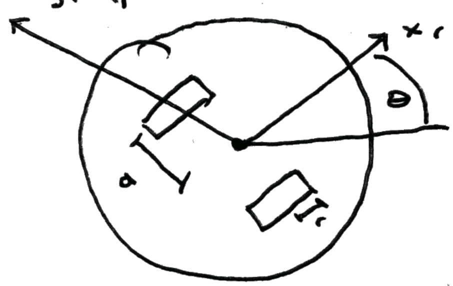
Differential drive!

- two wheels on common axis and can spin indep

but the wheels can rotate indep

turtlebot:

giant turtlebot!!



#topview

① what if both wheels spin at a velocity of $\mathcal{K}$ rad/s

So $v_L = \mathcal{K}$ $\quad v_R = \mathcal{K}$

So then look at motion in world

forward, sideways, any

$\dot{x}_R = r\mathcal{K}$ b/c wheels spin at

$\dot{y}_R = 0$

$\dot{\theta}_R = 0$

Angle	Length
2π rad ~	$2\pi r$ m
1 rad ~	r m

② Little bit harder. What if $v_R = \mathcal{K}$ $\quad v_L = -\mathcal{K}$

same speed diff directions

$\dot{x}_R = 0$

$\dot{y}_R = 0$

$\dot{\theta}_R =$

next page ⇒

$\frac{\mathcal{K}r}{d} = \theta$

more about $AA^T = A^TA = I$ & $A^{-1} = A^T$

basically the same info

mtx mult:

$$\underbrace{\underset{m\times n \; n\times k}{AB}} = \underbrace{\underset{m\times k}{C}}$$

So what about $(AB)^T \stackrel{?}{=} C^T$ ~~xxxx~~

$$\underset{m\times n \; n\times k}{AB} \neq \underset{n\times m \; k\times n}{A^T B^T}$$

$\neq$

$$\text{so } \underset{k\times m}{(AB)^T = C^T} = \underset{k\times n \; n\times m}{B^T A^T}$$

✓

identity matrix

$$I = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ & & \ddots & \\ 0 & & & 1 \end{bmatrix}$$

so $IA = A$

$Ix = x$

example of an orthogonal transformation:

ex. 2×2 Rotation matrix:

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Want to show perpendicular columns:

$$\vec{u} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

Need to show ~~3 things~~

① are they perpendicular?

take $\vec{u}\cdot\vec{v} = -\cos\theta\sin\theta + \sin\theta\cos\theta = 0$

so we know $\vec{u} \perp \vec{v}$

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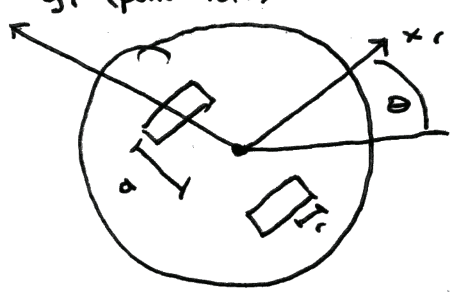

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① what if both wheels spin at a velocity of $\cancel{K}$ K rad/s

So $V_L = K$ $\quad V_R = K$

So then look at motion in world

forward, sideways, ang

Angle	Length
2π rad ~	$2\pi r$ m
1 rad ~	r m

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$\dot{y}_R = 0$ $\frac{m}{s}$

$\dot{\theta}_R = 0$ $\frac{m}{s}$

② Little bit harder. What if $V_R = K$ $\quad V_L = -K$

same speed diff directions

$\dot{x}_R = 0$

$\dot{y}_R = 0$

$\dot{\theta}_R =$

next ⇒ page

$\frac{x_r}{d} = \theta$

more about mtx mult: $AA^T = A^TA = I$ & $A^{-1} = A^T$

basically the same info

$$\underset{m\times n \;\; n\times k}{AB} = \underset{m\times k}{C}$$

So what about $(AB)^T \overset{?}{=} C^T$

$$\underset{m\times n \;\; n\times k}{AB} \neq \underset{n\times m \;\; k\times n}{A^TB^T}$$

