

WeBWorK assignment number Sec8.7 is due : 12/02/2023 at 11:59pm EST.

The Taylor Series of f at a is $f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$

The Maclaurin series of f is $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$

Some important Maclaurin series:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n, \quad (-1, 1).$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad (-\infty, \infty).$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \quad (-\infty, \infty).$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \quad (-\infty, \infty).$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)}, \quad [-1, 1].$$

The Binomial Series : If k is any real number and $|x| < 1$, then $(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \dots$

Problem 1. (1 point)

Book Problem 1

If $f(x) = \sum_{n=0}^{\infty} b_n(x-7)^n$ for all x , which of the following formulas for b_2 is true?

Enter T for true and F for false:

___1. $b_2 = \frac{f^{(2)}(0)}{2!}$

___2. $b_2 = \frac{f^{(2)}(7)}{2}$

___3. $b_2 = \frac{f^{(2)}(7)}{2!}$

Answer(s) submitted:

- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 2. (1 point)

Book Problem 3

If $f^{(n)}(0) = (n+4)!$ for $n = 0, 1, 2, \dots$, then the Maclaurin series for f is

$f(x) = \underline{\hspace{2cm}} + \dots$ (Enter only the first four non-zero terms.)

Answer(s) submitted:

- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 3. (1 point)

Book Problem 4

If $f^{(n)}(7) = \frac{(-1)^n n!}{2^n(n+3)}$ for $n = 0, 1, 2, \dots$, then the Taylor series for f centered at 7 is

$$f(x) = \sum_{n=0}^{\infty} \text{_____} (x-7)^n.$$

$$f(x) = \text{___} + \text{___} (x-7) + \text{___} (x-7)^2 + \text{___} (x-7)^3 + \dots$$

Answer(s) submitted:

- no response
- no response
- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 4. (1 point)

Book Problem 5

Find the Maclaurin series for $f(x) = \cos(6x)$ using the definition of a Maclaurin series:

$$f(x) = \text{_____, } f(0) = \text{_____,}$$

$$f'(x) = \text{_____, } f'(0) = \text{_____,}$$

$$f''(x) = \text{_____, } f''(0) = \text{_____,}$$

$$f^{(3)}(x) = \text{_____, } f^{(3)}(0) = \text{_____, } \dots$$

$$\cos(6x) = \text{___} + \text{___} x + \text{___} x^2 + \text{___} x^3 + \dots$$

Answer(s) submitted:

- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 5. (1 point)

Find the Taylor series for $f(x) = 8 + 7x + x^2$ centered at $a = 3$:

$$f(x) = \underline{\hspace{2cm}}, f(3) = \underline{\hspace{2cm}},$$

$$f'(x) = \underline{\hspace{2cm}}, f'(3) = \underline{\hspace{2cm}},$$

$$f''(x) = \underline{\hspace{2cm}}, f''(3) = \underline{\hspace{2cm}},$$

$$f^{(3)}(x) = \underline{\hspace{2cm}}, f^{(3)}(3) = \underline{\hspace{2cm}}, \dots$$

$$f(x) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} (x-3) + \underline{\hspace{1cm}} (x-3)^2 + \underline{\hspace{1cm}} (x-3)^3 + \dots$$

Answer(s) submitted:

- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 6. (1 point)

Find the Taylor series for $f(x) = 6\ln(5x)$ centered at $a = 3$:

$$f(x) = \underline{\hspace{2cm}}, f(3) = \underline{\hspace{2cm}},$$

$$f'(x) = \underline{\hspace{2cm}}, f'(3) = \underline{\hspace{2cm}},$$

$$f''(x) = \underline{\hspace{2cm}}, f''(3) = \underline{\hspace{2cm}},$$

$$f^{(3)}(x) = \underline{\hspace{2cm}}, f^{(3)}(3) = \underline{\hspace{2cm}}, \dots$$

$$6\ln(5x) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} (x-3) + \underline{\hspace{1cm}} (x-3)^2 + \underline{\hspace{1cm}} (x-3)^3 + \dots$$

Answer(s) submitted:

- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 7. (1 point)

