CAMBRIDGE UNIVERSITY ENGINEERING DEPARTMENT

Part IB Laboratory Report

I1 Spectrum Analysis

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1 Summary

This experiment aims to introduce techniques on analyzing a signal by spectrum analysis, during which signals are decomposed into their frequency components, revealing more information. This is done by a computer-based spectrum analyser during this experiment. Amplitude Modulation (AM) is also analysed later in the experiment. At the same time, demodulator design is also introduced.

2 Readings and Results

2.1 Basic operation of picoscope software

This section shows the basic operation methods of picoscope software. The waveform recorded by the picoscope might appear unstable. Triggering can solve this problem as shown in Figure 1. We can use the build-in spectrum analyser of picoscope software to obtain the spectrum of the recorded waveform as shown in Figure 2.

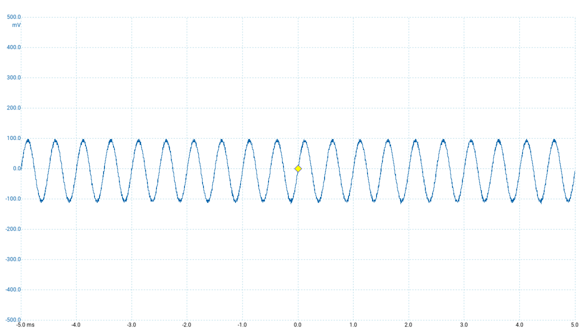

Figure 1: Time domain of the example wave

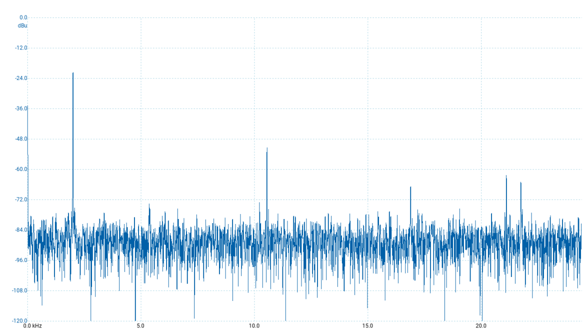

Figure 2: Frequency domain of the example wave

As we can see, frequency domain of the wave contains a peak which corresponds to the value of the frequency of wave in time domain. Thus, we can see that it successfully decompose a signal in time domain into its frequency components, and in this case, there is only one frequency component.

2.2 Spectra of Simple Periodic Signals

In this section, 2 iconic periordic signals is generated by the wave generator and recorded by the picoscope. Their spectrum is generated by the software and compared to Fourier Series theories.

2.2.1 Square wave

For a perfect square wave with peak-to-peak amplitude 2 and period T has a Fourier Series representation of:

$$x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \tag{1}$$

Where $\omega_0 = \frac{2\pi}{T}$ is the fundamental frequency and the coefficient is found by

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\sin n\omega_0 t dt = \begin{cases} \frac{4}{n\pi}, n = 1, 3, 5, \ldots \\ 0, \text{ otherwise} \end{cases} \tag{2}$$

2.2.2 Triangular wave

Fourier Series of a perfect triangular wave with peak-to-peak amplitude 2 and period T is:

$$x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \tag{3}$$

with coefficient

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\sin n\omega_0 t dt = \begin{cases} \frac{8}{\pi^2}\frac{(-1)^{(n-1)/2}}{n^2}, n = 1, 3, 5, \ldots \\ 0, \text{ otherwise} \end{cases} \tag{4}$$

