Identifying Quark and Neutron Stars from Gravitational Wave Data

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May-June 2023

Abstract

Introduction

This project was funded by a grant from the Haverford College Koshland Integrated Natural Sciences Center.

This paper relies heavily on the work of Ho et. al. [1] and Zhao and Lattimer [2], who will be referenced henceforth by name. This paper also rests on several assumptions about pulsar glitches, namely that pure quark stars do exist, that they experience glitches, and that these glitches are approximately the same as those observed in other pulsars, so that some portion of the existing population of recorded glitches were within quark stars. All of these assumptions are still unknown, and particularly the matter of whether quark stars exist at all is disputed. However, this paper will demonstrate that observation of gravitational waves generated by pulsar glitches could be a reliable way to answer these and other questions. This paper also uses data collected from the ATNF Pulsar Catalog and the JBCA Glitch Catalog. These databases were used to generate a combined data set of 650 glitches in a population of 195 pulsars. Due the variability of glitches and glitching pulsars, as well as the fact that some pulsars were only discovered and began to be observed recently, some pulsars have up to 20 glitches recorded in the data, while others have only one. The code and data sets used for this paper from start to finish can be found in this Github repository.

Pulsar and Pulsar Glitch Distribution

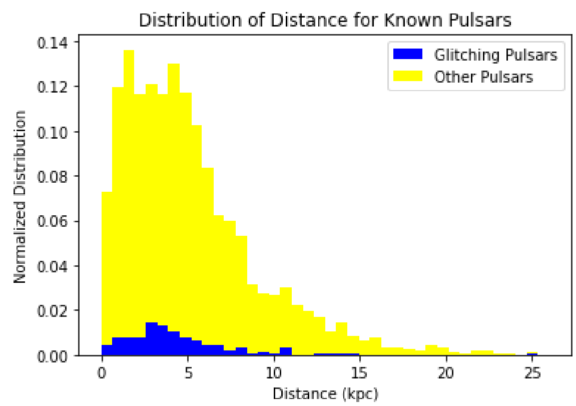


FIG. 1. Normalized distribution of the distances of known pulsars, with glitching and non-glitching pulsars visually separated. Pulsars in the Large Magellanic Cloud were omitted. Figure 1 displays the distance distribution of some 3141 pulsars in the ATNF Pulsar Catalog, with 195 glitching pulsars highlighted in blue. The vast majority of known pulsars are within a distance of 10 kpc, but this is almost entirely due to the fact that closer pulsars are easier to detect. GW strain (the primary characteristic influencing whether or not a wave is detected) is inversely proportional to distance, meaning that if and when detections of GWs generated by glitches do happen, the source pulsars will likewise be concentrated within a specific distance from Earth, with higher energy glitches being able to be detected at a greater distance.

