AD-AS model

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January 31, 2021

Abstract

I first derive the AD curve using the intuitive path through Keynes Cross model to the IS curve and then combine it with the LM/TR curve to build intuition for how the demand side of the economy functions. Then I derive the microfounded New Keynesian AD relation from household's problem. The AS curve is derived using staggered wage setting.

Introduction

The AD-AS model describes the functioning of the macro-economy. It presents the dynamics of the demand (AD) and supply (AS) sides of the economy, using graphical analysis as a tool for building qualitative intuitions. The graph combines the key real and nominal quantities, plotting output on the x-axis and inflation on the y-axis. The equilibrium of the economy is determined by the intersection of the AD and AS curves. More specifically, the natural output level, around which the economy fluctuates with business cycles, is determined by the long run features of the supply side (not visible in the graph below). The impact of fiscal and monetary policy, on the other hand, are inbuilt in the behavior of the AD curve.

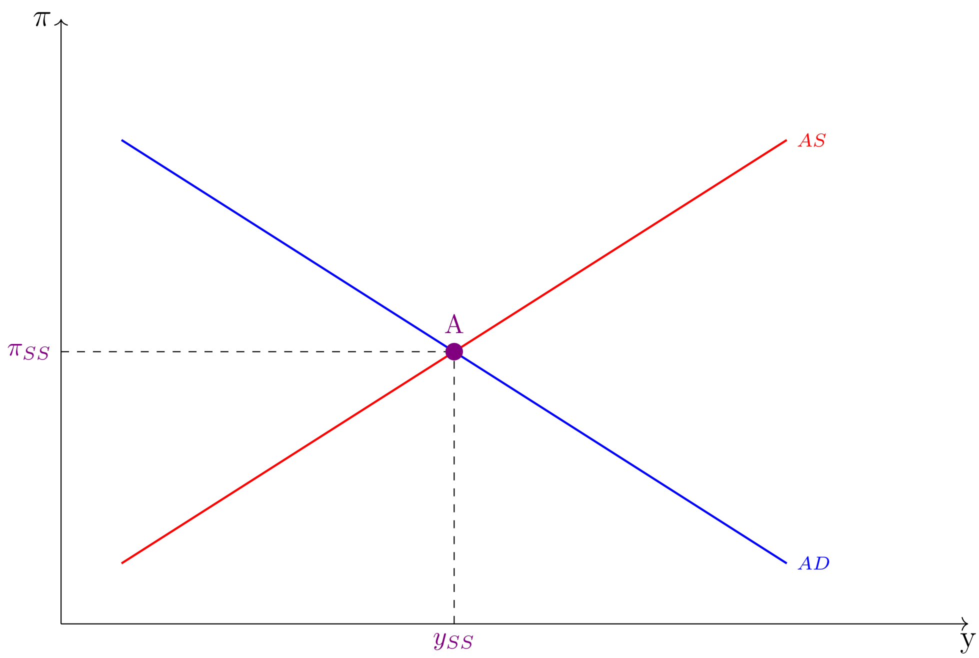

Figure 1: AD-AS model. Lower index SS denoting for steady state values.

Keynes Cross

Let the Desired Demand DD(Y) take the following form

$$DD(Y) = C(Y) + I_p + G + NX(Y) \tag{1}$$

$$\begin{aligned} &\text{with} \\ C(Y) &= a + b(Y - T) \\ I_p &= c - di \\ NX(Y) &= \underbrace{fY^f}_{Exports} - \underbrace{gY}_{Imports} \end{aligned}$$

The economy is always on the 45° line, where the aggregate supply equals aggregate (desired) demand.

If the output Y_{lower} is below the Desired Demand, $DD(Y_{lower}) > Y_{lower}$, the firms notice the excess demand, will hire more workers and increase the production until the next period. Likewise, if the output Y_{higher} is above the Desired Demand, $DD(Y_{higher}) < Y_{higher}$, the firms can't sell all the goods they produced (unplanned inventory investments), and they reduce the production for the next period.

There is a unique equilibrium where $DD(Y) = Y$, which is reached by altering the employment level.

