detection.

2.2 Signal Processing Techniques

2.2.1 Sampling Theorem

The sampling theorem is a consequence of digitizing analogue signals. Sampling an analogue signal stores quantized values of the amplitude of a continuous signal at regular intervals determined by the sampling rate.

The sampling theorem says that to avoid higher frequency components aliasing as lower frequencies components the following must be satisfied. Considering a sampling frequency F_s and Nyquist frequency F_N.

$$F_s > 2 \cdot F_N \tag{2.3}$$

Where B is the highest frequency expected in the signal. Frequently a sampling rate of 44.1 kHz is used in audio recording because the range of human hearing is from 20hertz-20kHz.

2.2.2 Discrete Fourier Transform

Consider a finite-length sequence $x[n]$ of length N samples such that $x[n] = 0$ outside the range $0 \leq n \leq N-1$. To each finite-length sequence of length N, it is possible to associate a periodic sequence.

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n - rN] \tag{2.4}$$

This is implied in the mathematics of the DTFT, that the signal of interest is periodic in nature even when it has a finite length. The Discrete Time Fourier Transform (DTFT) of such a signal is given by :

$$\tilde{X}[\omega] = \sum_{n=-\infty}^{n=\infty} \tilde{x}[n] \exp^{-j\omega n} \tag{2.5}$$

This sequence is itself periodic with a period N. The Discrete Fourier Transform (DFT) of the original signal finite length signal $x[n]$ can be found by sampling $\tilde{X}$ at $\omega = \frac{2\pi}{N}$ and only considering the values of k within $0 \leq k \leq N-1$:

$$X[k] = \sum_{n=0}^{n=N-1} x[n] \exp^{-j\frac{2\pi}{N}k} \tag{2.6}$$

The DFT is often implemented as the Fast Fourier Transform which reduces the order of complexity to $O(N \log N)$ by exploiting symmetries in the transformation. [11] Equation 2.6 will be used frequently throughout this project and extended upon in § 2.2.3.

2.2.3 Short-Time Fourier Transform

As in other audio-related applications, the most popular tool for describing the time-varying energy across different frequency bands is the short-time Fourier Transform (STFT), which, when visualized as its magnitude, is known as the spectrogram.

Formally, let x be a discrete-time signal obtained by uniform sampling a waveform at a sampling rate F_s Hz. Using a N-point tapered window w (eg. Hamming $w[n] = 0.5 - 0.46 \cdot cos(\frac{2\pi n}{N})$ for $n \in [0, N-1]$) and an overlap of half a window length we obtain the STFT.

$$X[m,k] = \sum_{n=0}^{N-1} w[n] \cdot x[n + m \cdot \frac{N}{2}] \cdot exp\{-j\frac{2\pi kn}{N}\} \tag{2.7}$$

With $m \in [0, T-1]$ and $k \in [0, K-1]$. Here, T determines the number of frames , $K = \frac{N}{2}$ is the index of the last unique frequency value as dictated by the Sampling Theorem. Thus $X[m,k]$ corresponds :

$$f_{coeff}(k) = \frac{k}{N} \cdot F_s \qquad \text{[Hz]} \tag{2.8}$$

$$t_{frame}(m) = t \cdot \frac{N}{2F_s} \qquad \text{[s]} \tag{2.9}$$

$X[m,k]$ is complex-valued, with the phase depending on the alignment of each short-time analysis window. Often it is only the amplitude $\mid X[m,k] \mid$ that is used. [11]

2.2.3.1 Log-Frequency Spectrogram

Note that the Fourier coefficients of $X[m,k]$ are linearly spaced on the frequency axis. Using suitable binning strategies, various approaches switch over to a logarithmically spaced frequency axis, by using mel-frequency bands or pitch bands as seen in Figure 2.3. Keeping the linear frequency axis puts greater emphasis on the high-frequency regions of the signal, thus accentuating the aforementioned noise bursts visible as high-frequency content. One simple yet important step often applied in the processing of music signals, is referred to as logarithmic compression. Such a compression not only accounts for the logarithmic nature that describes how humans perceive sound but also balances out the dynamic range of the signal. [1]

2.3 State of the Art Methods

Many approaches have been developed for AMT applied to polyphonic music. While the end goal of AMT is to convert an acoustic music recording to some