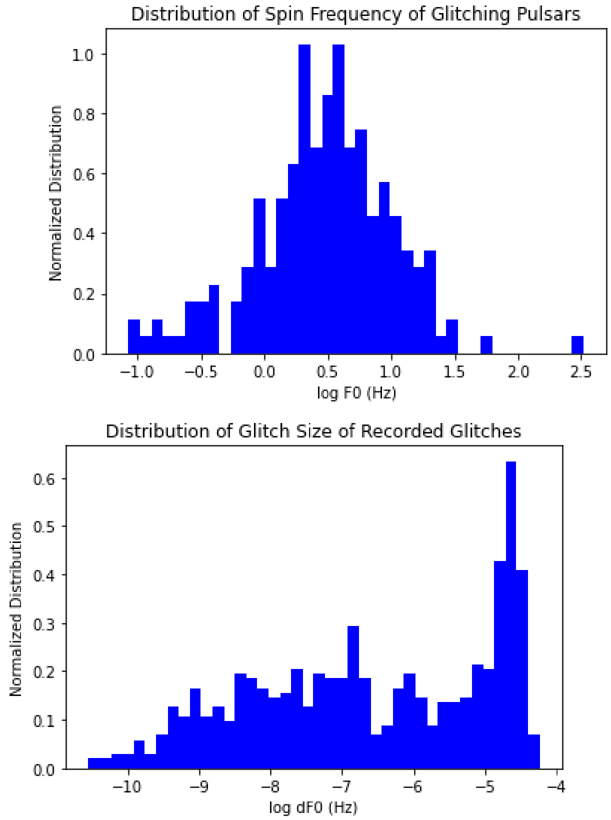


FIG. 2. Top: Normalized distribution of the log of the spin frequency of 155 known glitching pulsars, in Hz. Bottom: Normalized distribution of the log of the change in frequency for 650 recorded glitches in the JBCA catalog. Figure 2 shows the logs of the frequency and glitch size of the pulsars and glitches in the JBCA catalog. Spin frequency ranges from a little more than 0.08 Hz to 327 Hz, while glitch size ranges all the way from 2.83e-11 Hz to 4.37e-5 Hz. GW strain is proportional to the square root of both spin frequency and glitch size, so the ranges of 4 and 6 orders of magnitude means that GW strain will also have a wide distribution, even without accounting for other factors such as distance and mass.

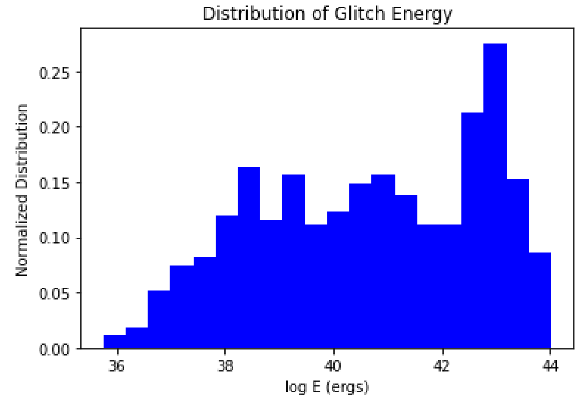


FIG. 3. Normalized distribution of glitch energy in ergs for the 650 glitches from the JBCA catalog. Figure 3 shows the log of the energy of each of the 650 glitches in the JBCA catalog. Glitch energy is proportional to the product of spin frequency and glitch size, so GW strain can also be expressed as being proportional to the square root of glitch energy. As can be observed on the histogram, glitch energy in the data ranges across 8 orders of magnitude, confirming the point above about contributing to a wide range of strains. Based on glitch energy alone, one would expect a range of at least 4 orders of magnitude for strain.

Modeling GWs from Quark and Neutron Star Glitches

In modeling GWs from quark and neutron star glitches, two primary characteristics of the wave were focused on. These are GW frequency and damping time, which, combined with the information generated by the above analysis of glitches, can be used to determine GW amplitude and strain. For quark stars, Zhao and Lattimer demonstrated that if M_{max} (the maximum mass for a non-rotating quark star) and c_s^2 (the speed of sound in the star squared, as a fraction of the speed of light) are parameterized, then GW frequency and damping time can be expressed as a function of the mass of the star. For this model, two parameterizations were chosen: $M_{max} = 2.2M_\odot$ and $c_s^2 = 1/3$, which will be called "orange," and $M_{max} = 2.2M_\odot$ and $c_s^2 = 1$, which will be called "purple." These parameterizations were chosen because M_{max} functions mostly as a scaling factor, and 1 and 1/3 were the upper and lower bounds of the speeds of sound modeled by Zhao and Lattimer. By choosing these two parameterizations we can observe the effect that different speeds of sound in the quark matter will have on the modeled GWs. The mass-frequency and mass-damping time relationships for these two parameterizations and for the N3LO EOS for NSs found in Zhao and Lattimer's paper were approximated, and masses were randomly assigned along a Gaussian with median $1.4M_{\odot}$ and width $0.15M_{\odot}$ to each of the 650 glitches from the JBCA catalog. This allowed a calculation of GW frequency and damping time for each glitch.

