

Process x

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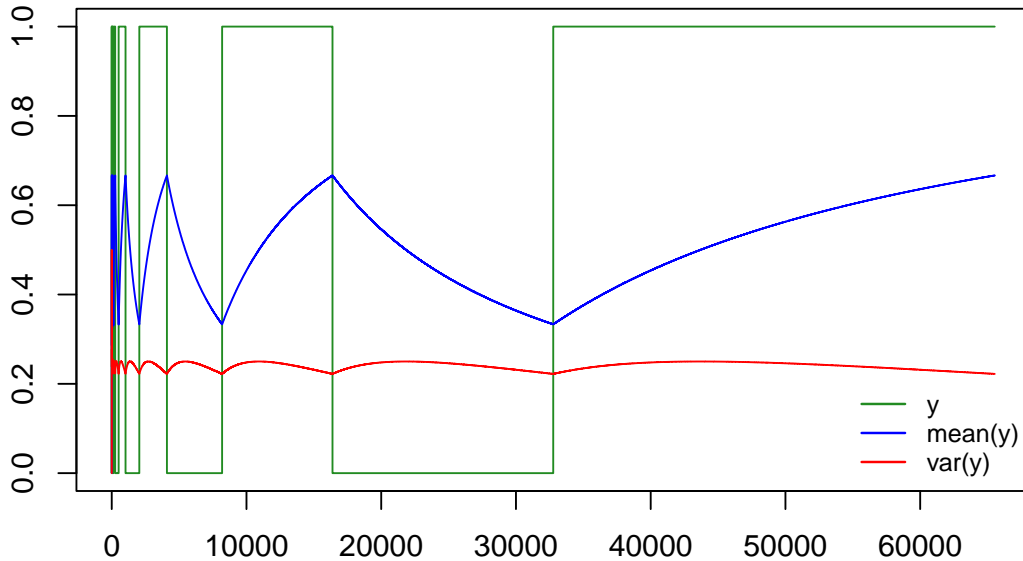
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With t-distribution, when shrinking the degrees of freedom towards 1, first the finite variance seizes to exist at $df \leq 2$ and then finite mean vanishes when $df \leq 1$. The latter case corresponds to the Cauchy distribution, in which case the sample average over n observations doesn't converge to the real mean when $n \rightarrow \infty$, because there is no real mean. The issue stems from the scaling of the tails of the distribution, allowing ever more extreme realizations.

But what would a process look like, if it didn't have a converging mean, but it did have a finite variance? In other words, the reason for the non-converging mean was not the exploding variance.

A process with finite variance but non-converging mean

1. Initialize as $x=0$, $y=0$ and $i=1$
2. If $x=1$, set $x=0$, $x=1$ otherwise
3. Assign 2^i times the value x to y
4. Set $i = i + 1$
5. Repeat steps 2-4 indefinitely

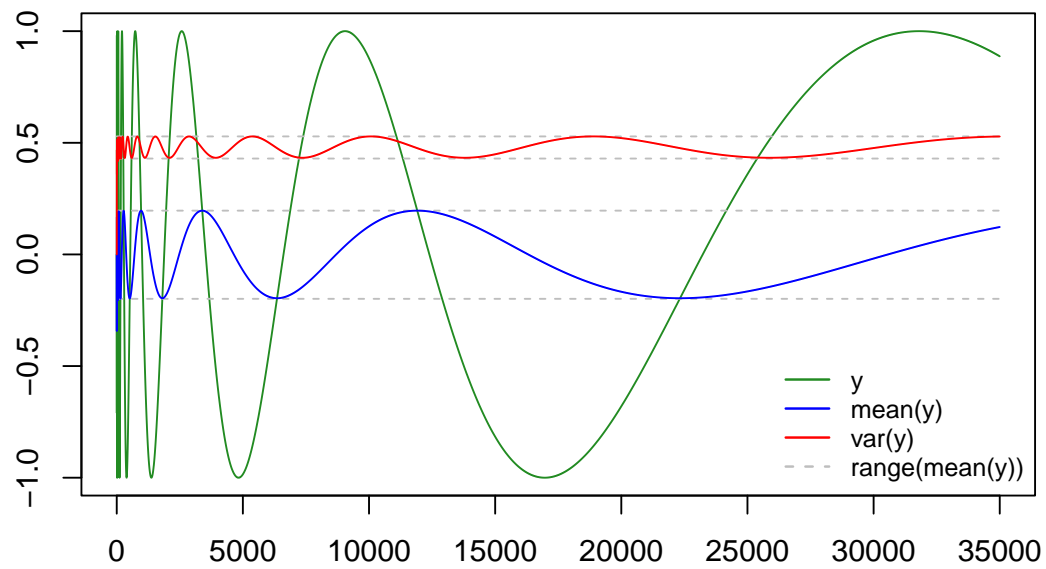


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Same with a continuous function\

$$y = \sin(\log(x^k))$$

where $k = 5$ only compresses the waves for visualization. The grey lines illustrate the infimum and supremum for the mean of y , which remain constant for all cutoffs of x .



It should be noted that also the variance diverges in these examples.