S2 Math Intensive Class (October)

S.2 Mathematics Intensive Class

Lesson 5: Use of Formula I (公式 I)

✵ Algebraic fraction 代數分式

✓ The form $\dfrac{P}{Q}$ where P and Q are polynomials, and Q is not a constant

代數式的分子分母都是多項式，且分母不等於常數。

例如： $\dfrac{1}{x+y}, \dfrac{2+b}{b}, \dfrac{x}{x^2-4x+4}, \dfrac{a+b}{2+b}$

denominator is not equal to zero
分母不可以等於 0

Example 1

化簡下列代數分式

$\dfrac{7}{14p-21q}$

Solution

$$\frac{7}{14p-21q}$$

$$=\frac{7}{7\times 2p-7\times 3q}$$

$$=\frac{\cancel{7}}{\cancel{7}(2p-3q)}$$

$$=\frac{1}{2p-3q}$$

By taking common factor to factorize it
利用**提取公因式的方法**把分母因式分解

When the numerator and denominator are unequal addition subtraction, they cannot be simplified.

For example, $\frac{x+2}{x+7}$ cannot be simplified as $\frac{2}{7}$.

當分子和分母的多項式是相加或減的關係，這是不能相約的。

例如：$\frac{x+2}{x+7}$不能化為$\frac{2}{7}$

Quick Practice 1

Simplify
化簡下列代數分式

$$\frac{5}{15a+100c}$$

✓ **Multiplication and Division of Algebraic Fraction 代數分式的乘法和除法**

Multiplication 乘法：

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$

Division 除法：

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$$

Example 2

Simplify the following.
化簡下列代數分式

$$\frac{a}{q^2}\times\frac{qy}{b}$$

Solution

$$\frac{a}{q^2}\times\frac{qy}{b}$$

$$=\frac{a}{\cancel{q}\times q}\times\frac{\cancel{q}y}{b}$$

$$=\frac{a}{q}\times\frac{y}{b}$$

$$=\underline{\underline{\frac{ay}{bq}}}$$

Example 3

Simplify the following.
化簡下列代數分式

$$\frac{1}{r^2}\div\frac{r+1}{r}$$

When doing fraction division, the numerator and denominator of the divisor should be upside down.
當分數相除時，記得除數的分子和分母上下顛倒。

Solution

$$\frac{1}{r^2}\div\frac{r+1}{r}$$

$$=\frac{1}{r\times r}\times\frac{r}{r+1}$$

$$=\frac{1}{\cancel{r}\times r}\times\frac{\cancel{r}}{r+1}$$

$$=\underline{\underline{\frac{1}{r(r+1)}}}$$

Quick Practice 2

Simplify the followings.
化簡下列代數分式

(a) $\frac{a}{b} \times \frac{2b}{3a}$

(b) $2cx^2 \div \frac{1}{3}c^2x$

✓ **Addition and Subtraction of Algebraic Fractions 代數分式的加法和減法**

Equal denominator 分母相同： Add or subtract each numerator directly
直接把分子相加或相減

Example 4

Simplify the following.
化簡下列代數分式

$\frac{x}{5y} + \frac{3x}{5y}$

Solution

$\frac{x}{5y} + \frac{3x}{5y}$

$= \frac{x+3x}{5y}$ Step 1 (步一)

$= \frac{4x}{5y}$

Unequal denominators 分母不相同

Step 1(步一)： Find L.C.M. of the denominators
先找出各分母的 L.C.M. (最小公倍數)

Step 2(步二)： Add or subtract the numerators
分子相加或相減

Example 5

Simplify the following.
化簡下列代數分式

$$\frac{2}{3b}+\frac{1}{4b}$$

Solution

$$\frac{2}{3b}+\frac{1}{4b}$$

$$=\frac{2\times 4}{12b}+\frac{1\times 3}{12b}$$

$$=\frac{8+3}{12b}$$

$$=\underline{\underline{\frac{11}{12b}}}$$

$3b = 3 \times b$
$\underline{4b = 4 \times b}$
L.C.M. $= 3\times 4\times b$
$= 12b$

Example 6

Simplify the following.

化簡下列代數分式

$$\frac{1}{x-1}+\frac{x}{1-x}$$

Solution

$$\frac{1}{x-1}-\frac{x}{1-x}$$

$1-x=-(x-1)$

$$=\frac{1}{x-1}-\frac{x}{-(x-1)}$$

$-\frac{a}{b}=\frac{-a}{b}=\frac{a}{-b}$

$$=\frac{1}{x-1}-\left[-\frac{x}{(x-1)}\right]$$

$$=\frac{1}{x-1}+\frac{x}{x-1}$$

$-(-)=+$

$$=\underline{\underline{\frac{1+x}{x-1}}}$$

Quick Practice 3

Simplify the followings.

