Income- and substitution effects

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The household often has two competing sources of utility in a trade-off relation, and it has to divide the income between them, for example

- Consumption of goods X and Y
- Consumption now C_t and later C_{t+1}
- Consumption and leisure

What happens if there is a change in prices, taxes, wages or other such, which makes one of the consumption choices relatively cheaper or more desirable than the other?

In the last example the household must sacrifice some of the leisure time in order to earn income by working, which is then used for consumption. Household can therefore choose how much it acquires leisure and consumption, where they are interchangeable by altering the hours worked h. In the optimum the household is indifferent between enjoying an extra unit of leisure or an extra unit of consumption.

Let there then be a change, for example, the income tax increases. This is what happens in the next exercise. The price of consumption in units of leisure increases, because the household now has to give up more leisure to earn the same income as before, to get the same amount of consumption as before. In other words, consumption becomes relatively more expensive than leisure.

Notice, that if the HH were to work full hours, it could not anymore acquire the same amount of consumption as it was able to with the full budget before the tax increase. However, the HH can still enjoy the same maximum amount of leisure as before, by not working at all. We could re-phrase this by saying, that after the tax increase the HH can afford less consumption, but it can still afford the same amount of leisure. This is an important feature, as the less affordable variable (consumption in this case) will have a straight forward response: it will decrease after an increase in the income tax. But the response of the other variable, which still has the same affordability as before, leisure in this case, depends on the substitution and income effects. Leisure can either decrease or increase in the aftermath of the tax increase depending on the HH's utility function.

Substitution effect refers to the situation, where a change in the relative prices causes the household to increase the consumption of the cheaper source of utility and decrease the more expensive one. Using the example of an increase in the income tax, if the substitution effect dominates, then the household wants to decrease consumption and increase leisure, $\frac{C\downarrow}{l\uparrow}$.

Does this substitution effect always take place? In other words, the amount consumed in this example does decrease in all cases, but does the amount of leisure have to increase? Not necessarily, because at the same time the increasing taxes decrease the disposable income and make the household feel poorer than before. This is because it can no longer afford the same amount of consumption and leisure with the same budget as before (with same number of hours worked).

This is called the **Income effect**, which means that as the household feels poorer, it tends to decrease all spending, including on the now "less expensive" leisure, $\frac{C\downarrow}{l\downarrow}$. Hence, now that the household gets a smaller income than before with the same amount of work, it decides to cut down both consumption and leisure.

What determines which effect dominates? Some changes in the economic landscape can only lead to the income effect, such as a lump sum tax in the case that we have been considering (it equally reduces the affordability of both leisure and consumption, as the household must work for certain hours to pay the tax). Other changes can also give rise to substitution effects, such as increasing the prices of consumption goods. Pure substitution effect says that the suddenly more favorable leisure will increase and pure income effect says that it will decrease. Typically both of the effects are at play simultaneously, with one being stronger than the other.

Also the utility function of the household has a crucial impact, on whether the substitution or income effect dominates. More specifically, this is controlled by the σ in the isoelastic utility function.

The substitution and income effects cancel out perfectly when we use the logarithmic utility function ($\sigma = 1$), as in the following exercise. A tax increase with log-utility only reduces the consumption, but leisure (and hours worked) remain unaltered (in absence of transfers $v = 0$).

Let's take a closer look at how σ influences the decisions of Households, and thereby the behavior of the Demand side at the Aggregate level. The size of the σ determines which one is stronger, substitution or income effect.

I have collected the relevant equations below. Equation 1 depicts the isoelastic utility function with only consumption-related terms included. Equation 2 expresses the log-linearized Euler equation. Figure 1 shows, how the isoelastic utility function transforms input (consuption) into utility, with varying values of σ. Smaller the σ, the less sensitive the utility becomes to the changes in the level of C.[1]

$$U(C,\bar{C}) = \frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \beta\left[\frac{C_{t+1}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right] \qquad \text{Isoelastic utility function} \qquad (1)$$

$$c_t = c_{t+1} - \sigma[\bar{i} - p_{t+1} + p_t] \qquad \text{log-linear Euler} \qquad (2)$$

[1] All three curves have the same range of inputs C in (0,1). The output values have been normalized to fit in the same plot, without changing the qualitative interpretation of the results (we are interested in the ordinal utility, not cardinal). The curves were approximated in R using $\sigma = 0.01$ for the red curve, $\sigma = 0.99$ for the blue curve and $\sigma = 1000$ for the black curve.

