Avalanche Search and Rescue using UAV

Yiyang Li
Mechatronics and Robotics
TUM
Munich, Germany
ge89yes@tum.de

Tao Ma
Mechatronics and Robotics
TUM
Munich, Germany
ge43kop@mytum.de

Chenming Wang
Mechatronics and Robotics
TUM
Munich, Germany
ge47vas@mytum.de

Yiwei Wang
Mechatronics and Robotics
TUM
Munich, Germany
yiwei.wang@tum.de

***Abstract*—This report introduces the student project: Avalanche Search and Rescue using UAV, for the course Autonomous System. This project deals with a scenario of locating avalanche victims by utilizing unmanned aerial vehicles (here a Quadcopter) in order to accelerate the searching phase and therefore increase the survival rate of the victim. To explore the feasibility of this approach, a simulation of the whole avalanche victim searching process is build using Robot Operating System (ROS) and Unity, including the following three main aspects: environment and signal simulation, global search, and local search. By evaluating the simulation results, we compare two types of local searching algorithm under different sensing conditions. Pros and cons of these two UAV based methods, as well as the traditional human search method, are then discussed.**

***Index Terms*—Avalanche rescue, search and rescue robotics, unmanned aerial vehicles (UAVs), ROS**

I. Introduction

Snow avalanches are responsible for over 200 fatalities each year worldwide, and traditional avalanche search and rescue methods have remained largely unchanged since the introduction of the electromagnetic signal transceiver (known as ARVA), which is commonly used in search and rescue operations. The ARVAs are carried both by the victim, working as signal emitter by setting to the send mode, and the rescuers/companions, working as receiver by setting to the search mode.

Traditionally, the process of rescuing avalanche victims involves various steps: Once local rescuers have been notified of an incident, they head to the scene using a helicopter or snowmobile. The team leader organises the search, marks the search area and decides on a search strategy based on the last known location of the victim. During the primary (global) search stage, rescuers use rectangular or zigzags search strips to cover the search area horizontally or vertically. The width of the search strip is a key parameter and must be chosen carefully according to the maximum detection range of the signal, since the trade-off between reducing the search time and the minimizing the probability of missing victims should be considered.

Once a signal from the victim's transmitter is detected, the rescuers use their hand-held receiver to locate the victim by gradually approaching the source of the signal during the fine (local) search phase. With the distance and direction information deduced automatically from electromagnetic signal by receiver, the Rescuers should be able to pinpoint the victim's location. As soon as the victim is located, rescuers use shovels to dig up the victim and administer first aid.

Research shows that about 90 % of victims survive after burring by the avalanche if they are buried for less than 10 minutes. The survival rate drops to 30 % after 30 minutes. This corresponds to death by asphyxiation due to insufficient amount of air under the snow. After this, the survival rate mortality does not decrease much and death is caused by trauma [1]. Those statistics emphasize that adequately short search and rescue time is the key to increase the survival rate. (see Figure 1)

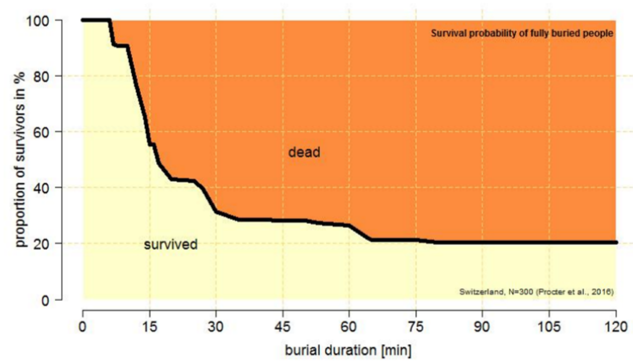

Fig. 1. Survival probability of avalanche victims depending on buried time. [1]