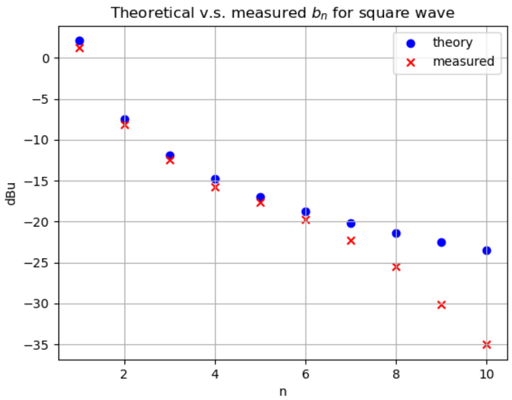

Figure 3: Comparison of b_n of square wave

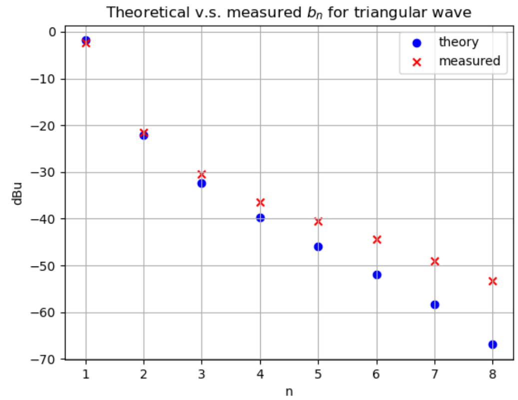

Figure 4: Comparison of b_n of triangular wave

As shown in Figure 3 and 4, the first several harmonics have measured value highly agree to theoretical values. However, the plot seems to deviate a lot from n = 6 in square wave plot and n = 4 in triangular wave. This is because a decibel scale is used and when numerical values gets small, the error is exponentially enlarged. When n goes up, as given by the expression of coefficients, b_n will become smaller value and is more prone to be affected by noise present. Thus, this deviation is within expectation.

To provide more insight on the hidden relations between square wave and triangular wave, it can be stated from the fact that two square waves can be convolved to be a triangular wave.

2.3 Amplitude Modulation

Assume the information signal is:

$$x(t) = E\cos(\omega t) \tag{5}$$

This signal can be modulated by a carrier signal:

$$x_c(t) = E_c\cos(\omega_c t) \tag{6}$$

The resultant modulated signal is composed of carrier signal and a offsetted information signal.

$$x_m(t) = x_c(t) + x(t)\cos(\omega_c t) = E_c(1 + m\cos\omega t)\cos\omega_c t \tag{7}$$

In which m is the modulation ratio defined by $\frac{E}{E_c}$. This expression can be expanded to show its frequency components:

$$x_m(t) = E_c\cos\omega_c t + \frac{mE_c}{2}\cos{(\omega_c + \omega)t} + \frac{mE_c}{2}\cos{(\omega_c - \omega)t} \tag{8}$$

Amplitgude modulation is widely used in signal transmission, especially for many band-limited signals over common mediums. Band-limited signals have such a small bandwidth that they are hard to be transmitted over a long distance and have much higher chance to interfere with other signals with close range of frequency.

However, AM provides a tool to change the frequency content of information signal to a higher value determined by carrier wave. Besides, demodulation of AM is simple to be conducted.

