AD-AS model

Oliver Snellman
oliver.snellman@helsinki.fi

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Abstract

The AD-AS is a pedagogical model, which uses graphical analysis as a tool for building qualitative intuitions about the macro economy. In this document I show how to derive the model step by step, starting from the Keynes Cross model to get the IS-LM/TR model, from which the demand side AD curve arises from. I also derive the microfounded New Keynesian AD relation from household's problem. Finally, I derive the supply side AS curve using staggered wage setting.

Introduction

The AD-AS model takes into account the key aspects of the macro economy, and describes the economic relations between the central real and nominal quantities, the prices and the output. The model can be used to study, how the **supply** and **demand** side of the economy and the **central bank** respond to many interesting phenomena, such as changes in inflation, output gap, interest rate, expectations about the future, unemployment, net exports, government spending, consumption behavior etc.

We start by considering a simple Keynes Cross model, and then add more and more features into it, until we have something potentially useful.

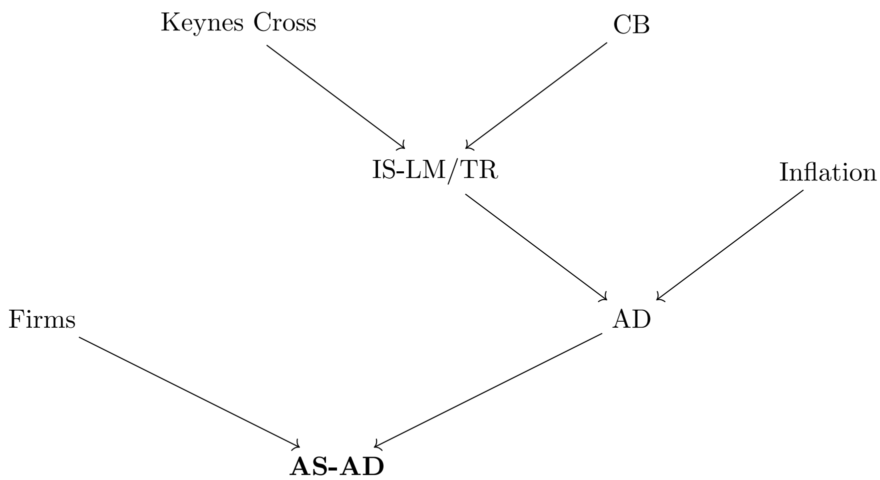

Figure 1: Construction of the AD-AS model

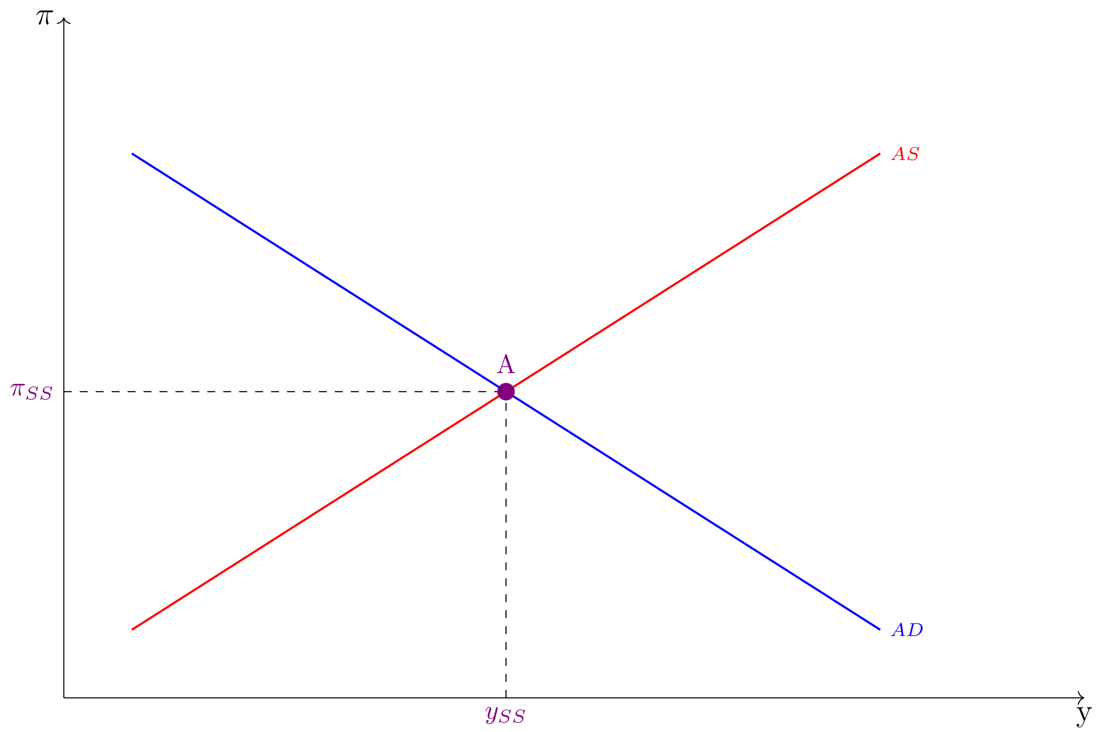

Figure 2: AD-AS model. Lower index SS denoting for steady state values.