Due to the difficulties of utilizing large machinery like excavators in the mountainous terrain, the digging process could not be accelerated simply and remains manually in the most cases. However, the traditional approach has clear drawbacks in the search phase: a large amount of time is required for the global search by foot due to the physically exhausting walk on thick snow and steep slopes in typical avalanche terrains. Furthermore, the rescuers walk on unstable snow with the potential risk of inducing a second avalanche event, which threats the safety of both victims and rescuers. In this context, small unmanned aerial vehicles, which are increasingly being recognized as an agile, cost-effective platform for searching missions, represent a valid alternative to humans. Additionally, UAVs can be equipped with a variety of sensors, enabling them to scan large areas of terrain and identify obstacle on complex terrain. This can be particularly useful in remote or hard-to-reach locations, such as mountainous areas or dense forests. Indeed, modern UAVs are sufficiently intelligent to fly autonomously in the avalanche region to perform both global and local search pattern to locate the victim, thus resulting in a faster and safer research.

This report provides a comprehensive overview of our group project, exploring the avalanche search and rescue using UAV in simulation. It is organized as follows. Section II gives an introduction of the simulated avalanche scenario and the generation of beacon signal. Section III and IV describe the implemented global and two different local search patterns, which provides desired path for the UAV. Section V briefly states the used methods for trajectory generation and control. Finally, in Section VI we evaluate and discuss the simulation results and possible future improvements.

II. Environment

A. Avalanche Scenario Simulation

In this project, we used ROS, Unity, and RViz for joint simulation. In Unity, a mountain terrain covered with snow is created to represent the avalanche scenario and the quadcopter model is also included, visualizing the flying process of the UAV. The Unity environment is connected with ROS through unity bridge for transmitting information such as the UAV's current state and unity map, which describes the shape of the terrain. Due to unknown issues with the signal reception of the victim beacon which is originally provided in unity, our team chose to generate a similar signal ourselves, which will be introduced in section II B. To clearly illustrate the searching path and the relative position of the UAV and the victims, an additional visualization is created using RViz.

B. Beacon Signal Generation

As the information provided by a typical ARVA transceiver, two signals are generated: **beaconSignalStrength** and **beaconAngle**. The maximum detectable range of the signal is set to 15 meters, using the Euclidean distance in the 3-D space. When the distance between the UAV and the victims' position is smaller than 15 m, signals will be generated to corresponding topics (controlled by a callback function). For the first signal beaconSignalStrength, its value is linear to the distance and a noise is also added as:

$$\text{SignalStrength} = 1 - \frac{\text{space\_distance}}{15} + \text{noise} \tag{1}$$

The beaconAngle signal is generated that gives the angle between two vectors. One vector points from the UAV's current position to the victim's position in the x-y plane, and another vector is the current velocity vector in the x-y plane. With this angle information, we simulate the direction arrows which are displayed on the hand-hold transceiver. Noise will also be added for the angle. The calculation formula is as the following:

$$\alpha_{vel} = \tan^{-1}\left(\frac{\text{v\_y}}{\text{v\_x}}\right) \cdot \frac{180}{\pi} \tag{2}$$

$$\alpha_{rel} = \tan^{-1}\left(\frac{\text{y\_vic} - \text{y\_uav}}{\text{x\_vic} - \text{x\_uav}}\right) \cdot \frac{180}{\pi} \tag{3}$$

$$\text{beaconAngle} = \alpha_{rel} - \alpha_{vel} + \text{Angle\_noise} \tag{4}$$

Considering the real-world situation, two types of noise for the signal strength are simulated: The first one is a constant negative offset added on the strength term, for the shielding effect in case in the beacon is located under relative thick snow. Another type is a random noise for both strength and angle terms, which aims to simulate the random noise caused eventually by the outdoor environment and the non-ideal emitter/receiver.

III. Global Search

In search and rescue operations, it is critical to cover the entire search area efficiently to locate missing individuals. UAV can serve as an effective tool, as they can cover vast search areas in a short time. For this task, a well-designed global search strategy is needed aiming at discovering the victims' signal. Since we could not utilize any signal in this phase, a simple zigzags search strips in the x-direction are used. However, it is essential to choose reasonable y-distance between the search paths according to the size of the search area and the maximal detectable range of signal. The goal of the global search phase is to detect signals from all victims as soon as possible and meanwhile ensure that no victim is missed.