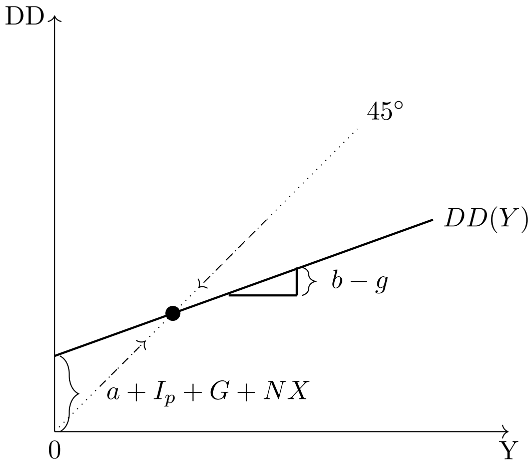

Figure 2: Keynes cross

Parameter b is the fraction of additional unit of disposable income spent on consumption, and g is the fraction of consumption that is spent on imports. The slope of the DD(Y) -curve is therefore b-g, the marginal propensity to consume domestically produced output.

IS-curve

Now we introduce the interest rate, which impact the goods market equilibrium negatively through planned investments $I_p = c - di = 25 - 100i$.

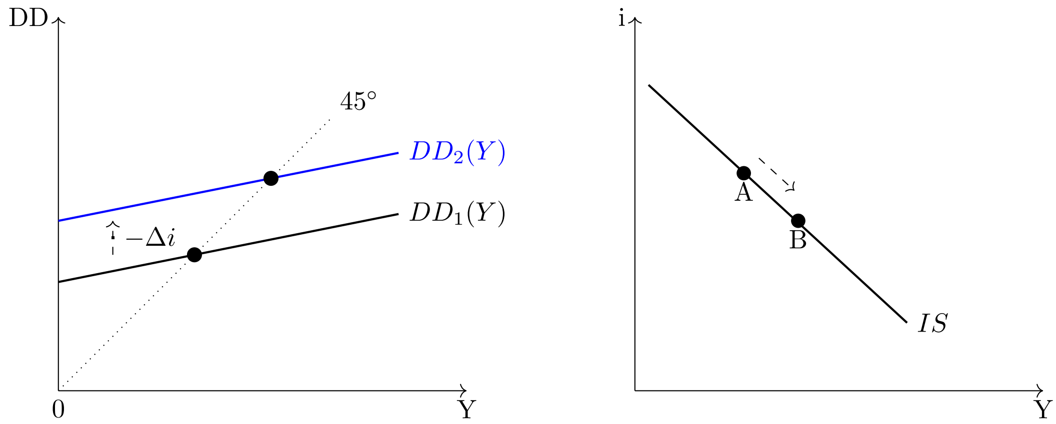

Figure 3: Deriving the IS curve

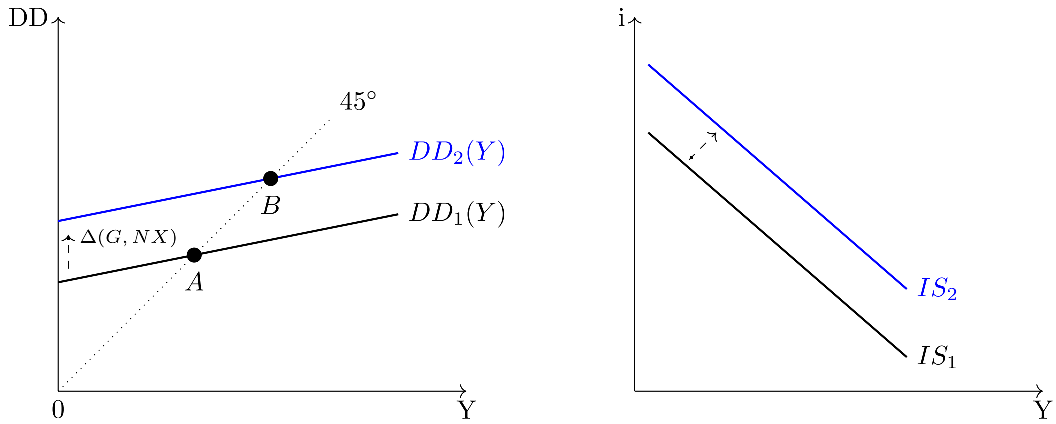

Figure 4: Shifting the IS curve

G ↑	IS ↑
T ↓	IS ↑
NX ↑	IS ↑

Table 1: Shifting the IS-curve

The slope of the IS curve depends on the sensitivity of investments to interest rate, d, which tells how much the DD(Y) -curve shifts when the interest rate changes. The slope of the IS curve also depends on the slope of the DD(Y) -curve, b-g, - how much the impact of Δi is multiplied.

