

Also sharpie

Red pen

Black pen

Green pen

Mechanical pencil

Dull Pencil

Orthogonal Transformations

- spatial relationships in 3D

Def - An $n \times n$ matrix A is an orthogonal transformation I.F.F (if and only if)

- It has n mutually perpendicular rows or columns with unit length

- \perp rows must be independent (can't be multiples of each other)

ex $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \rightarrow$ linearly dependent

$\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix} \rightarrow$ independent but not \perp

- to be perpendicular, the dot product must be 0

dot product: $x \cdot y = \sum_{i=1}^n x_i y_i$

$x \cdot y = 0 \iff x \perp y$ (perp.)

- rows/columns must have unit length

$\hookrightarrow \|x\| = \sqrt{\sum_i x_i^2} = \sqrt{x \cdot x}$

- The rows or columns of A form an orthonormal basis of \mathbb{R}^n

- basic for space - set of vectors that can combine to create any vector in a space

- basically first point with more words

- $AA^T = A^T A = I \rightsquigarrow$ transpose $[]^T$
switches the rows and columns

- $A^{-1} = A^T$

\downarrow
ex: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

* More about transpose on next page

more about $AA^T = A^TA = I$ & $A^{-1} = A^T$
 mtr mult: $AB = C$
 $m \times n$ $n \times k$ $m \times k$
 basically the same info

So what about $(AB)^T \stackrel{?}{=} C^T$ ~~XXXXX~~

$$AB \neq A^TB^T \quad \text{sad face}$$

$m \times n$ $n \times k$ $n \times m$ $k \times n$
 \neq

$$\text{so } (AB)^T = C^T = B^TA^T$$

$k \times m$ $k \times n$ $n \times m$
 \checkmark

identity matrix

$$I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

$$\text{so } IA = A$$

$$IX = X$$

example of an orthogonal transformation:

ex • 2×2 Rotation matrix:

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Want to show perpendicular columns:

$$\vec{u} = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \quad \vec{v} = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$

Need to show ~~3 things~~...

① are they perpendicular?

$$\text{take } \vec{u} \cdot \vec{v} = -\cos \theta \sin \theta + \sin \theta \cos \theta = 0$$

so we know $\vec{u} \perp \vec{v}$