About modeling

The key criterion, according to which we judge the validity of our models in modern economics, is how **useful** they are. The aim of model building is to help us understand the world, and to survive and thrive in it. Mathematical methods also allow us to communicate our ideas in exact and clear manner. Some models are mainly used for pedagogical purposes, like AD-AS - to build intuition about a complex phenomenon and to soften the students' landing to the subject. It is then easier to proceed to study more nuanced and mathematically challenging models, like DSGE, which are actually used in the cutting edge research.

The central aim of these models is therefore not to represent the world realistically in all levels and aspects. Naturally, it is hard for a model to be useful, if it doesn't bare any resemblance to the real world, so some degree of realism will and should be present. But typically the mathematical complexity increases when the model is made more realistic. It might not always be worth while obtaining a small increase in the realism or explanatory power of the model, if it requires increasing the complexity by manyfold.

In philosophical terms, this means that the epistemological justification for economic modelling stems from pragmatism ($\sim$ that is true enough, which works well enough). We can't access the reality in itself without interpretation, but we can try to represent it with different kind of verbal and mathematical structures, i.e. theories or models. As we can't compare the models to the reality as such (noumenal), we should concentrate on what's the result of using the models, i.e. their usefulness. As the famous saying goes: All models are false in the sense, that they are not the same as the reality, but some models can be useful.

Keynes Cross

Let the Desired Demand DD(Y) take the following form

$$DD(Y) = C(Y) + I_p + G + NX(Y) \tag{1}$$

$$\begin{aligned} &\text{with} \\ C(Y) &= a + b(Y - T) \\ I_p &= c - di \\ NX(Y) &= \underbrace{fY^f}_{Exports} - \underbrace{gY}_{Imports} \end{aligned}$$

The economy is always on the 45° line, where the aggregate supply equals aggregate (desired) demand.

If the output Y_{lower} is below the Desired Demand, $DD(Y_{lower}) > Y_{lower}$, the firms notice the excess demand, will hire more workers and increase the production until the next period. Likewise, if the output Y_{higher} is above the Desired Demand, $DD(Y_{higher}) < Y_{higher}$, the firms can't sell all the goods they produced (unplanned inventory investments), and they reduce the production for the next period.

There is a unique equilibrium where $DD(Y) = Y$, which is reached by altering the employment level.

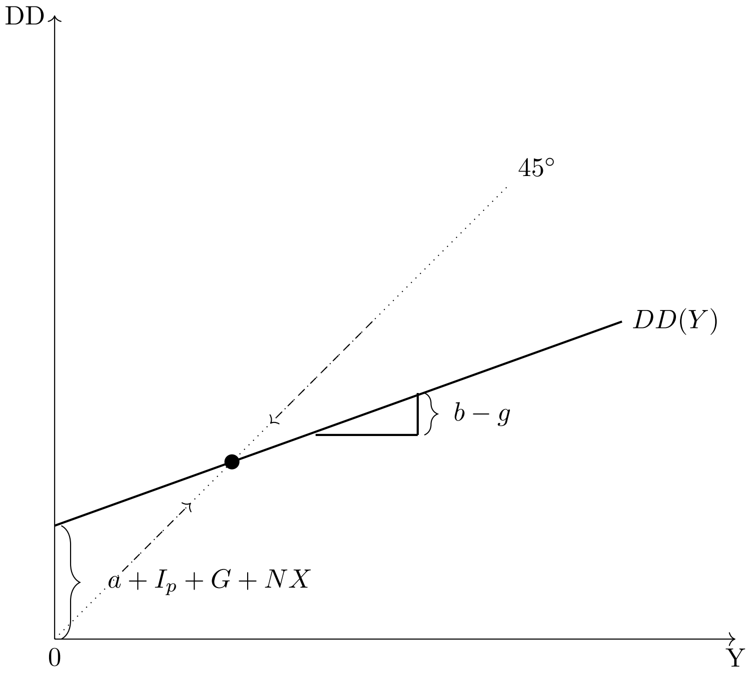

Figure 3: Keynes cross

Parameter b is the fraction of additional unit of disposable income spent on consumption, and g is the fraction of consumption that is spent on imports. The slope of the DD(Y) -curve is therefore b-g, the marginal propensity to consume domestically produced output.

IS-curve

Now we introduce the interest rate, which impact the goods market equilibrium negatively through planned investments $I_p = c - di = 25 - 100i$.