The marginal propensity of domestic consumption, b-g, also impacts on how much the IS-curve shifts due to Demand shocks (other than i); Larger the b-g, larger the shift.

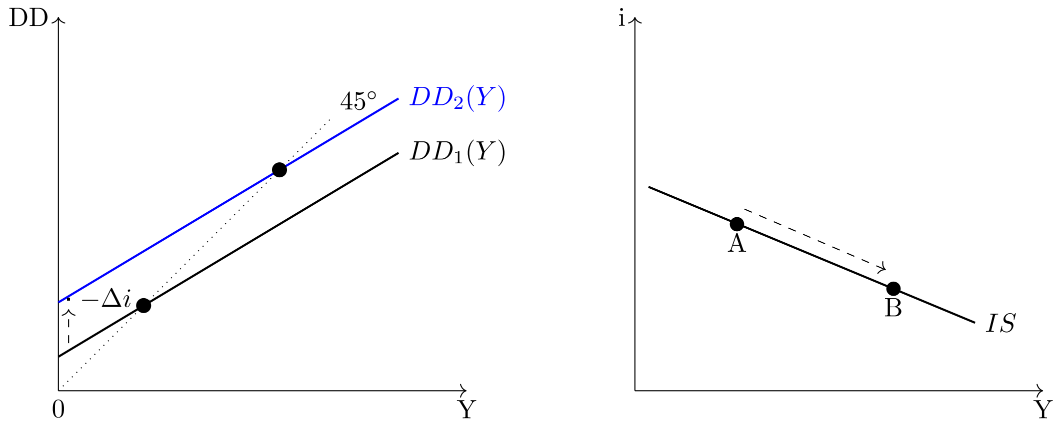

Figure 5: Higher b flattens the slope of the IS curve

LM/TR-curve

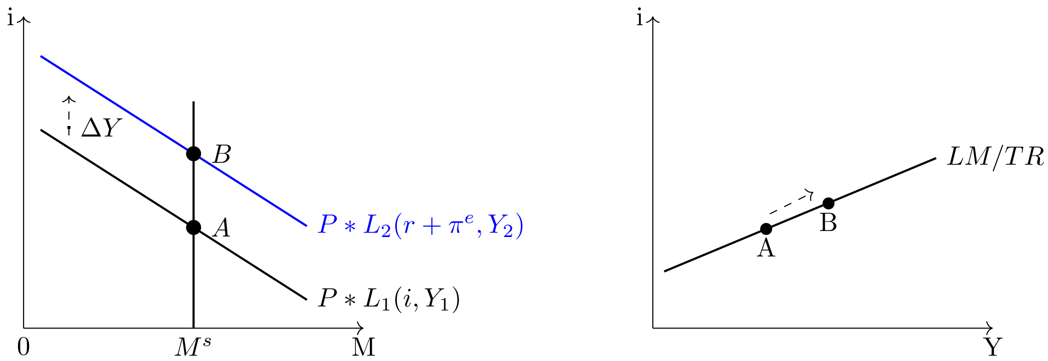

Figure 6: Deriving the LM curve

$M^s \uparrow$	LM/TR $\downarrow$
P $\uparrow$	LM/TR $\uparrow$
$\pi^e \uparrow$	LM $\downarrow$
$\pi^* \uparrow$	TR $\downarrow$

Table 2: Shifting the LM-curve

An increase in the real money balances $\frac{M}{P}$ raises the LM-curve. Increase in inflation expectation π^e raises the nominal interest rate via Fisherian relation $i = r + \pi^e$ (given a real interest rate), which in turn shifts the LM-curve down.