化簡下列代數分式

1) $\frac{1}{x-1}-\frac{x}{x-1}$

2) $\frac{3}{4-3y}-\frac{2y}{3y-4}$

Exercise 1 練習一

Simplify the followings.

試化簡下列各數式。

1) (a) $-\frac{2}{a}-\frac{2}{a}$

(b) $-\frac{2}{r-2}+\frac{7r}{r-2}$

(c) $\frac{h}{h+k}+\frac{k}{h+k}$

(d) $\frac{7}{3r-2}+\frac{4}{2-3r}$

2) (a) $\frac{1}{a}-\frac{1}{a+b}$

(b) $\frac{2c-d}{c^2d}+\frac{c+d}{cd^2}$

(c) $4-\frac{a-2}{5a}$

(d) $\frac{2}{3x}+\frac{1-4y}{6xy}$

3) (a) $\frac{-3}{6x}$

(b) $\frac{-48x^2}{-144x^3}$

(c) $\frac{4h^2-6h}{2h}$

(d) $\frac{p}{p^2r-p^2s}$

4) (a) $\frac{am-2an}{6mn-3m^2}$

(b) $\frac{a}{6}\div\frac{a^2}{3}$

(c) $\frac{a+b}{a-b}+\frac{1}{2}$

(d) $\frac{9y+1}{3y}+\frac{5y-1}{2y}$

5) $\dfrac{p^2-pq}{q}\times\dfrac{p^2q}{pq-q^2}$

6) $\dfrac{9u^2v}{5u-2v}\times\dfrac{10v-25u}{3v^2}$

7) $\dfrac{9x}{(x-y)^2}-\dfrac{8}{y-x}$

8) $\dfrac{1}{r+3s}+\dfrac{r}{3rs+9s^2}$

9) $\dfrac{4ax^3+12ax^2}{6ax^2}$

10) (a) Find L.C.M. of $2xy^2$, $3y$ and $6xy^2$
求 $2xy^2$, $3y$,及 $6xy^2$ 的 L.C.M.

(b) Simplify $\dfrac{3xy-1}{2xy^2}-\dfrac{5}{3y}-\dfrac{7xy-3}{6xy^2}$
化簡$\dfrac{3xy-1}{2xy^2}-\dfrac{5}{3y}-\dfrac{7xy-3}{6xy^2}$

Extra Questions 額外題目

Simplify the following expressions. 試化簡下列各數式。

1. $\dfrac{256a^3b^4}{144a^5b^2}$

2. $\dfrac{-(5x+3)^2}{15ax+9a}$

3. $\dfrac{12a-4ab}{2b^2-6b}$

4. $\dfrac{x^2+2xy+y^2}{x^2-y^2}$

5. $\dfrac{pq+qr+rs+ps}{-pq-qr+rs+ps}$

6. $\dfrac{8ab}{72(ab)^3}\times\dfrac{48a^5b}{64b^3}$

7. $\dfrac{4a^4}{5b^2}\div\dfrac{8a^2}{10ab^3}$

8. $\dfrac{a^2-b^2}{a^2-2ab+b^2}\times\dfrac{a-b}{a+b}$

9. $\dfrac{1}{a+1}-\dfrac{1}{a+2}$

10. $\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}$

11. $\dfrac{1}{x^2+7x+12}+\dfrac{1}{x^2+5x+6}$

Challenging Questions 挑戰題

Simplify the following expressions.
簡化以下各題。

1. $\dfrac{256a^3b^4}{144a^5b^2}$

2. $\dfrac{-(5x+3)^2}{15ax+9a}$

3. $\dfrac{12a-4ab}{2b^2-6b}$

4. $\dfrac{x^2+2xy+y^2}{x^2-y^2}$

5. $\dfrac{pq+qr+rs+ps}{-pq-qr+rs+ps}$

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Lesson 6: Use of Formula 2 (公式 2)

Formulae (公式)

✓ To represent the relation among variable (表示兩個或以上變數關係的等式)

例如： Ohm's Law (歐姆定律)： $V = IR$

✓ Using substitution to find the unknowns (利用代入法求出公式中的未知數)

Step 1 (步一)： 將已知的未知數的數值代入公式。

Step 2 (步二)： 求餘下的未知數。

Example 1

Given the formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, if $f = 3$ and $u = 4$, find the value of v.

已知公式 $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$，若 $f = 3$ 及 $u = 4$，求 v 的值。

Solution

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{3}=\frac{1}{4}+\frac{1}{v}$$

Step 1 (步一)： 把 *f* = 3, *u* = 4 代入公式中

Step 2 (步二)： 求餘下的未知數(*v*)

$$\frac{1}{3}-\frac{1}{4}=\frac{1}{v}$$

$$\frac{4-3}{12}=\frac{1}{v}$$

$$\frac{1}{12}=\frac{1}{v}$$

$$v=\underline{\underline{12}}$$

✓ <u>Change of subject in a formula (變換主項)</u>

The subject of the formula = when a variable is expressed by other variables.

