CUED - Engineering Tripos Part IIA 2025-2026

Module Laboratory Report

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Feedback to the student

☐ **See also comments in the text**

		Very good	**Good**	Needs improvmt
CONTENT	**Completeness, quantity of content:** Has the report covered all aspects of the lab? Has the analysis been carried out thoroughly?			
	Correctness, quality of content Is the data correct? Is the analysis of the data correct? Are the conclusions correct?			
	Depth of understanding, quality of discussion Does the report show a good technical understanding? Have all the relevant conclusions been drawn?			
	Comments:			
PRESENTATION	**Attention to detail, typesetting and typographical errors** Is the report free of typographical errors? Are the figures/tables/references presented professionally?			
	Comments:			

Raw report mark	**/ 5**
Penalty for lateness	

The weighting of comments is not intended to be equal, and the relative importance of criteria may vary between modules. A good report should attract 4 marks.

1 mark / week or part week.
Please refer to the online information regarding our extension policy.

Marker: Date:

1 Lab Worksheet

1.1 Simplified Aircraft Model

Transfer function = $\frac{10}{s^2+10s}$

num = [10], den = [1,10,0]

1.2 Modelling Manual Control

Controller transfer function = ke^{-Ds}

k = 1.56, D = 0.57, Phase margin = 30°

Amount of extra time delay which can be tolerated = 0.338s

1.3 Pilot Induced Oscillation

Period of oscillation (observed) = 4.44s

Period of oscillation (theoretical) = 4.488s

1.4 Sinusoidal Disturbances

Maximum stabilising gain = 0.422

Gain at 0.66 Hz = 9dB

Phase at 0.66 Hz = -240°

1.5 Unstable Aircraft

Fastest pole at T = 0.6s

1.6 Autopilot with Proportion Control

Proportional gain K_C = 16, Period of oscillation T_C = 2s

1.7 Autopilot with PID Control

Transfer function of PID controller = $K_P(1 + \frac{1}{sT_i} + sT_d)$

PID constants: $K_P = 9.6$, $T_i = 1$, $T_d = 0.25$

Adjusted value of $T_d = 0.35$

1.8 Integrator Wind-up

Integrator bound Q = 0.2

2 Lab Report

2.1 (§2.2 Modelling Manual Control) Nyquist diagram (from Bode diagram) for controller in series with plant.

From Figure 4b, in which the upper graph is the magnitude plot while the lower graph is the phase plot. Information contained in this two plot are combined to give the shape of Nyquist Diagram of $K(j\omega)G(j\omega)$, which is given in Figure 1.

It is clear that the Nyquist Diagram behaves in a spiral shape approaching the origin point of the Argand Diagram. It is due to $e^{-j\omega D}$ term which rises an exponential decay of magnitude while keeping the curve "rotating" as $\omega \to \infty$. Besides, it is worth noting that the Nyquist diagram does not encircle $(-1, j0)$ according to the magnitude plot in Figure 4b. By Nyquist Stability Criteria, this closed loop system is stable.

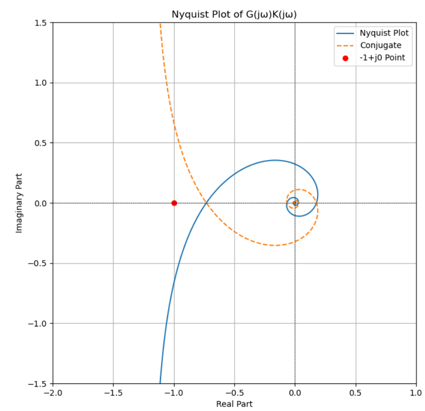


Figure 1: Step response of manual control

2.1.1 (§2.2 Modelling Manual Control) Are you using any integral action? Give a brief explanation. What does this imply about the accuracy of the model of the human controller?

As shown by Figure 4c, it is clear that an integral action was used. Starting from around 4.7s, the orange $u(t)$ curve, which represents the manual control signal, maintains at around 5 to attempting to stabilize $e(t)$ to 0. This is an integral control because both the $e(t)$ and $\frac{de(t)}{dt}$ are zero afterwards. $u(t) = k_i \int_0^t e(s)ds$ for some k_i here.

2.2 (§2.3 Pilot Induced Oscillation) Explain the oscillation of the feedback loop. How does your observed period of oscillation compare to the theoretical prediction?

Due to the poorly designed controller, there is a large phase lag at a certain frequency. When the phase lag reaches $-\frac{\pi}{2}$, the closed loop becomes marginally stable, and oscillations occur. This frequency at which the aircraft oscillates can be found by looking at the Bode plot in Figure 4f. It is clear that when the frequency is around 1.4 rad/s, the phase lag reaches $-\frac{\pi}{2}$ and the magnitude is around 0dB. From this, the theoretical period of oscillation can be calculated as: $T = \frac{2\pi}{\omega} = \frac{2\pi}{1.4} = 4.488s$.