The TR-curve has the same properties as the LM-curve, but the idea is more modern: Instead of influencing the interest rate through controlling money supply, the CB sets the interest rate directly and uses it to influence the inflation (and money supply indirectly).

$$i = \bar{i} + \alpha(\pi - \pi^*) + \beta Y_{gap} \qquad | \quad \text{Taylor rule}$$

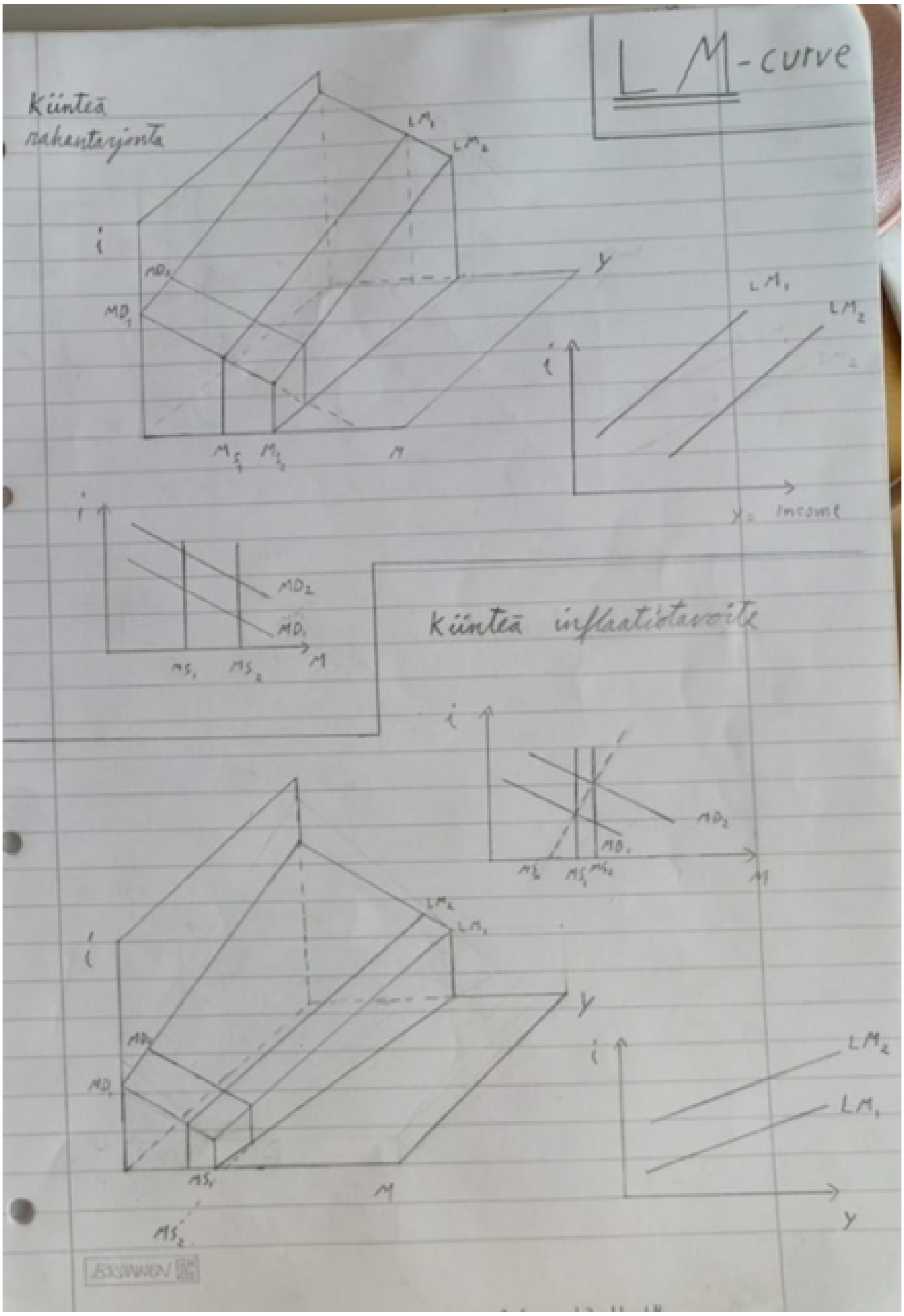

Figure 7: 3D interpretation of the LM-curve

AD-curve

Increase in inflation (increase in prices) reduces the real money balances $\frac{M}{P}$ (or deviations from the inflation target cause CB to raise the interest rate) and shifts the LM/TR -curve up. The new intersection with the IS-curve occurs at higher interest rate and reduced output. The AD-curve maps the (π, Y) points which are consistent with both goods- and money market equilibria.