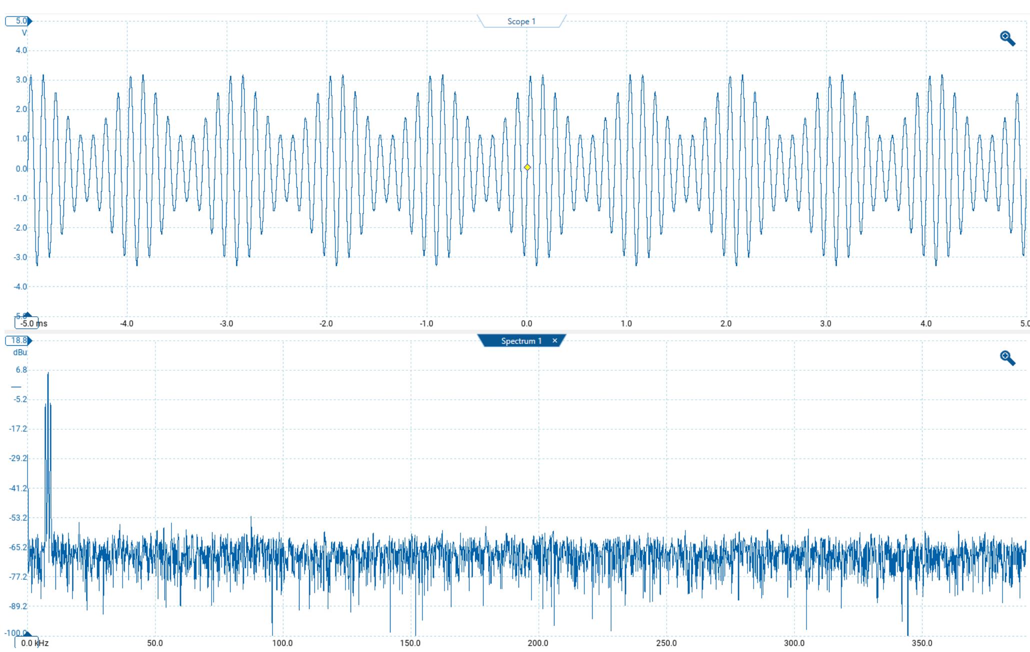

Figure 5: Time and frequency domain of AM signal

To adjust the waveform to be similar to the shape as shown in Figure 4 in the handout, the modulator settings are:

1. carrier amplitude = 4.3V

2. AM depth = 50%

To find the modulation index, it can be noted that the ratio of maximum amplitude of the envelope to the minimum is $\frac{1+m}{1-m}$. In this case, the ratio is approximately 3. Thus, $m = \frac{1}{2}$.
For the case $m = 1$, $E_{min} = min_t\{E_c(1 + m\cos\omega t)\} = 0$. So we can see how value of m affects the shape of the envelope of the modulated wave. We can adjust parameters of the modulator so that this wave can be simulated. In order to achieve the value of m to be 1, we can adjust the modulator parameter to:

1. carrier level = 3V

2. modulation depth = 100%

The resulting wavefrom is shown in Figure 6.
However, this simple AM can not represent the original information signal well since there is a carrier frequency component in the spectrum. To eliminate this carrier frequency component, Double-SideBand Suppressed Carrier (DSB-SC) wave can be used. This wave is shown in Figure 7. As shown in Figure 7, the carrier frequency component is eliminated. In this case, $\frac{1+m}{1-m} = \frac{3}{0.15}$, m is found to be $\frac{19}{21} = 0.905$.

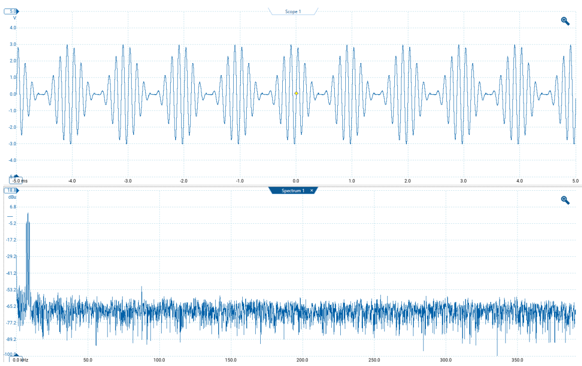

Figure 6: AM wave with m=1

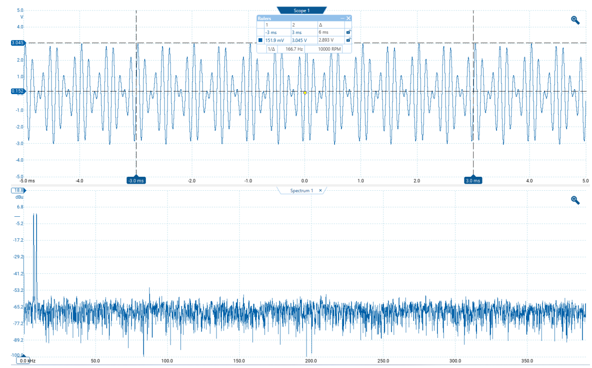

Figure 7: DSB-SC wave

2.4 Amplitude Demodulation

A simple demodulator circuit is shown in Figure 8, which is composed of a diode, a RC charging and discharging circuit.

