# S2 Math Intensive Class (October)

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# **S.2** Mathematics Intensive Class

# Lesson 5: Use of Formula I (公式 I)

- \* Algebraic fraction (代數分式)
- ✓ P and Q are polynomials, where Q is not a constant, the form  $\frac{P}{O}$

代數式的分子分母都是多項式,且分母不等於常數。

例如:  $\frac{1}{x+y}$ ,  $\frac{2+b}{b}$ ,  $\frac{x}{x^2-4x+4}$ ,  $\frac{a+b}{2+b}$ 

denominator is not equal to zero 分母不可以等於 0



# Example 1

化簡下列代數分式

$$\frac{7}{14p - 21q}$$

**Solution** 

$$\frac{7}{14p - 21q}$$

$$= \frac{7}{7 \times 2p - 7 \times 3q}$$

$$= \frac{\cancel{1}}{\cancel{1}(2p - 3q)}$$



When the numerator and denominator are unequal addition subtraction, they cannot be simplified.

For example,  $\frac{x+2}{x+7}$  cannot be simplified as  $\frac{2}{7}$ .

當分子和分母的多項式是相加或減的關係,這是不能相約的。

例如: $\frac{x+2}{x+7}$ 不能化為 $\frac{2}{7}$ 



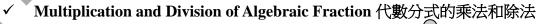
# Quick Practice 1

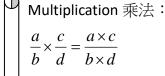
## Simplify

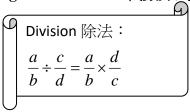
化簡下列代數分式

$$\frac{5}{15a+100c}$$

$$1/(3a + 20c)$$









# Example 2

Simplify the following. 化簡下列代數分式

$$\frac{a}{q^2} \times \frac{qy}{b}$$



### **Solution**

$$\frac{a}{q^2} \times \frac{qy}{b}$$

$$= \frac{a}{\cancel{q} \times q} \times \frac{\cancel{q}y}{b}$$

$$= \frac{a}{q} \times \frac{y}{b}$$

$$= \frac{ay}{bq}$$



# Example 3

Simplify the following. 化簡下列代數分式

$$\frac{1}{r^2} \div \frac{r+1}{r}$$

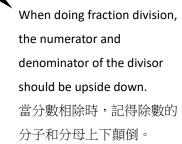


$$= \frac{1}{r^2} \times \frac{r}{r}$$

$$= \frac{1}{r \times r} \times \frac{r}{r+1}$$

$$= \frac{1}{\cancel{x} \times r} \times \frac{\cancel{x}}{r+1}$$

$$= \frac{1}{r(r+1)}$$





# **Quick Practice 2**

### Simplify

化簡下列代數分式

- (a)  $\frac{a}{b} \times \frac{2b}{3a}$
- (b)  $2cx^2 \div \frac{1}{3}c^2x$
- (a) 2/3
- (b) 6x/c

# ✓ Addition and Subtraction of Algebraic Fractions 代數分式的加法和減法

Equal denominator 分母相同: Add or subtract each numerator directly 直接把分子相加或相减



# Example 4

### Simplify the following.

化簡下列代數分式

$$\frac{x}{5y} + \frac{3x}{5y}$$

$$\frac{x}{5y} + \frac{3x}{5y}$$

$$x + 3x$$

$$= \frac{x+3x}{5y}$$



$$= \frac{4x}{5y}$$

### Unequal denominators 分母不相同

Step 1(步一): Find L.C.M. of the denominators

先找出各分母的 L.C.M. (最小公倍數)

Step 2(步二): Add or subtract the numerators

分子相加或相減



### Example 5

### Simplify the following.

化簡下列代數分式

$$\frac{2}{3b} + \frac{1}{4b}$$

### 

$$\frac{2}{3b} + \frac{1}{4b}$$

$$= \frac{2 \times 4}{12b} + \frac{1 \times 3}{12b}$$

$$= \frac{8+3}{12b}$$

$$= \frac{8+3}{12b}$$

$$= \frac{3b=3}{12b} \times b$$

$$= \frac{4b=4 \times b}{1.C.M.} = 3x \times 4x \times b$$



# Example 6

Simplify the following.

化簡下列代數分式

$$\frac{1}{x-1} + \frac{x}{1-x}$$

# **Solution**

$$\frac{1}{x-1} - \frac{x}{1-x}$$

$$= \frac{1}{x-1} - \frac{x}{-(x-1)}$$

$$= \frac{1}{x-1} - \left[ -\frac{x}{(x-1)} \right]$$

$$= \frac{1}{x-1} + \frac{x}{x-1}$$

$$= \frac{1+x}{x-1}$$



# Quick Practice 3

Simplify the followings.

化簡下列代數分式

$$1) \quad \frac{1}{x-1} - \frac{x}{x-1}$$

$$2) \quad \frac{3}{4 - 3y} - \frac{2y}{3y - 4}$$

$$2. (3-2y)/(4-3y)$$

# ✓ Exercise 1 練習─

Simplify the followings.

試化簡下列各數式。

- 1) (a)  $-\frac{2}{a} \frac{2}{a}$ 
  - (b)  $-\frac{2}{r-2} + \frac{7r}{r-2}$
  - (c)  $\frac{h}{h+k} + \frac{k}{h+k}$
  - (d)  $\frac{7}{3r-2} + \frac{4}{2-3r}$
- 2) (a)  $\frac{1}{a} \frac{1}{a+b}$ 
  - (b)  $\frac{2c-d}{c^2d} + \frac{c+d}{cd^2}$
  - (c)  $4 \frac{a-2}{5a}$
  - $(d) \quad \frac{2}{3x} + \frac{1 4y}{6xy}$
- 3) (a)  $\frac{-3}{6x}$ 
  - (b)  $\frac{-48x^2}{-144x^3}$
  - (c)  $\frac{4h^2 6h}{2h}$
  - (d)  $\frac{p}{p^2r p^2s}$
- $4) \quad (a) \quad \frac{am 2an}{6mn 3m^2}$ 
  - (b)  $\frac{a}{6} \div \frac{a^2}{3}$
  - (c)  $\frac{a+b}{a-b} + \frac{1}{2}$
  - (d)  $\frac{9y+1}{3y} + \frac{5y-1}{2y}$

