Risk aversion

Oliver Snellman
oliver.snellman@gmail.fi

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What is risk aversion?

We typically want to be compensated for taking risks. Classic example: Consider a bet involving a coin toss, where heads leads to winning 10 euros and tails leads to gaining 0 euros. The average payoff, or the expected value, of the bet is $\mathbb{E}$ [bet]$= 0.5 * 0 + 0.5 * 10 = 5$ euros. How much would you be willing to pay for the possibility to participate in such a bet?

Most people would accept this bet only, if the participation cost was less than 5 euros, meaning that they require a risk premium. In other words, the utility that they get from the bet with expected value of 5 euros is lower than the utility that they would get from receiving 5 euros for certain.

Most people would also want to take this bet if the cost of participating was small enough, even if it was non-zero. Would you rather receive 1 cent for certain or get the opportunity to win 10e with 50 % chance? Therefore there must be a sweet spot c between 0e and 5e, called Certainty Equivalent, where the certain outcome of size c and the bet produce the same utility.

Certainty Equivalent (CE) is the lowest amount of money, that you would be willing to settle instead of the bet. Let there be two options, a certain outcome and a gamble. Your CE for that gamble is the value of the certain outcome, which would make you indifferent between the options. Your risk premium is the difference between the expected value of the bet and the CE.

The expected utility of a bet is calculated as the combination of the utilities of both (or all) options, weighted by the probability of the options occurring, as can be seen in Figure 1. The CE of a bet can be found by moving horizontally on the same utility level until hitting the utility function.

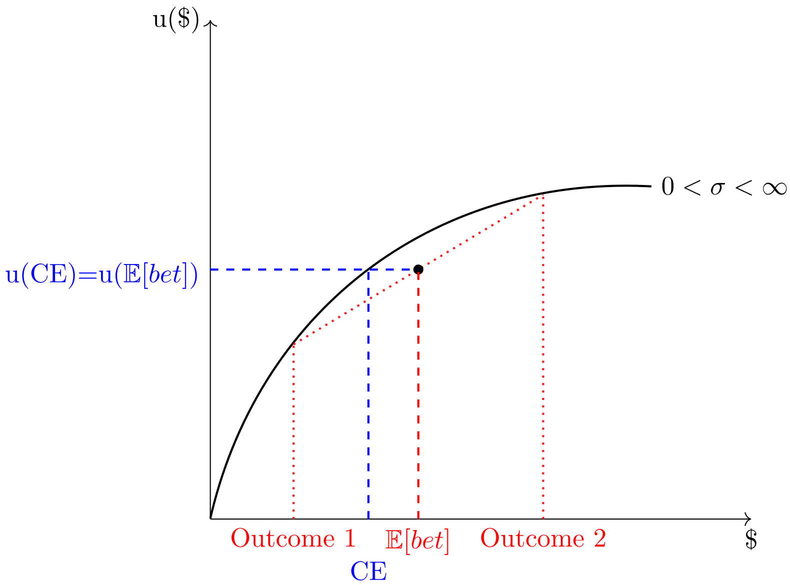

Figure 1: Using a concave utility function to express risk aversion.

Mindset of a risk averse household

A typical household's problem is to allocate expected future earnings into consumption at different stages of life. The household makes decisions based on its utility function, and executes the allocation by saving or borrowing. The household wants to enjoy a certain standard of living throughout the life. Because there is uncertainty about future income, the household can end up in a better or worse situation.

A risk averse household is more worried about the possibility of a bad situation, whereas a risk loving household would be more intrigued by the upside. By saving, the household can guarantee a certain level of consumption for the future, regardless of the future income. A risk averse household wants to save more to guarantee a higher living standard for the future, whereas a risk neutral (or risk loving) household is more willing to gamble.[1]

Expressing risk preferences mathematically.

It is typical to use the isoelastic utility function to model behavior under risk, because it has nice properties, namely Constant Relative Risk Aversion (CRRA).

$$u(c) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \qquad \text{Isoelastic function} \tag{1}$$

We can use the isoelastic function to accommodate many different risk preferences. The curvature of the utility function tells us, how much more does the household appreciate a certain outcome c over the mixture of possibilities with the same expected value $\mathbb{E}[\text{gamble}] = c$. The curvature, and thereby the risk preference of the household, can be controlled by varying the sigma parameter of the isoelastic utility function.