A. Search Area

In our simulated system, we obtained key information about the size and shape of the search area. Specifically, the area spans 300 meters in the x-direction and 70 meters in the y-direction on a mountain slope with a relative altitude difference of about 67m in the x-direction. This area corresponds to typical size and terrain of a avalanche. (see figure 2)In addition, as mentioned in section II B, the UAV can detect signals from the victim within a range of 15 meters.

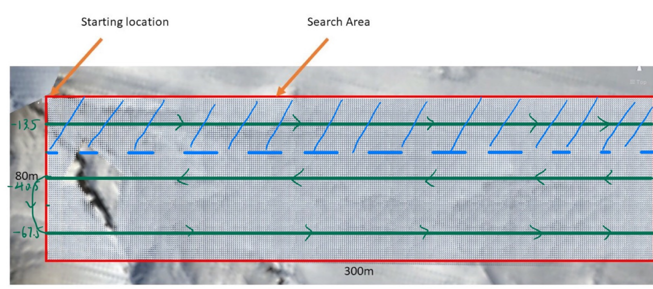

Fig. 2. Search area of size 300m x 80m. The search paths are coloured in green and the blue shadowed area corresponds to the covered area by the first search path.

B. Path Planning

In order to use UAVs in search and rescue operations, various factors that affect the efficiency and effectiveness of the search must be considered when developing a path planning strategy. Specifically, factors such as UAV's operation altitude, maximal speed as well as the aforementioned y-distance between search path must be carefully considered in order to develop an optimized algorithm that suits the specific needs of the operation.

In our simulation, the UAV's flight height is set to 5 meters, which is an appropriate altitude for conducting a safe and efficient search, according to some experiments from the simulation. On the one hand, the trajectory planning and flight control algorithm may cause some deviation from the desired constant altitude of the waypoint, especially during the accelerating and turning phase, so a certain safety margin should be added to prevent crash into terrain. On the other hand, a higher operation altitude causes less search coverage in the x-y plane, since the maximal detectable range is constant at 15m, calculated by 3-D euclidean distance. Based on this, the UAV's flight path has been meticulously planned:

First of all, we choose to search along paths parallel to x-axis due to the fact that our area is narrower in y-direction than in x-direction. Such paths reduces the U-turns the UAV should perform and thus decreases the searching time. Also, according to the chosen operation altitude of 5m, the maximal covered range in the searching plane (parallel to the x-y plane of the terrain) is approximately equal to 14.142m. That means, any point on the mountain surface, where the victim could occur, should have a distance less than 14.142m to the search paths. Consider that we search the entire x-direction, our y-distance between the search lines are chosen as 27m (along each line, the rectangular area of 13.5m on both sides are covered), which not only leave a certain safety margin for possible noise, but also reduce the total search paths to 3, covering the 80m wide area.

Specifically, after takeoff, the UAV follows a fixed path, starting from point (0, -13.5) and moving towards the +x direction while keeping the y = -13.5 constant. When it reaches x = 300m, the UAV moves 27 meters towards the -y direction and makes a U turn to fly along the line of y = -40.5m until reaching the point (0, -40.5). The UAV then moves again to the -y direction for 27m to (0, -67.5) and then continues to fly along y = -67.5m until it reaches x = 300m, thus completing the entire search area. Along each of the three search path, we set a way point for every 30m, which help to keep the actual flight path to be smooth, straight and keep a suitable altitude while climbing the mountain slope. In total, 39 way points are set for the whole global search path.

During the global search process, the UAV continuously attempts to receive signals from the victim within a range of 15 meters. This ensures that any signal emitted by the victim is detected and the UAV can quickly interrupt its global search and switch to a local search strategy.