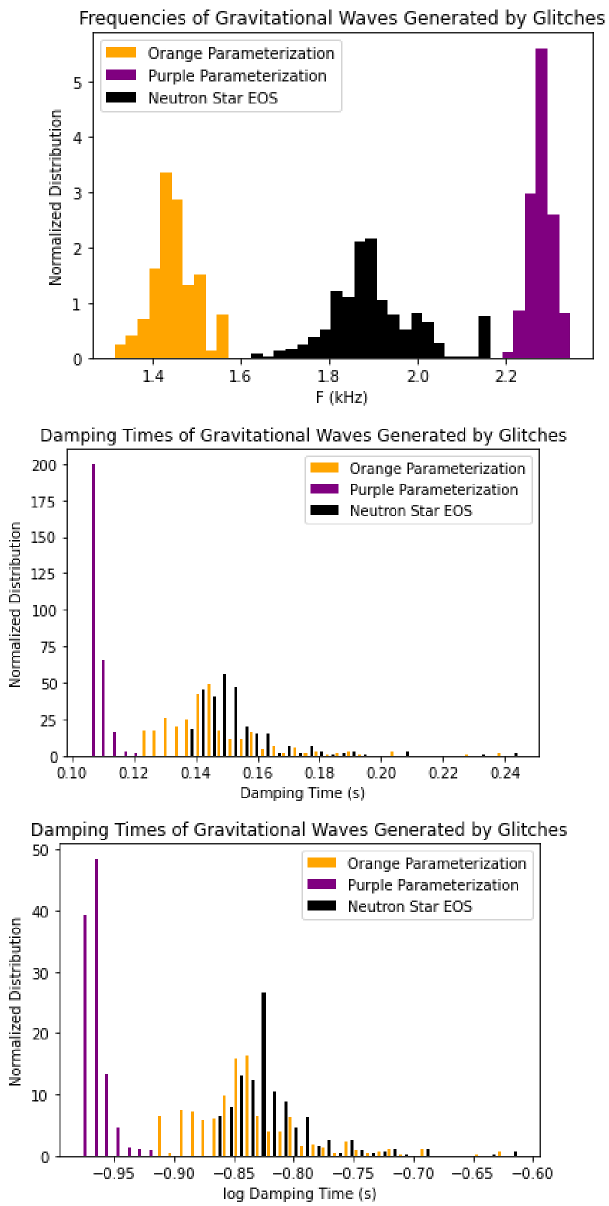
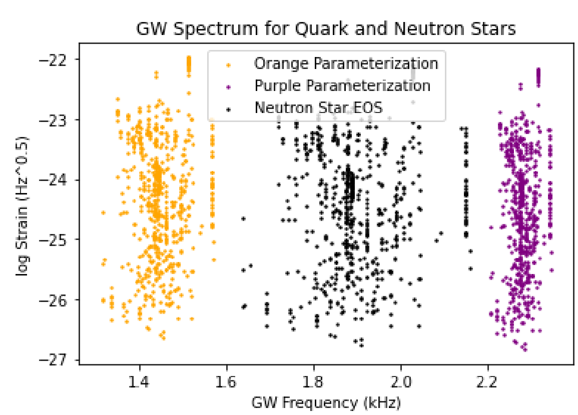


FIG. 4. Top: Normalized distribution of GW frequency for the glitches. Middle: Normalized distribution of GW damping times for the glitches. Bottom: Normalized distribution of the log of GW damping times for the glitches. Figure 4 demonstrates a clear separation between the expected damping times and especially frequencies for the two QS parameterizations. $c_s^2 = 1$ and $c_s^2 = 1/3$ are on the extreme ends of the parameterizations laid out by Zhao and Lattimer, and therefore the real speed of sound would likely fall somewhere between these two values, creating a distribution of frequencies and damping times between the two peaks. However, the distinct separation and approximately normal distribution that Figure 4 does show indicates that as data is collected, the speed of sound in quark matter would be able to be narrowed down. Somewhat unfortunately, the NS frequencies fall between the two QS parameterizations, meaning that we might expect GWs from QS and NS glitches to have significant overlap in the frequencies produced. Along the same lines, there is visible overlap between the distribution of damping times for the orange parameterization for QSs and the N3LO EOS for NSs. Therefore, especially given the uncertainty in the EOS for quark stars, it would not be easy to distinguish between GWs from QS or NS glitches based on either their damping time or frequency individually. However, if both factors are considered simultaneously, it becomes significantly easier to distinguish, as we will explore in the next section.

Shifting our focus to the actual detection of these GWs in GW observatories, the most relevant factor in whether a GW can be detected is its strain, which is the product of its amplitude and the square root of its damping time.