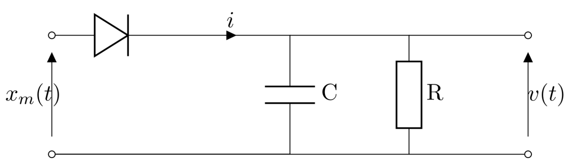

Figure 8: A demodulator circuit

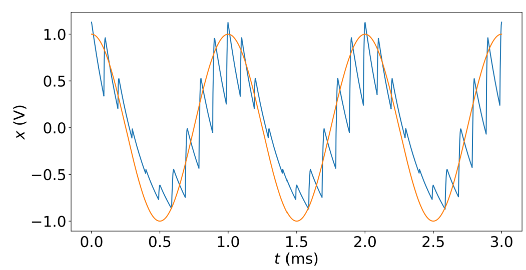

Figure 9: Resulting demodulator waveform

During demodulation process, the RC keep switching between charging and discharging state. The output voltage is given by:

$$v(t) = \begin{cases} v(\tau)e^{-\frac{t-\tau}{RC}}, & \text{if } x_m(t) < v(\tau)e^{-\frac{t-\tau}{RC}} \\ x_m(t), & \text{otherwise} \end{cases} \tag{9}$$

The charging and discharging process rates are dependent on the value of time constant of the RC circuit, the product of resistance and capacitance, RC. When RC has a large value, the process will be really slow. Vice versa, the process is really fast. In order to achieve effective reconstruction, the value of RC should be as small as possible so that charging and discharging processes are fast and the resultant curve is close to $x_m(t)$.

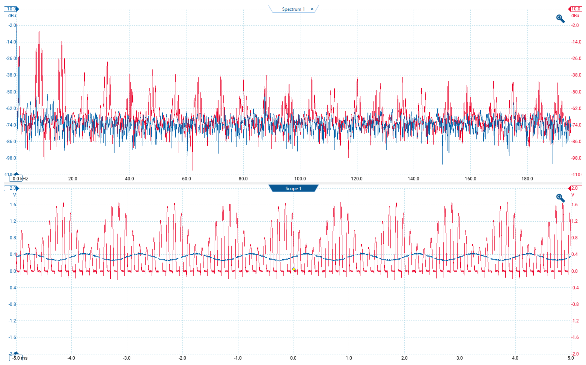

Figure 10: Waveform at TP1

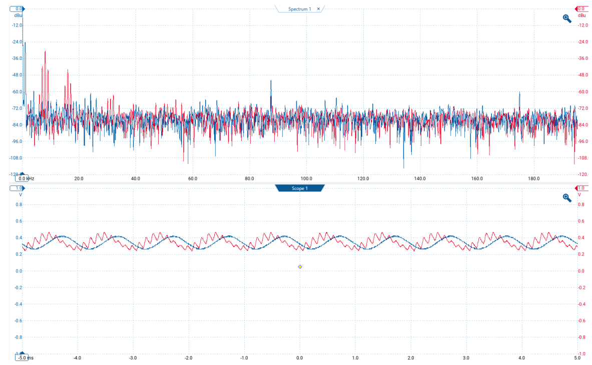

Figure 11: Waveform at TP2

The recorded waveforms at Test Point 1 and Test Point 2 are shown in Figure 10 and 11.

3 Conclusion

In the experiment, theories on spectrum analysis and amplitude modulation are derived and tested over a picoscope, a wave generator, and a modulator. From the experiment results, the observations agree well to the theories and provide more insight to future applications of spectrum analysis.