- 1) (a)  $-\frac{4}{a}$ 
  - (b)  $\frac{7r-2}{r-2}$
  - (c) 1
  - (d)  $\frac{3}{3r-2}$
- $2)(a) \frac{b}{a(a+b)}$
- (b)  $\frac{c^2 + 3cd d^2}{c^2 d^2}$ 
  - (c)  $\frac{19a+2}{5a}$
  - (d)  $\frac{1}{6xy}$
- 3)(a)  $\frac{-1}{2x}$ 
  - (b)  $\frac{1}{3x}$
  - (c) 2h 3
- (d)  $\frac{1}{p(r-s)}$
- 4)(a)  $-\frac{a}{3m}$ 
  - (b)  $\frac{1}{2a}$
  - (c)  $\frac{3a-b}{2(a-b)}$
  - $(d) \qquad \frac{33y 1}{6y}$

$$5) \quad \frac{p^2 - pq}{q} \times \frac{p^2 q}{pq - q^2}$$

6) 
$$\frac{9u^2v}{5u - 2v} \times \frac{10v - 25u}{3v^2}$$

7) 
$$\frac{9x}{(x-y)^2} - \frac{8}{y-x}$$

8) 
$$\frac{1}{r+3s} + \frac{r}{3rs+9s^2}$$

9) 
$$\frac{4ax^3 + 12ax^2}{6ax^2}$$

10) (a) Find L.C.M. of  $2xy^2$ , 3y and  $6xy^2$ 求 2xy², 3y,及 6xy² 的 L.C.M.

(b) Simplify 
$$\frac{3xy-1}{2xy^2} - \frac{5}{3y} - \frac{7xy-3}{6xy^2}$$
  

$$\text{Liff } \frac{3xy-1}{2xy^2} - \frac{5}{3y} - \frac{7xy-3}{6xy^2}$$

$$\frac{p^3}{q}$$

$$6) \quad \frac{-15u^2}{v}$$

7) 
$$\frac{17x - 8y}{(x - y)^2}$$

8) 
$$\frac{1}{3s}$$

9) 
$$\frac{2x+6}{3}$$
  
10 (a)  $6xy^2$   
(b)  $-\frac{4}{3y}$ 

10 (a) 
$$6xy^2$$

(b) 
$$-\frac{4}{3y}$$

# Extra Questions 額外題目

Simplify the following expressions. 試化簡下列各數式。

$$1. \ \frac{256a^3b^4}{144a^5b^2} = \frac{16b^2}{9a^2}$$

$$\begin{array}{ccc}
1 & 144a^5b^2 & 9a^2 \\
2 & \frac{-(5x+3)^2}{15ax+9a} & = \frac{-(5x+3)}{3a}
\end{array}$$

3. 
$$\frac{12a - 4ab}{2b^2 - 6b} = -\frac{2a}{b}$$

4. 
$$\frac{x^2 + 2xy + y^2}{x^2 - y^2} = \frac{x + y}{x - y}$$

5. 
$$\frac{pq+qr+rs+ps}{-pq-qr+rs+ps} = \frac{s+q}{s-q}$$

6. 
$$\frac{8ab}{72(ab)^3} \times \frac{48a^5b}{64b^3} = \frac{a^3}{12b^4}$$

7. 
$$\frac{4a^4}{5b^2} \div \frac{8a^2}{10ab^3} = a^3b$$

8. 
$$\frac{a^2 - b^2}{a^2 - 2ab + b^2} \times \frac{a - b}{a + b} = 1$$

9. 
$$\frac{1}{a+1} - \frac{1}{a+2} = \frac{1}{(a+1)(a+2)}$$

10. 
$$\frac{x+y}{x-y} - \frac{x-y}{x+y} = \frac{4xy}{x^2 - y^2}$$

11. 
$$\frac{1}{x^2 + 7x + 12} + \frac{1}{x^2 + 5x + 6} = \frac{2}{(x+4)(x+2)}$$



# Challenging Questions 挑戰題

Simplify the following expression

簡化以下各題

$$1. \quad \frac{256a^3b^4}{144a^5b^2}$$

$$2. \quad \frac{-(5x+3)^2}{15ax+9a}$$

$$3. \quad \frac{12a - 4ab}{2b^2 - 6b}$$

4. 
$$\frac{x^2 + 2xy + y^2}{x^2 - y^2}$$

$$5. \quad \frac{pq+qr+rs+ps}{-pq-qr+rs+ps}$$

1. 
$$\frac{16b^2}{9a^2}$$

2. 
$$\frac{-5(5x+3)^2}{3a}$$

3. 
$$\frac{-2a}{b}$$

$$4. \qquad \frac{x+y}{x-y}$$

5. 
$$\frac{s+q}{s-q}$$

## **S.2 Mathematics Intensive Class**

# Lesson 6: Use of Formula 2 (公式 2)

### Formulae (公式)

✓ To represent the relation among variable (表示兩個或以上變數關係的等式)

例如: Ohm's Law (歐姆定律): V = IR

✓ Using substitution to find the unknowns (利用代入法求出公式中的未知數)

Step 1 (步一): 將已知的未知數的數值代人公式。

Step 2 (步二): 求餘下的未知數。



### Example 1

Given the formula  $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ , if f = 3 and u = 4, find the value of v.

已知公式  $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ ,若f = 3 及 u = 4,求v的值。



### Solution

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{v}$$

$$\frac{1}{3} - \frac{1}{4} = \frac{1}{v}$$

$$\frac{4-3}{12} = \frac{1}{v}$$

$$\frac{1}{12}$$
  $\frac{1}{v}$ 

$$v = \underline{12}$$



Step 1 (步一): 把 *f = 3, u = 4 代入公式中* 

Step 2 (步二): 求餘下的未知數(v)

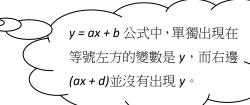
# ✓ Change of subject in a formula (變換主項)

The subject of the formula = when a variable is expressed by other variables.

公式的主項 = 在一個公式中,單獨出現在等號一方(多是左方)的變數,而另一邊並沒有該變數出現。

例如: y is a subject of the formula y = ax + d.

y = ax + d 這條公式中,y 是主項。



y = mx + c公式中,y是主項,但現在變為x作主項,即將x單獨放在等號的一方,而另一方

並沒有 x 出現。



## Example 2

Change x as the subject.

把公式 y = mx + c 的主項變為 x



### Solution

y = mx + c

$$\frac{y-c}{m} = \frac{y / x}{y / x}$$

y - c = mx

$$x = \frac{y - c}{m}$$

把 c 移到左方

左右兩方除以 m, m≠0



# Example 3

Make the letter in the bracket the subject of each of the formulae.