公式的主項 = 在一個公式中，單獨出現在等號一方(多是左方)的變數，而另一邊並沒有該變數出現。

例如： y is a subject of the formula $y = ax + d$.

$y = ax + d$ 這條公式中，y 是主項。

$y = ax + b$ 公式中，單獨出現在等號左方的變數是 y，而右邊 $(ax + d)$並沒有出現 y。

Example 2

Change x as the subject.

把公式 $y = mx + c$ 的主項變為 x

$y = mx + c$ 公式中，y 是主項，但現在變為 x 作主項，即將 x 單獨放在等號的一方，而另一方並沒有 x 出現。

Solution

$$y = mx + c$$

把 c 移到左方

$$y - c = mx$$

左右兩方除以 m, $m\neq 0$

$$\frac{y-c}{m}=\frac{\cancel{m}x}{\cancel{m}}$$

$$\underline{\underline{x=\frac{y-c}{m}}}$$

Example 3

Make the letter in the bracket the subject of each of the formulae.
把公式的主項變換為括號的字母

(a) $t = \dfrac{a-n}{n+a}$ [n]

(b) If t = -3 and a = -1,find the value of n.
若 t = -3 及 a = -1, 求 n 的值。

Solution

(a)

$$t = \frac{a-n}{n+a}$$

both sides multiply *(n + a)* to eliminate the fraction.
於兩邊同時相乘*(n + a)*，從而取消右方分數的*(n + a)*。

$$t(n+a) = \frac{a-n}{n+a}(n+a)$$

Expand *t(n + a)*
展開 *t(n + a)*

$$t(n + a) = a - n$$

$$nt + at = a - n$$

move –*n* to the left side and *a to the right side.*
把-*n* 移到左方，*at* 移到右方

$$nt + n = a - at$$

$$n(t + 1) = a(1 - t)$$

提取公式 *n* 和 *a*
左右兩方同時除以 *t + 1*

$$n = \frac{a(1-t)}{t+1}$$

(b) Using substitution to substitute t = -3and a = -1 into the result of (a) $n = \dfrac{a(1-t)}{t+1}$

利用代入法把 t = -3 及 a = -1 代入(a)的結果 $n = \dfrac{a(1-t)}{t+1}$

$$n = \frac{-1\times[1-(-3)]}{(-3)+1}$$

$-(-) = +$

$$n = \frac{-1\times(4)}{-2}$$

$\dfrac{(-)}{(-)} = +$

$n = \underline{\underline{+2}}$

即時練習 1

1) (a) If $V=\frac{1}{3}\pi r^2 h$, change r as the subject.

若$V=\frac{1}{3}\pi r^2 h$ ，則改 r 為主項

(b) By the result of (a), if $V = 66$ and $\pi=\frac{22}{7}$, find the value of r.

利用結果(a)，若 $V = 66$, h = 7 和 $\pi=\frac{22}{7}$ ，求 r 的值。

Exercise 1 (練習一)

1) Make the letter in the brackets the subject of each of the following formulae.
把公式的主項變換為括號的字母

(a) $d = b + c$ [c] (b) $a + b + c = 180^{O}$ [a]
(c) $F = ma$ [m] (d) $PV = RT$ [R]
(e) $a = \dfrac{b-4}{3}$ [b] (f) $a = \dfrac{6-b}{5}$ [b]

2) Make the letter in the brackets the subject of each of the following formulae.
把公式的主項變換為括號的字母

(a) $y = \dfrac{2}{3x} + \dfrac{1}{x}$ [x] (b) $S = \dfrac{1}{2}(v+u)t$ [t]
(c) $y = \dfrac{2x}{x-2}$ [x] (d) $y = 3 + \sqrt{x-1}$ [x]
(e) $p = \dfrac{A}{1+ni}$ [n] (f) $q = \dfrac{2}{rp} - \dfrac{1}{p}$ [p]

3) If $F = \dfrac{9}{5}C + 32$, change C as the subject.
若 $F = \dfrac{9}{5}C + 32$，試改 C 為主項。

4) Make v the subject of the formula $k = \dfrac{1}{2}mv^2$
已知公式 $k = \dfrac{1}{2}mv^2$，將 v 變為主項。

5) If $y = \dfrac{a+x}{a-x}$ change x as the subject.
若 $y = \dfrac{a+x}{a-x}$, 則改 x 變為主項。

6) Make h the subject of the formula $A = \pi r^2 + 2\pi rh$
已知公式 $A = \pi r^2 + 2\pi rh$，試將 h 變為主項。

7) If $T = 2\pi\sqrt{\dfrac{l}{g}}$, find g in terms of T and l.
若 $T = 2\pi\sqrt{\dfrac{l}{g}}$，則求 g，以 T 及 l 來表達。

8) If $1-\frac{a}{b}=\frac{1+ax}{1+bx}$ 若$1-\frac{a}{b}=\frac{1+ax}{1+bx}$

(a) Express x in terms of a and b.
試用 y 和 b 表示 a

(b) If $a = 2, b = 3$, find the value of x.
若 $a = 2, b = 3$, 求 x *的值*

9) The curved surface area of a cone is given by 圓錐體之曲面面積

$$S=\pi r\sqrt{h^2+r^2}$$

(a) If $S = 330m^2$, $\pi=\frac{22}{7}$ and $r = 7m$, find the value of h.

