

$$\sum F_y = 0 = V(x) - w(x) dx - \left(V(x) + \frac{dV}{dx} dx \right) = 0$$

divide by dx as $dx \rightarrow 0$

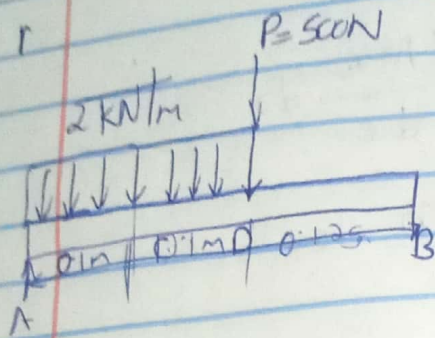
$$\frac{dV}{dx} = -w(x)$$

take moments about the Right hand edge of the element.

$$\sum M_{RH \text{ edge}} = 0 = -M(x) - V(x) dx + w(x) dx \frac{dx}{2} + \left(M(x) + \frac{dM}{dx} dx \right) = 0$$

divide by dx as $dx \rightarrow 0$.

$$\frac{dM}{dx} = V(x)$$



$$\sum F_v = 0$$

$$250 + 500 + 400 - F_{Av} = 0$$

$$F_{Av} = 1150 \text{ N}$$

$$M_D = 250 \times 0.125 = -31.25$$

$$M_C = (-500 \times 0.1) = -81.25$$

$$M_A = 400 \times 0.1 + (-81.25) = -121.25 \text{ N.m}$$

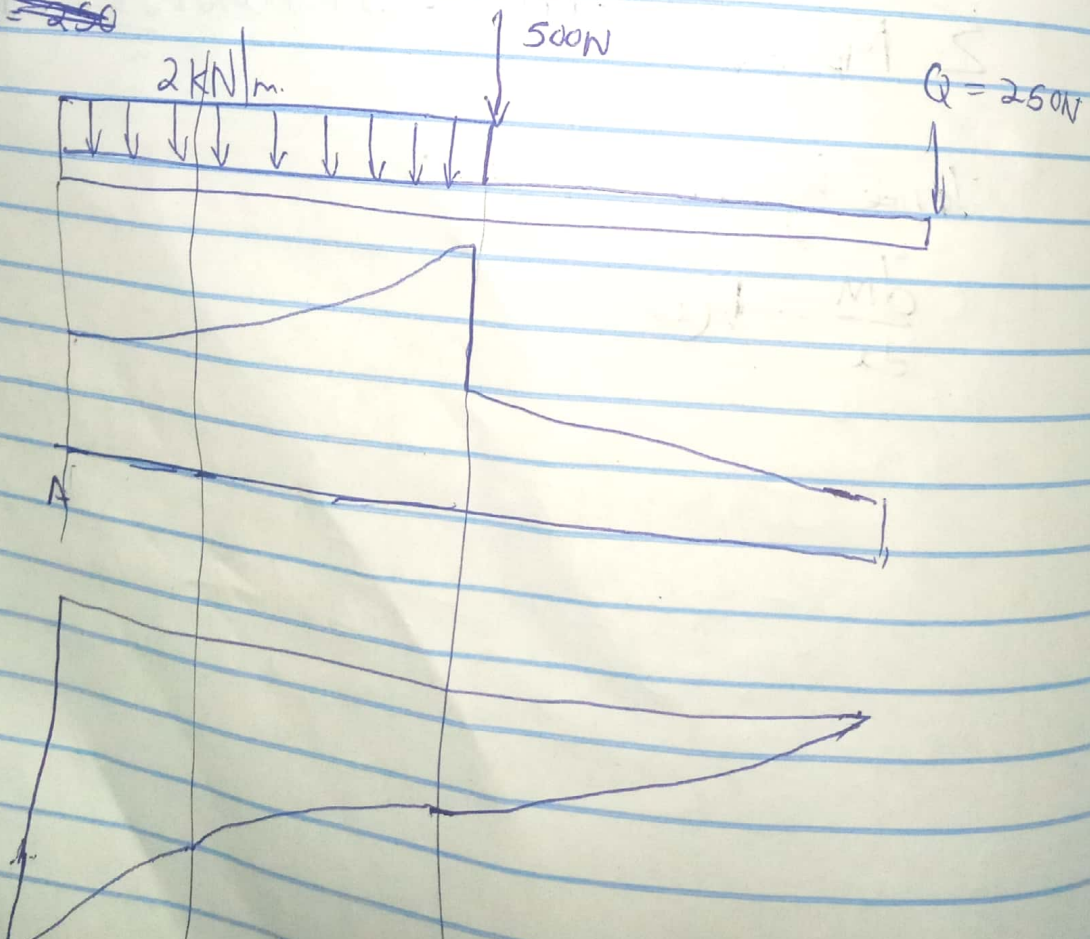
$$\text{at } B = 250$$

$$\text{at } D = 250 + 500 = 750 \text{ N}$$

$$\text{after } D = 750 + 2 \text{ kN} = 2750$$

$$A = 2750 - 1150 = 1600 \text{ N}$$

$$M_D = -250$$



Beam AB two concentrated load P & Q
 normal stress due to bending on bottom edge
 = + 55 MPa at D & + 37.5 MPa at F.
 @ Draw shear & bending-moment diagrams