把公式的主項變換為括號的字母

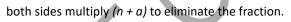
(a) 
$$t = \frac{a-n}{n+a}$$

(b) If 
$$t = -3$$
 and  $a = -1$ , find the value of  $n$ . 若  $t = -3$  及  $a = -1$ ,求  $n$  的值。



### Solution

(a) 
$$t = \frac{a - n}{n + a}$$



於兩邊同時相乘(n+a),從而取消右方分數的(n+a)。

$$t(n+a) = \frac{a-n}{n+a}(n+a)$$

Expand 
$$t(n + a)$$

$$t(n+a)=a-n$$

展開 t(n + a)

$$nt + at = a - n$$

move –n to the left side and a to the right side.

$$nt + n = a - at$$

把-n 移到左方,at 移到右方

$$n(t+1) = a(1-t)$$

提取公式
$$n$$
和 $a$ 

$$n = \frac{a(1-t)}{t+1}$$

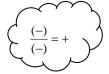
左右兩方同時除以 t+1

(b) Using substitution to substitute t = -3 and a = -1 into the result of (a)  $n = \frac{a(1-t)}{t+1}$ 

利用代入法把 t = -3 及 a = -1 代入(a)的結果  $n = \frac{a(1-t)}{t+1}$ 

$$n = \frac{-1 \times [1 - (-3)]}{(-3) + 1} \quad - \quad - \quad - \quad - \quad - \quad - \quad -$$

$$n = \frac{-1 \times (4)}{-2} \qquad \circ \quad \bigcirc \quad \bigcirc$$







# 即時練習 1

- - (b) By the result of (a), if V=66 and  $\pi=\frac{22}{7}$ , find the value of r. 利用結果(a),若 V=66, h=7 和  $\pi=\frac{22}{7}$ ,求 r 的值。
    - 1. (a)  $\sqrt{\frac{3v}{\pi h}}$

# Æ Exercise 1 (練習一)

1) Make the letter in the brackets the subject of each of the following formulae. 把公式的主項變換為括號的字母

(a) 
$$d = b + c$$
 [c]

(b) 
$$a + b + c = 180^{\circ}$$

(c) 
$$F = ma$$
 [m]

(d) 
$$PV = RT$$
 [R]

(e) 
$$a = \frac{b-4}{3}$$
 [b]

(f) 
$$a = \frac{6-b}{5}$$
 [b]

2) Make the letter in the brackets the subject of each of the following formulae. 把公式的主項變換為括號的字母

(a) 
$$y = \frac{2}{3x} + \frac{1}{x}$$
 [x]

(b) 
$$S = \frac{1}{2}(v+u)t$$
 [t]

(c) 
$$y = \frac{2x}{x-2}$$
 [x]

(d) 
$$y = 3 + \sqrt{x-1}$$
 [x]

(e) 
$$p = \frac{A}{1+ni}$$
 [n]

(f) 
$$q = \frac{2}{rp} - \frac{1}{p}$$
 [p]

- 4) Make v the subject of the formula  $k = \frac{1}{2}mv^2$  已知公式  $k = \frac{1}{2}mv^2$ ,將 v 變為主項。
- 5) If  $y = \frac{a+x}{a-x}$  change x as the subject. 若  $y = \frac{a+x}{a-x}$ ,則改 x 變為主項。
- 6) Make h the subject of the formula  $A = \pi r^2 + 2\pi r h$  已知公式  $A = \pi r^2 + 2\pi r h$  ,試將 h 變為主項。

8) If 
$$1 - \frac{a}{b} = \frac{1 + ax}{1 + bx}$$
  $\stackrel{\text{#}}{=} 1 - \frac{a}{b} = \frac{1 + ax}{1 + bx}$ 

- (a) Express x in terms of a and b. 試用y和b表示a
- (b) If a = 2, b = 3, find the value of x. 若 a = 2, b = 3, 求 x 的值
- The curved surface area of a cone is given by 圓錐體之曲面面積 9)  $S = \pi r \sqrt{h^2 + r^2}$ 
  - (a) If  $S = 330m^2$ ,  $\pi = \frac{22}{7}$  and r = 7m, find the value of h. 若  $S = 330m^2$ ,  $\pi = \frac{22}{7}$  及 r = 7m,求 h 之值。
  - (b) Make h the subject of the formula. 試將h變為主項。
- 10) The length L of a spring of natural length l hanging vertically is given by the formula

彈簧在沒有懸掛物件時的長度是l,懸掛物件後,它的長度L可用以下公式

$$L = l + \frac{W}{k}$$

where W is the weight of the hanging object and k is a constant.

- 其中W是懸掛物的重量,k是常數。 (a) (i) Make W the subject of the formula.
  - 把公式的主項變換為 W。 (ii) Find the value of W when L = 13, l = 8 and k = 1.5. 若 L = 13, l = 8 和 k = 1.5, 求 W 的值。
- (b) (i) Make k the subject of the formula. 把公式主項變換為k。
  - (ii) Find the value of k when L = 30, l = 26 and W = 4. 若L = 30, l = 26和W = -4,求k的值。

1)(a) 
$$c = d - b$$
  
(b)  $a = 180 - b - c$  2)(a)  $x = \frac{5}{3y}$  3)  $C = \frac{5}{9}(F - 32)$ 

(b) 
$$a = 180 - b - c$$
 (b)  $t = 2s/(v + u)$  (c)  $m = \frac{F}{a}$  (d)  $R = \frac{PV}{T}$  (e)  $b = 3a + 4$  (f)  $b = 6 - 5a$  (e)  $t = 2s/(v + u)$  (f)  $t = 2s/(v + u)$  (g)  $t = 2s/(v + u)$  (h)  $t = 2s/(v + u)$ 

(d) 
$$R = \frac{PV}{T}$$
 (d)  $x = (y - 3)^2 + 1$  5)  $x = \frac{a(y-1)}{y+1}$ 

(e) 
$$b = 3a + 4$$
 (e)  $n = \frac{A - p}{ip}$  (f)  $b = 6 - 5a$  (e)  $n = \frac{A - p}{2\pi r}$ 

(f) 
$$p = \frac{2-r}{qr}$$
 7) 
$$g = \frac{4\pi^2 l}{T^2}$$

8) (a) 
$$x = \frac{a}{b^2 - 2ab}$$

(b) 
$$x = -\frac{2}{3}$$

9) (a) 
$$h = 13.27m$$

(b) 
$$h = \frac{\sqrt{s^2 - \pi^2 r^4}}{\pi r}$$

10. ai) 
$$w = k (L - 1)$$

aii) 
$$w = 7.5$$

$$bi) k = w/(L - l)$$

$$ii) - 1$$

### More Questions(額外題目)

In each of the following, make the letter in the square bracket as the subject of the formula. 於下列各題中,將方括號中的字母變換成公式的主項。

1. 
$$Ax + By + C = 0$$

$$\therefore y = -\frac{Ax + C}{B}$$

$$2. V = \frac{IB}{nOt}$$

$$\therefore B = \frac{n V Q}{I}$$

3. 
$$p(x + v) + q(x - v) + r = 0$$

$$\therefore x = \frac{qy - py - r}{p + q}$$

4. 
$$y = \frac{x-1}{x+1} + \frac{1}{x}$$

[x]

[B]

$$\therefore z = \frac{x+1}{y(x+1)-x+1}$$

$$5. \ \frac{x-a}{x+a} = b$$

$$\therefore x = \frac{a + ab}{1 - b}$$

In each of the following, find the value of the unknown by changing the subject of the formula to a suitable variable.