若 $S = 330m^2$, $\pi=\frac{22}{7}$ 及 $r = 7m$，求 h 之值。

(b) Make h the subject of the formula.
試將 h 變為主項。

10) The length L of a spring of natural length l hanging vertically is given by the formula
彈簧在沒有懸掛物件時的長度是l，懸掛物件後，它的長度 L 可用以下公式求得：

$$L=l+\frac{W}{k}$$

where W is the weight of the hanging object and k is a constant.
其中 W 是懸掛物的重量，k 是常數。

(a) (i) Make W the subject of the formula.
把公式的主項變換為 W。

(ii) Find the value of W when $L = 13$, $l = 8$ and $k = 1.5$.
若 $L = 13$, $l = 8$ 和 $k = 1.5$，求 W 的值。

(b) (i) Make k the subject of the formula.
把公式主項變換為 k。

(ii) Find the value of k when $L = 30$, $l = 26$ and $W = 4$.
若 $L = 30$, $l = 26$ 和 $W = -4$，求 k 的值。

More Questions(額外題目)

In each of the following, make the letter in the square bracket as the subject of the formula.
於下列各題中，將方括號中的字母變換成公式的主項。

1. Ax + By + C = 0 [y]

2. V = $\frac{IB}{nQt}$ [B]

3. p(x + y) + q(x – y) + r = 0 [x]

4. y = $\frac{x-1}{x+1}+\frac{1}{z}$ [z]

5. $\dfrac{x-a}{x+a}=b$ [x]

In each of the following, find the value of the unknown by changing the subject of the formula to a suitable variable.
於下列各題中，先將公式的主項變換為合適的變數，然後求未知數的值。

6. $\dfrac{1}{u}+\dfrac{1}{v}=\dfrac{1}{f}$ If $u=-20$ and $f=-40$, find v. 如果$u=-20$而 $f=-40$，求 v。

7. $S=\dfrac{a(1-R^n)}{1-R}$ If $S=8, n=2$ and $R=3$, find a. 如果$S=8, n=2$而$R=3$, 求 a。

8. If $a=\dfrac{1}{1+b}-1$, express b in terms of a.

若 $a=\dfrac{1}{1+b}-1$，試以 a 表示 b。

9. If $(x-a)^2=(x-1)(x-b)$, express x in terms of a and b.
若$(x-a)^2=(x-1)(x-b)$ ，試以 a 及 b 表示 x。

10. If $y=1+\sqrt{2x-1}$, express x in terms of y.

若$y=1+\sqrt{2x-1}$ ，試以 y 表示 x。

Harder Questions:

1. Given $a^2 + b^2 = c^2$. Which of the following is/are true ?
   A. $b^2 = a^2 - c^2$
   B. $c = \sqrt{a+b}$
   C. $b = \sqrt{a^2 + c^2}$

   A. I only
   B. III only
   C. I and II only
   D. None of them

   已知 $a^2 + b^2 = c^2$，下列何者為正確?
   I. $b^2 = a^2 - c^2$
   II. $c = \sqrt{a+b}$
   III. $b = \sqrt{a^2 + c^2}$

   A. 只有 I
   B. 只有 III
   C. 只有 I 及 II
   D. 無一正確

2. Given $I = \dfrac{V}{R+r}$, which of the following is true?

   已知 $I = \dfrac{V}{R+r}$,下列何者為正確?

   A. $R = \dfrac{V - Ir}{I}$
   B. $r = \dfrac{V + I}{R}$
   C. $V = IR + r$
   D. $I = \dfrac{V - R}{r}$

3. If $x - a = p$ and $y - p = a$, then $x + y =$
   若 $x - a = p$ 及 $y - p = a$，則 $x + y =$

A. 2a + p

B. 2p + a

C. 2(a + p)

D. 2(a - p)

4. Given that $T = \dfrac{a - 4b}{b}$.

(a) Find the value of T when a=4 and b=-2.

(b) Make b as the subject of the formula.

(c) Hence find the value of b when T= -1 and a= 15.

已知 $T = \dfrac{a - 4b}{b}$

(a) 若 a=4 及 b=-2, 求 T 的值;

(b) 把公式的主項變換為 b;

(c) 由此, 若 T= -1 及 a= 15, 求 b 的值

5. Given that $x = y^2 + 1$ and $y = \dfrac{3z}{4}$.

(a) Write a formula relating x and z only. (without y)

(b) Change z as the subject of formula.