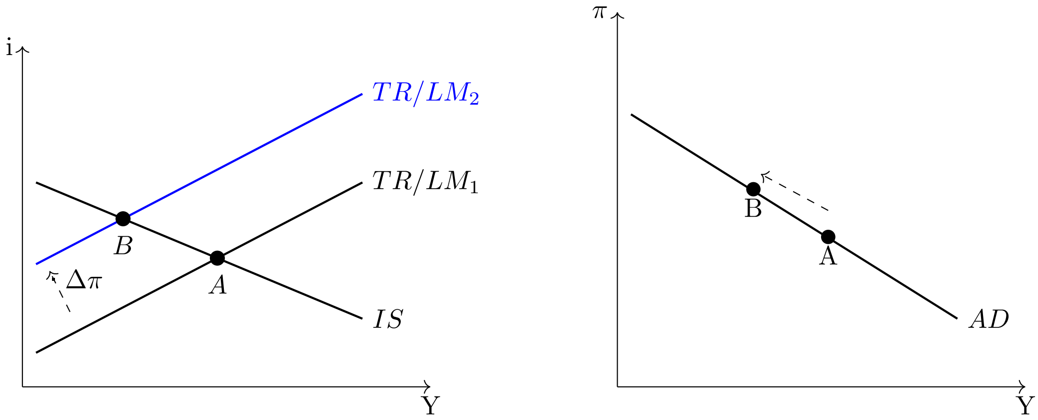

Figure 8: Deriving the AD curve

The AD-curve shifts up, when either the IS-curve shifts up, or the LM-curve shifts down (their intersection moves to the right).

G $\uparrow$	IS $\uparrow$	AD $\uparrow$
T $\downarrow$	IS $\uparrow$	AD $\uparrow$
NX $\uparrow$	IS $\uparrow$	AD $\uparrow$
M^s $\uparrow$	LM $\downarrow$	AD $\uparrow$
P $\downarrow$	LM $\downarrow$	AD $\uparrow$
π^e $\uparrow$	LM $\downarrow$	AD $\uparrow$

Table 3: Shifting the AD-curve

New Keynesian AD-curve

Now that we have established the intuitive understanding of how the demand side of the economy functions, let's derive the same relation from microfoundations. This means replacing the simple postulated consumption equation in the Keynes Cross, $C = a + bY$, with the consumption rule obtained from solving the optimization problem of the household.

To get the New Keynesian AD-curve, we need to log-linearize and combine the consumption Euler-equation and the aggregate resource constraint.

Let's start by solving the Euler from the two period household's problem

$$\max_{C,L,C,L} \quad \underbrace{u(C)}_{\text{Utility from C}} \quad - \quad \underbrace{v(L)}_{\text{Disutility from L}} \quad +\beta \quad \underbrace{\left[u(C) - v(L)\right]}_{\text{utility of next period}} \qquad \text{Objective function}$$

s.t.

$$\underbrace{PC + \frac{PC}{1+i}}_{\text{Total (discounted) lifetime spending}} \quad = \quad \underbrace{WL + \frac{WL}{1+i} + T}_{\text{Total (discounted) lifetime wealth}} \qquad \text{Lifetime constraint}$$

where the bar above the variables indicate them being the second period values; $C =: C_t$ and $C =: C_{t+1}$.

Solve using the Lagrangian

$$\mathcal{L} = \max_{C,L,C,L} \; u(C) - v(L) + \beta\left[u(C) - v(L)\right] + \lambda\left[WL + \frac{WL}{1+i} + T - PC - \frac{PC}{1+i}\right]$$

As we are only interested in the Euler equation, we need just the optimality conditions for the consumption choices. I'll leave the working decisions and the constraint out of the FOC's for the sake of brevity.

FOC:

$$\frac{\partial \mathcal{L}}{\partial C} = u'(C) - \lambda P = 0 \qquad \iff \qquad \lambda = \frac{u'(C)}{P} \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial C} = \beta u'(C) - \lambda \frac{P}{1+i} = 0 \qquad \iff \qquad \lambda = \beta(1+i)\frac{u'(C)}{P} \tag{3}$$

Combine Equations 2 and 3 to find the consumption Euler equation

$$\lambda = \lambda$$

$$\frac{u'(C)}{P} = \beta(1+i)\frac{u'(C)}{P}$$

$$\frac{u'(C)}{\beta u'(C)} = (1+i)\frac{P}{P} \qquad \text{Generic Euler}$$

To proceed, we need to postulate a functional form for the utility-function of the households. Let's go with the isoelastic-utility function.