於下列各題中,先將公式的主項變換為合適的變數,然後求未知數的值。

6. 
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

6. 
$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
 If  $u = -20$  and  $f = -40$ , find v. 如果 $u = -20$ 而  $f = -40$ ,求 v。

7. 
$$S = \frac{a(1-R^n)}{1-R}$$

7. 
$$S = \frac{a(1-R^n)}{1-R}$$
 If  $S = 8, n = 2$  and  $R = 3$ , find a. 如果  $S = 8, n = 2$  而  $R = 3$ , 求 a。

8. If 
$$a = \frac{1}{1+b} - 1$$
, express b in terms of a.

若 
$$a = \frac{1}{1+b} - 1$$
,試以 a 表示 b。

$$\therefore b = \frac{-a}{a+1}$$

9. If 
$$(x-a)^2 = (x-1)(x-b)$$
, express x in terms of a and b.

若
$$(x-a)^2 = (x-1)(x-b)$$
,試以a及b表示x。

$$\therefore x = \frac{b - a^2}{b + 1 - 2a}$$

10. If 
$$y = 1 + \sqrt{2x - 1}$$
, express x in terms of y.

若 
$$y = 1 + \sqrt{2x - 1}$$
, 試以 y 表示 x。

$$\therefore x = \frac{y^2 - 2y + 2}{2}$$



### Harder Questions:

1. Given  $a^2 + b^2 = c^2$ . Which of the following is/are true?

A. 
$$b^2 = a^2 - c^2$$

B. 
$$c = \sqrt{a+b}$$

C. 
$$b = \sqrt{a^2 + c^2}$$

- A. I only
- B. III only
- C. I and II only
- D. None of them

已知
$$a^2 + b^2 = c^2$$
,下列何者為正確?

I. 
$$b^2 = a^2 - c^2$$

II. 
$$c = \sqrt{a+b}$$

III. 
$$b = \sqrt{a^2 + c^2}$$

- A. 只有 I
- B. 只有 III
- C. 只有 I 及 II
- D. 無一正確
- 2. Given  $I = \frac{V}{R+r}$ , which of the following is true?

已知
$$I = \frac{V}{R+r}$$
,下列何者為正確?

A. 
$$R = \frac{V - Ir}{I}$$

B. 
$$r = \frac{V + I}{R}$$

C. 
$$V = IR + r$$

D. 
$$I = \frac{V - R}{r}$$

A. 
$$2a + p$$

B. 
$$2p + a$$

C. 
$$2(a + p)$$

D. 
$$2(a - p)$$

- 4. Given that  $T = \frac{a-4b}{b}$ .
  - (a) Find the value of T when a=4 and b=-2.
  - (b) Make b as the subject of the formula.
  - (c) Hence find the value of b when T=-1 and a=15.

已知
$$T = \frac{a-4b}{b}$$

- (a) 若 a=4 及 b=-2, 求 T 的值;
- (b) 把公式的主項變換為 b;
- (c) 由此, 若 T=-1 及 a=15, 求 b 的值

4. (a) -6  
(b) 
$$b = \frac{a}{T+4}$$
  
(c) 5

- 5. Given that  $x = y^2 + 1$  and  $y = \frac{3z}{4}$ .
  - (a) Write a formula relating x and z only. ( without y)
  - (b) Change z as the subject of formula.
  - (c) Find the value of z when x=10.

已知 
$$x = y^2 + 1$$
 及  $y = \frac{3z}{4}$ 

- (a) 試寫出一條只有 x 及 z 的公式
- (b) 把公式的主項變換為 z
- (c) 若 x= 10, 求 z 的值

5. (a) 
$$\frac{9z^2}{16} + 1$$
  
(b)  $\pm \frac{4}{3}\sqrt{x-1}$   
(c)  $\pm 4$ 

## **S.2 Mathematics Intensive Class**

# Lesson 7: Factorization 1 (因式分解) 1

- \* Factorization by Identity (利用恆等式進行因式分解)
- ✓ In previous chapter, we had learnt three useful identities. They are Identity of Difference of Two Squares and Identity of Perfect Square.

在之前的課題中,我們學習過三個常用的恆等式。它們分別是兩平方的差及 完全平方。

Difference of two square:

兩平方的差:

$$a^2 - b^2 = (a + b)(a - b)$$

Perfect square:

完全平方:

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$



### Example 1

Factorize the following expression.

因式分解下列表達式。

(a) 
$$16x^2 - 9y^2$$

(b) 
$$25(a + b)^2 - 4$$

(c) 
$$(c + d)^2 - (e - f)^2$$

(a) 
$$16x^2 - 9y^2 = (4x)^2 - (3y)^2$$
  
=  $(4x + 3y)(4x - 3y)$ 

(b) 
$$25(a + b)^2 - 4 = [5(a + b)]^2 - 2^2$$
  
=  $[5(a + b) + 2][5(a + b) - 2]$   
=  $(5a + 5b + 2)(5a + 5b - 2)$ 

(c) 
$$(c + d)^2 - (e - f)^2 = [(c + d) + (e - f)][(c + d) - (e - f)]$$
  
=  $(c + d + e - f)(c + d - e + f)$ 



### Example 2

Factorize the following expressions. 因式分解下列表達式

(a) 
$$2x^2 - 8$$

(b) 
$$18p - 2p(1 - d)^2$$

# Solution

(a) 
$$2x^2 - 8 = 2(x^2 - 4)$$
  
=  $2(x^2 - 2^2)$   
=  $2(x + 2)(x - 2)$ 

(b) 
$$18p - 2p(1-d)^2 = 2p[9 - (1-d)^2]$$
  
=  $2p[3^2 - (1-d)^2]$   
=  $2p[3 + (1-d)][3 - (1-d)]$   
=  $2p(3 + 1 - d)(3 - 1 + d)$   
=  $2p(4 - d)(2 + d)$ 



