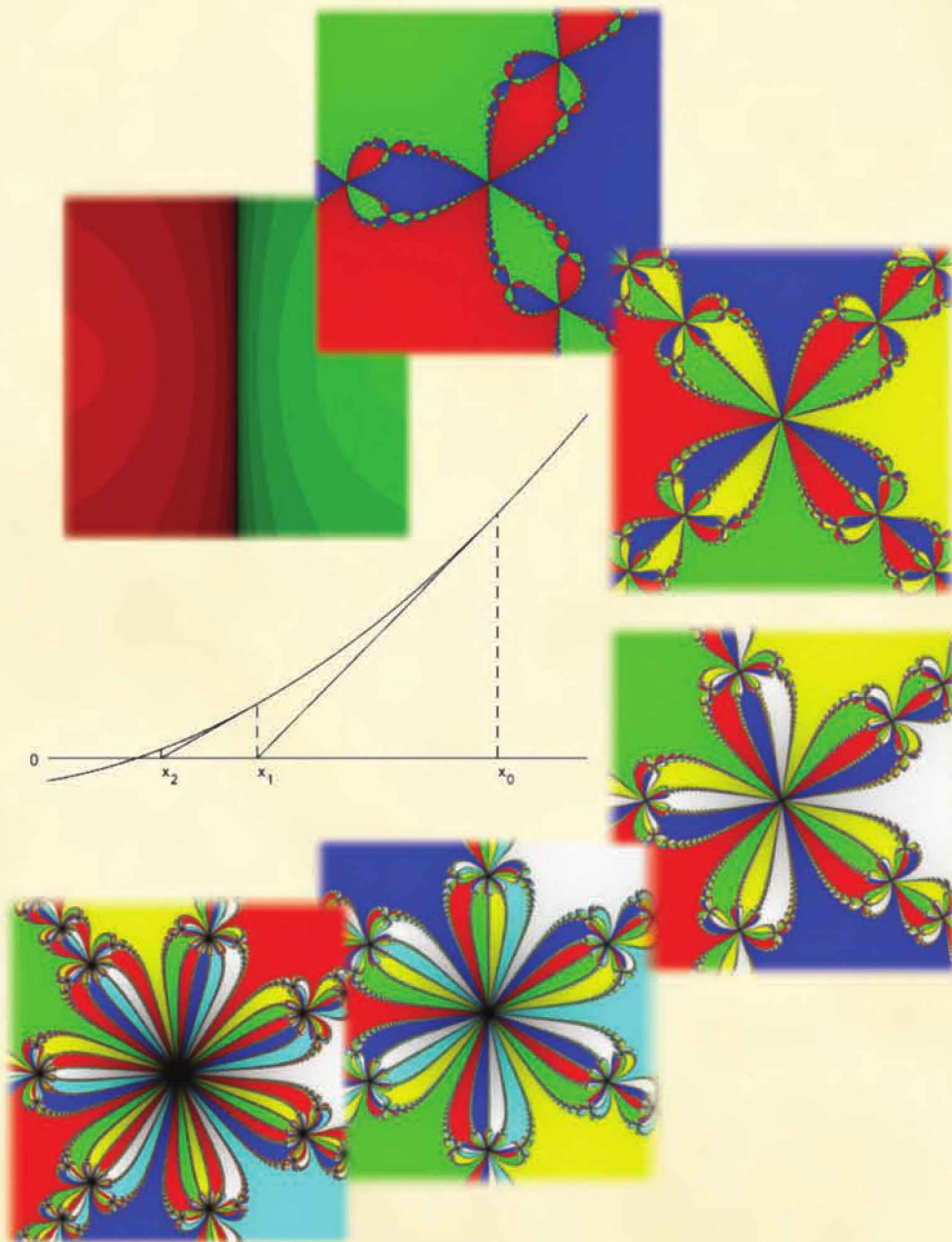


A Concise Introduction to Numerical Analysis

A. C. Faul



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