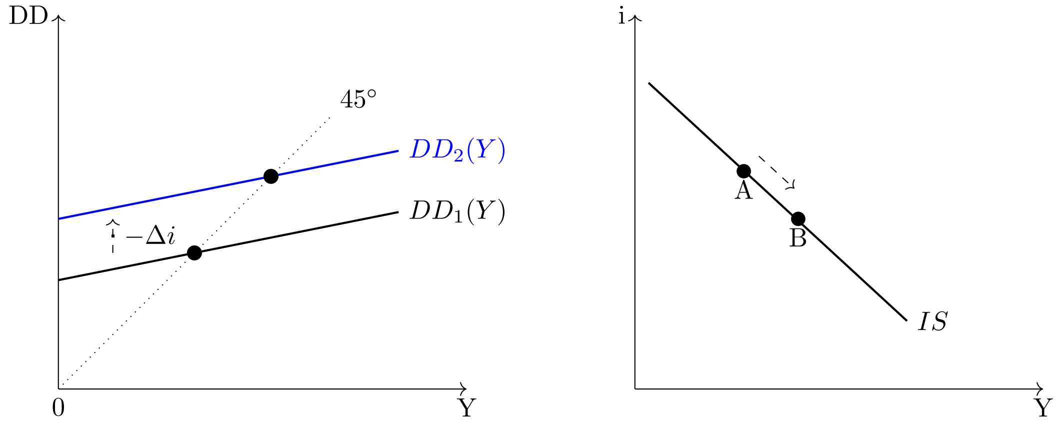

Figure 4: Deriving the IS curve

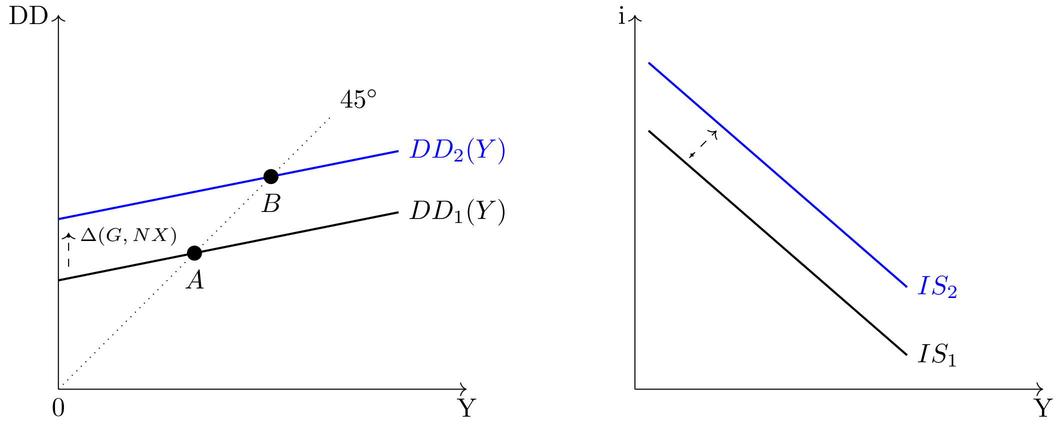

Figure 5: Shifting the IS curve

G ↑	IS ↑
T ↓	IS ↑
NX ↑	IS ↑

Table 1: Shifting the IS-curve

The slope of the IS curve depends on the sensitivity of investments to interest rate, d, which tells how much the DD(Y) -curve shifts when the interest rate changes. The slope of the IS curve also depends on the slope of the DD(Y) -curve, b-g, - how much the impact of Δi is multiplied.

The marginal propensity of domestic consumption, b-g, also impacts on how much the IS-curve shifts due to Demand shocks (other than i); Larger the b-g, larger the shift.

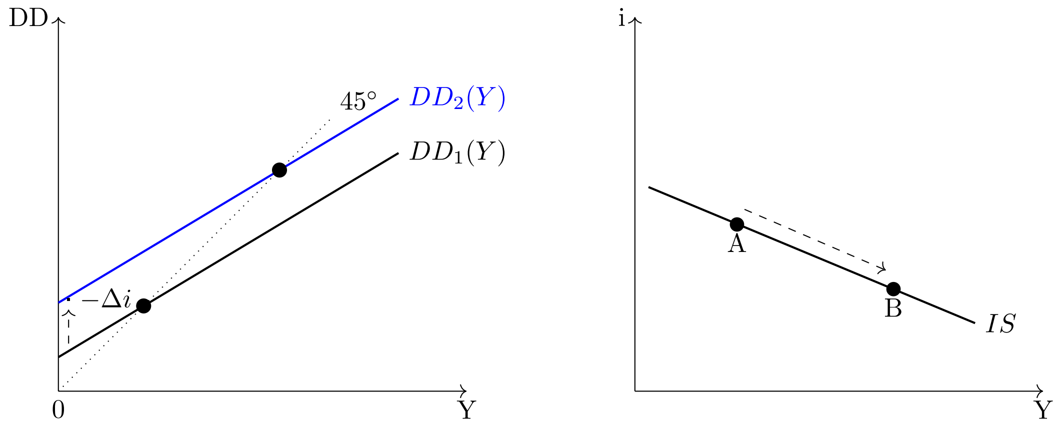

Figure 6: Higher b flattens the slope of the IS curve

LM/TR-curve

Let's bring the Central Bank (CB) into the picture. The CB has an important stabilizing role in the model economy. There are two interpretations for how the CB functions, but both lead to the same results. In the LM-approach, the CB controls the money supply and influences the nominal interest rate as a side effect. In the TR-approach, the CB sets the nominal interest rate directly according to its objectives.