IV. Local Search

In this project, we implement two local search strategies: predicting the victim's position using both signal strength and angle, and conducting a two-step gradient search using only signal strength. For the former, we assume that the UAV can obtain both the strength and direction of the victim's signal with relatively high accuracy simultaneously. With this information, we can predict the victim's position and plan the path directly pointing to the expected location accordingly. In the practice however, even the ARVA receiver displays the magnitude and direction, the direction information corresponds to the tangent to the EM field flux line at the operator location. Since the tangent of the flux line does not necessarily point to the signal source, especially at the near field, the direction information is not reliable [2]. Thus, we also implement a pure signal strength based two-step gradient search approach which is introduced in part B.

By using above local search strategies, we can locate victim more accurately for the rescuers, in order to reduce the excavation effort. When conducting local searches, we also need to consider factors such as UAV height and speed to ensure that search process can proceed smoothly and achieve optimal results.

A. Local Search with Angle and Strength

Real-time signal strength and UAV's position information is received during the global search. Due to the signal area being a spherical shape with the victim's location as the center in space, the projection onto the xy plane is a circular shape. As the UAV flies along the x-direction, the absolute value of the signal angle increases as it approaches the victim in the x-direction, reaching a maximum value of 90 degrees.

Therefore, the algorithm is as follows: When the UAV receives a signal strength greater than 0, it enters the victim signal area. At this time, the absolute value of the signal angle, signal strength, UAV velocity direction, and corresponding UAV coordinates are constantly recorded. When the angle reaches the maximum value, the UAV has the same x-coordinate as the victim, and the distance between the UAV and the victim in the z-direction is about 5 meters, enabling the calculation of the victim's y-coordinate as follows:

$$\text{distance} = 15 \cdot (1 - \text{max\_strength}) \tag{5}$$

$$\text{y\_distance} = \sqrt{|\text{distance}^2 - 5^2|} \tag{6}$$

$$\text{y} = \text{max\_angle\_y} \pm \text{y\_distance} \tag{7}$$

Once the coordinates are obtained, they are published to the relevant topic, and subscribers who have subscribed to this topic receive the message and call the corresponding function. The global search process is interrupted, and a new trajectory is created to make the UAV fly directly to the predicted victim location from its current position. When the UAV reaches the target position, the signal strength will be outputted in the terminal. A higher signal strength indicates a more accurately predicted position. Then, based on the current coordinates of UAV, a new global search path is replanned to continue searching for the next victim.

The proposed algorithm has significant practical implications for search and rescue operations. By recording and analyzing signal strength, angle, and UAV velocity data, the algorithm can make efficient and accurate predictions about the victim's location, which minimizes the search time and increases the probability of successful rescue.

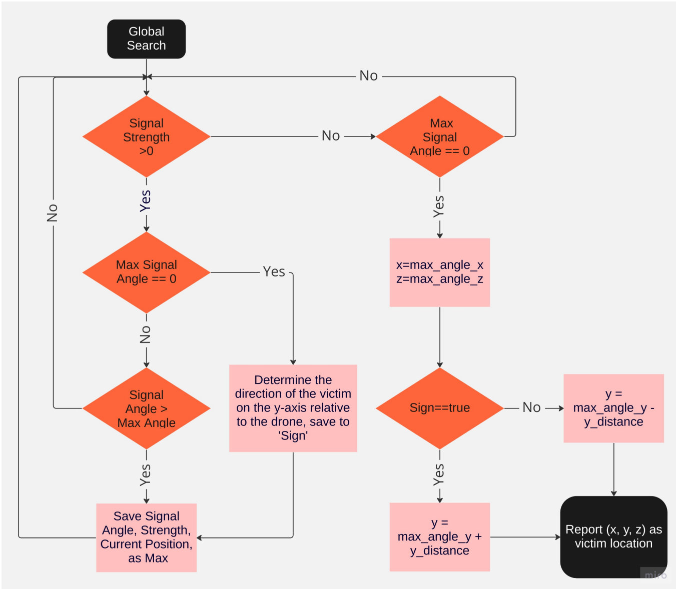

Fig. 3. Flow chart of the local search with angle and strength.