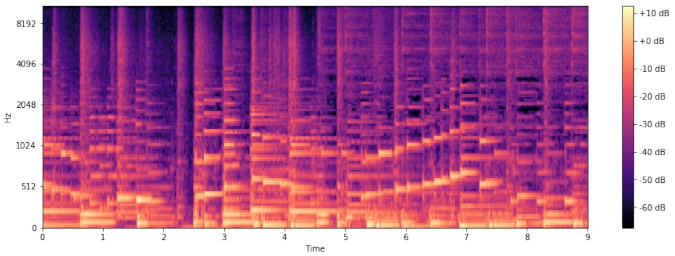

Figure 2.3: STFT of a 10s excerpt from Blues in F - Bill Evans Trio recording

form of music notation, most approaches are aimed at achieving an intermediate goal. Some commercial applications provide the capability of converting a piano-roll representation into typeset music notation. However, the end results are generally musically illogical especially in genres like jazz where notes often fall on the upbeat and rythms are highly syncopated.

AMT approaches can generally be organized into four categories : frame-level, note level, stream level and notation level. Frame level transcription which is also known as *Multipitch estimation* aims at identifying the number and pitch of notes that are present in a frame of music. A frame is generally on the time scale of 10ms depending on the type of analysis window. Note-level transcription not only estimates the pitch in each time frame but also the onset and offset times. Stream level transcription of *instrument tracking* targets the grouping of estimated notes into streams. These groupings typically correspond to different instruments or timbres. Notation level transcription or *audio-to-note transcription* aims to transcribe the music audio into a musical score such as that seen on staff notation. Harmonic and rhythmic structures have to be incorporated into the modelling and as a result the complexity is monumentally higher then MPE approaches. [12]

Readers interested in a comparison of the performance of different approaches are referred to the Multiple Fundamental Frequency Estimation and Tracking task of the annual Music Information Retrieval Evaluation eXchange (MIREX) (`http://www.music-ir.org/mirex`).

2.3.1 Non-negative Matrix Factorization

A large subset of transcription systems employ methods stemming from spectrogram factorization techniques, which exploit the redundancies found in music

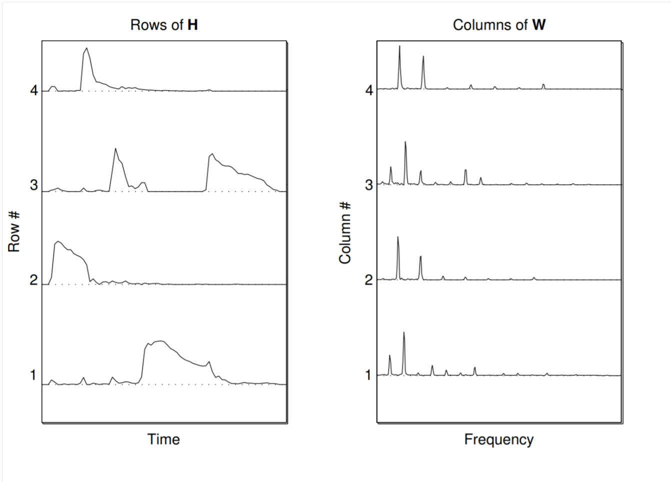

Figure 2.4: Example of NMF decomposition taken from Smaragdis 2003 [14]

spectrograms. Non-negative matrix factorization (NMF) was first proposed by Lee and Seung [13]

Starting with a non-negative M by N matrix $\mathbf{X}$ the goal of NMF is to approximate it as a product of two non-negative matrics $\mathbf{W}_{M\times R}$ and $\mathbf{H}_{R\times N}$, where $R \leq M$ such the cost function is minimized :

$$C = |\ \mathbf{X} - \mathbf{W} \cdot \mathbf{H}\ |_F \tag{2.10}$$

where $|\ .\ |_F$ is the Frobenius norm. This is actually equivalent to Gradient Descent based minimization of divergence. [14] There are a number of algorithms for finding the appropriate values of $\mathbf{W}$ and $\mathbf{H}$. For example, the generalized Kullback-Leibler divergence between $\mathbf{X}$ and $\mathbf{W} \cdot \mathbf{H}$ is non-increasing under the following updates and guarantees the nonnegativity of both $\mathbf{W}$ and $\mathbf{H}$:

$$H \Leftarrow H \odot \frac{W^T \frac{X}{WH}}{W^T J} \text{ and } W \Leftarrow W \odot \frac{\frac{X}{WH} H^T}{J H^T} \tag{2.11}$$

where the $\odot$ operator denotes pointwise multiplication, $J \in \mathbb{R}^{M\times N}$ denotes the matrix of ones, and the divison is pointwise. [15]

In the context of time-frequency representations and AMT both unknown matrices have an intuitive interpretation. $\mathbf{X}$ in the most basic cases in time-frequency analysis is a STFT of the audio signal. $\mathbf{W}$ encodes the spectral profiles of the R components and is commonly referred to as the dictionary matrix. $\mathbf{H}$ encodes the temporal activity of the each of those components and named the activation matrix. This is more clear by inspecting Figure 2.4 which shows an example decomposition.