soln

$$I = \frac{1}{12} (24)(60)^3 = 432 \times 10^3 \text{ mm}^4 \quad C = 30$$

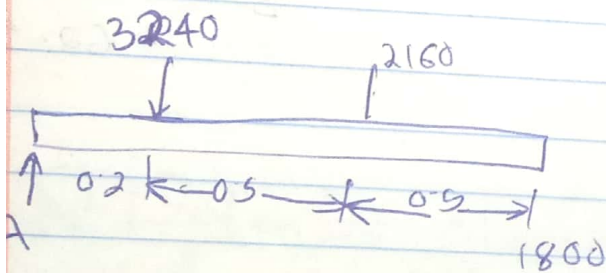
$$S = \frac{I}{C} = 14.4 \times 10^3 \text{ mm}^3 = 14.4 \times 10^{-6} \text{ m}^3 \quad M = S \sigma$$

At D. $M_D = (14.4 \times 10^{-6})(55 \times 10^6) = 792 \text{ N}\cdot\text{m}$
 At F $M_F = (14.4 \times 10^{-6})(37.5 \times 10^6) = 540 \text{ N}\cdot\text{m}$

using free body FB $\uparrow \sum M_F = 0 \quad -540 + 0.3B = 0$
 $B = \frac{540}{0.3} = 1800 \text{ N}$

using free body DE FB $\uparrow \sum M_D = 0 \quad -792 - 3Q(0.8)(1800) = 0$
 $Q = 2160 \text{ N}$

using entire beam $\uparrow \sum M_A = 0 \quad -0.2P(0.7)(2160) + (1.2)(1800) = 0$
 $P = 3240 \text{ N}$



$\uparrow \sum F_y = 0 \quad A - 3240 - 2160 + 1800 = 0$
 $A = 3600 \text{ N}$

A to C $V = 3600$

C to E $V = 3600 - 3240 = 360$

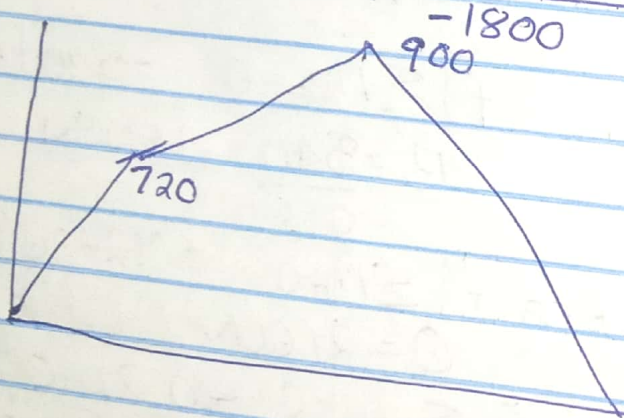
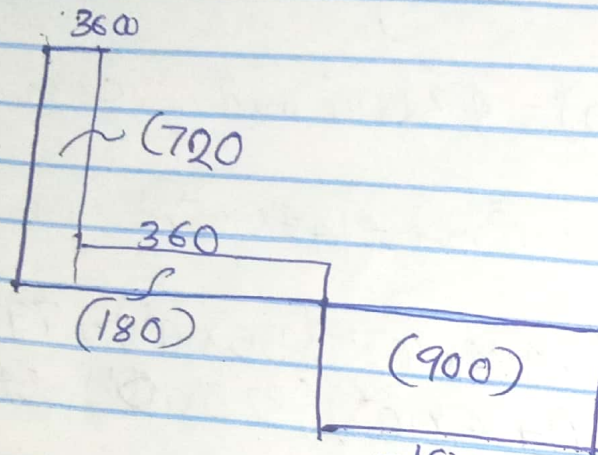
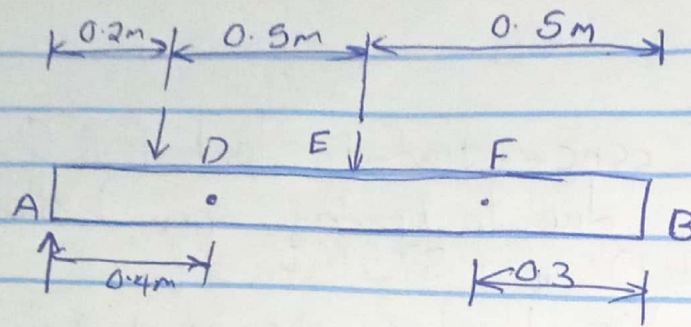
E to B $= -180 \text{ N}$