$$u(C) = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{4}$$

$$u'(C) = (1-\frac{1}{\sigma})\frac{C^{1-\frac{1}{\sigma}-1}}{1-\frac{1}{\sigma}} = C^{-\frac{1}{\sigma}} \qquad \text{Similarly for } C$$

where σ is the intertemporal elasticity of substitution of consumption.
Plug these derivatives for C and C into the Euler

$$\frac{C^{-\frac{1}{\sigma}}}{\beta C^{-\frac{1}{\sigma}}} = (1+i)\frac{P}{P} \qquad \text{Euler for isoelastic utility}$$

Log-linearize the LHS (lower index SS denoting for the steady state value)

$$\begin{aligned}
log\left(\frac{C^{-\frac{1}{\sigma}}}{\beta C^{-\frac{1}{\sigma}}}\right) - log\left(\frac{C_{SS}^{-\frac{1}{\sigma}}}{\beta C_{SS}^{-\frac{1}{\sigma}}}\right) &= log(C^{-\frac{1}{\sigma}}) - log(\beta) - log(C^{-\frac{1}{\sigma}}) - log(C_{SS}^{-\frac{1}{\sigma}}) + log(\beta) + log(C_{SS}^{-\frac{1}{\sigma}}) \\
&= -\frac{1}{\sigma}log(C) + \frac{1}{\sigma}log(C) + \frac{1}{\sigma}log(C_{SS}) - \frac{1}{\sigma}log(C_{SS}) \\
&= \frac{1}{\sigma}\left[log(C) - log(C_{SS}) - log(C) + log(C_{SS})\right] \\
&= \frac{1}{\sigma}\left[\underbrace{log(C) - log(C_{SS})}_{=:c} - (\underbrace{log(C) - log(C_{SS})}_{=:c})\right] \\
&= \frac{1}{\sigma}[c - c]
\end{aligned}$$

and the RHS

$$\begin{aligned}
log\left((1+i)\frac{P}{P}\right) - log\left((1+i_{SS})\frac{P_{SS}}{P_{SS}}\right) =& log(1+i) + log(P) - log(P) \\
& - log(1+i_{SS}) - log(P_{SS}) + log(P_{SS}) \\
=& \underbrace{log(1+i) - log(1+i_{SS})}_{=:\tilde{i}} + \underbrace{log(P) - log(P_{SS})}_{=:p} \\
& - \underbrace{(log(P) - log(P_{SS}))}_{=:p} \\
=& \tilde{i} + p - p
\end{aligned}$$

Notice, that p here is the inflation π at time t and p can be thought of as the inflation expectation.

Put the LHS and RHS back together to get the log-linearized Euler equation

$$\frac{1}{\sigma}[c - c] = \tilde{i} - p + p$$
$$c - c = \sigma[\tilde{i} - (p - p)] \qquad \text{Log-linearized Euler} \qquad (5)$$

Next we turn to the simplified Aggregate resource constraint, which expresses that all income is used either for private consumption or government spending.

$$Y = C + G$$
$$Y = C + G$$

Log linearizing the sum on the RHS needs an additional step. Also remember, that $\frac{b - b_{SS}}{b_{SS}} \approx log(b) - log(b_{SS})$ for small deviation $b - b_{SS}$.

$$Y - Y_{SS} = C + G - (C_{SS} + G_{SS})$$
$$Y - Y_{SS} = C - C_{SS} + G - G_{SS} \qquad || \frac{1}{Y_{SS}}$$
$$\frac{Y - Y_{SS}}{Y_{SS}} = \frac{C - C_{SS}}{Y_{SS}} + \frac{G - G_{SS}}{Y_{SS}}$$

$$log(Y) - log(Y_{SS}) \approx \underbrace{\frac{C_{SS}}{C_{SS}}}_{=1} \frac{(C - C_{SS})}{Y_{SS}} + \underbrace{\frac{G_{SS}}{G_{SS}}}_{=1} \frac{(G - G_{SS})}{Y_{SS}}$$

$$y \approx \underbrace{\frac{C_{SS}}{Y_{SS}}}_{=:s_c} \underbrace{\frac{(C - C_{SS})}{C_{SS}}}_{\approx log(C) - log(C_{SS})} + \underbrace{\frac{G_{SS}}{Y_{SS}}}_{=:s_g} \underbrace{\frac{(G - G_{SS})}{G_{SS}}}_{\approx log(G) - log(G_{SS})} \qquad || \log(C) - \log(C_{SS}) =: \tilde{c}$$

$$y \approx \underbrace{s_c \tilde{c}}_{=:c} + \underbrace{s_g \tilde{g}}_{=:g}$$

$$y = c + g$$

In the last line we have defined c and g as the percentage deviations of consumption and government spending (approximated with logarithmic differences) from their steady state values, $\tilde{c}$ and $\tilde{g}$, weighted by their share of the output, s_c and s_g.