### Example 3

Factorize m² – n² – an + am. 因式分解 m² – n² – an + am。

# Solution

$$m^2 - n^2 - an + am = (m^2 - n^2) + (am - an)$$
  
=  $(m + n)(m - n) + a(m - n)$   
=  $(m - n)(m + n + a)$ 

# 🥜 🖠

### 堂上練習1

Factorize the following expression. 因式分解下列各表達式。

1.  $49 - 16k^2$ 

### (7 + 4k)(7 - 4k)

2. 
$$(x + 1)^2 - 36$$

$$(x + 7)(x - 5)$$

3. 
$$3b^2 - 27y^2z^2$$

$$3(b + 3yz)(b - 3yz)$$

4. 
$$64 (a + b)^2 - (b - a)^2$$

5. 
$$50x^2 - 2(y + z)^2$$

$$2(5x + y + z)(5x - y - z)$$

6. 
$$pr + qr - 4p^2 + 4q^2$$

$$(p + q)(r - 4p + 4q)$$

# 🕜即時練習1

Factorize the following expression. 因式分解下列各表達式。

1. 
$$x^2 - 25$$

$$(x + 5)(x - 5)$$

2. 
$$16a^2 - 81$$

3. 
$$100r^2 - 49x^2$$

$$(10r + 7x)(10r - 7x)$$

4. 
$$(x-1)^2-144$$

$$(x + 11)(x - 13)$$

5. 
$$3t^2 - 75$$

6. 
$$27a^2 - 12b^2$$

7. 
$$-6x^2y^2 + 96$$

$$6(4 + xy)(4 - xy)$$

8. 
$$-243ac^2 + 75b^2a$$

$$3a(5b + 9c)(5b - 9c)$$

9. 
$$p^2 - (p + q)^2$$

$$-q(2p + q)$$

10. 
$$25(x + 1)^2 - 16(x - 1)^2$$

$$(9x + 1)(x + 9)$$

11. 
$$9p^2 - q^2 - 12p - 4q$$

$$(3p + q)(3p - q - 4)$$

12. 
$$x^2y^4 - 16x^2$$

$$x^2(y^2 + 4)(y + 2)(y - 2)$$



# Example 4

Factorize x<sup>2</sup> + 4xy + 4y<sup>2</sup>. 因式分解 x<sup>2</sup> + 4xy + 4y<sup>2</sup>。

# Solution

$$x^{2} + 4xy + 4y^{2} = x^{2} + 2(x)(2y) + (2y)^{2}$$
  
=  $(x + 2y)^{2}$ 



### Example 5

Factorize 16(m-1)<sup>2</sup>-8(m-1)+1. 因式分解 16(m-1)<sup>2</sup>-8(m-1)+1。

# Solution

$$16(m-1)^{2} - 8(m-1) + 1 = [4(m-1)]^{2} - 2[4(m-1)](1) + 1^{2}$$
$$= [4(m-1) - 1]^{2}$$
$$= (4m-5)^{2}$$

**Alternative Solution** 

Let 
$$x = m - 1$$

$$16(m-1)^{2} - 8(m-1) + 1 = 16x^{2} - 8x + 1$$

$$= (4x)^{2} - 2(4x)(1) + 1^{2}$$

$$= (4x - 1)^{2}$$

$$= [4(m-1) - 1]^{2}$$

$$= (4m - 5)^{2}$$



# Example 6

Factorize -27x<sup>2</sup> + 36xy - 12y<sup>2</sup>.因式分解-27x<sup>2</sup> + 36xy - 12y<sup>2</sup>。

# Solution

$$-27x^{2} + 36xy - 12y^{2} = -3(9x^{2} - 12xy + 4y^{2})$$
$$= -3[(3x)^{2} - 2(3x)(2y) + (2y)^{2}]$$
$$= -3(3x - 2y)^{2}$$



# 堂上練習2

Factorize the following expression. 因式分解下列各表達式。

1. 
$$k^2 - 12k + 36$$

$$(k - 6)^2$$

2. 
$$25m^2 + 4n^2 + 20mn$$

 $(5m + 2n)^2$ 

3. 
$$(x + y)^2 - 22(x + y) + 121$$

$$(x + y - 11)^2$$

4. 
$$-18a^2 - 48ab - 32b^2$$

$$-2(3a + 4b)^2$$

✓ Using Identities of Sum and Difference of Two Cubes 利用恆等式中兩立方的和與差

The following are another two algebraic identities:

下列展出數為另外兩條代數式的恆等式。

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
  
 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ 

They are called the identities of the sum and the difference of two cubes.

它們稱為恆等式中兩立方的和與差。



### Example 7

Factorize the following expression. 因式分解下列各表達式。

(a) 
$$8x^3 + 1$$
 (b)  $27p^3 - 64q^3$ 

(a) 
$$8x^3 + 1 = (2x)^3 + 1^3$$
  
=  $(2x + 1)([(2x)^2 - (2x)(1) + 1^2]$   
=  $(2x + 1)(4x^2 - 2x + 1)$ 

(b) 
$$27p^3 - 64q^3 = (3p)^3 - (4q)^3$$
  
=  $(3p - 4q)[(3p)^2 + (3p)(4q) + (4q)^2]$   
=  $(3p - 4q)(9p^2 + 12pq + 16q^2)$ 



### Example 8

Factorize the following expression. 因式分解下列各表達式。

(a) 
$$(x-1)^3 - 27$$
 (b)  $-2mn^3 - 16m$ 

# Solution

(a) 
$$(x-1)^3 - 27 = (x-1)^3 - 3^3$$
  

$$= [(x-1)-3][(x-1)^2 + (x-1)(3) + 3^2]$$

$$= (x-4)[(x^2-2x+1) + (3x-3) + 9]$$

$$= (x-4)(x^2+x+7)$$

(b) 
$$-2mn^3 - 16m = -2m(n^3 + 8)$$
  
=  $-2m(n^3 + 2^3)$   
=  $-2m(n + 2)[n^2 - (n)(2) + 2^2]$   
=  $-2m(n + 2)(n^2 - 2n + 4)$ 