$M_A = 0$

$M_C = 0 + 720 = 720$

$M_E = 720 + 180 = 900$

$M_B = 900 - 900 = 0$



$$\sigma_{\max} = \frac{(M)_{\max}}{S} = \frac{900}{14.4 \times 10^{-6}} = 62.5 \times 10^6$$

Shear $V = 80 \text{ kN}$
 @plot sheart stress distribution acting on beam.

$$I = \frac{1}{12} (0.015)(0.200^3) + 2 \left[\frac{1}{12} (0.3)(0.02)^3 + 0.3(0.02)(0.11)^2 \right]$$

$$I = 155.6(10^{-6}) \text{ m}^4$$

$$Q_b = \bar{y}' A' = (0.110)(0.3)(0.02) = 0.66(10^{-3}) \text{ m}^3$$

$$\tau_b = \frac{V Q_b}{I t_b} = \frac{80 \text{ kN} (0.66)(10^{-3}) \text{ m}^3}{155.6(10^{-6}) \text{ m}^4 (0.300)} = 1.13 \text{ MPa}$$

From t $t_b = 0.015$ & $Q_b = Q_b$

$$\tau_b = \frac{V Q_b}{I t_b} = \frac{80 \text{ kN} (0.66)(10^{-3}) \text{ m}^3}{155.6(10^{-6}) \text{ m}^4 (0.015)} = 22.6 \text{ MPa}$$

$$Q_c = \sum y' A' = (0.110)(0.300)(0.02) + (0.05)(0.15)(0.1) = 0.735$$

$$\tau_{\max} = \frac{V Q_c}{I t_c} = \frac{80 \text{ kN} (0.735)(10^{-3}) \text{ m}^3}{155.6(10^{-6}) \text{ m}^4 (0.015)} = 25.2 \text{ MPa}$$

(b) Determine shear force by the web

$$I = 155.6(10^{-6}) \text{ m}^4$$

$$t = 0.3 \text{ m}$$

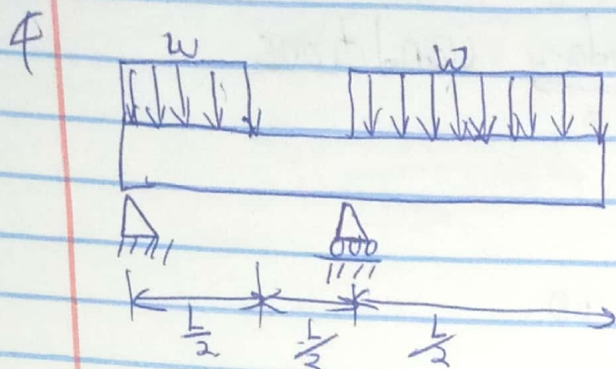
$$A' = 0.3(0.12 - y) \text{ m}^2$$

$$\bar{y}' = y + \frac{1}{2} (0.12 - y)$$

$$= \frac{1}{2} (0.12 + y) \text{ m}$$

$$Q = \bar{y}' A' = (0.15)(0.120 - y^2) \text{ m}^2$$

$$\tau = \frac{V Q}{I t} = \frac{80 \text{ kN} (0.15)(0.120 - y^2) \text{ m}^3}{(155.6)(10^{-6})(0.3)} = 257(0.12 - y^2) \text{ MPa}$$