Similarly we have

$$y = c + g$$

Solve for c and c

$$c = y - g$$
$$c = y - g$$

Plug these into the log-linearized Euler, Equation 5, to finally arrive at the log-linearized New-Keynesian AD equation

$$\begin{aligned} y - g - y - g &= \sigma[\tilde{i} - (p - p)] \\ y &= y + (g - g) - \sigma[i - (p - p)] && \textbf{Aggregate Demand} \end{aligned}$$

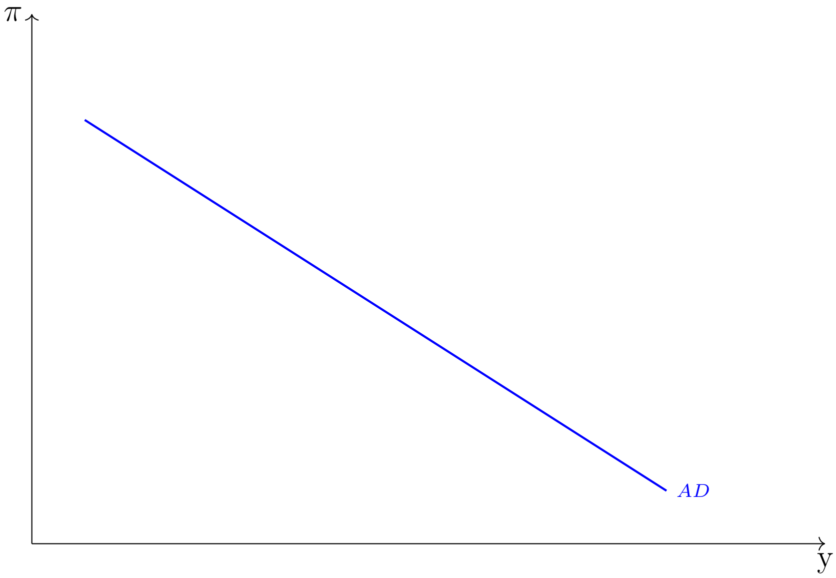

Figure 9: AD curve

AS-curve

We start deriving the AS-curve from the fundamentals of production. The frictions are brought about by staggered wage negotiations. Other way would be to introduce price frictions.

Production function. We assume, that capital is predetermined in the time spans we are interested in, and firms can only alter the number of workers on payroll, L.[1]

$$Y = f(L, K^*) = L \qquad \text{Production function}$$

Wage is the price of using labor in production, and it influences directly the unit cost of firms. Wages are set in negotiations, based on the bargaining power of the labor unions, γ, which depends on the unemployment level.

$$w = (1 + \gamma)\frac{s_L P^e Y}{L} \qquad \text{Wage per unit of labor} \tag{6}$$

$$\frac{wL}{Y} \qquad \text{Unit cost of production}$$

[1] Using for example the Cobb-Douglas production function, $Y = K^\alpha L^{1-\alpha}$, we can set the predetermined capital $K^* = 1$ and then $(1 - \alpha) = 1$ to get Y=L and simplify the math, without loss of generality.

Because wages can't be re-negotiated daily, the expected price level P^e is used instead of the current price level P. This is one of the important factors in constructing the non-vertical AS-curve - the price of the factor of production (wage) doesn't react to changes in the actual prices of outputs immediately. The "staggerdness" of the wage negotiations depends on the speed by which P^e converges towards the actual prices.

The fraction s_L tells, how much of the total income, $PY = wL + rK$, goes to labor (wL) instead of capital based on the production function.

