8 out of 10 - well done

ELEC6228 Iterative Learning Control

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***Abstract*—When errors and delays are added to real-world control systems, model predictive control preforms poorly at managing these errors. This paper investigates iterative learning control, which adjusts control inputs to specifically minimize error on the output.**

I. A.1

The system to be controlled has the state-space form:

$$x_k(t+1) = x_k(t) + u_k(t)$$
$$y_k(t) = 2x_k(t)$$

Where 'k' is the iteration number and 't' is the time taken inside each iteration. From this the values A=1, B=1, C=2, D=0 can be found. The input and output vectors can be related by a matrix G shown below.

$$\begin{bmatrix} y_k(1) \\ \vdots \\ y_k(N) \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ \vdots & \ddots & 0 & \vdots \\ 2 & 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} u_k(0) \\ \vdots \\ u_k(N) \end{bmatrix}$$

As A and B are both 1, the values for CAB for all powers of A are equal to C=2 in this case.

II. A.2

Following a gradient based algorithm to minimize a certain cost, we can imp

the error returne

recursive equatio

Actually showing convergence to zero is quite simple: you can either use optimality to show || ek+1 ||^2 +w gamma k^2 <= || ek ||^2. Or you can compute that || ek+1 ||^2 +w gamma k^2 + gamma_k || GT e_k ||^2 = || ek ||^2 using the objective function, the plant update eqn, and the gamma definition. Then from the answer to A.2 we know gamma k > iff ek not equal to zero (since GGT positive definite) , i.e. it must converge to limit ek = 0.

Where e_k is the

system cost is:

$$||e_{k+1}||^2 + \omega {\gamma_k}^2$$

ω is a small weighting scalar. To solve for the optimal learning gain γ_k, the error for the next iteration is defined as the reference action minus the current output action:

$$e_{k+1} = r - y_{k+1}$$
$$= r - Gu_{k+1}$$
$$= r - G(u_k + \gamma_k G^T e_k)$$
$$= e_k - \gamma_k G G^T e_k$$

Expanding the error component present in the cost:

$$||e_{k+1}||^2 = {e_{k+1}}^T e_{k+1}$$
$$= (e_k - \gamma_k G G^T e_k)^T (e_k - \gamma_k G G^T e_k)$$
$$= e_k^T e_k - 2\gamma_k e_k^T G G^T e_k + \gamma_k^2 e_k^T (GG^T)^2 e_k$$

Now both terms present in the cost function can be expressed in terms of γ_k. To find the value of γ_k that minimizes the cost, compute the derivative with respect to γ_k and set it equal to 0.

$$\frac{\delta J}{\delta \gamma_k} = 2\gamma_k e_k^T (GG^T)^2 e_k - 2e_k^T G G^T e_k + 2\omega\gamma_k$$

$$if\ \frac{\delta J}{\delta \gamma_k} = 0:$$

$$\gamma_k = \frac{2e_k^T G G^T e_k}{2e_k^T (GG^T)^2 e_k + 2\omega}$$

III. A.3

From this equation for γ_k it shows that $||e_{k+1}|| \leq ||e_k||$. This inequality can also be written as $||e_{k+1}||^2 - ||e_k||^2 \leq 0$:

$$||e_{k+1}||^2 - ||e_k||^2 = -2\gamma_k e_k^T G G^T e_k + \gamma_k^2 e_k^T (GG^T)^2 e_k$$

$$\therefore$$

$$2\gamma_k e_k^T G G^T e_k \geq \gamma_k^2 e_k^T (GG^T)^2 e_k$$

$$\cancel{2GG^T \geq \gamma_k (GG^T)^2}$$

why?
second equality involves matrices and so isn't well defined. Here you just need to subs in gamma definition and you would have finished your proof

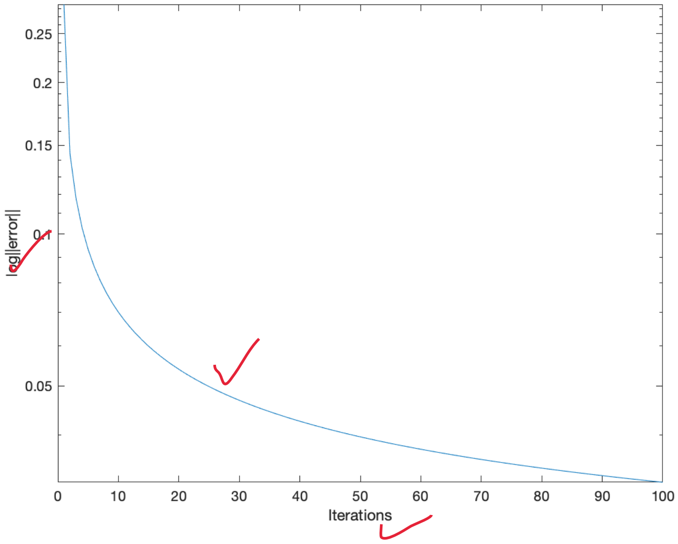
Given the shape and size

e axes are orthogonal. An example of

e reference given is specified as:

$$r_x(t) = 0.035t\ \ 0 \leq t \leq 1$$
$$r_x(t) = 0.07 - 0.035t\ \ 1 < t \leq 2$$

A model can be made to track a single axis using this 'vector of scalars' reference. The sampling time used was 0.01 seconds. The following graph shows the log of the error norm with the choice of $\omega = 10^{-8}$.



