

Mechanical pencil

Dull Pencil

Orthogonal Transformations

- spatial relationships in 3D

Def - An $n \times n$ matrix A is an orthogonal transformation I.F.F (if and only if)

- It has n mutually perpendicular rows or columns with unit length

- \perp rows must be independent (can't be multiples of each other)

ex $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \rightarrow$ linearly dependent

$\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix} \rightarrow$ independent but not \perp

- to be perpendicular, the dot product must be 0

dot product: $x \cdot y = \sum_{i=1}^n x_i y_i$

$$x \cdot y = 0 \iff x \perp y \text{ (perp.)}$$

- rows/columns must have unit length

$$\hookrightarrow \|x\| = \sqrt{\sum_i x_i^2} = \sqrt{x \cdot x}$$

- The rows or columns of A form an orthonormal basis of \mathbb{R}^n

- basic for space - set of vectors that can combine to create any vector in a space

- basically first point with more words

- $AA^T = A^T A = I \rightsquigarrow$ transpose $[]^T$
switches the rows and columns

\downarrow
ex: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

* More about transpose on next page