The observed period of oscillation from the simulation is 4.44s, which is very close to the theoretical prediction of 4.488s.

2.3 (§2.3 Pilot Induced Oscillation) Can you give a rough guideline to the control designer to make PIO less likely?

For designers to make PIO less likely, they should make sure the phase margin is sufficiently large at the frequency which gives the magniude to be 0dB. According to relevant literatures[2], the average phase gradient is more important to avoid PIO than the phase margin itself. This gradient must be small to allow the pilot to react to the oscillations.

2.4 (§2.4 Sinusoidal Disturbances) Was your manual input able to reduce the error (as compared to providing no input)?

It is very hard to keep the aircraft stable with or without manual control. By comparing Figure 4h and Figure 5b, it is clear that the error $e(t)$ barely reduced by manual control, even under manual control with a small movement.

2.5 (§2.5 An Unstable Aircraft) Nyquist diagram for G2(s).

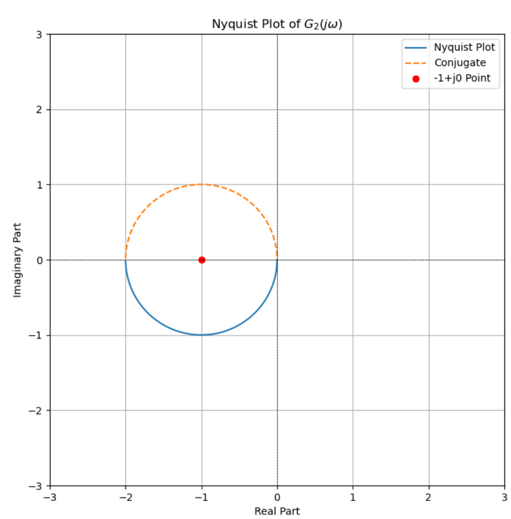


Figure 2: Nyquist diagram for G2(s)

2.6 (§2.5 An Unstable Aircraft) Explain, using the Nyquist criterion, why the feedback system is stable with a proportional gain greater than 0.5.

For a system with proportional gain K, the Nyquist stability criterion states that the closed-loop system is stable if the Nyquist plot of $G(s)$ does not encircle $(-\frac{1}{K}, j0)$. From Figure 2, it is clear that when $K > 0.5$, the system is stable as the plot intersect the negative real axis at (2,0).

2.7 (§2.5 An Unstable Aircraft) Sketch of a Nyquist diagram for G2(s). with a small time delay D.

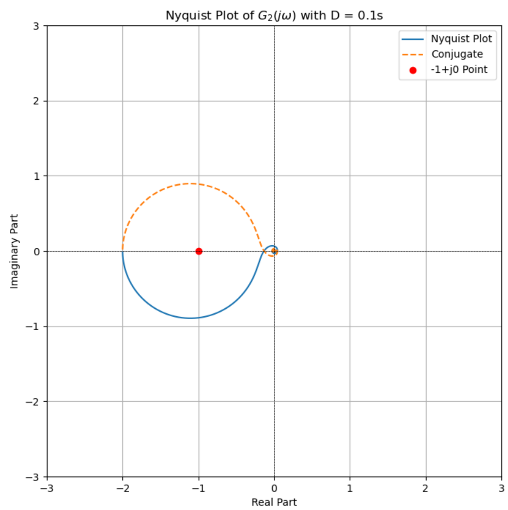


Figure 3: Nyquist diagram for G2(s) with time delay D

2.8 (§3.4 Integrator Wind-up) Explain how you calculated the bound on Q.

The time domain expression of the PID controller is:

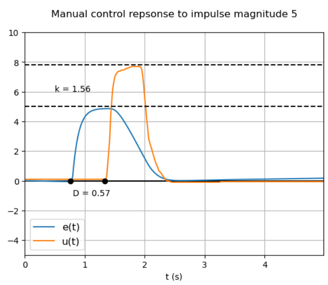
$$u(t) = K_P \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right)$$

And the bound Q is defined to be the maximum value of $\int_0^t e(\tau)d\tau$. We can see that at steady state, $e(t) = 0$ and $\frac{de(t)}{dt} = 0$, so the only term that contributes to $u(t)$ is the integral term. Therefore, we have: $u(t) = \frac{k_P}{T_i} \int_0^t e(\tau)d\tau$, and Q $= \frac{T_i}{K_P} u_{max} \approx 0.2$.

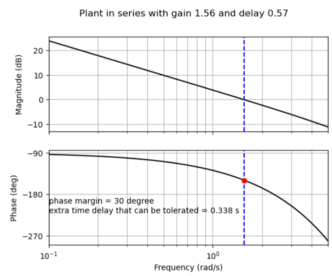
A Link to Jupyter Notebook

This link goes to the worked Jupyter Notebook finished on the lab session.

