ENGINEERING TRIPOS PART IIA

EIETL MODULE EXPERIMENT 3F3

RANDOM VARIABLES and RANDOM NUMBER GENERATION
Short Report

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1. **Uniform and normal random variables.**

   Histogram of Gaussian random numbers overlaid on exact Gaussian curve (scaled):

   Include your graphic here

   Histogram of Uniform random numbers overlaid on exact Uniform curve (scaled):

   Include your graphic here

Kernel density estimate for Gaussian random numbers overlaid on exact Gaussian curve:

Include your graphic here

Kernel density estimate for Uniform random numbers overlaid on exact Gaussian curve:

Include your graphic here

Comment on the advantages and disadvantages of the kernel density method compared with the histogram method for estimation of a probability density from random samples:

Text answer here

Theoretical mean and standard deviation calculation for uniform density as a function of N:

Text/maths answer here

Explain behaviour as N becomes large:

Text/maths answer here

Plot of histograms for $N = 100$, $N = 1000$ and $N = 10000$ with theoretical mean and ± 3 standard deviation lines:

Include your graphic here

Are your histogram results consistent with the multinomial distribution theory?

Text/maths answer here

2. **Functions of random variables** For normally distributed $\mathcal{N}(x|0,1)$ random variables, take $y = f(x) = ax + b$. Calculate $p(y)$ using the Jacobian formula:

   Text/maths answer here

   Explain how this is linked to the general normal density with non-zero mean and non-unity variance:

   Text/maths answer here

Verify this formula by transforming a large collection of random samples $x^{(i)}$ to give $y^{(i)} = f(x^{(i)})$, histogramming the resulting y samples, and overlaying a plot of your formula calculated using the Jacobian:

Include your graphic here

Now take $p(x) = \mathcal{N}(x|0,1)$ and $f(x) = x^2$. Calculate $p(y)$ using the Jacobian formula:

Text/maths answer here

Verify your result by histogramming of transformed random samples:

Include your graphic here

3. **Inverse CDF method**

   Calculate the CDF and the inverse CDF for the exponential distribution:

   The pdf of an exponential distribution Y with mean 1 is:

   $$f_Y(y) = e^{-y}$$

   The corresponding cdf is found by integration:

   $$F_Y(y) = \int_0^y f_Y(t)dt = \int_0^y e^{-t}dt = 1 - e^{-y}$$

   The inverse of this function is found by: $x = F_Y(y), y = F_Y^{-1}(x)$.

   $$F_Y^{-1}(x) = -\ln(1-x)$$

   Matlab/Python code for inverse CDF method for generating samples from the exponential distribution:

   Listing 1: MATLAB

```
x_data = np.random.uniform(0,1,1000)
y_data = -np.log(1-x_data)

y = np.linspace(0,5,1000)
exp_theoretical = np.e**(-y)
```

   Plot histograms/ kernel density estimates and overlay them on the desired exponential density:

   Include your graphic here

4. **Simulation from a 'non-standard' density.**

   Matlab/Python code to generate N random numbers drawn from the distribution of X:

Plot some histogram density estimates with $\alpha = 0.5$, 1.5 and several values of β.

Hence comment on the interpretation of the parameters α and β.

A Link to worked files

The worked python files are uploaded to a repository which can be found at: `https://github.com/OliverJiang2025/3F3_lab.git`