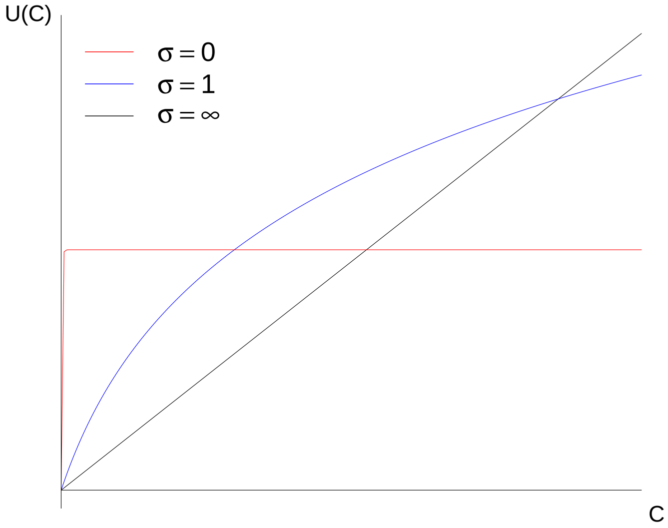

Figure 1: Mapping the consumption level into utility using the isoelastic function, with different σ's

When the **elasticity of intertemporal substitution is large**, $1 < \sigma < \infty$, the consumption is easily moved across the periods. This is because changing the consumption level has a large impact on the utility, see Figure 1. **Hence, the substitution effect dominates.** In the extreme case of $\sigma = \infty$, Equation 1 becomes linear $U(C, \bar{C}) = C + \beta\bar{C}$, making the current and future consumption perfect substitutes to each other. Therefore, if there is a price difference $P/\bar{P}$ and/or the interest rate differs from the household's time preferences $(1+i)\beta \neq 1$, all the consumption will be concentrated to the current or future period(s). This is easiest to see by checking what happens in Equation 2, if $[i - \bar{p} + p] \neq 0$, the HH increases or decreases the current period consumption as much as possible (note, that the log approximation becomes inaccurate with large deviations).

When the **elasticity of intertemporal substitution is small**, $0 < \sigma < 1$, the allocation of consumption for different periods doesn't change much in response to relative changes in prices or interest rates. As can be seen from the Figure 1, changes in consumption don't impact the utility much at all (as long as the consumption is positive) and therefore there's no reason to change the original consumption plan. **Now the income effect dominates.** At the extreme, when $\sigma \approx 0$, regardless of the conditions, i.e. even if $[i - \bar{p} + p] \neq 0$ in Equation 2, the household still consumes the same fraction of income each period as before, $c = \bar{c}$. Hence, the current and future consumption are regarded as complements to each other. Naturally the actual amounts consumed does change by income.

At $\sigma = 1$ the utility function becomes logarithmic, and the substitution and income effects cancel out perfectly. You can easily see this by checking the derivative

$$\begin{aligned} \frac{\partial}{\partial C}\left[\frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right] &= C^{-\frac{1}{\sigma}} && \text{Set } \sigma = 1 \\ &= C^{-1} \\ &= \frac{\partial}{\partial C}(log(C)) \end{aligned}$$

Caveat 1. When the household chooses between two goods, the affordability of both options can change independently. The same applies, when the HH chooses between consumption now and later. By this I mean the following: If the household was to use its full income only to buy one of the goods (whichever), it would be possible to change the number of units affordable of that good, while keeping the number of units affordable fixed for the other good (blue and red lines in Figure 2). When this kind of change in affordability happens to one of the goods, then also the relative price between the goods change: how much of one do you need to give up to acquire the other. However, keep in mind that changes in income don't impact the relative prices, and does keep the proportions consumed the same (the whole budget line shifts in parallel).

The rule of thumb is: when one of the options becomes more (or less) affordable while the other stays the same, the consumption of the changed option will go up (or down), but the impact on the other option depends on the utility function and which one is stronger; substitution or income effect (value of σ). Hence, it is important, from which one of the options does the change in relative prices stem from.

In the figure below, Y stands for income, and the curves are budget lines familiar from intermediate micro. The blue line refers to situation, where only the price of C_1 or C_t has reduced, and the red line where the price of C_2 or C_{t+1} has reduced.