# 堂上練習3

Factorize the following expression. 因式分解下列各表達式。

1. 
$$x^3 - 125$$
  
 $(x - 5)(x^2 + 5x + 25)$ 

2. 
$$64m^3 + 1$$
  
 $(4m + 1)(16m^2 - 4m + 1)$ 

3. 
$$8m^3 + 27n^3$$
  $(2m + 3n)(4m^2 - 6mn + 9n^2)$ 

4. 
$$192 - 3a^3$$

$$3(4 - a)(16 + 4a + a^2)$$

5. 
$$-z^4 - 27z$$

$$-z(z+3)(z^2-3z+9)$$

6. 
$$p^3 - (1 - p)^3$$

$$(2p-1)(p^2-p+1)$$



# 即時練習 2

1. 
$$x^2 + 8x + 16$$

$$(x + 4)^2$$

2. 
$$4x^2 + 12x + 9$$

$$(2s + 3)^2$$

$$(10 - 7m)^2$$

4. 
$$f^2 - 6fg + 9g^2$$

$$(f - 3g)$$

5. 
$$49r^2 + 28ts + 4s^2$$

$$17r + 2c1^2$$

6. 
$$-16p^2 + 56pq - 49q^2$$

$$-(4p - 7q)^2$$

7. 
$$8m^2 - 24m + 18$$

$$2(2m-3)^2$$

8. 
$$m^3 - 8$$

$$(m-2)(m^2+2m+4)$$

9. 
$$x^3y^3 + 64$$

$$(xy + 4)(x^2y^2 - 4xy + 16)$$

$$10.343r^3 - 1000s^3$$

$$(7r - 10s)(49r^2 + 70rs + 100s^2)$$

11. 
$$a^3 - 8a^2 + 16a$$

$$a (a - 4)^2$$

12. 
$$4 - 28(r - 3) + 49(r - 3)^2$$

$$(23 - 7r)^2$$

13. 
$$2(a + 2b)^2 - 24(a + 2b) + 72$$

$$2(a + 2b - 6)^2$$

$$14.\ 5x^2 - 100xy^2 + 500y^4$$

$$5(x - 10v^2)^2$$

$$a(2a + b - 3)(2a - b + 3)$$

16. 
$$1 - x^2 + 4xy - 4y^2$$

$$(1 + x - 2y)(1 - x + 2y)$$

17. 
$$a^6k^6 - 1$$

$$(ak + 1)(ak - 1)(a^2k^2 - ak + 1)(a^2k^2 + ak + 1)$$

18. 
$$2x + 4y + x^3 + 8y^3$$

$$(x + 2y)(x^2 - 2xy + 4y^2 + 2)$$

### More Questions(額外題目)

Factorize the following polynomials by using the difference of two squares.

運用平方差的恆等式將下列各多項式分解為因式。

1. 
$$a^2 - 16$$

### =(a+4)(a-4)

2. 
$$25b^2 - 16$$

$$=(5b-4)(5b+4)$$

3. 
$$81p^2 - 4q^2$$

$$=(9p-2q)(9p+2q)$$

Factorize the following expressions.

因式分解下列各式

4. (a) 
$$18a^2 + 48ab + 32b^2$$

$$=2(3a+4b)^{2}$$

(b) 
$$50a^2 + 60ab + 18b^2$$

$$=2(5a+3b)^2$$

(c) 
$$20x^2 + 60xy + 45y^2$$

$$=5(2x+3y)^2$$

5. (a) 
$$(x+y)^2 - 49$$

$$=(x+y-7)(x+y+7)$$

(b) 
$$81 - (2x - 3y)^2$$

### =(9-2x+3y)(9+2x-3y)

(c) 
$$(2x-y)^2-x^2$$

$$= (x - y)(3x - y)$$

6. (a) 
$$(a-b)^2 - (a+b)^2$$

### =-4ab

(b) 
$$(2a+b)^2 - (a-b)^2$$

# =3a(a+2b)

(c) 
$$(a-2b)^2 - (a+b)^2$$

### =-3b(2a-b)

(d) 
$$(2x+y)^2 - (x+2y)^2$$

$$=3(x-y)(x+y)$$

7. (a) 
$$(a+b)^2 + 4(a+b) + 4$$
  
=  $(a+b+2)^2$ 

(b) 
$$(x-y)^2 - 8(x-y) + 16$$
  
=  $(x-y-4)^2$ 

(c) 
$$(x+3y)^2 + 6(x+3y) + 9$$

$$=(x+3y+3)^2$$

(d) 
$$4(2x+y)^2 + 4(2x+y) + 1$$

$$=(4x+2y+1)^2$$

8. (a) 
$$x^2 - y^2 + 4x + 4y$$

# = (x+y)(x-y+4)

(b) 
$$4x^2 - y^2 - 8x - 4y$$

### =(2x+y)(2x-y-4)

(c) 
$$9x^2 - y^2 + 6x - 2y$$

# =(3x-y)(3x+y+2)

(d) 
$$8x^2 - 18y^2 - 6x + 9y$$

$$=(2x-3y)(4x+6y-3)$$

9. (a) 
$$(x^2 + 4x + 4) - 9y^2$$

$$=(x+2-3y)(x+2+3y)$$

(b) 
$$25x^2 - (9y^2 - 6y + 1)$$

$$= (5x - 3y + 1)(5x + 3y - 1)$$

(c) 
$$(x^2 + 6xy + 9y^2) - 16z^2$$

$$=(x+3y-4z)(x+3y+4z)$$

(d) 
$$4x^2 - (4y^2 - 4yz + z^2)$$

$$=(2x-2y+z)(2x+2y-z)$$

10. Using identities to find the values of the following.



試應用恆等式求下列各式的值。

(a)  $81^2 - 19^2$ 

### = 6,200

(b)  $275^2 - 225^2$ 

### **= 25,000**

(c)  $345^2 - 344^2$ 

### = 689

(d)  $538^2 - 462^2$ 

### <del>= 76,000</del>

(e)  $1001^2$ 

### = 1,002,001

(f)  $999^2$ 

### <del>= 998,001</del>

### **Harder Questions:**

- 1. Factorize  $(x+4)^2 18x$ 
  - A. (x-2)(x-8)
  - B. (x-2)(x+8)
  - C. (x+4)(x+4)
  - D. (x+2)(x-10)
- 2. Which of the following is/are true?

以下何者正確?

- I. 4a-2b=-2(b-2a)
- II. (2a-1) is a factor 因式 of (b-2ab)
- III.  $a^2 + b^2 = (a+b)^2$

- A. I only
- B. II only
- C. I and II only
- D. I, II and III
- 3. Which of the following is a common factor of  $x^2 + 2x + 8$  and  $x^2 6x + 8$  以下何者是  $x^2 + 2x 8$  及  $x^2 6x + 8$  的公因式?
  - A. X+2
  - B. X-2
  - C. X+4
  - D. X-4
- 4. Which of the following is/are factors of  $(a^3 a^2)$ ?

以下何者是 $(a^3 - a^2)$ 的因式?