Book Problem 23

Use the binomial series to expand the following functions as a power series. Give the first 3 non-zero terms.

$$f(x) = \sqrt[7]{1+x} = \underline{\hspace{2cm}} + \dots$$

$$g(x) = \sqrt{1+3x} = \underline{\hspace{2cm}} + \dots$$

$$h(x) = \frac{1}{(1-x)^5} = \underline{\hspace{2cm}} + \dots$$

Answer(s) submitted:

- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 8. (1 point)

Use the binomial series to expand the following function as a power series. Give the first 3 non-zero terms.

$$h(x) = \frac{1}{(5+x)^7} = \text{---} + \text{---} x + \text{---} x^2 + \dots$$

Answer(s) submitted:

- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 9. (1 point)

Book Problems 27 - 31

Use a Maclaurin series derived in this section to obtain the Maclaurin series for the given functions. Enter the first 3 non-zero terms only.

$$f(x) = \cos(4x^4) = \text{---} + \dots$$

$$f(x) = \sin(-\pi x) = \text{---} + \dots$$

$$f(x) = x \tan^{-1}(4x) = \text{---} + \dots$$

$$f(x) = x^4 e^{-x/2} = \text{---} + \dots$$

Answer(s) submitted:

- no response
- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 10. (1 point)

Book Problem 41

a) Use the binomial series to give the first 3 non-zero terms of the power series for

$$\frac{1}{\sqrt{1-9x^2}} = \text{---} + \dots$$

b) Evaluate the indefinite integral $\int \frac{1}{\sqrt{1-9x^2}} = \text{---} + C$.
(your answer should be an antiderivative not a power series)

c) Using parts (a) and (b), give the first 3 non-zero terms of the power series for

$$f(x) = \sin^{-1}(3x) = \text{---} + \dots$$

Answer(s) submitted:

- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 11. (1 point)

Book Problem 43

Evaluate the indefinite integral as an infinite series. Give the first 3 non-zero terms only.

$$\int x \cos(x^6) dx = \int (\text{---} + \dots) dx = C + \text{---} + \dots$$

Answer(s) submitted:

- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 12. (1 point)

Book Problems 51 - 53

Use series to evaluate the following limits. Give only two non-zero terms in the power series expansions.

$$\frac{1+4x-e^{4x}}{x^2} = \text{_____} + \dots, \quad \lim_{x \rightarrow 0} \frac{1+4x-e^{4x}}{x^2} = \text{_____}$$

$$\frac{3x - \tan^{-1} 3x}{x^3} = \text{_____} + \dots, \quad \lim_{x \rightarrow 0} \frac{3x - \tan^{-1} 3x}{x^3} = \text{_____}$$

$$\frac{\sin x - x + \frac{1}{6}x^3}{4x^5} = \text{_____} + \dots, \quad \lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3}{4x^5} = \text{_____}$$

Answer(s) submitted:

- no response
- no response
- no response
- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 13. (1 point)

Book Problem 59

MIX and MATCH

—1. $\sum_{n=0}^{\infty} (-1)^n \frac{4x^{2n+1}}{(2n+1)!}$

—2. $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n}}{(2n)!}$

—3. $\sum_{n=0}^{\infty} \frac{(-1)^n 4x^{2n+1}}{2n+1}$

—4. $\sum_{n=0}^{\infty} \frac{4^n x^n}{n!}$

—5. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{n!}$

A. $4 \arctan(x)$

B. e^{4x}

C. $\cos(4x)$

D. e^{-x^4}

E. $4 \sin(x)$

Answer(s) submitted:

- no response
- no response
- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

Problem 14. (1 point)

Give the first 5 non-zero terms in the power series expansion and evaluate the derivative of the following functions:

$$f(x) = e^{4x^2} = \text{_____} + \dots, \quad f^{(6)}(0) = \text{_____}.$$

$$g(x) = \tan^{-1} 4x = \text{_____} + \dots, \quad g^{(7)}(0) = \text{_____}.$$

Answer(s) submitted:

- no response
- no response
- no response
- no response

submitted: (incorrect)

recorded: (incorrect)