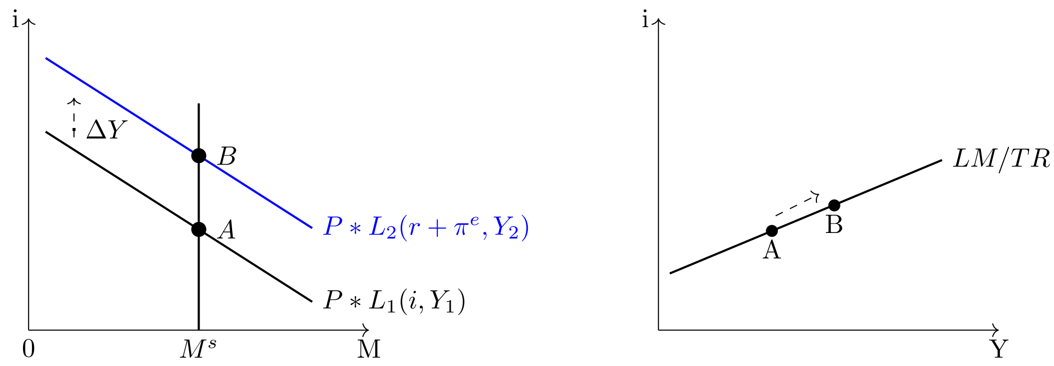

Figure 7: Deriving the LM curve

$\text{M}^s \uparrow$	LM/TR $\downarrow$
P $\uparrow$	LM/TR $\uparrow$
$\pi^e \uparrow$	LM $\downarrow$
$\pi^* \uparrow$	TR $\downarrow$

Table 2: Shifting the LM-curve

An increase in the real money balances $\frac{M}{P}$ raises the LM-curve. Increase in inflation expectation π^e raises the nominal interest rate via Fisherian relation $i = r + \pi^e$ (given a real interest rate), which in turn shifts the LM-curve down.

The TR-curve has the same properties as the LM-curve, but the idea is more modern: Instead of influencing the interest rate through controlling money supply, the CB sets the interest rate directly and uses it to influence the inflation (and money supply indirectly).[1]

$$i = \bar{i} + \alpha(\pi - \pi^*) + \beta Y_{gap} \qquad | \quad \text{Taylor rule}$$

AD-curve

Increase in inflation π (increase in prices P) reduces the real money balances $\frac{M}{P}$ (or deviations from the inflation target cause CB to raise the interest rate) and shifts the LM/TR -curve up. The new intersection with the IS-curve occurs at higher interest rate and reduced output. The AD-curve maps the (π, Y) points which are consistent with both goods- and money market (or CB's policy rule) equilibria.

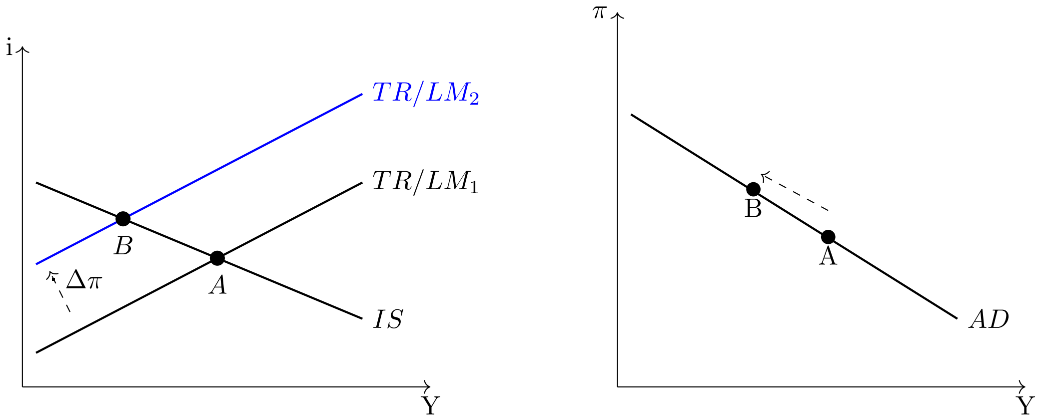

Figure 8: Deriving the AD curve

The AD-curve shifts up, when either the IS-curve shifts up, or the LM-curve shifts down (their intersection moves to the right).

G $\uparrow$	IS $\uparrow$	AD $\uparrow$
T $\downarrow$	IS $\uparrow$	AD $\uparrow$
NX $\uparrow$	IS $\uparrow$	AD $\uparrow$
M^s $\uparrow$	LM $\downarrow$	AD $\uparrow$
P $\downarrow$	LM $\downarrow$	AD $\uparrow$
π^e $\uparrow$	LM $\downarrow$	AD $\uparrow$

Table 3: Shifting the AD-curve

[1] More specifically, the CB sets the nominal interest rate, which influences the real interest rate via Fisher equation ($i = \pi^E + r$), which in turn impacts the aggregate demand. The change in output demanded then influences the employment, wages and prices through the labor markets, resulting in the new inflation rate and output. Nowadays the CB has also many other tools to conduct the monetary policy, additional to setting the nominal interest rate.