B. Local Search with Strength

As long as the receiver detects a positive signal, the UAV automatically change its mode from global to local search, in which the flight path is planned in a much closer distance of 1m. With the planning frequency of 10Hz, the UAV evaluates the current signal strength and compares it to the last one. As the global search is conducted along the x-axis, the two-step gradient search also begins first with x-direction, until the signal strength drops for the first time, which indicates that the UAV has reached approximately the same x-coordinate as the maximum of the signal (victim).

The second step is to search in the y-direction while holding the estimated optimal x-coordinate as constant. Without any direction information, the UAV can not determine the right turning direction. Thus, a step to the positive y-direction is attempted firstly. If the signal strength increases, this direction is considered to be correct, otherwise, it will make 2 steps in the opposite direction.

Notice that as the path during local search deviates from the original global search path, the flight altitude should also be considered, since the terrain is not even in both x and y directions. The UAV will plan a climb for 0.2m, once the flight altitude is below the 5m threshold.

The stopping criteria of the search in y-direction is set as the following: a maximal reachable signal strength is calculated as:

$$\text{max\_strength} = 1 - \frac{\text{altitude}}{15} \tag{8}$$

which corresponds the signal strength at the current altitude, in case the victim is directly located at the same x and y coordinates of the UAV. The UAV searches in the y-direction until the absolute value of the difference between the maximum reachable and the current received signal strength is less than 0.02. This parameter is tuned with experiments considering the trade-off between search time and accuracy.

This algorithm also intended to deal with the two typical noise types mentioned in section II. For the negative offset of the strength, it probably causes the failure of the aforementioned stopping criteria,, because no information of noise is considered in the maximal reachable signal strength. The UAV performs often an endless oscillation in the y-direction without fulfilling the stopping criteria, although the UAV has already reached a acceptable small distance to the victim in the x-y plane. An intuitive strategy is to add an upper limit to the search steps. During the simulation, an upper limit of 25 steps is set for the search in y-direction, due to the fact that the actual flying distance for every planned step is about 0.6m and the coverage in y-direction for each global search path is set to $\pm$ 13.5m.

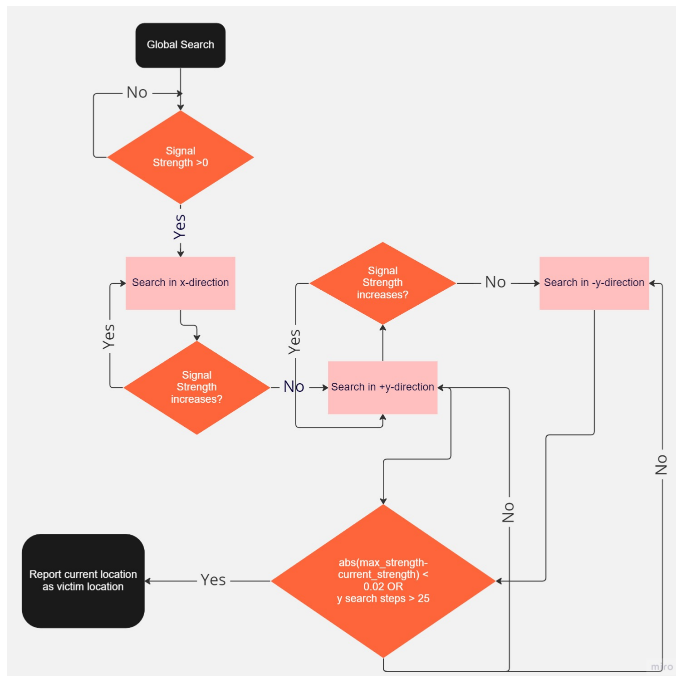

Fig. 4. Flow chart of the local search with strength.

To compensate the random noise caused eventually by the outdoor environment and the non-ideal emitter/receiver, the average signal strength of the last 5 signals are calculated and used as the current signal strength.

V. Trajectory Generation

With the planned path for both global and local search, concrete trajectory is generated for the UAV to follow. This task is completed using the **mav_trajectory_generation** package provided by ETH [3]. This package contains tools for polynomial trajectory generation and optimization based on methods described in [4]. These techniques are especially suitable for rotary-wing micro aerial vehicles.

