Very good - 7.25 out of 10

more detail next time in abstract would be great. Fine for now.

Also short conclusion and a ref would help reader - we want a self-contained report

ELEC6228 Linear Quadratic Regulator

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...is the work for the first piece of coursework on ...le.

We want report to be concise, but to make sense to a non expert reader. So please introduce system, objective and solution form first (and reference where more info can be found)

I. A.1

...n:

$$x_{k+1} = x_k + u_k$$
$$y_k = x_k$$
$$= \sum_{t=0}^{\infty} {y_t}^2 + \rho {u_t}^2$$

...s of the standard Linear Quadratic ...or I(N), B = 1 or $\begin{bmatrix} 1 \\ \vdots \\ 1_N \end{bmatrix}$, R=$\rho *$ I(N), Q = ...for when the state can be represented by a singular value, substituting into the discrete ARE gives:

You don't give the answers needed here: you need to compute the values of P and K. They should be functions of rho

$$P = 1 + P - P(\rho + P)^{-1}P$$
$$= 1 + P - \frac{P^2}{P + \rho}$$
$$= \frac{P + \rho + \rho P}{P + \rho}$$

Need to add uk = K xk explicitly

II. A.2

...d loop pole(s) of the system, solve:

$$Eig(z\mathrm{I} - (A - BK))$$

...y first calculating K according to the ARE and ...the pole:

$$K = -(\rho + P)^{-1}P = -\frac{P}{\rho + P}$$
$$A - BK = 1 + \frac{P}{\rho + P}$$
$$Eig\left(z - \left(1 + \frac{P}{\rho + P}\right)\right) = 1 + \frac{P}{\rho + P}$$
$$CLP = \frac{2P + \rho}{\rho + P}$$

define CLP

should be function of rho

III. A.3

$\rho = 0$: The state goes immediately to zero. $P_t = Q$ for all t and $u_t = -\frac{A}{B}x_t$. Therefore $u_t = -x_t$ leading to $x_{t+1} = 0$. The pole of the system, as K = -1 in this case, would be equal to $Eig(z - (1 + 1)) = pole\ at\ 2$.

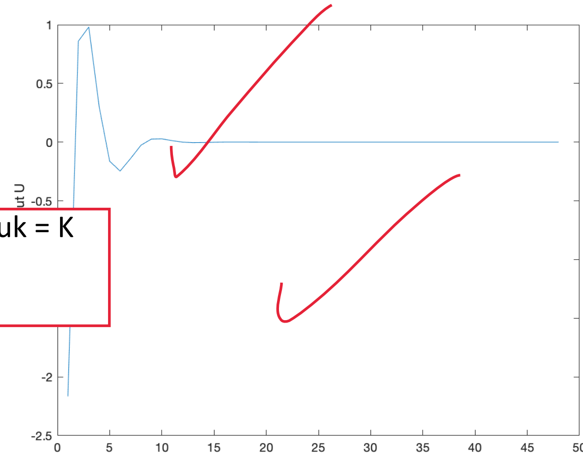
$\rho = \infty$: Costly control. In this case, control is ... zero:$K_t = 0, u_t = 0x_t$. This leads to x_{t+1} ... $A^2P_{t+1} + Q, P_N = 1$. The pole of the system, a... case, would be equal to $Eig(z - (1)) = pole\ a$...

partially correct: Cl pole = 1 + K
K tends to -1 as rho tends to 0
K tends to 0 as rho tends infty
Cl pole tends to 0 as rho tends to 0
Cl pole tends to 1 as rho tends infty

IV. B.1

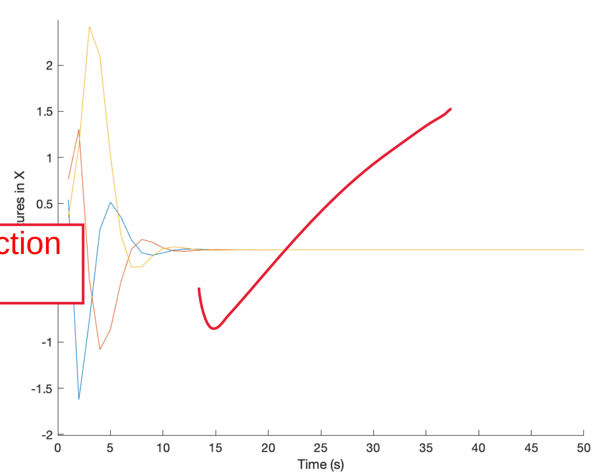
For this problem, the values of A, B, C and D w... along with the transfer function dependent on A and B. From the cost function $\rho \sum_{t=0}^{N-1} {u_t}^2 + \sum_{t=0}^{N} {y_t}^2$ and the output function $y_k = Cx_k, Q_f = C^T C$. From this value of $Q_f = P_N$ previous P matrices were calculated from the recurrence relationship in the Riccati Equation.