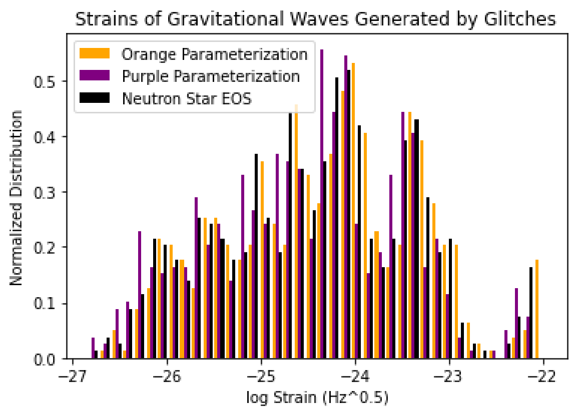


FIG. 5. Top: GW spectrum of frequency vs strain for the orange and purple parameterizations. Bottom: Distribution of the log of the strain for the GW generated by each glitch. Figure 5 lays out the GW spectrum and strain distribution for the 650 quark star glitches which were modeled, for both parameterizations. The very highest strains are mostly attributable to Vela, which at between 0.24 and 0.37 kpc is one of the closest and best-understood pulsars. Vela currently has a total of 20 glitches recorded in the JBCA catalog. LIGO has not been sensitive enough in the past to detect GWs from NS or QS glitches, but the most recent upgrade increased its sensitivity to around 10^{-23} $Hz^{1/2}$ for the relevant frequencies, while the future Cosmic Explorer and Einstein Telescope observatories will have sensitivities up to 10^{-24} $Hz^{1/2}$. Therefore, aLIGO may be able to detect a GW from a glitch in Vela or a similarly proximate pulsar (including a yet-undiscovered pulsar) if one takes place during the current run of data collection. Furthermore, over the next two decades, upgrades to existing detectors and new third-generation detectors will be have sensitivities well into the predicted strain distribution for both quark and neutron star glitches. Observed glitches of the required proximity are still relatively infrequent, however current glitch detection is dependent on the source being a pulsar which is "pointed" towards Earth, whereas GWs propagate in all directions, so GWs generated by glitches could reveal yet-undetected neutron stars and quark stars.

Differentiating between GWs from Quark and Neutron Star Glitches

As mentioned previously, it cannot be determined if a GW is from a QS or NS glitch based only on its frequency or damping time. However, QSs and NSs have *combinations* of these two values which have minimal overlap. One of many ways to do this is to multiply the frequency and damping time of a gravitational wave by each other to form a unitless differentiation metric.

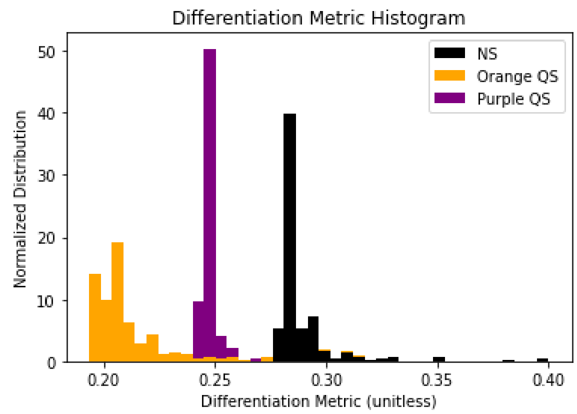


FIG 6. Distribution of the Differentiation Metric (the product of GW frequency and damping time) for the 650 glitches using the N3LO NS EOS and the orange and purple QS parameterizations Figure 6 demonstrates that there is minimal overlap between QS and NS gravitational waves if this differentiation metric is used. Interestingly, only the orange QS parameterization overlaps at all with NSs, despite having a median further from that of the NSs than the purple QS parameterization. This is because the orange parameterizaton has a much wider distribution than the purple, so given its status as a bound for the expected EOS one might expect the real distribution for QSs to have a more constrained and therefore even more easily differentiable distribution. Furthermore, the NS median is more than 4.4 standard deviations from the orange median, while the orange median is more than 5.3 standard deviations from the NS median, meaning if we assume an approximately normal distribution they have an essentially negligible overlap. Taking all this into account, assuming the N3LO EOS is approximately correct for this purpose and that the orange and purple parameterizations are indeed the bounds for a reasonable QS EOS, one should be able to determine whether a glitch-generated GW originated in a quark or neutron star with considerable certainty, and a population of even a handful of detected glitches should be sufficient to determine whether a given star or group of stars are QSs or NSs. Therefore, if as few as 2 or 3 confirmed glitch detections in LIGO or another detector were characteristic of QSs, that would provide considerable evidence that QSs do in fact exist, and in doing so, move towards confirming the strange matter hypothesis.

[1] Ho, Wynn C.G., et al. “Gravitational waves from transient neutron star f-mode oscillations.” Physical Review, vol. 101, no. 10, 2020, p. 103009.

[2] Zhao, Tianqi, and Lattimer, James M. “Universal relations for neutron star f-mode and g-mode oscillations.” Physical Review, vol. 103, no. 12, 2022, p. 123002.