- I. a
- II.  $a^2$
- III. a-1
- A. II only
- B. III only
- C. II and III only
- D. I and II and III
- 5. Given that  $P(x) = x^2 + 2x$  and  $Q(x) = x^3 + 2x^2 3x 6$ 
  - (a) Factorize 因式分解 P(x) and Q(x)
  - (b) Simplify 化簡  $\frac{P(x)}{Q(x)}$
  - (a) P(x) = x(x+2) $Q(x) = (x^2 - 3)(x + 2)$
  - (b)  $\frac{x}{x^2 3}$

# S.2 Mathematics Intensive Class Lesson 8: Factorization 2 (因式分解 2)

- \* Factorization by Grouping Terms (利用併項法作因式分解)
- ✓ In previous lesson, we had learnt to factorize an algebraic expression by taking out the common factors. For example,

在以前的課中,我們已經學習過透過抽取同類項而對代數表達式進行因式分解。例如:

$$5x - 15y = 5(x - 3y)$$
  
 $6a^3 + 15a^2 = 3a^2 (2a + 5)$ 

Consider the polynomial ax + ay + bx + by. We cannot factorize it by taking out the common factors since there are no common factors among the terms.

考慮多項式 ax + ay + bx + by。我們並不能把它進行因式分解,因為在這四項中,並沒有同一的同類項。

However, if we group the terms into two parts properly, we can factorize the polynomials as follow:

但是,若果我們把這四項分成兩部份來看,我們可以把這多項式分解如下:

1st pair 2nd pair
$$\overline{ax + ay} + \overline{bx + by} = a(x + y) + b(x + y)$$

$$= (a + b)(x + y)$$

This is known as the grouping terms method.

這種進行因式分解的辦法稱為「併項法」。

There may be different ways to group the terms. For example, we can group the terms of ax + ay + bx + by as follows:

除了上述的方法外,還有其他方式去組合各項。例如,我們可以把 ax + ay + bx + by 的各項組合如下:

$$ax + ay + bx + by = \underbrace{ax + bx}_{1} + \underbrace{ay + by}_{2}$$

$$= x(a + b) + y(a + b)$$

$$= (a + b)(x + y)$$

Note that the same result can be obtained.

注意上述兩個答案是一樣的。



### Example 1

Factorize  $a^2 + cd + ad - ca$  by the grouping terms methods. 利用併合法因式分解  $a^2 + cd - ad - ca$ 。

## Solution

$$a^{2} + cd - ad - ca$$

$$= (a^{2} - ca) + (cd - ad)$$

$$= a (a - c) + d(c - a)$$

$$= a (a - c) - d (a - c)$$

$$= (a - d)(a - c)$$
OR 
$$a^{2} + cd - ad - ca$$

$$= (a^{2} - ad) + (cd - ca)$$

$$= a (a - d) + c(d - a)$$

$$= a (a - d) - c(a - d)$$

$$= (a - d)(a - c)$$



### Example 2

Factorize  $3p - 9q - 2pq + 6q^2$  by the grouping terms methods. 利用併合法因式分解  $3p - 9q - 2pq + 6q^2$ 。

### Solution

$$3p - 9q - 2pq + 6q^2 = (3p - 9q) - (2pq - 6q^2)$$
  
=  $3(p - 3q) - 2q(p - 3q)$   
=  $(p - 3q)(3 - 2q)$ 

In this question, do we have another way for group terms? 在這條題目中,我們作有第二種方法進行併合法嗎?



### Example 3

Factorize mx + nx - px + py - ny - my by the grouping terms method. 利用併合法因式分解 mx + nx - px + py - ny - my。

### Solution

$$mx + nx - px + py - ny - my = mx - my + nx - ny - px + py$$
  
=  $(mx - my) + (nx - ny) - (px - py)$   
=  $m(x - y) + n(x - y) - p(x - y)$   
=  $(x - y)(m + n - p)$ 

**Alternative Solution** 

$$mx + nx - px + py - ny - my = (mx + nx - px) + (py - ny - my)$$
  
=  $x(m + n - p) - y(m + n - p)$   
=  $(m + n - p)(x - y)$ 



# 堂上練習1

Factorize the following expression by the grouping terms method.

利用併合法因式分解下列各題。

1. 
$$2p + ps + 2q + qs$$

$$(2 + s)(p + q)$$

2. 
$$x^2 + xz - xy - yz$$

$$(x + z)(x - y)$$

3. 
$$4ab - 3c - 4ac + 3b$$

$$(b-c)(4a+3)$$

4. 
$$Ax + By + Cx + Ay + Bx + Cy$$

$$(x + y)(A + B + C)$$



# ●即時練習1

Factorize the following expression by the grouping terms method.

利用併合法因式分解下列各題。

1. 
$$ax + a + x + 1$$

$$(x + 1)(a + 1)$$

2. 
$$3a + 3b + ac + bc$$

$$(a + b)(3 + c)$$

3. 
$$4y - xy + 4z - xz$$

$$(4 - x)(y + z)$$

4. 
$$ax - ay - bx + by$$

$$(x - y)(a - b)$$

5. 
$$2ac + 2bc - 3ad - 3bd$$

$$(a + b)(2c - 3d)$$

6. 
$$p + q + qs + ps$$

$$(p + q)(1 + s)$$

7. 
$$bc + 7c - ab - 7a$$

$$(b + 7)(c - a)$$

8. 
$$x^2 + xy + ax + ay$$

$$(x + y)(x + a)$$

9. 
$$5x^2 - 5xy - 2x + 2y$$

$$(x - y)(5x - 2)$$

10. 
$$4b + 7a - 7ab - 4b^2$$

$$(x + 3)(x^2 + 2)$$

12. 
$$a^2 - b + ab - a$$

$$(a + b)(a - 1)$$

13. 
$$2b^2$$
 -6ab - b + 3a

$$(b - 3a)(2b - 1)$$

14. 
$$5p^2 - 5px - 3x + 3p$$

$$(p-x)(5p+3)$$

15. 
$$a^3 - ab + 5b - 5a^2$$

$$(a^2 - b)(a - 5)$$

16. 
$$3a^2 - b + ab - 3a$$

$$(3a + b)(a - 1)$$

17. 
$$4x^3 + 2x^2 + 16x + 8$$

$$2(2x + 1)(x^2 + 4)$$

18. 
$$(b + a)^2 - ap - bp$$

$$(b + a)(b + a - p)$$

19. 
$$a^3 + a^2b - a^2c - abc$$

$$a(a + b)(a - c)$$

20. 
$$r^3 - 6x^3 - 3r^2x^2 + 2rx$$

$$(r-3x^2)(r^2+2x)$$

21. 
$$3r^2st + 9rstx - 36rx^2 - 12r^2x$$

$$3r(r + 3x)(st - 4x)$$

22. 
$$-ax - bx - cx + ay + by + cy$$

$$(y - x)(a + b + c)$$

23. 
$$ax - bx - ay + by + ax - bz$$

$$(a-b)(x-y+z)$$

24. 
$$ax - ay + cx - cy$$

$$(x - y)(a + c)$$

$$(c + d)(e - f)$$

26. 
$$a^2 + ab + ac + bc$$

$$(a + b)(a + c)$$

$$27.8c^2 - 4c + 4cd - 2d$$

$$2(2c-1)(2c+d)$$

$$(5b - 3c)(a + 1)$$

29. 
$$p^3 + p - p^2 - 1$$

$$(p^2 + 1)(p - 1)$$

$$30. x^2 - xt + 2t^2 - 2xt$$

$$(x-t)(x-2t)$$



### More Questions(額外題目)

Factorize the following expressions.

