Vehicle Dynamics

Week 3 - Velocity-Acceleration Plot

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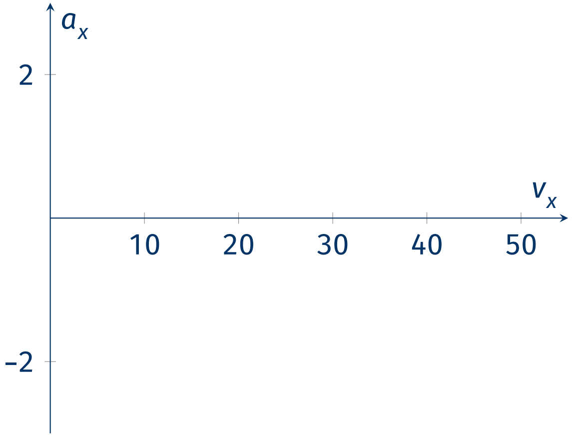
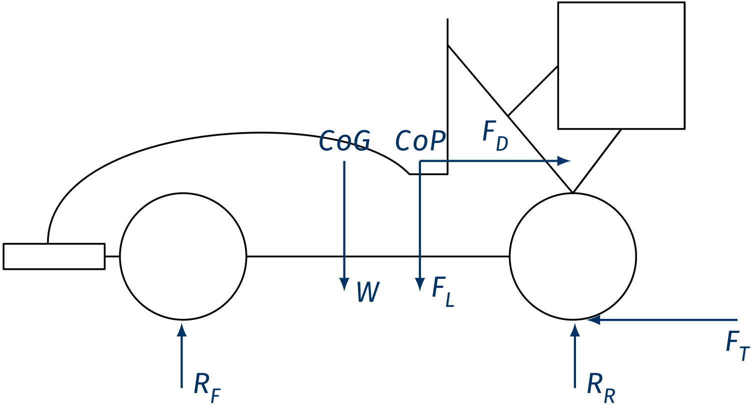
Velocity-Acceleration Plot

The velocity-acceleration plot is a diagram with velocity on the x-axis and (longitudinal) acceleration on the y-axis.



This is known as a **state space** diagram.

Lets add some limits to the graph. We will begin by drawing a free-body diagram of a car.



Consider the forces acting horizontally (the x-direction):

$$F_T = \frac{P}{v}$$

$$F_D = \frac{1}{2}\rho C_D A v^2$$

The acceleration of the vehicle is the sum of these forces:

$$\begin{aligned} a_x &= \frac{1}{m}\sum F_x \\ &= \frac{1}{m}\left(\frac{P}{v} - \frac{1}{2}\rho C_D A v^2\right) \\ &= \frac{P}{mv} - \frac{\rho C_D A v^2}{2m} \end{aligned}$$

This is known as the **power limit** of the vehicle.

Acceleration is also limited by the grip available from the tyres. First, we calculate the normal force on the tyres:

$$N = W + F_L = mg + \frac{1}{2}\rho C_L A v^2$$

For a tyre with a constant coefficient of friction μ, the maximum grip available is:

$$F_f = \mu N = \mu\left(mg + \frac{1}{2}\rho C_L A v^2\right)$$

Therefore, the **traction-limited** acceleration is:

$$a_x = \frac{F_f}{m} = \frac{\mu}{m}\left(mg + \frac{1}{2}\rho C_L A v^2\right) = \mu g + \frac{\mu \rho C_L A}{2m} v^2$$

Real tyres are **load sensitive**, meaning that μ decreases as N increases. This means that adding more downforce has diminishing returns.

Finally, the top speed of the car is limited by the top speed of the motor, ω_{max}, which can be found on the motor's datasheet (try searching for *'Emrax 228 datasheet'*).

This is divided by the final drive ratio to find the rotational velocity of the wheels, and multiplied by the tyre radius to find the linear velocity of the car.

$$v_{\text{max}} = \frac{\omega_{\text{max}} R_0}{\text{FDR}}$$

Lets plot these three lines on the velocity-acceleration diagram:

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