New Keynesian AD-curve

Now that we have established the intuitive understanding of how the demand side of the economy functions, let's derive the same relation from microfoundations. This means replacing the simple postulated consumption equation in the Keynes Cross, $C = a + bY$, with the consumption rule obtained from solving the optimization problem of the household.

To get the New Keynesian AD-curve, we need to log-linearize and combine the consumption Euler-equation and the aggregate resource constraint.

Let's start by solving the Euler from the two period household's problem

$$\max_{C,L,C,L} \quad \underbrace{u(C)}_{\text{Utility from C}} \quad - \quad \underbrace{v(L)}_{\text{Disutility from L}} \quad +\beta \; \underbrace{\big[u(C) - v(L)\big]}_{\text{utility of next period}} \qquad \text{Objective function}$$

s.t.

$$\underbrace{PC + \frac{PC}{1+i}}_{\text{Total (discounted) lifetime spending}} \quad = \quad \underbrace{WL + \frac{WL}{1+i} + T}_{\text{Total (discounted) lifetime wealth}} \qquad \text{Lifetime constraint}$$

where the bar above the variables indicate them being the second period values; $C =: C_t$ and $C =: C_{t+1}$.

Solve using the Lagrangian

$$\mathcal{L} = \max_{C,L,C,L} \; u(C) - v(L) + \beta\big[u(C) - v(L)\big] + \lambda\Big[WL + \frac{WL}{1+i} + T - PC - \frac{PC}{1+i}\Big]$$

As we are only interested in the Euler equation, we need just the optimality conditions for the consumption choices. I'll leave the working decisions and the constraint out of the FOC's for the sake of brevity.

FOC:

$$\frac{\partial \mathcal{L}}{\partial C} = u'(C) - \lambda P = 0 \qquad \Longleftrightarrow \qquad \lambda = \frac{u'(C)}{P} \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial C} = \beta u'(C) - \lambda \frac{P}{1+i} = 0 \qquad \Longleftrightarrow \qquad \lambda = \beta(1+i)\frac{u'(C)}{P} \tag{3}$$

Combine Equations 2 and 3 to find the consumption Euler equation

$$\lambda = \lambda$$

$$\frac{u'(C)}{P} = \beta(1+i)\frac{u'(C)}{P}$$

$$\frac{u'(C)}{\beta u'(C)} = (1+i)\frac{P}{P} \qquad \text{Generic Euler}$$

To proceed, we need to postulate a functional form for the utility-function of the households. Let's go with the isoelastic-utility function.

$$u(C) = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{4}$$

$$u'(C) = (1-\frac{1}{\sigma})\frac{C^{1-\frac{1}{\sigma}-1}}{1-\frac{1}{\sigma}} = C^{-\frac{1}{\sigma}} \qquad\qquad \text{Similarly for } C$$

where σ is the intertemporal elasticity of substitution of consumption.
Plug these derivatives for C and C into the Euler

$$\frac{C^{-\frac{1}{\sigma}}}{\beta C^{-\frac{1}{\sigma}}} = (1+i)\frac{P}{P} \qquad\qquad \text{Euler for isoelastic utility}$$

Log-linearize the LHS (lower index SS denoting for the steady state value)

$$\begin{aligned}
log\left(\frac{C^{-\frac{1}{\sigma}}}{\beta C^{-\frac{1}{\sigma}}}\right) - log\left(\frac{C_{SS}^{-\frac{1}{\sigma}}}{\beta C_{SS}^{-\frac{1}{\sigma}}}\right) &= log(C^{-\frac{1}{\sigma}}) - log(\beta) - log(C^{-\frac{1}{\sigma}}) - log(C_{SS}^{-\frac{1}{\sigma}}) + log(\beta) + log(C_{SS}^{-\frac{1}{\sigma}}) \\
&= -\frac{1}{\sigma}log(C) + \frac{1}{\sigma}log(C) + \frac{1}{\sigma}log(C_{SS}) - \frac{1}{\sigma}log(C_{SS}) \\
&= \frac{1}{\sigma}\big[log(C) - log(C_{SS}) - log(C) + log(C_{SS})\big] \\
&= \frac{1}{\sigma}\big[\underbrace{log(C) - log(C_{SS})}_{=:c} - \underbrace{(log(C) - log(C_{SS}))}_{=:c}\big] \\
&= \frac{1}{\sigma}[c - c]
\end{aligned}$$

and the RHS

$$\begin{aligned}
log\left((1+i)\frac{P}{P}\right) - log\left((1+i_{SS})\frac{P_{SS}}{P_{SS}}\right) =& log(1+i) + log(P) - log(P) \\
& - log(1+i_{SS}) - log(P_{SS}) + log(P_{SS}) \\
=& \underbrace{log(1+i) - log(1+i_{SS})}_{=:\tilde{i}} + \underbrace{log(P) - log(P_{SS})}_{=:p} \\
& - \underbrace{(log(P) - log(P_{SS}))}_{=:p} \\
=& \tilde{i} + p - p
\end{aligned}$$

Notice, that p here is the inflation π at time t and p can be thought of as the inflation expectation.