Before take-off, the whole global search trajectory is generated using all way points defined in global search. However, for our application, this planned trajectory should be interrupted in time, if we detect signal from the victims. Also, for local search, we need to plan the path and generate trajectory according to the current signal information step by step, which makes the provided planner node not completely suitable. Base on that, we modify the trajectory generation algorithm in order to enable the interruption of the global search and step-by-step planning by integrating the two main function of the planner **planTrajectory** and **publishTrajectory** into our node for victim searching. Specifically, if the signal is detected, the first local search trajectory will be generated and published to the controller of the UAV and thus overlap the currently conducted global search.

During the local search, new desired way points are provided at each step by one of the two local search algorithm. The implemented function of step-wise planning takes current position of the UAV and the planned goal position, as input. Since the aforementioned trajectory generation method needs at least three points to do optimization, otherwise, the middle points of start position and goal position is calculated and added to the trajectory.

Furthermore, to prevent redundant trajectory generation during each local search step, a wait function is needed, in order to wait the drone arrive in a certain range around the goal position and then the next local search step is started.

After each local search, the UAV should back to the global search mode so that the drone can continue to search victims. But the signal of last victim will have a impact on the drone which will mislead it back to the same position. In order to eliminate its influence, the UAV will calculate automatically the distance between its current position and the last found victicm. If the drone is close to last victim, it will ignore current signal and continue to execute the local search. Also, because of the safety margin of the y-distance between the search paths in global search, the UAV may encounter the same victim twice. So, every time the drone finds a new victim's signal, it will try to at first compare the distance between current position and all found victim positions from take-off, only for distance bigger than 15 meters, new local search will be conducted.

The generated trajectory is then followed by the geometric controller implemented in the assignment 2 of the course, using the method introduced by "Geometric tracking control of a quadrotor UAV on SE(3)". Further details can be found in [5].

VI. Evaluation and Discussion

To evaluate the performance of our proposed search algorithm, we conduct four different simulation scenarios. In each scenario, we randomly generated five victims in the 300m x 80m search area and use the two in section Iv mentioned local search strategies, each repeated for three times. Note that in the following tables I-VII, search mode 1 refers to section IV A.Local Search with Angle and Strength, while mode 2 refers to B.Local Search with Strength.

A. Simulation scenarios

1) No signal noise: In this scenario, we simulated a noise-free environment to test the baseline performance of our search algorithm.

TABLE I
Scenario 1 Mode 1

Information	Test 1	Test 2	Test3
global search time	282.94791	283.198152	281.305594
average local search time	5.077891	5.098514	5.075022
standard deviation (local)	0.002029	0.001958	0.001405
average distance error	0.407882	0.826086	0.312919

TABLE II
Scenario 1 Mode 2

Information	Test 1	Test 2	Test3
global search time	463.256347	421.315445	446.976246
average local search time	47.121739	40.101409	46.028033
standard deviation (local)	10.47674	11.024213	10.879889
average distance error	1.471985	2.922929	1.443801

2) Signal strength random noise ± 0.02, signal angle random noise $\pm 10°$: In this scenario, we introduced random noise in signal strength and angle which aims to simulate the random noise caused eventually by the outdoor environment and the non-ideal emitter/receiver behavior, to test the robustness of our search algorithm.

TABLE III
Scenario 2 Mode 1

Information	Test 1	Test 2	Test3
global search time	290.698381	273.59735	256.29099
average local search time	5.146234	5.083524	5.094643
standard deviation (local)	0.138154	0.005491	0.003964
average distance error	0.706028	0.963725	0.653671

TABLE IV
Scenario 2 Mode 2

Information	Test 1	Test 2	Test3
global search time	405.359339	413.126449	426.346457
average local search time	35.818823	39.55433	40.001796
standard deviation (local)	5.762559	9.959535	7.515077
average distance error	2.255175	1.780168	2.065041

3) Signal angle random noise $\pm 60°$: In this scenario, we set a large angle noise to simulate the situation in reality where the signal angle is deduced from the tangent direction of the flux line of the magnetic field when the UAV is close to the victim. Here, depending on the orientation of the magnetic field generated by the emitter and the position of the UAV, the tangent of the flux line may vary in a large range, even pointing away from the signal source.

