Process x

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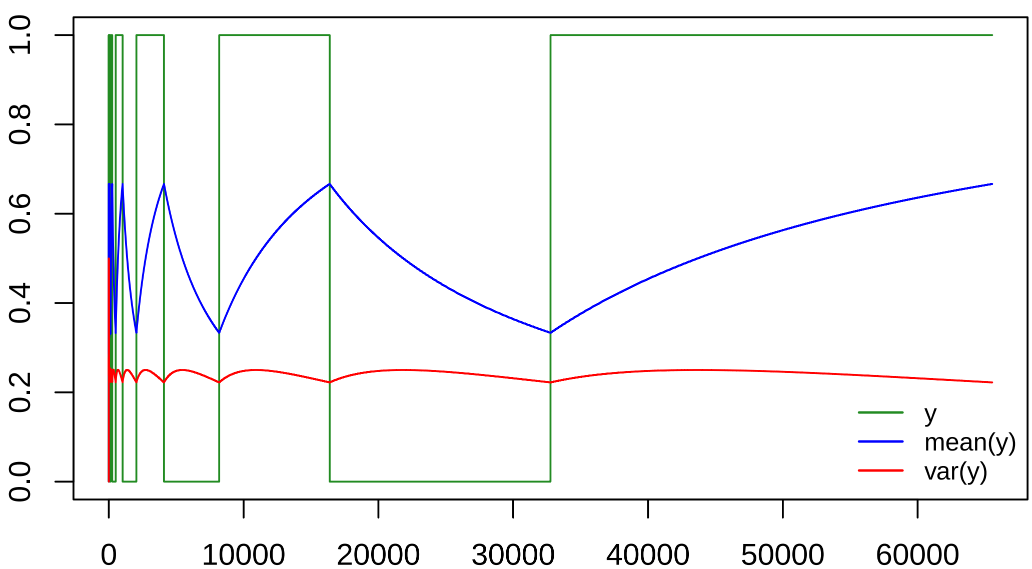
With t-distribution, when shrinking the degrees of freedom towards 1, first the finite variance seizes to exist at df ≤ 2 and then the finite mean vanishes when df ≤ 1. The latter corresponds to the Cauchy distribution, in which case the sample average over n observations doesn't converge to the real mean when $n \to \infty$, because there is no real mean. The issue stems from the scaling of the tails of the distribution, allowing ever more extreme realizations for the random variables.

But what would a process look like, if it didn't have a converging mean, but it did have a finite variance? In other words, what if the reason for the non-converging mean was not the exploding variance?

A process with finite variance but non-converging mean

1. Initialize as x=0, y=0 and i=1
2. If x==0, set x=1, x=0 otherwise
3. Assign 2^i times the value x to y
4. Set i = i + 1
5. Repeat steps 2-4 indefinitely



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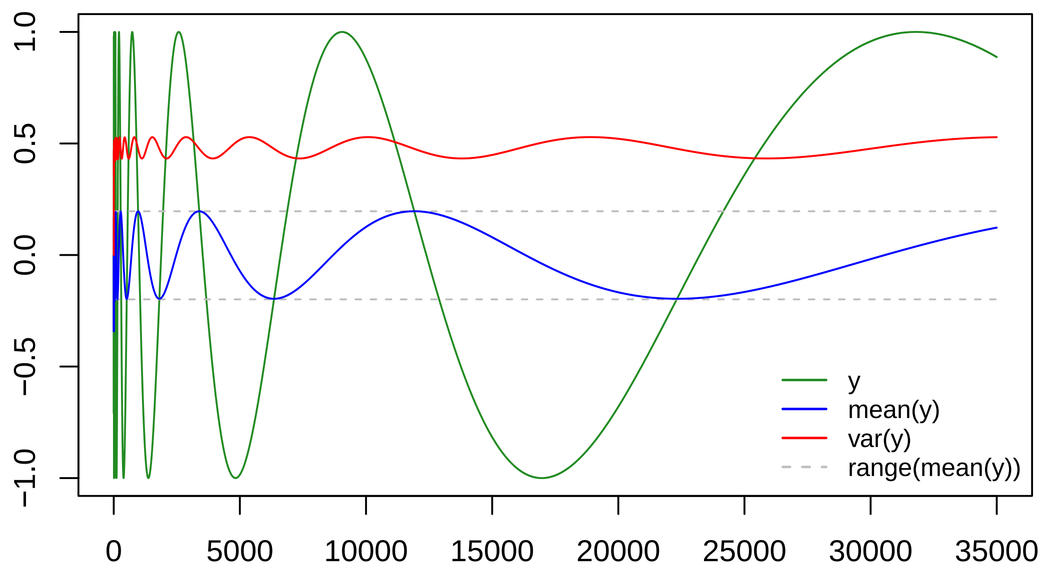
Same with a continuous function

$$y = \sin(\log(x^k))$$

where $k = 5$ only compresses the waves for visualization. The grey lines illustrate the infimum and supremum for the mean of y, which remain constant for all cutoffs of x.



It should be noted that also the variance diverges in these examples.