Plot of the optimal input as a function of time:

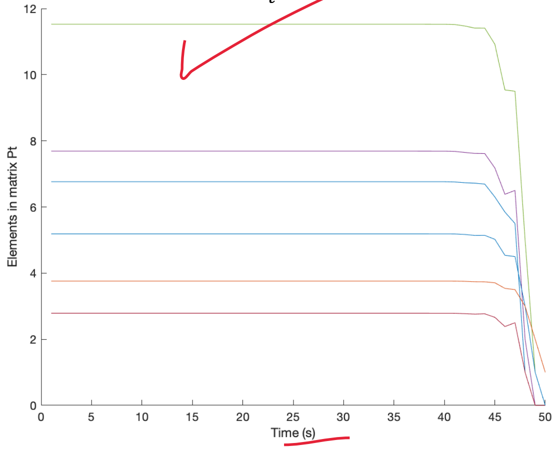


actually sample number

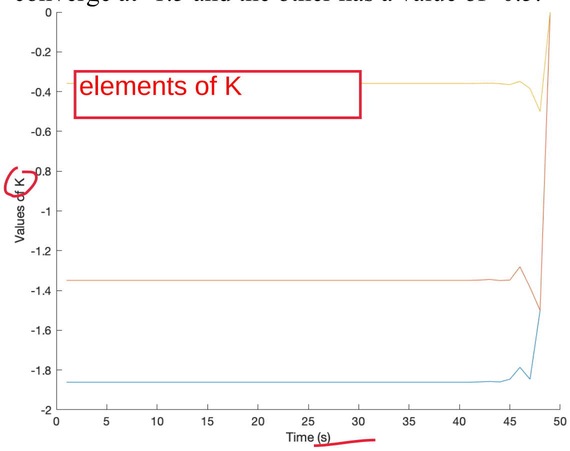
Plot of the three state fea...



Plot of the elements of P_t:



Plot of the values of K over time. For the final Q matrix Q_f, the calculation of K equals 0. Before this, two of the values converge at -1.5 and the other has a value of -0.5.

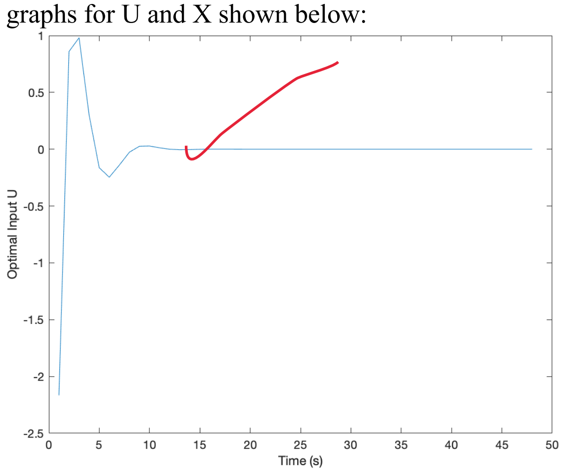


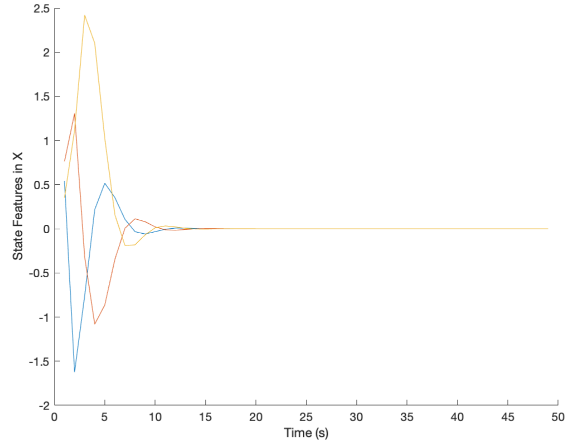
elements of K

I found that the system becomes stable after 20 samples, shown in the first two figures. The elements of P and K stay constant for most of the control but do converge close to/on 0 as the control sequence comes to its termination point.