Ground Reactions

$$\sum M_A = 0$$

$$0 = \left(\frac{wL}{2} \times \frac{L}{4} \right) - R_R(L) + \left(\frac{wL}{2} \times \frac{5L}{4} \right)$$

$$R_R = \frac{wL^2}{8} + \frac{5wL^2}{8}$$

$$R_R = \frac{3}{4} wL$$

$$\sum F_y = 0$$

$$R_{Ay} + R_{Ry} - \frac{wL}{2} - \frac{wL}{2} = 0$$

$$R_{Ay} = \frac{1}{4} wL$$

Bending moment equation

$$\sum M_o = 0$$

$$M(x) = R_{Ay}(x)' + R_{Ry}(x-L)' - \frac{wL}{2} \left(x - \frac{L}{4} \right) - w(x-L) \left(\frac{x-L}{2} \right)$$

$$M(x) = \frac{1}{4} wL(x)' + \frac{3}{8} (wL) (x-L)' - \frac{wL}{2} \left(x - \frac{L}{4} \right) - \frac{w}{2} (x-L)^2$$

Elastic curve equation

$$\sum I \theta = \frac{1}{8} wL(x)^3 + \frac{3}{8} (wL) (x-L)^2 - \frac{wL}{4} \left(x - \frac{L}{4} \right)^2 - \frac{w}{6} (x-L)^3 + C$$

$$\sum I \theta = \frac{1}{24} (wL) (x)^3 + \frac{3}{24} wL(x-L)^2 - \frac{wL}{12} \left(x - \frac{L}{4} \right)^2 - \frac{w}{24} (x-L)^4 + Cx + D$$

Kinematic Boundary Conditions

At $x=0, V=0$

At

$$EI \times 0 = 0 + 0 - 0 - 0 + 0$$

$$D=0$$

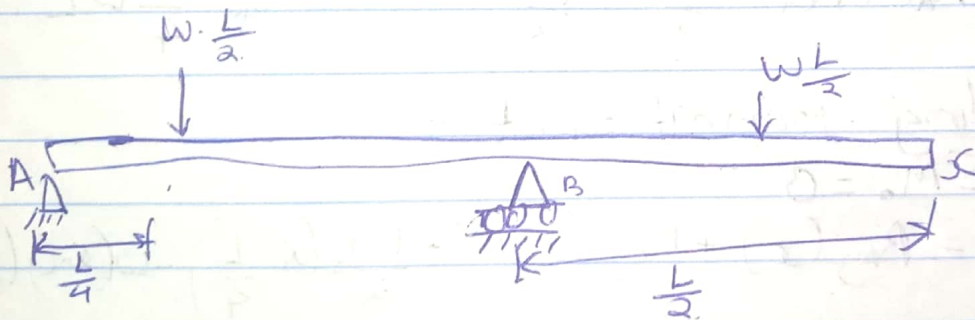
At $x=L, V=0$ at point C

$$EI \times 0 = \frac{1}{24} WL^4 + \frac{0.9}{256} WL^4 - 0 + 0 - 0$$

$$CL = -\frac{5}{768} WL^4 \quad C = -\frac{5WL^3}{768}$$

Elastic curve equation

$$\frac{WL(x)^3}{24} - \frac{3WL}{24} (x-L)^3 - \frac{WL(x-L)^3}{12} - \frac{W}{24} (x-L)^4 - \frac{5WL^3}{768} x$$



At B

$$V_B = \frac{WL}{24} \left(\frac{L}{2}\right)^3 - \frac{3WL}{24} \left(\frac{L}{2} - L\right)^3 - \frac{WL}{12} \left(\frac{L}{2} - \frac{L}{4}\right)^3 - \left(\frac{W}{24} \left(\frac{L}{2} - L\right)^4 \right) - \frac{5WL^4}{768 \times 2} + \frac{WL^4}{196} + \frac{WL^4}{64} - \frac{WL^4}{768} - \frac{WL^4}{384} - \frac{5WL^4}{1536}$$

$$V_B = \frac{1021}{75264} WL^4$$

At point D

$$\frac{WL}{24} \left(\frac{3}{2}L\right)^3 - \frac{3WL}{24} \left(\frac{3}{2}L - L\right)^3 - \frac{WL}{12} \left(\frac{3}{2}L - \frac{L}{2}\right)^3 - \frac{W}{24} \left(\frac{3}{2}L - L\right)^4 - \frac{5WL^3}{768} \left(\frac{3}{2}L\right)$$

$$V_c = \frac{9 w L^4}{64} - \frac{w L^4}{64} - \frac{125 w L^4}{768} - \frac{w L^4}{384} - \frac{5 w L^4}{512}$$

$$V_c = \frac{-77}{1536} w L^4$$