Put the LHS and RHS back together to get the log-linearized Euler equation

$$\begin{aligned} \frac{1}{\sigma}[c - c] &= \tilde{i} - p + p \\ c - c &= \sigma[\tilde{i} - (p - p)] && \text{Log-linearized Euler} \qquad (5) \end{aligned}$$

Next we turn to the simplified Aggregate resource constraint, which expresses that all income is used either for private consumption or government spending.

$$\begin{aligned} Y &= C + G \\ Y &= C + G \end{aligned}$$

Log linearizing the sum on the RHS needs an additional step. Also remember, that $\frac{b-b_{SS}}{b_{SS}} \approx log(b) - log(b_{SS})$ for small deviation $b - b_{SS}$.

$$\begin{aligned} Y - Y_{SS} &= C + G - (C_{SS} + G_{SS}) \\ Y - Y_{SS} &= C - C_{SS} + G - G_{SS} && || \frac{1}{Y_{SS}} \\ \frac{Y - Y_{SS}}{Y_{SS}} &= \frac{C - C_{SS}}{Y_{SS}} + \frac{G - G_{SS}}{Y_{SS}} \end{aligned}$$

$$log(Y) - log(Y_{SS}) \approx \underbrace{\frac{C_{SS}}{C_{SS}}}_{=1} \frac{(C - C_{SS})}{Y_{SS}} + \underbrace{\frac{G_{SS}}{G_{SS}}}_{=1} \frac{(G - G_{SS})}{Y_{SS}}$$

$$y \approx \underbrace{\frac{C_{SS}}{Y_{SS}}}_{=:s_c} \underbrace{\frac{(C - C_{SS})}{C_{SS}}}_{\approx log(C) - log(C_{SS})} + \underbrace{\frac{G_{SS}}{Y_{SS}}}_{=:s_g} \underbrace{\frac{(G - G_{SS})}{G_{SS}}}_{\approx log(G) - log(G_{SS})} \qquad || \log(C) - \log(C_{SS}) =: \tilde{c}$$

$$y \approx \underbrace{s_c \tilde{c}}_{=:c} + \underbrace{s_g \tilde{g}}_{=:g}$$

$$y = c + g$$

In the last line we have defined c and g as the percentage deviations of consumption and government spending (approximated with logarithmic differences) from their steady state values, $\tilde{c}$ and $\tilde{g}$, weighted by their share of the output, s_c and s_g.

Similarly we have

$$y = c + g$$

Solve for c and c

$$\begin{aligned} c &= y - g \\ c &= y - g \end{aligned}$$

Plug these into the log-linearized Euler, Equation 5, to finally arrive at the log-linearized New-Keynesian AD equation

$$
\begin{aligned}
y - g - y - g &= \sigma[\tilde{i} - (p - p)] \\
y &= y + (g - g) - \sigma[i - (p - p)] \qquad \textbf{Aggregate Demand}
\end{aligned}
$$

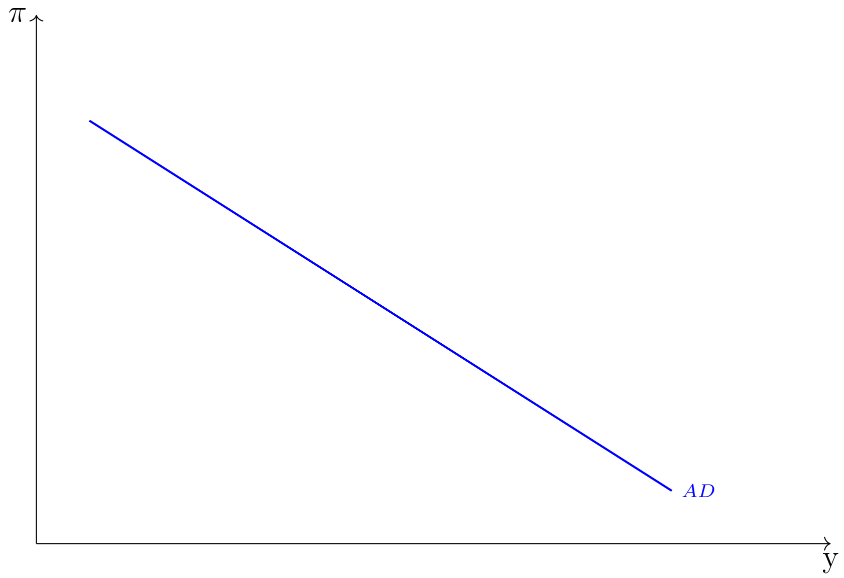

Figure 9: AD curve

AS-curve

We start deriving the AS-curve from the fundamentals of production, firm's production function. The key arena behind the AS-curve is the labor market, where the price of labor, nominal wages, are negotiated between firms and workers. Keep in mind througout the following steps, that the aim is to build a positive supply side relation between output Y and prices P (inflation).