(c) Find the value of z when x=10.

已知 $x = y^2 + 1$ 及 $y = \dfrac{3z}{4}$

(a) 試寫出一條只有 x 及 z 的公式

(b) 把公式的主項變換為 z

(c) 若 x= 10, 求 z 的值

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Lesson 7: Factorization 1 (因式分解) 1

❋ Factorization by Identity (利用恆等式進行因式分解)

✓ In previous chapter, we had learnt three useful identities. They are Identity of Difference of Two Squares and Identity of Perfect Square.

在之前的課題中，我們學習過三個常用的恆等式。它們分別是兩平方的差及完全平方。

Difference of two square:
兩平方的差：

$$a^2 - b^2 = (a + b)(a - b)$$

Perfect square:
完全平方：

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

Example 1

Factorize the following expression.
因式分解下列表達式。

(a) $16x^2 - 9y^2$

(b) $25(a + b)^2 - 4$

(c) $(c + d)^2 - (e - f)^2$

Solution

(a) $16x^2 - 9y^2 = (4x)^2 - (3y)^2$

$\quad = (4x + 3y)(4x - 3y)$

(b) $25(a+b)^2 - 4 = [5(a+b)]^2 - 2^2$

$= [5(a+b)+2]\,[5(a+b)-2]$

$= (5a+5b+2)(5a+5b-2)$

(c) $(c+d)^2 - (e-f)^2 = [(c+d)+(e-f)][(c+d)-(e-f)]$

$= (c+d+e-f)(c+d-e+f)$

Example 2

Factorize the following expressions. 因式分解下列表達式。

(a) $2x^2 - 8$ (b) $18p - 2p(1-d)^2$

Solution

(a) $2x^2 - 8 = 2(x^2 - 4)$

$= 2(x^2 - 2^2)$

$= 2(x+2)(x-2)$

(b) $18p - 2p(1-d)^2 = 2p[9 - (1-d)^2]$

$= 2p\,[3^2 - (1-d)^2]$

$= 2p\,[3+(1-d)][3-(1-d)]$

$= 2p\,(3+1-d)(3-1+d)$

$= 2p(4-d)(2+d)$

Example 3

Factorize $m^2 - n^2 - an + am$. 因式分解 $m^2 - n^2 - an + am$。

Solution

$m^2 - n^2 - an + am = (m^2 - n^2) + (am - an)$

$= (m+n)(m-n) + a(m-n)$

$= (m-n)(m+n+a)$

堂上練習 1

Factorize the following expression. 因式分解下列各表達式。

1. $49 - 16k^2$

2. $(x + 1)^2 - 36$

3. $3b^2 - 27y^2z^2$

4. $64\,(a + b)^2 - (b - a)^2$

5. $50x^2 - 2(y + z)^2$

6. $pr + qr - 4p^2 + 4q^2$

即時練習 1

Factorize the following expression. 因式分解下列各表達式。

1. $x^2 - 25$

2. $16a^2 - 81$

3. $100r^2 - 49x^2$

4. $(x - 1)^2 - 144$

5. $3t^2 - 75$

6. $27a^2 - 12b^2$

7. $-6x^2y^2 + 96$

8. $-243ac^2 + 75b^2a$

9. $p^2 - (p + q)^2$

10. $25(x + 1)^2 - 16(x - 1)^2$

11. $9p^2 - q^2 - 12p - 4q$

12. $x^2y^4 - 16x^2$

Example 4

Factorize $x^2 + 4xy + 4y^2$. 因式分解 $x^2 + 4xy + 4y^2$。

Solution

$$\begin{aligned} x^2 + 4xy + 4y^2 &= x^2 + 2(x)(2y) + (2y)^2 \\ &= (x + 2y)^2 \end{aligned}$$

Example 5

Factorize $16(m - 1)^2 - 8(m - 1) + 1$. 因式分解 $16(m - 1)^2 - 8(m - 1) + 1$。

Solution

$$16(m-1)^2 - 8(m-1) + 1 = [4(m-1)]^2 - 2[4(m-1)](1) + 1^2$$
$$= [4(m-1) - 1]^2$$
$$= (4m-5)^2$$

Alternative Solution

Let $x = m - 1$

$$16(m-1)^2 - 8(m-1) + 1 = 16x^2 - 8x + 1$$
$$= (4x)^2 - 2(4x)(1) + 1^2$$
$$= (4x-1)^2$$
$$= [4(m-1) - 1]^2$$
$$= (4m-5)^2$$

Example 6

Factorize $-27x^2 + 36xy - 12y^2$.因式分解$-27x^2 + 36xy - 12y^2$。

Solution

$$-27x^2 + 36xy - 12y^2 = -3(9x^2 - 12xy + 4y^2)$$
$$= -3[(3x)^2 - 2(3x)(2y) + (2y)^2]$$
$$= -3(3x-2y)^2$$