4) Fixed signal strength offset -0.02: In this scenario, we set a fixed signal strength offset to simulate the situation where the signal strength is weakened due to the victim being covered by snow.

TABLE V
SEARCH MODE 1

Information	Test 1	Test 2	Test3
global search time	321.135525	305.926991	322.010947
average local search time	5.101830	5.130575	5.061489
standard deviation (local)	0.029894	0.024743	0.011140
average distance error	1.630328	1.496154	2.307477

TABLE VI
SEARCH MODE 2

Information	Test 1	Test 2	Test3
global search time	493.202272	422.510094	456.446520
average local search time	48.875459	36.342176	40.729325
standard deviation (local)	6.532863	4.698671	8.733278
average distance error	1.629906	1.851704	1.224592

B. Analysis of Simulation Result

In the simualtaion, the average speed of the UAV is about 6m/s during global searc. Based on the simulation results, it can be concluded that in all four test cases, the completion time for the global search with local search mode 1 (Local search with angle and strength) is approximately 4-5 minutes, while local search mode 2 (Local search with strength) requires about 7-8 minutes. Both algorithms are proven to be able to finish the global and local search for 5 victims in a 300m x 80m area much faster than searching by human rescuers: According to the Naismith's rule, normal human walking speed decreases from 5km/h to about 1.8 km/h (0.5556m/s) on a slope of 13 degrees [6], which corresponds to our terrain. To complete the 3 x 300m global search path with aforementioned speed, human rescuers would need approximately 27 min. Additional time will be needed for local search and orientation with the receiver (we assume that the speed on the second path downwards the mountain is not much faster.) In comparison to searching using snowmobile, the UAV approach does not cause the risk of new avalanche. Thus it is much safer for both victims and rescuers. For the visualization of the two search methods in Rviz, see figure 5,6 and the provided videos on github.

TABLE VII
SEARCH MODE 1

Information	Test 1	Test 2	Test3
global search time	295.481816	309.883425	316.827245
average local search time	4.963251	5.199940	5.127926
standard deviation (local)	0.008449	0.070794	0.017093
average distance error	0.765452	0.985674	0.668210

TABLE VIII
SEARCH MODE 2

Information	Test 1	Test 2	Test3
global search time	511.627403	526.570747	483.963965
average local search time	53.886679	52.572318	45.771698
standard deviation (local)	7.433911	10.729464	11.397666
average distance error	2.764052	1.747292	1.955542

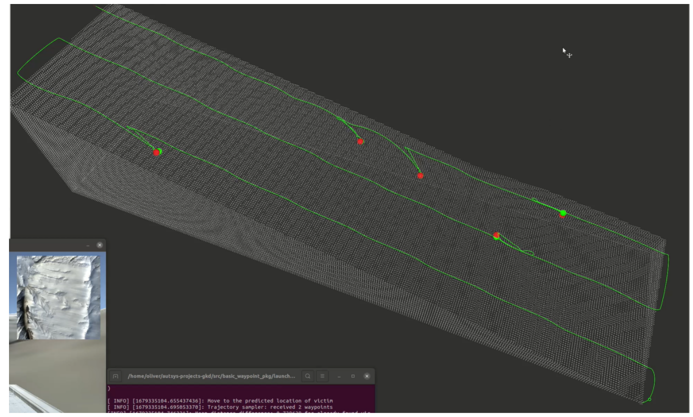

Fig. 5. Visualization of the search process using local search mode 1. The green line illustrate the search path of the UAV, the green points are the actual victim locations, and the red points are the estimated victim locations by the algorithm.