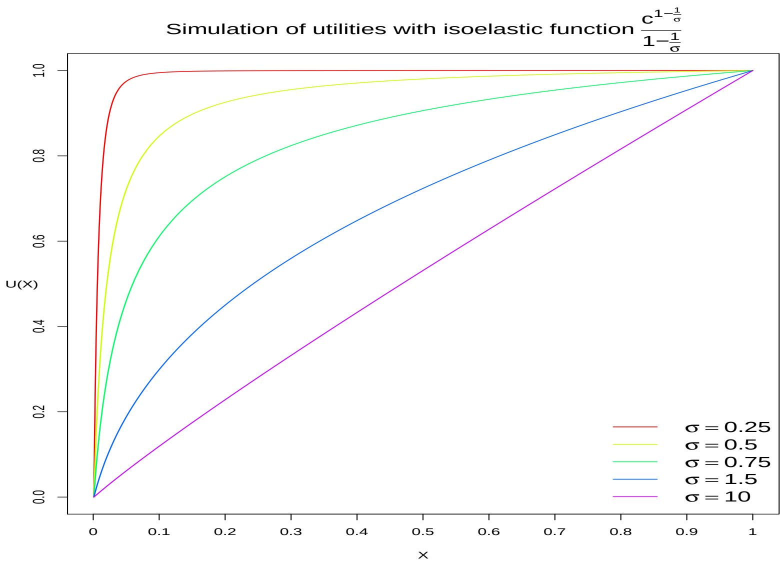

Figure 2: Mapping of utility with the isoelastic function with different σs

[1] We tend to label some behavior as risk aversion (or even some kind of cognitive biases), if they deviate from something that our models would deem as being risk neutral behavior. There is an interesting, albeit quite technical argument, that sometimes the models might not produce correct estimates for neutral behavior, due to an underlying mathematical assumption called ergodicity. This assumption might not be warranted in all cases, and letting it go might produce different neutral behavior in the models, and explain away some of the postulated biases.

The mappings of the isoelastic utility function from consumption to utility are illustrated in Figure 2, with several values of σ. The actual utility levels have been normalized to fit in the same figure. Remember that we are only interested in ordinal utility, not cardinal, so the actual levels of utility are not important (they can just as well be negative). The table below gives the extremes of isoelastic function.[2]

Perfect risk neutrality	$\sigma = \infty$	u(c) = $\mathbb{E}$ [u(gamble)]
Extreme risk aversion	$\sigma = 0$	u(c) > $\mathbb{E}$ [u(gamble)]

The size of the CE depends on the risk preferences, described by σ. Figure 3 illustrates how the CE is determined for three agents with different risk preferences (σs), for the same coin toss described above. This means, that the more risk averse the agent is, the smaller is the certain outcome she's willing to settle down, instead of taking the bet.

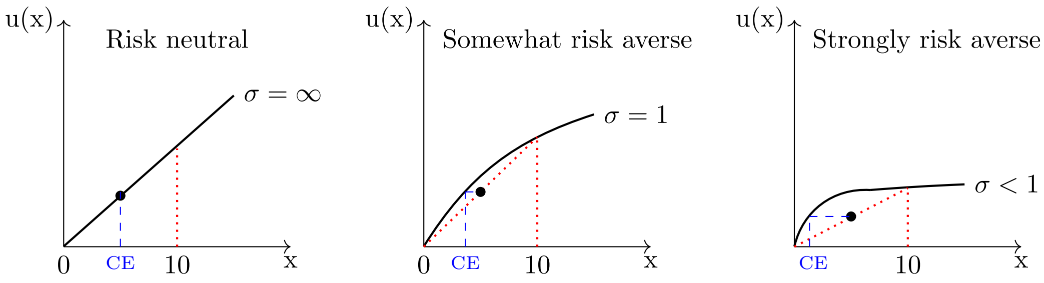

Figure 3: CE's with different σs

Some of you may notice an oddity in the highly risk averse case in the rightmost panel of Figure 3. There seems to be a flat area after the initial increase of the function, where the utility function becomes a straight line. Does the agent become risk neutral with these preferences, after the initial increase in utility? No, as the curve doesn't really become flat. Zooming into the curve (anywhere) reveals the same curvature, when the scale of the y-axis is adjusted properly.

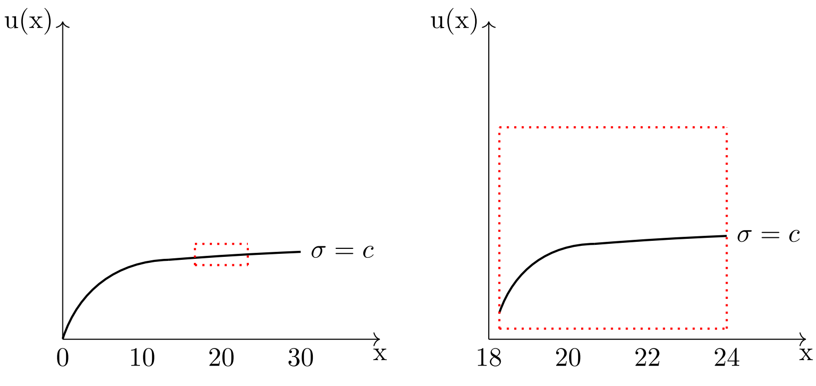

Figure 4: Zooming in shows the curvature

[2]It is also technically possible but uncommon to model risk loving behavior with this function. Then it must hold that $\sigma = \frac{1}{X}$, $s.t. X \in \mathbb{Z}$, to guarantee that the exponent of the input c is an integer. The inputs also have to be transformed.