因式分解下列各式。

1. (a) 
$$2a + 6b + 4$$

$$=2(a+3b+2)$$

(b) 
$$14r - 7s + 28t$$

$$=7(2r+4t-s)$$

(c) 
$$8t + 32s - 24st$$

$$=8(t+4s-3st)$$

(d) 
$$ac + ad - ae$$

$$=a(c+d-e)$$

2. (a) 
$$10a^3 - 2a^2 + 6a$$

$$=2a(5a^2-a+3)$$

(b) 
$$21b^4 + 15b^3 - 24b$$

$$=3b^2(7b^2+5b-8)$$

(c) 
$$7c - 49c^2 - 28c^3$$

$$=7c(1-7c-4c^2)$$

(d) 
$$20m^3 + 35m^2 - 10m^5$$

$$=5m^2(4m+7-2m^3)$$

3. (a) 
$$ab^2 - 2ab + a^2b$$

$$= ab(b-2+a)$$

(b) 
$$14x^2y - 21xy^2 - 7yx$$

$$=7xy(2x-3y-1)$$

(c) 
$$4pq^2 + 16q^3p - 12p^3q$$

$$=4pq(q+4q^2-3p^2)$$

(d) 
$$8a^3c - 2ac^4 + 6ac^2$$

$$=2ac(4a^2-c^3+3c)$$

(e) 
$$-5n^2m + 15m^2n - 10nm^4$$

$$=-5mn(n-3m+2m^3)$$

(f) 
$$38h^2k - 57kh^3 - 76hk^4$$

$$=19hk(2h-3h^2-4k^3)$$

4. (a) 
$$6(r-1)^2 + 8(r-1)^2$$

$$=2(r-1)(3r+1)$$

(b) 
$$12(p+q)r^2 - 16(p+q)r$$

$$=4r(p+q)(3r-4)$$

(c) 
$$9(a+b)^2 y - 15(a+b)y^2$$

$$=3y(a+b)(3a+3b-5y)$$

(d) 
$$6(a+1)x^3 + (a+1)x^2 + 2(a+1)x$$

$$= x(a+1)(6x^2+x+2)$$

(e) 
$$24(x+y)z^2 + 16(x+y)^2z - 8(x+y)z^3$$

$$=8z(x+y)(3z+2x+2y-z^2)$$

(f) 
$$10(h-k)mn-6(h-k)mn^3+12(h-k)m^2n$$

$$=2mn(h-k)(5-3n^2+6m)$$

5. (a) 
$$ac - ad + bc - bd$$

$$=(a+b)(c-d)$$

(b) 
$$a-b+ac-bc$$

$$= (a-b)(1+c)$$

(c) 
$$rs - rt + ws - wt$$

$$=(r+w)(s-t)$$

(d) 
$$ax - ay - by + bx$$

$$=(a+b)(x-y)$$

6. (a) 
$$xy + xz + 6y + 6z$$

$$= (x+6)(y+z)$$

(b) 
$$pm - mn - 4p - 4n$$

### 不能進行因式分解

(c) 
$$3xy - 3xz + 5y - 5z$$

$$= (3x+5)(y-z)$$

(d) 
$$4m - 4n + mp - np$$

### = (4+p)(m-n)

(e) 
$$2am - 2an - 7bm + 7bn$$

### = (2a-7b)(m-n)

(f) 
$$2ab + ac + 14b + 7c$$

$$= (2b+c)(a+7)$$

7. (a) 
$$6ac - bd - 6ad + bc$$

$$=(6a+b)(c-d)$$

(b) 
$$5b^2 + 6cd + 3bd + 10bc$$

$$=(5b+3d)(b+2c)$$

(c) 
$$2a^2 - 9bc - 6ab + 3ac$$

$$= (2a+3c)(a-3b)$$

(d) 
$$14rx - 40sy + 35sx + 16ry$$

### 不能進行因式分解

8. (a) 
$$r^2 - rs - rt + st$$

$$= (r-s)(r-t)$$

(b) 
$$mh - mk + h^2 - hk$$

$$= (h-k)(m+h)$$

(c) 
$$6g - g^2 + 6h - gh$$

$$= (g+h)(6-g)$$

(d) 
$$x^2y + ax + bxy + ab$$

$$=(x+b)(xy+a)$$

9. (a) 
$$y^3 + y^2 + y + 1$$

$$=(y+1)(y^2+1)$$

(b) 
$$9x^3 - 5x^2 - 18x + 10$$

$$=(x^2-2)(9x-5)$$

(c) 
$$2+3a-14a^2-21a^3$$

$$=(2+3a)(1-7a^2)$$

(d) 
$$4ab - 6ab^2 - 2 + 3b$$

$$= (2ab-1)(2-3b)$$

**10.** (a) 
$$x^3 + yz + xy + x^2z$$

$$=(x^2+y)(x+z)$$

(b) 
$$p^3 + a^2p + bp^2 + a^2b$$

$$= (p+b)(p^2+a^2)$$

(c) 
$$p^2r^2 + p^2s^2 + q^2s^2 + q^2r^2$$

$$=(r^2+s^2)(p^2+q^2)$$

(d) 
$$b + a^3 - a + ba^2$$

### 不能進行因式分解

#### **Harder Questions:**

1. 
$$y(a+b)-x(a+b)$$

$$(a+b)(y-x)$$

2. 
$$(a+b)^2 - 2(a+b)$$

$$(a+b)(a+b-2)$$

3. 
$$2x(a-b) + 3y(b-a)$$

$$(a-b)(2x-3y)$$

$$4. \quad ab^2 + 3ab + a^2b$$

$$ab(b+3+a)$$

5. 
$$2b + 4b^2 - 16b^3$$

$$2b(1+2b-8b^2)$$