堂上練習 2

Factorize the following expression. 因式分解下列各表達式。

1. $k^2 - 12k + 36$

2. $25m^2 + 4n^2 + 20mn$

3. $(x + y)^2 - 22(x + y) + 121$

4. $-18a^2 - 48ab - 32b^2$

✓ Using Identities of Sum and Difference of Two Cubes
利用恆等式中兩立方的和與差

The following are another two algebraic identities:
下列展出數為另外兩條代數式的恆等式。

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

They are called the identities of the sum and the difference of two cubes.
它們稱為恆等式中兩立方的和與差。

Example 7

Factorize the following expression. 因式分解下列各表達式。

(a) $8x^3 + 1$ (b) $27p^3 - 64q^3$

Solution

(a) $8x^3 + 1 = (2x)^3 + 1^3$
$= (2x + 1)([(2x)^2 - (2x)(1) + 1^2]$
$= (2x + 1)(4x^2 - 2x + 1)$

(b) $27p^3 - 64q^3 = (3p)^3 - (4q)^3$

$= (3p - 4q)[(3p)^2 + (3p)(4q) + (4q)^2]$

$= (3p - 4q)(9p^2 + 12pq + 16q^2)$

Example 8

Factorize the following expression. 因式分解下列各表達式。

(a) $(x - 1)^3 - 27$ (b) $-2mn^3 - 16m$

Solution

(a) $(x - 1)^3 - 27 = (x - 1)^3 - 3^3$

$= [(x - 1) - 3][(x - 1)^2 + (x - 1)(3) + 3^2]$

$= (x - 4)[(x^2 - 2x + 1) + (3x - 3) + 9]$

$= (x - 4)(x^2 + x + 7)$

(b) $-2mn^3 - 16m = -2m(n^3 + 8)$

$= -2m(n^3 + 2^3)$

$= -2m(n + 2)[n^2 - (n)(2) + 2^2]$

$= -2m(n + 2)(n^2 - 2n + 4)$

堂上練習 3

Factorize the following expression. 因式分解下列各表達式。

1. $x^3 - 125$

2. $64m^3 + 1$

3. $8m^3 + 27n^3$

4. $192 - 3a^3$

5. $-z^4 - 27z$

6. $p^3 - (1 - p)^3$

即時練習 2

1. $x^2 + 8x + 16$

2. $4x^2 + 12x + 9$

3. $100 - 140m + 49m^2$

4. $f^2 - 6fg + 9g^2$

5. $49r^2 + 28ts + 4s^2$

6. $-16p^2 + 56pq - 49q^2$

7. $8m^2 - 24m + 18$

8. $m^3 - 8$

9. $x^3y^3 + 64$

10. $343r^3 - 1000s^3$

11. $a^3 - 8a^2 + 16a$

12. $4 - 28(r - 3) + 49(r - 3)^2$

13. $2(a + 2b)^2 - 24(a + 2b) + 72$

14. $5x^2 - 100xy^2 + 500y^4$

15. $4a^3 - (b^2 - 6b + 9)a$

16. $1 - x^2 + 4xy - 4y^2$

17. $a^6k^6 - 1$

18. $2x + 4y + x^3 + 8y^3$

More Questions(額外題目)

Factorize the following polynomials by using the difference of two squares.
運用平方差的恆等式將下列各多項式分解為因式。

1. $a^2 - 16$

2. $25b^2 - 16$

3. $81p^2 - 4q^2$

Factorize the following expressions.
因式分解下列各式。

4. (a) $18a^2 + 48ab + 32b^2$

(b) $50a^2 + 60ab + 18b^2$

(c) $20x^2 + 60xy + 45y^2$

5. (a) $(x+y)^2-49$

(b) $81-(2x-3y)^2$

(c) $(2x-y)^2-x^2$

6. (a) $(a-b)^2-(a+b)^2$

(b) $(2a+b)^2-(a-b)^2$

(c) $(a-2b)^2-(a+b)^2$

(d) $(2x+y)^2-(x+2y)^2$

7. (a) $(a+b)^2+4(a+b)+4$

(b) $(x-y)^2-8(x-y)+16$

(c) $(x+3y)^2+6(x+3y)+9$

(d) $4(2x+y)^2+4(2x+y)+1$

8. (a) $x^2-y^2+4x+4y$

(b) $4x^2-y^2-8x-4y$

(c) $9x^2-y^2+6x-2y$

(d) $8x^2-18y^2-6x+9y$

9. (a) $(x^2+4x+4)-9y^2$

(b) $25x^2-(9y^2-6y+1)$

(c) $(x^2+6xy+9y^2)-16z^2$

(d) $4x^2-(4y^2-4yz+z^2)$

10. Using identities to find the values of the following.

試應用恆等式求下列各式的值。

(a) $81^2 - 19^2$

(b) $275^2 - 225^2$

(c) $345^2 - 344^2$

(d) $538^2 - 462^2$

(e) 1001^2

(f) 999^2

Harder Questions:

1. Factorize $(x+4)^2 - 18x$
   A. (x-2)(x-8)
   B. (x-2)(x+8)
   C. (x+4)(x+4)
   D. (x+2)(x-10)

2. Which of the following is/are true?
   以下何者正確?
   I. 4a-2b=-2(b-2a)
   II. (2a-1) is a factor 因式 of (b-2ab)
   III. $a^2 + b^2 = (a+b)^2$

A. I only

B. II only

C. I and II only

D. I, II and III

3. Which of the following is a common factor of x^2+2x+8 and x^2-6x+8

以下何者是x^2+2x-8及x^2-6x+8的公因式?

A. X+2

B. X-2

C. X+4

D. X-4

4. Which of the following is/are factors of (a^3-a^2) ?

以下何者是(a^3-a^2)的因式?

I. a

II. a^2

III. $a-1$

A. II only

B. III only

C. II and III only

D. I and II and III

5. Given that $P(x)=x^2+2x$ and $Q(x)=x^3+2x^2-3x-6$

(a) Factorize 因式分解 P(x) and Q(x)

(b) Simplify 化簡 $\dfrac{P(x)}{Q(x)}$

S.2 Mathematics Intensive Class

Lesson 8: Factorization 2 (因式分解 2)

✵ Factorization by Grouping Terms (利用併項法作因式分解)

✓ In previous lesson, we had learnt to factorize an algebraic expression by taking out the common factors. For example,

在以前的課中，我們已經學習過透過抽取同類項而對代數表達式進行因式分解。例如：

$5x - 15y = 5(x - 3y)$
$6a^3 + 15a^2 = 3a^2(2a + 5)$

Consider the polynomial $ax + ay + bx + by$. We cannot factorize it by taking out the common factors since there are no common factors among the terms.

考慮多項式 $ax + ay + bx + by$。我們並不能把它進行因式分解，因為在這四項中，並沒有同一的同類項。
However, if we group the terms into two parts properly, we can factorize the polynomials as follow:

但是，若果我們把這四項分成兩部份來看，我們可以把這多項式分解如下：

$\overbrace{ax+ay}^{\text{1st pair}} + \overbrace{bx+by}^{\text{2nd pair}}$ = a(x + y) + b(x + y)

= (a + b)(x + y)

This is known as the grouping terms method.
這種進行因式分解的辦法稱為「併項法」。

There may be different ways to group the terms. For example, we can group the terms of ax + ay + bx + by as follows:
除了上述的方法外，還有其他方式去組合各項。例如，我們可以把 ax + ay + bx + by 的各項組合如下：

ax + ay + bx + by = $\overbrace{ax+bx}^{\text{1st pair}} + \overbrace{ay+by}^{\text{2nd pair}}$

= x(a + b) + y(a + b)

= (a + b)(x + y)

Note that the same result can be obtained.
注意上述兩個答案是一樣的。

Example 1

Factorize a^2 + cd + ad – ca by the grouping terms methods.
利用併合法因式分解 a^2 + cd – ad – ca。

Solution

a^2 + cd - ad – ca
= (a^2 – ca) + (cd – ad)
= a (a – c) + d(c – a)
= a (a – c) – d (a – c)
= (a – d)(a – c)

OR a^2 + cd – ad – ca
= (a^2 - ad) + (cd – ca)
= a (a - d) + c(d – a)
= a (a – d) – c(a – d)
= (a – d)(a – c)

Example 2

Factorize $3p - 9q - 2pq + 6q^2$ by the grouping terms methods.
利用併合法因式分解 $3p - 9q - 2pq + 6q^2$。

Solution

$$\begin{aligned} 3p - 9q - 2pq + 6q^2 &= (3p - 9q) - (2pq - 6q^2) \\ &= 3(p - 3q) - 2q(p - 3q) \\ &= (p - 3q)(3 - 2q) \end{aligned}$$

In this question, do we have another way for group terms?
在這條題目中，我們作有第二種方法進行併合法嗎？

Example 3

Factorize $mx + nx - px + py - ny - my$ by the grouping terms method.
利用併合法因式分解 $mx + nx - px + py - ny - my$。

Solution

$$\begin{aligned} mx + nx - px + py - ny - my &= mx - my + nx - ny - px + py \\ &= (mx - my) + (nx - ny) - (px - py) \\ &= m(x - y) + n(x - y) - p(x - y) \\ &= (x - y)(m + n - p) \end{aligned}$$

Alternative Solution

$$\begin{aligned} mx + nx - px + py - ny - my &= (mx + nx - px) + (py - ny - my) \\ &= x(m + n - p) - y(m + n - p) \\ &= (m + n - p)(x - y) \end{aligned}$$