There are two classes of NMF approches that fall into supervised and unsupervised approaches. In supervised approaches the dictionary matrix is prextracted. For explanatory purposes, one can imagine the applicaiton of such an NMF AMT system. To compile the dictionary matrix a recording of each note played in isolation is recorded and concatenated. Thus each component can be thought of corresponding to individual pitches with their associated harmonic profiles. The NMF-based decomposition is then performed by applying the update rules in Equation 2.11 to find $\mathbf{H}$ to minimize the cost function.

The unsupervised approach involves hyperparameter tuning to discover the optimal value for the number of components. This can be achieved by grid search methods cv-fold testing by splitting up the audio signal into smaller segments. Both approaches are widely used and there have been many studies based on improving performance and accuracy. For a comprehensive overview of a number of these techniques refer to [16–18].

2.3.2 Neural Networks

Neural networks (NN) are systems that are vaguely inspired by biological neural networks. They are based on a collection of connected units or nodes called artifical neurons. They are able to learn non-linear functions from input to output via an optimization algorithm. The goal of a network is to learn the weights w_{ij} by minimizing the cost function with respect to the training data. [19]

There are a number of important parameters which have to be tuned in a neural network through a process known as hyperparameter tuning. This is typically done using a grid-search algorithm to find a set of parameters which optimizes the cost function. [20]

In recent years NNs have had a considerable impact on the problem of music transcription and on music signal processing in general. However, compared to other fields progress on NNs for music transcription has been slower due to a lack of annotated data with labels which is essential for training the models appropriately. [21]

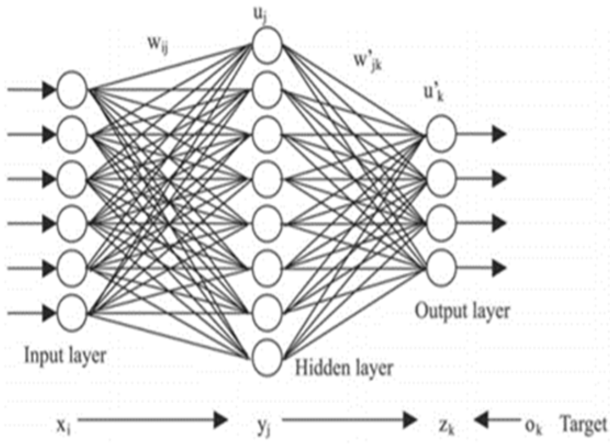

Figure 2.5: An example of a feed forward architecture neural network appropriate for classification tasks. [2]

The current state of the art method for piano transcription was proposed in research completed by Google Brain [22]. This approach combines two networks, one which detects onsets and one which finds note lengths. The output from the note onset network is used to inform the second network calculating note lengths.

Despite the appeal of NNs and the promises they hold they are often still outperformed by NMF-based methods for a number of reasons :

1. Lack of annotated labelled datasets - neural networks rely on data to be effective. There are only a a small number of annotated datasets which in themselves are restricted to certain types of instruments and genres of music. [3]

2. Adaptablity to new conditions - there are currently no methods to retrain or adapt an NN-based AMT systems on only a few seconds of audio. As such NMF-based systems can perform considerably better with less data and are easier to adapt.

2.4 Summary

In general there are drawbacks and advantages to both types of approaches outlined in this chapter. NNs can be more effective in context dependent environments whilst NMFs show more adaptablity. Currently there are no methods which are favoured in all situations.

Chapter 3 will introduce an AMT system which uses NMF to transcribe single note melodies and discuss the system architecture. Chapter 3 will also discuss how to apply a NN to musical data obtained from 2018 MusicNet database.

Chapter 3

System Design

Chapter 4

Results

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4.1 Preliminaries

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4.2 Methodology

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4.3 Evaluation

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4.4 Summary

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Chapter 5

Conclusion and Future Work

5.1 Conclusion

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Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

5.2 Future Research Directions

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5.2.1 NMF

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5.2.2 NN

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5.2.3 Evaluation Metrics

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5.2.4 Non-western music

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Appendix A

Signal Processing Methods

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A.1 Euler identity

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A.2 Fast Fourier Transform

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A.3 LTI systems

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Appendix B

Neural Networks

B.1 Architecture

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B.2 Loss Functions

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B.3 Performance metrics

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B.4 Resources

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