$\neq$

so $(AB)^T = C^T = B^T A^T$

$$\underset{k\times m}{C^T} \quad \underset{k\times n \;\; n\times m}{B^T A^T}$$

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Also sharpie

Red pen

Black pen

Green pen

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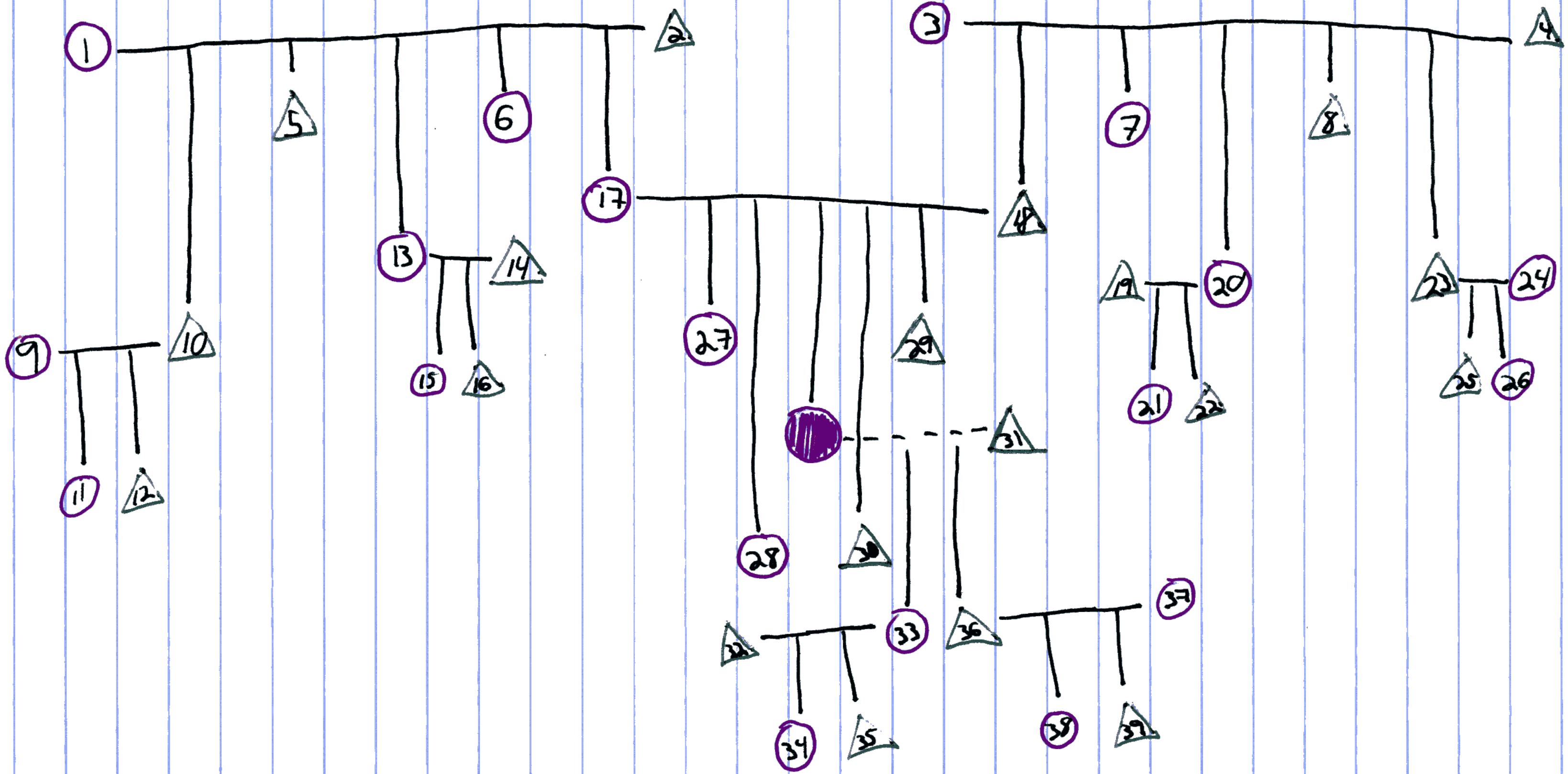
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- then let's look at $\dot{\theta}_R$

  suggestion 1: krd

  $$\frac{\frac{rad}{s}}{1/s} = \text{DIMENSIONLESS}$$

  dimensional analysis gives $\frac{1}{s} \cdot m \cdot m$

  answer: $\frac{Kr}{d}$

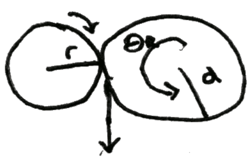
let's derive this:

- think about proportionality
  - $\dot{\theta}_R \propto K$

    b/c more speed means quicker

  if r is big, linear motion is going to be faster

  therefore
  - directly $\propto$ to r
  - indirectly related to d



Kr

$$\dot{\theta}_R d = Kr$$

$$\boxed{\dot{\theta}_R = \frac{Kr}{d}}$$

③ $v_R = R$ $\quad v_L = 0$

this one is tricky!

Orthogonal Transformations

- spatial relationships in 3D

Def – An $n \times n$ matrix A is an orthogonal transformation I.F.F (if and only if)

- It has n mutually perpendicular rows or columns with unit length
  - $\perp$ rows must be independent (can't be multiples of each other)

    ex $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ → linearly dependent

    $\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$ → independent but not $\perp$
  - to be perpendicular, the dot product must be 0

dot product: $x \cdot y = \sum_{i=1}^{n} x_i y_i$

$x \cdot y = 0 \longleftrightarrow x \perp y$ (perp.)

  - rows/colums must have unit length

    ↳ $\|x\| = \sqrt{\sum_i x_i^2} = \sqrt{x \cdot x}$

- The rows or columns of A form an orthonormal basis of $\mathbb{R}^n$
  - basic for space – set of vectors that can combine to create any vector in a space
  - basically first point with more words
- $AA^T = A^TA = I$ ⇝ transpose $[\,]^T$ switches the rows and columns

  ex: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$
- $A^{-1} = A^T$

* More about transpose on next page

Mechanical pencil

Dull Pencil

This is written in pencil

Also sharpie

Red pen

Black pen

Green pen

Mechanical pencil

Dull pencil