If we want to model short run business cycles and the impact of shocks and policies, we need to introduce **Nominal frictions**. Without frictions, the model economy's nominal variables adjust immediately to changes for example in aggregate demand, leaving the real quantities untouched. Typically frictions are modelled by preventing either nominal prices or wages from being immediately updated after changes occur.

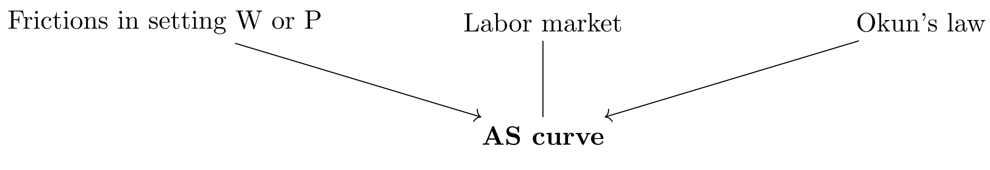

Figure 10: Construction of the short run AS curve

Frictions and order of events

Here we assume **frictions in wage setting**: The wages can only be re-negotiated with a delay. Prices, on the other hand, are fully flexible, as they are set with all of the relevant information at hand. This has a nice interpretation in terms of the order in which the variables are defined.

In the beginning of a new period, the agents form the expectation regarding the key variables with uncertain values, namely the expected price level P_t^e[2]. Then the nominal wage W is negotiated based on the expected prices. Then the exogenous shocks impact the economy. Finally, the firms set their prices P_t optimally, with the information about the nominal wages and shocks.

$$\longrightarrow \boxed{t} \longrightarrow \boxed{\text{Set } P_t^e} \longrightarrow \boxed{W_t} \longrightarrow \boxed{\text{Shocks}} \longrightarrow \boxed{P_t} \longrightarrow \boxed{t+1} \longrightarrow$$

Figure 11: An example of timing the setting of W and P, assuming that the nominal wages can only be adjusted with a lag of one period.

Production function. We assume, that capital is predetermined in the time spans we are interested in, and firms can only alter the number of workers on payroll, L.[3]

$$Y = f(L, K^*) = L \qquad \text{Production function}$$

Wage is the price of using labor in production, and it influences directly the unit cost of firms. Wages are set in negotiations, based on the bargaining power of the labor unions, γ, which depends on the unemployment level.

$$W = (1+\gamma)\frac{s_L P^e Y}{L} \qquad \text{Wage per unit of labor} \tag{6}$$

$$\frac{WL}{Y} \qquad \text{Unit cost of production}$$

The workers are not interested in the nominal wages W, because what actually matters for their purchasing power are the real wages $w = \frac{W}{P}$. The problem is that the nominal wage W_t has to be negotiated before the prices P_t are known.

Because wages can't be re-negotiated daily, the expected price level P^e is used instead of the current price level P. This is one of the important factors in constructing the non-vertical AS-curve - the price of the factor of production (wage) doesn't react to changes in the actual prices of outputs immediately. The "staggerdness" of the wage negotiations depends on the speed by which P^e converges towards the actual prices.

The fraction s_L tells, how much of the total income, $PY = WL + rK$, goes to labor (WL) instead of capital, based on the production function.

Prices. The firm sets a price (when possible), based on the costs of production and the firm's market power, θ.

$$P = (1+\theta)\frac{WL}{Y} \qquad \text{Unit price of output} \tag{7}$$

[2] How are these expectations formed? Typically we assume either adaptive or rational expectation formation. The simplest version of adaptive expectations is $\pi^e = \pi_{t-1}$. A general specification is $\pi_{t+1}^e = (1-a)\pi_t^e + a\pi_t$, where the speed by which the expectation adapt to changes in realized inflation is given by the parameter $a \in [0,1]$. We encounter an example of this in Exercise 5. **Rational expectations** mean, that the agents' expectations adjust immediately to changes in inflation, $\pi_t^e = \pi_t$, and they also have a full understanding of the functioning of the model they are inside of.

[3] Using for example the Cobb-Douglas production function, $Y = K^\alpha L^{1-\alpha}$, we can set the predetermined capital $K^* = 1$ and then $(1-\alpha) = 1$ to get Y=L and simplify the math, without loss of generality.

Inserting Equation 6 into 7 gives

$$P = (1+\theta)(1+\gamma)s_L P^e \qquad \text{Unit price of output} \tag{8}$$

Notice, that here the firms setting the prices P take into account the nominal wage W, negotiated earlier. Hence, we now have inserted the staggered wage setting into the equations.