堂上練習 1

Factorize the following expression by the grouping terms method.
利用併合法因式分解下列各題。

1. $2p + ps + 2q + qs$

2. $x^2 + xz - xy - yz$

3. $4ab - 3c - 4ac + 3b$

4. $Ax + By + Cx + Ay + Bx + Cy$

即時練習 1

Factorize the following expression by the grouping terms method.
利用併合法因式分解下列各題。

1. $ax + a + x + 1$

2. $3a + 3b + ac + bc$

3. $4y - xy + 4z - xz$

4. $ax - ay - bx + by$

5. $2ac + 2bc - 3ad - 3bd$

6. $p + q + qs + ps$

7. $bc + 7c - ab - 7a$

8. $x^2 + xy + ax + ay$

9. $5x^2 - 5xy - 2x + 2y$

10. $4b + 7a - 7ab - 4b^2$

11. $x^3 + 3x^2 + 2x + 6$

12. $a^2 - b + ab - a$

13. $2b^2 - 6ab - b + 3a$

14. $5p^2 - 5px - 3x + 3p$

15. $a^3 - ab + 5b - 5a^2$

16. $3a^2 - b + ab - 3a$

17. $4x^3 + 2x^2 + 16x + 8$

18. $(b + a)^2 - ap - bp$

19. $a^3 + a^2b - a^2c - abc$

20. $r^3 - 6x^3 - 3r^2x^2 + 2rx$

21. $3r^2st + 9rstx - 36rx^2 - 12r^2x$

22. $-ax - bx - cx + ay + by + cy$

23. $ax - bx - ay + by + ax - bz$

24. $ax - ay + cx - cy$

25. $ec + ed - fc - fd$

26. $a^2 + ab + ac + bc$

27. $8c^2 - 4c + 4cd - 2d$

28. $5ab - 3c - 3ac + 5b$

29. $p^3 + p - p^2 - 1$

30. $x^2 - xt + 2t^2 - 2xt$

31. $pqs - 3at - pqt + 3as$

More Questions(額外題目)

Factorize the following expressions.

因式分解下列各式。

1. (a) $2a+6b+4$

   (b) $14r-7s+28t$

   (c) $8t+32s-24st$

   (d) $ac+ad-ae$

2. (a) $10a^3-2a^2+6a$

   (b) $21b^4+15b^3-24b^2$

   (c) $7c-49c^2-28c^3$

   (d) $20m^3+35m^2-10m^5$

3. (a) $ab^2 - 2ab + a^2b$

(b) $14x^2y - 21xy^2 - 7yx$

(c) $4pq^2 + 16q^3p - 12p^3q$

(d) $8a^3c - 2ac^4 + 6ac^2$

(e) $-5n^2m + 15m^2n - 10nm^4$

(f) $38h^2k - 57kh^3 - 76hk^4$

4. (a) $6(r-1)^2 + 8(r-1)$

(b) $12(p+q)r^2 - 16(p+q)r$

(c) $9(a+b)^2y - 15(a+b)y^2$

(d) $6(a+1)x^3+(a+1)x^2+2(a+1)x$

(e) $24(x+y)z^2+16(x+y)^2z-8(x+y)z^3$

(f) $10(h-k)mn-6(h-k)mn^3+12(h-k)m^2n$

5. (a) $ac-ad+bc-bd$

(b) $a-b+ac-bc$

(c) $rs-rt+ws-wt$

(d) $ax-ay-by+bx$

6. (a) $xy+xz+6y+6z$

(b) $pm-mn-4p-4n$

(c) $3xy-3xz+5y-5z$

(d) $4m-4n+mp-np$

(e) $2am-2an-7bm+7bn$

(f) $2ab+ac+14b+7c$

7. (a) $6ac-bd-6ad+bc$

(b) $5b^2+6cd+3bd+10bc$

(c) $2a^2-9bc-6ab+3ac$

(d) $14rx-40sy+35sx+16ry$

8. (a) $r^2 - rs - rt + st$

(b) $mh - mk + h^2 - hk$

(c) $6g - g^2 + 6h - gh$

(d) $x^2 y + ax + bxy + ab$

9. (a) $y^3 + y^2 + y + 1$

(b) $9x^3 - 5x^2 - 18x + 10$

(c) $2 + 3a - 14a^2 - 21a^3$

(d) $4ab - 6ab^2 - 2 + 3b$

10. (a) $x^3 + yz + xy + x^2 z$

(b) $p^3 + a^2 p + bp^2 + a^2 b$

(c) $p^2r^2 + p^2s^2 + q^2s^2 + q^2r^2$

(d) $b + a^3 - a + ba^2$

Harder Questions:

1. $y(a+b) - x(a+b)$

2. $(a+b)^2 - 2(a+b)$

3. $2x(a-b) + 3y(b-a)$

4. $ab^2 + 3ab + a^2b$

5. $2b + 4b^2 - 16b^3$