V. B.2

Using the steady-state LQR solution instead of the optimal time-varying solution, the value of K remains constant due to the solving of the ARE using a single value of P. This gave the graphs for U and X shown below:





These were the same as the time-varying, but now with constant K. The costs were also the same:

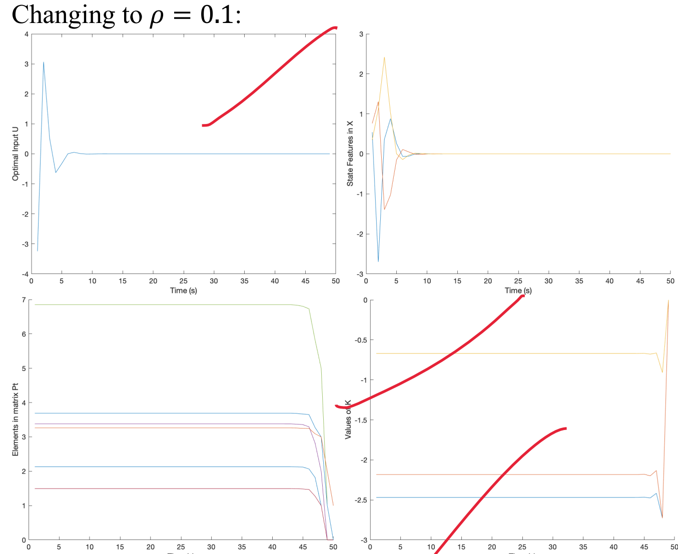
$$J = V_0(x_0) = x_0^T P x_0$$

actually they are very slightly different

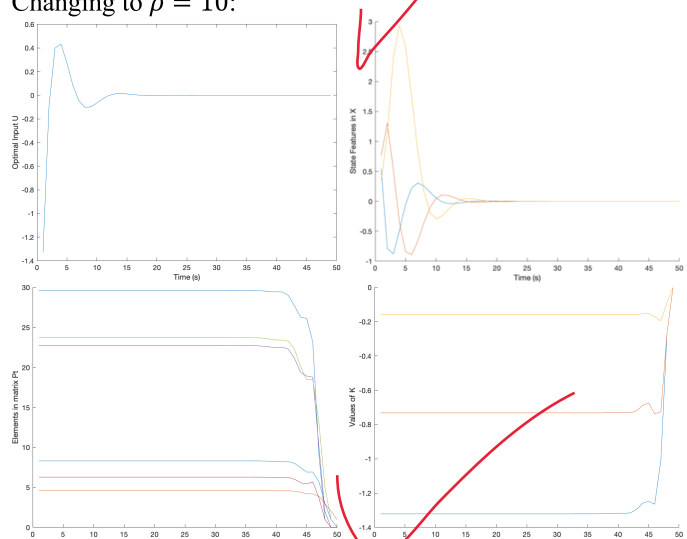
control and the steady state solution, this 19.3759.

VI. B.3

Changing to $\rho = 0.1$:



Changing to $\rho = 10$:



font size change?

Changing the value of ρ changes how long the system takes to reach a stable state. If ρ is large, the state of the system will have less of an impact on the cost. This therefore leads to a less sensitive system, reacting slower to each change in parameter leading to a slower rate of convergence. For large ρ, P values must be larger but K values are smaller (the P values also change more gradually for large ρ). This makes sense as the overall cost of a larger ρ is higher, with ρ=10 giving a total cost of 48.7794.

Therefore, one would assume that to improve performance one would use the smallest value of ρ possible, as shown previously when $\rho = 0.1$ the system converges faster. However, in real applications this would not stay the case as outside noise and uncertainty would cause a system that is too dependent on its current state to regulate its next input to over-correct too often.

As with the previous B2, the steady state analysis gives the same convergence behavior and total cost.

Nice discussion of trends – I think you have mentioned all the main points: As rho increases:
Costs increases
P values increase
K value decreases
Input peak decreases
Less aggressive