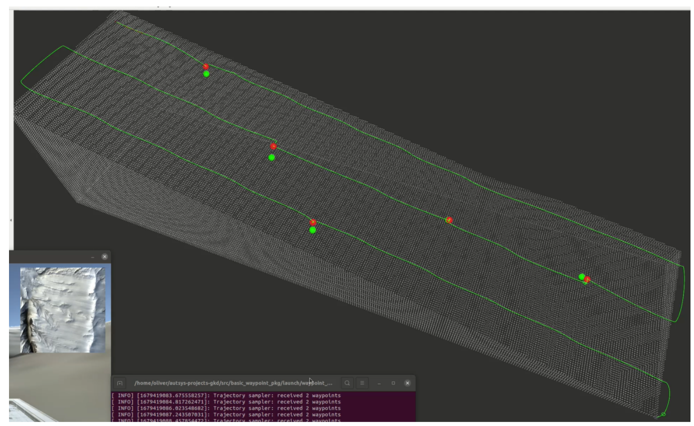

Fig. 6. Visualization of the search process using local search mode 2.

In terms of location accuracy, the method using angle information provided location estimation with distance error in x-y plane less than 1m. The error while performing the method using only strength varies from 1.4m to 2.9m. The accuracy is sufficient for the human rescuers to perform manually excavation. In general, Such search time and accuracy achieved by the UAV leaves relatively enough time for the rescuers to head directly to the estimated position and the following excavation operation.

For the four test settings, there are some difference between the two local search mode to be discussed. In test case s 1), 2) and 4), mode 1, which utilizes the angle information, clearly outperforms the mode 2, both in search time and accuracy. However, in scenario 3, where the angle noise is significant, the deviation of mode 1 search increases significantly to a level which is similar or higher than mode 2, while mode 2 remains more stable. As mentioned above in scenario 3, the tangent of the flux line can even point at the opposite direction in the reality, which corresponds to an error of 180 degrees, larger error may occur while using the mode 1 with emitter generating real magnetic field. Above all, large angle error increases the possibility of discriminating the y-direction incorrectly while searching with the mode 1: during testing, mode 1 could give estimation in the opposite y-direction. Such large location error causes useless excavation operation and even a complete new search operation. This case is illustrated by figure 7.

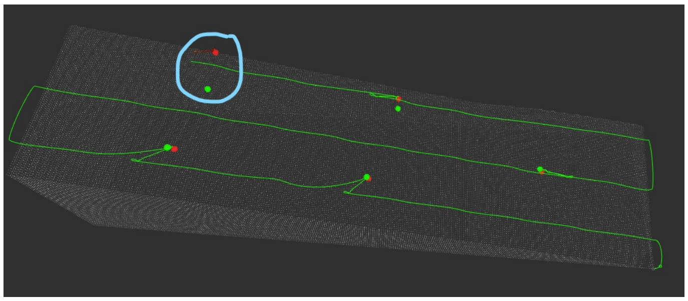

Fig. 7. Estimating the victim in the opposite y-direction using local search mode 1 under test case 3 (of the fifth victim marked in the blue circle) with large angle signal noise.

In summary, mode 1 has a higher search speed and accuracy, but may result in larger deviations when there is significant angle noise, while mode 2 has a lower search speed but is less affected by signal noise.

C. Future work

In this project, we implement one global and two types of local search algorithms for avalanche search and rescue using UAV. While the advantages of the UAV based method compared to manual search has proven by simulation, we still see some insufficiency of our project. Firstly, the signal generation process and the assumed noise types are simulated relatively simple. A more accurate magnetic field simulation of the emitter could be done to get a more realistic model of the signal properties. Also the noise types and strength may be gained through real-world measurements. Moreover, more complex signal processing and noise eliminating methods could be utilized. Secondly, more advanced local search methods can be designed or implemented, which may provide faster and more stable local search operation. Thirdly, to ensure the acceptable search time also in larger area, multi-UAV collaboration could be studied to increase the searching efficiency. Last but not least, in the real-world application, more interfering effects from the environment could occur during the search and rescue process, which may not be able to discover using test in simulation.

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