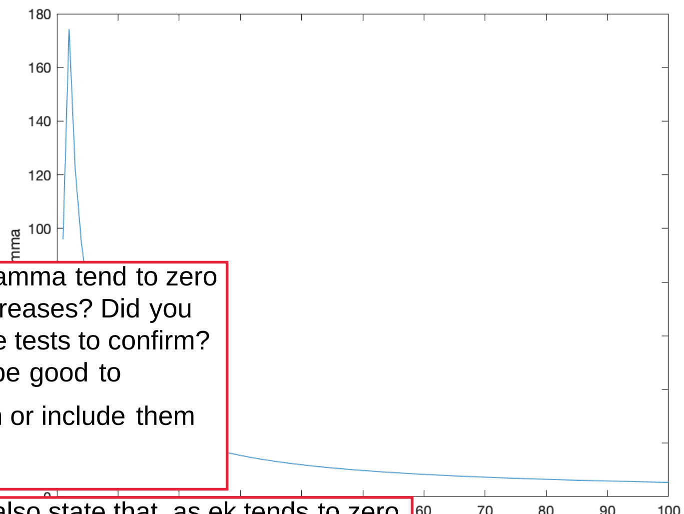
This simulation result does seem to show the properties from

is it strictly monotonic?

A.3. As it continually decreases with ality in task A.3 is satisfied.

V. B.2

Using a value of $\omega = 10^{-8}$, the value of γ_k changes over iterations like so:



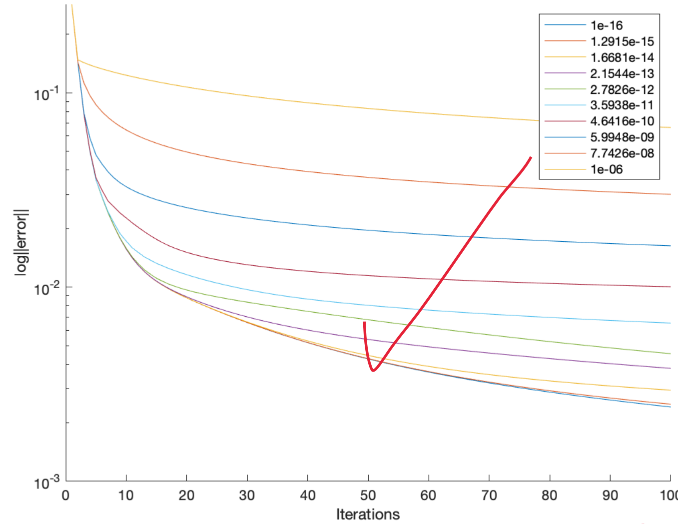
Does gamma tend to zero as k increases? Did you do more tests to confirm? Would be good to mention or include them

Ideally also state that, as ek tends to zero, gammak must also tend to zero, and again the property that GGT is positive definite is needed for this to hold

error is very large, so the s even though increasing γ_k has a negative effect on the cost, it is a worthwhile tradeoff to reduce the $||e_{k+1}||^2$ term. Then, as the value of the second term $\omega {\gamma_k}^2$ becomes more significant with decreasing error, the system works to decrease the value of γ_k.

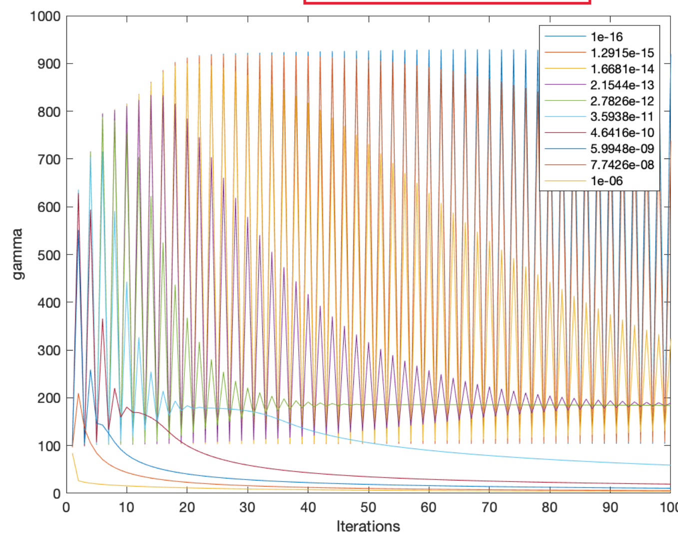
VI. B.3

Tasks B.1 and B.2 were repeated using a range of ω values. The resulting figures are shown in the following graphs.



This graph shows that the s better the performance of the system wer than 10^{-17} will give negligible i

faster? do they all converge to 0 error norm



The graphs for γ_k show an increasingly erratic and high magnitude value choice for lower values of ω. This makes sense considering the cost function $||e_{k+1}||^2 + \omega {\gamma_k}^2$, which basically has the second term eliminated at very low ω values. This means that the system worries less about the value of γ_k as it has almost no effect on the cost and will instead put all of its efforts towards minimizing error.

Good. The key points to include are: Gamma always converges to zero, Error always monotonically converges to zero, Convergence faster with smaller w, Gamma larger with smaller w