To approximate the relation in Eq 8 using rates of change, we need the following **technical steps**

$$\begin{aligned}
A_t &= B_t C_t \\
&\Longleftrightarrow \\
\frac{A_{t+1} - A_t}{A_t} &= \frac{B_{t+1}C_{t+1} - B_t C_t}{B_t C_t} && \mid \quad A_{t+1} - A_t =: \Delta A_t \\
\frac{\Delta A_t}{A_t} &\approx \frac{\Delta B_t}{B_t} + \frac{\Delta C_t}{C_t}
\end{aligned}$$

and

$$\begin{aligned}
\frac{(1+\theta_{t+1}) - (1+\theta_t)}{1+\theta_t} &= \frac{1+\theta_{t+1} - 1 - \theta_t}{1+\theta_t} \\
&= \frac{\theta_{t+1} - \theta_t}{1+\theta_t} \\
&= \frac{\Delta\theta_t}{1+\theta_t}
\end{aligned}$$

Continuing from Equation 8 (dropping the time indices for notational simplicity), the change in prices is approximated by (assuming that the market power of firms θ and the labor share s_L are constant in the short run)

$$\frac{\Delta P}{P} \approx \underbrace{\frac{\Delta\theta}{1+\theta}}_{=0} + \frac{\Delta\gamma}{1+\gamma} + \underbrace{\frac{\Delta s_L}{s_L}}_{=0} + \frac{\Delta P^e}{P^e} \qquad \mid \quad \frac{\Delta P}{P} =: \pi$$

$$\pi \approx \frac{\Delta\gamma}{1+\gamma} + \pi^e \tag{9}$$

We now have a link between inflation and labor union's bargaining power. Remember, that the aim was to connect inflation with output Y. We need two additional steps for that.

First we make an additional assumptions, that the labor union's bargaining power is a decreasing function of the unemployment, $\frac{\Delta\gamma}{1+\gamma} = -aU_{gap}$[4]. Inserting this to Eq 9 gives ()

$$\pi = -aU_{gap} + \pi^e \tag{10}$$

[4]This makes sense, as when the economy is close to full employment, the firms have hard time finding new workers, raising the labor unions' power. On downturn the situation is opposite, there are many unemployed people who are trying to find work, even with smaller salaries.

Now we have a connection between inflation and unemployment. The last step is to replace the unemployment with output. Remember, that we started with the firm's production function with labor inputs. Quite naturally, the output of the firm increases when the labor inputs (the employment level) are increased. This relation is expressed by **Okun's law** on the aggregate level, binding U_{gap} and Y_{gap} together

$$U_{gap} = -hY_{gap}$$

Using this in Equation 10 we get the **Expectations augmented Phillips curve**

$$\pi = bY_{gap} + \pi^e$$

where $b = (-a) * (-h)$. Replacing the inflation expectation π^e with the "underlying" inflation $\tilde{\pi}$ finally gets us to the AS-curve, which describes the supply side relationship between output gap and inflation

$$\pi = \tilde{\pi} + bY_{gap} + s \qquad \textbf{AS-curve} \qquad (11)$$

where s is a mean zero cost-push shock, which increases the production costs exogenously. Both $\tilde{\pi}$ and s shift the AS-curve, whereas the parameter b gives the slope (note, that b is a different parameter than the marginal propensity to consume in demand side).

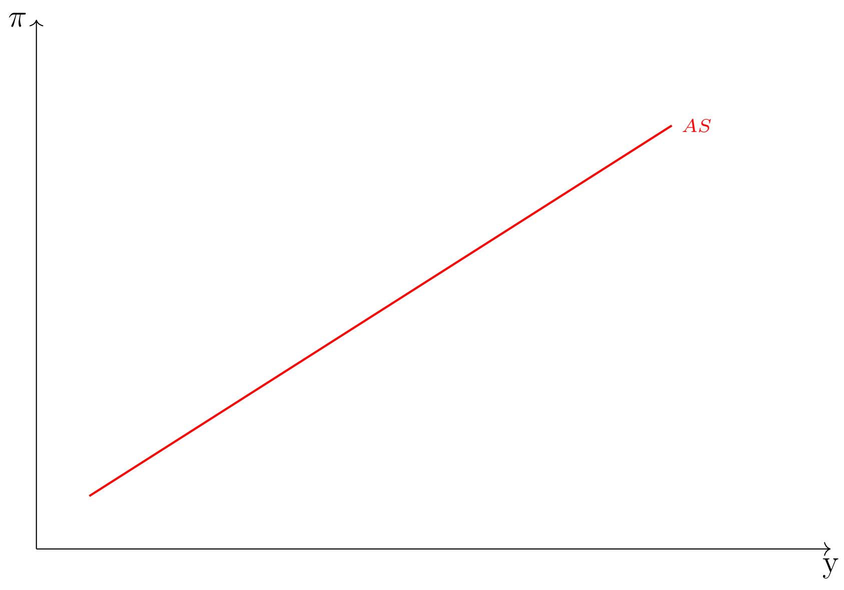

Figure 12: AS curve