Tobin's Q

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Empirical observation confirms, that investments are more volatile than GDP. Tobin's Q is a theoretical attempt to reconcile this fact. It suggests, that investment depends on expected after-tax profits and is very dependent on how optimistic firms are, so it tends to flourish in boom periods and collapse in downturns, increasing its volatility with respect to the other components of GDP and GDP itself. Moreover, investment can be delayed during recessions more easily than other components of GDP, such as consumption.

More specifically, there are two options for someone considering making an investment:

a) **Invest in new capital.** This means buying capital goods[1], which increase the capital stock K and in turn increase the output Y by the marginal product of capital F_K. The benefit of doing this is the expected additional future revenue stream coming from selling the extra output at price P. Buying one unit of new capital costs P_K euros and it produces revenue of PF_K. The part of the new capital which do not depreciate, $(1-\delta)$, can be used again in the next period, yielding even more return to investment. We assume, that it takes one period to install the new capital, so that the revenue stream starts running from the second period onwards. This results in the following expected stream of revenue:

$$\overbrace{0}^{\text{New output} =: \Delta y \text{ at t}} + P\overbrace{F_K}^{\Delta y \text{ at t+1}} + P\overbrace{(1-\delta)F_K}^{\Delta y \text{ at t+2}} + P\overbrace{(1-\delta)^2 F_K}^{\Delta y \text{ at t+3}} + \dots$$
$$\Longleftrightarrow$$
$$\underbrace{0}_{\text{return on t}} + \underbrace{PF_K}_{\text{return on t+1}} + \underbrace{(1-\delta)PF_K}_{\text{return on t+2}} + \underbrace{(1-\delta)^2 PF_K}_{\text{return on t+3}} + \dots$$
$$\Longleftrightarrow$$
$$[0 + 1 + (1-\delta) + (1-\delta)^2 + \dots]PF_K$$

[1] All final goods produced by firms are either consumption goods or capital (investment) goods. Whereas consumption goods are consumed by households for utility, capital goods are installed as K to be used by firms to produce more of either type of goods in the future.

b) **Buy already existing capital.** The opportunity cost for investing in new capital is what you could get with the same money elsewhere.We don't necessarily need to define this opportunity cost further, only that it pays expected return based on the real interest rate r. For intuition, you can think of this as buying stocks in the asset market, which means buying parts of existing firms (and their capital). The amount of money we are considering here is P_K, the price of acquiring one additional unit of capital. This money would yield a return of

$$\underbrace{P_K}_{\text{return on t}} + \underbrace{(1+r)P_K}_{\text{return on t+1}} + \underbrace{(1+r)^2 P_K}_{\text{return on t+2}} + \underbrace{(1+r)^3 P_K}_{\text{return on t+3}} + \dots$$
$$\iff$$
$$[1+(1+r)+(1+r)^2+\dots]P_K$$

We can compare these two options by evaluating their relative profitability separately at each period. Summing over these comparisons results in one number, q. This number tells us whether it is profitable to invest in new capital K or not, taking into account the opportunity costs of doing so.

Let's divide the expected return to investment, a), by the expected return from the asset markets, b), separately for each period.[2]

$$q = \frac{0}{p_K} + \frac{1}{(1+r)}\frac{PF_K}{P_K} + \frac{(1-\delta)}{(1+r)^2}\frac{PF_K}{P_K} + \frac{(1-\delta)^2}{(1+r)^3}\frac{PF_K}{P_K} + \dots$$
$$\iff$$
$$q = \left[\frac{1}{1+r} + \frac{1-\delta}{(1+r)^2} + \frac{(1-\delta)^2}{(1+r)^3} + \dots\right]\frac{PF_K}{P_K}$$
$$\iff$$
$$q = \frac{1}{1+r}\left[1 + \frac{1-\delta}{1+r} + \left(\frac{1-\delta}{1+r}\right)^2 + \dots\right]\frac{PF_K}{P_K}$$

We can express the coefficient inside the large parenthesis with the help of geometric sum as

$$1 + \frac{1-\delta}{1+r} + \left(\frac{1-\delta}{1+r}\right)^2 + \dots = \sum_{i=0}^{\infty}\left(\frac{1-\delta}{1+r}\right)^i = \frac{1}{1-\left(\frac{1-\delta}{1+r}\right)}$$

[2]Another way of thinking about this is that the we re-write the option a) again but in present value, meaning that we discount the future revenue stream with the interest rate r, and then divide each term with P_K to get $\frac{PF_K}{P_K}$. This gives the revenue produced by one unit of currency invested in the new capital.

The q becomes

$$
\begin{aligned}
q &= \frac{1}{1+r}\left[\frac{1}{1-\left(\frac{1-\delta}{1+r}\right)}\right]\frac{PF_K}{P_K} \\
&\iff \\
q &= \left[\frac{1}{(1+r)-\frac{\cancel{(1+r)}(1-\delta)}{\cancel{1+r}}}\right]\frac{PF_K}{P_K} \\
&\iff \\
q &= \left[\frac{1}{1+r-1+\delta}\right]\frac{PF_K}{P_K} \\
&\iff \\
q &= \frac{PF_K}{P_K(r+\delta)}
\end{aligned}
$$

q denotes the additional value investment creates relative to the costs of that investment. These costs take into account the return that the money could have made in asset market with real interest rate r, if it wasn't tied into the new capital K (The opportunity cost). If $q > 1$ new investments increase, if $q < 1$ investments shrink (no new investment, some replacement investment not made).

There are terms in both numerator and denominator, which actually are expected values, like the future price level P^E_{t+i}.

How does q work in practice? When economy is booming, there typically also tends to be low interest rates r and high inflation P. Both increase q, incentivizing higher investments I. The q is also increased, if the depreciation rate δ decreases or the marginal productivity of capital F_K increases.