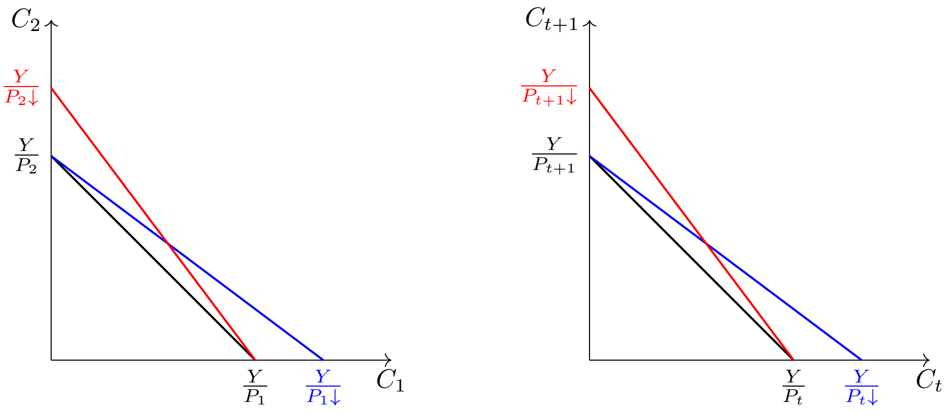

Figure 2: Affordability of both consumption choices can be changed independently

However, **there is an asymmetry in the case where the household chooses between consumption and leisure.** The HH has a time endowment $L = l + h$ and values leisure l and consumption $C = wh$, with unit price of C being P and hourly wage w earned for hours worked h. Think of the household first working full hours to get the maximum income wL, and then spending this income on leisure and consumption, with prices w and P, respectively. The largest possible amount of consumption is attained by spending the whole income on it, $C_{max} = wL/P$. The maximum amount of leisure is attained, when the whole income is spent to acquire leisure, $l_{max} = wL/w = L$.

We can make consumption more or less affordable by altering either P or w (or income tax), without having any impact on the maximum amount of leisure attainable L, blue curve in the Figure 3.

On the other hand, if we want to change the amount of leisure affordable, we necessarily also change the amount of C affordable. This is because the only way to increase the maximum affordable amount of leisure is to increase the total time available L, red curve in the Figure 3. This additional time is also available for work, and therefore increases the total income (wL). Increasing the income increases the units of C affordable.

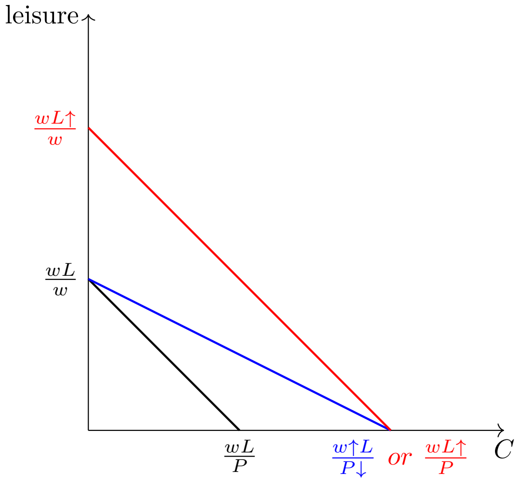

Figure 3: Increasing the maximum possible amount of leisure has an automatic income effect

Caveat 2. Some changes only effect the affordability of one of the options (like ΔP_1 or ΔP_t), and the relative prices between the options change as a consequence. But some changes have an impact on both at the same time. A raise in the interest rate r tilts the bugdet line around the point where saving/borrowing is zero. The impact of this on consumption depends on whether the household is a borrower or a saver (I wrote Y(r) to denote, that the life-cycle income depends on r).

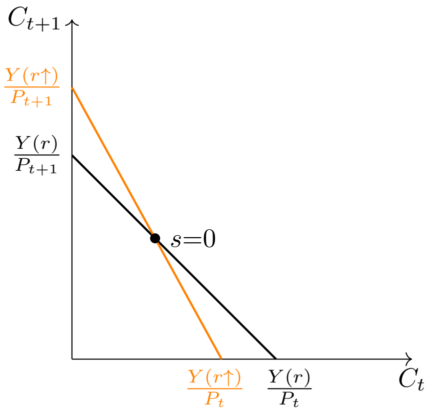

Figure 4: Interest rate tilts the intertemporal budget line around a point, where s=0