Prices. The firm sets a price (when possible), based on the costs of production and the firm's market power, θ.

$$P = (1+\theta)\frac{wL}{Y} \qquad \text{Unit price of output} \qquad (7)$$

Inserting Equation 6 into 7 gives

$$P = (1+\theta)(1+\gamma)s_L P^e \qquad \text{Unit price of output} \qquad (8)$$

To approximate this relation using rates of change, remind that

$$\begin{aligned} A_t &= B_t C_t \\ &\Longleftrightarrow \\ \frac{A_{t+1} - A_t}{A_t} &= \frac{B_{t+1}C_{t+1} - B_t C_t}{B_t C_t} \qquad \Big| \quad A_{t+1} - A_t =: \Delta A_t \\ \frac{\Delta A_t}{A_t} &\approx \frac{\Delta B_t}{B_t} + \frac{\Delta C_t}{C_t} \end{aligned}$$

and that

$$\begin{aligned} \frac{(1+\theta_{t+1}) - (1+\theta_t)}{1+\theta_t} &= \frac{1+\theta_{t+1} - 1 - \theta_t}{1+\theta_t} \\ &= \frac{\theta_{t+1} - \theta_t}{1+\theta_t} \\ &= \frac{\Delta\theta_t}{1+\theta_t} \end{aligned}$$

Continuing from Equation 8 (dropping the time indices for notational simplicity), the change in prices is approximated by (assuming that the market power of firms θ and the labor share s_L are constant in the short run)

$$\frac{\Delta P}{P} \approx \underbrace{\frac{\Delta\theta}{1+\theta}}_{=0} + \frac{\Delta\gamma}{1+\gamma} + \underbrace{\frac{\Delta s_L}{s_L}}_{=0} + \frac{\Delta P^e}{P^e} \qquad \Big| \quad \frac{\Delta P}{P} =: \pi$$

$$\pi \approx \frac{\Delta\gamma}{1+\gamma} + \pi^e$$

Next we need to make some additional assumptions to get the relation between π and Y_{gap}: The labor union's bargaining power is a decreasing function of the unemployment, $\frac{\Delta\gamma}{1+\gamma} = -aU_{gap}$.

$$\pi = -aU_{gap} + \pi^e \tag{9}$$

Remind, that **Okun's law** binds U_{gap} and Y_{gap} together

$$U_{gap} = -hY_{gap}$$

Using this in Equation 9 we get the **Expectations augmented Phillips curve**

$$\pi = bY_{gap} + \pi^e$$

where $b = (-a) * (-h)$. Replacing the inflation expectation π^e with the "underlying" inflation $\tilde{\pi}$ finally gets us to the AS-curve, which describes the supply side relationship between output gap and inflation

$$\pi = \tilde{\pi} + bY_{gap} + s \qquad \textbf{AS-curve} \tag{10}$$

where s is a mean zero cost-push shock, which increases the production costs exogenously. Both $\tilde{\pi}$ and s shift the AS-curve, whereas the parameter b gives the slope (note, that b is a different parameter than the marginal propensity to consume in demand side).

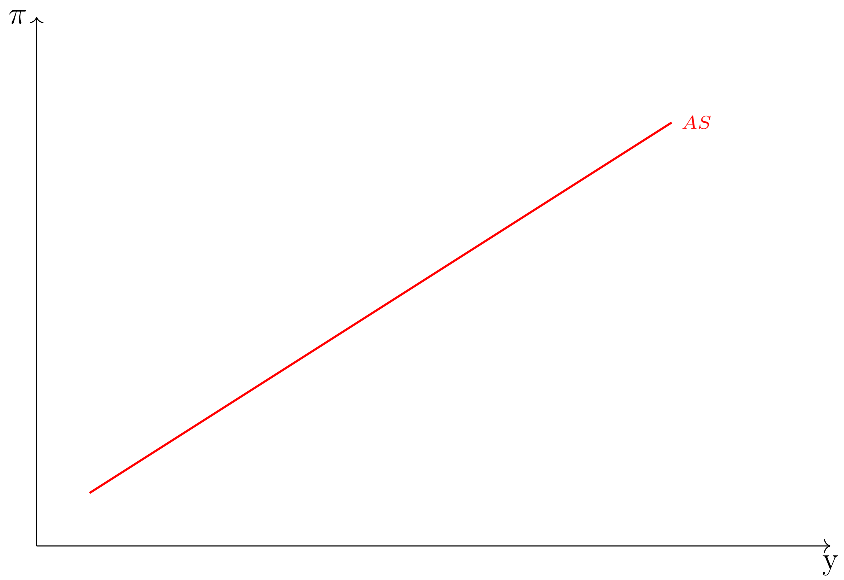

Figure 10: AS curve