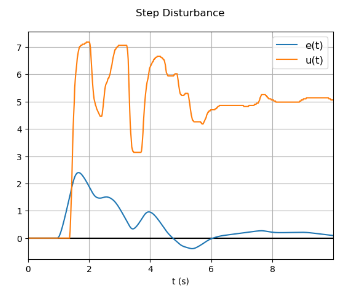
B Figures from Lab Session



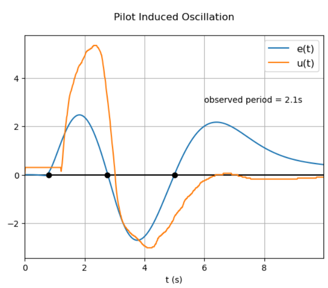
(a) Supplementary Figure 1



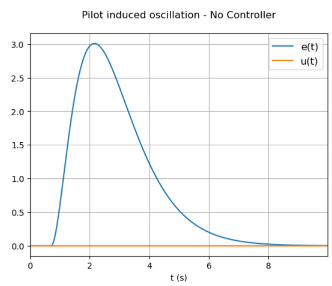
(b) Supplementary Figure 2



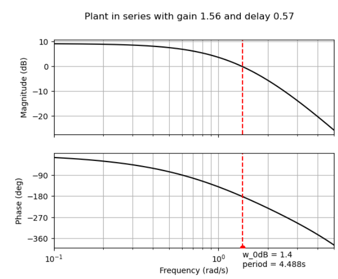
(c) Supplementary Figure 3



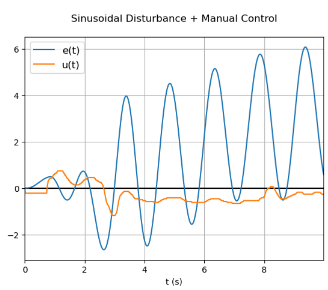
(d) Supplementary Figure 4



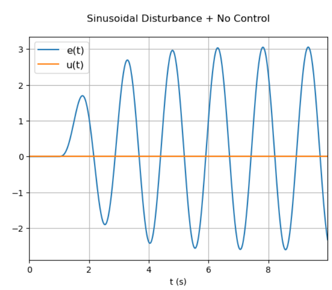
(e) Supplementary Figure 5



(f) Supplementary Figure 6

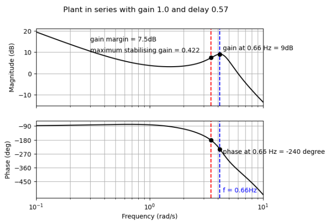


(g) Supplementary Figure 7

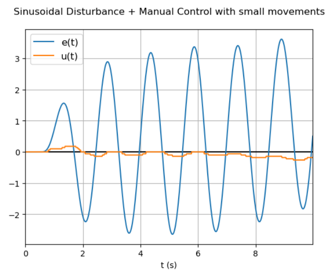


(h) Supplementary Figure 8

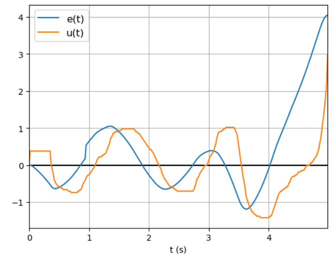
Figure 4: Supplementary figures 1–8 (continued on next page).



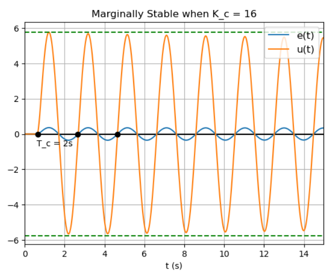
(a) Supplementary Figure 9



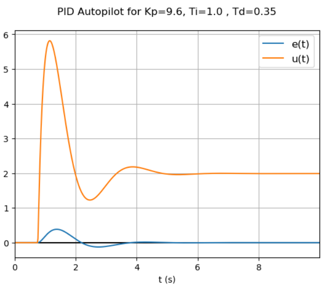
(b) Supplementary Figure 10



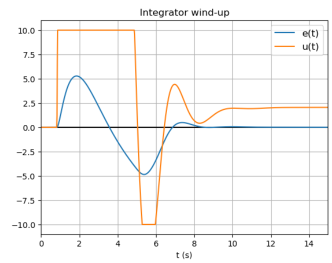
(c) Supplementary Figure 11



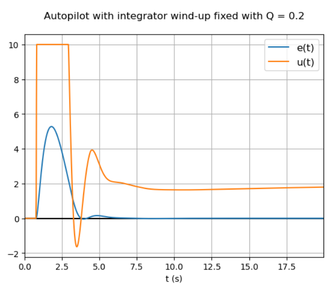
(d) Supplementary Figure 12



(e) Supplementary Figure 13



(f) Supplementary Figure 14



(g) Supplementary Figure 15

Figure 5: Supplementary figures 9–15 (continued from previous page).

References

[1] EIETL, CUED. *3F1 Flight Control Lab Guide*.

[2] U.S. Department of Defense. *Flying Qualities of Piloted Aircraft*. MIL-STD-1797A. Department of Defense, Washington, D.C., Jan 1990. (Military Standard).