Probability ; Conditional probability ; Independence.

1. <u>Probability</u>: A probability is how likely an event is to occur.

Between 0 and 1

The higher the probability, the more likely event will occur.

Example 1: flip a fair coin. 2 possible outcomes: {Head ; Tail}.

$S = \{ Head ; Tail \}$

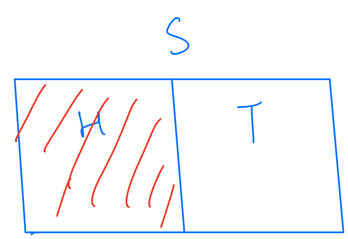
we call S is a "sample space"

※: An event is a subset of sample space.

Compute probability of an event:

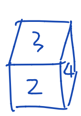
$$\text{prob} = \frac{\# \text{ outcomes in event}}{\# \text{ total outcomes}}$$

Ex 1: $p(Head) = \frac{1}{2}$



$P(H) =$ ratio of red area to the whole S

Example 2: roll a dice



$S = \{1, 2, 3, 4, 5, 6\}$

event: $\{1\}$, $\{1,2\}$, $\{3\}$, ... $\{1, 2, 3, 4, 5, 6\}$.

2. Conditional probability:

Conditional prob is the measure of prob A occuring given another B has occured.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$A \cap B$: A and B both happen

$$P(A \cap B) = \frac{\#\text{ outcomes of A and B both happen}}{\#\text{ total outcomes}}$$

$$P(B) = \frac{\#\text{ outcomes B happen}}{\#\text{ total outcomes.}}$$

$$P(A|B) = \frac{\#\text{ outcomes A and B both happen}}{\#\text{ outcomes B happen.}}$$

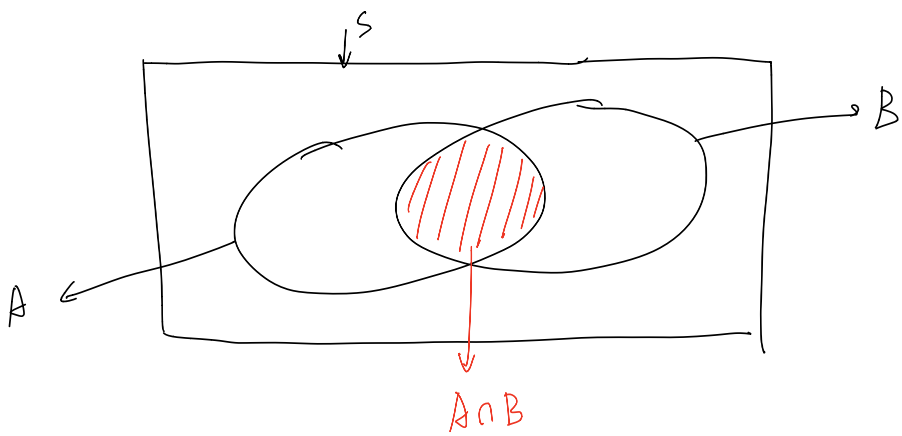
Ex: roll a dice

A: 1, 2, 3 appear

B: 3, 4, 5 appear

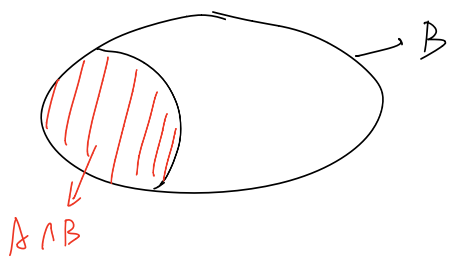
$$P(A|B) = \frac{\{3\}}{\{3,4,5\}} = \frac{1}{3}$$

Explain in diagram:



The even B already occured means event outside B is not possible

sample space reduce to



$P(A|B)$ ratio of red area to B

3. Independent:

A and B independent $\Longleftrightarrow$ ① $P(A \cap B) = P(A) \cdot P(B)$

The probability of A is not affected by whether B occured.
(stays constant)

② $P(A|B) = P(A)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A) \Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

② $P(B|A) = P(B)$