

The method of Proper orthogonal Decomposition for Dynamical Characterization and Order Reduction of Mechanical Systems: Discussion

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Basic idea

- Purpose: Project high-dimensional data into low-dimensional space; feature extraction by revealing data structure. (i.e. find some modes that can best depict the process characters).
- For a dynamic process $v(x, t)$, take several snapshots at different time point, obtain the most characteristic structure $\varphi(x)$ of an ensemble of snapshots.
- Find a series of such orthogonal structure $\varphi(x)$ as basis function that depict the motions of x , which can be used to form the process in a new way: $v(x, t) = \sum_{i=1}^{\infty} a_i(t) \varphi_i(x)$.
- Maximize $\langle |v^k, \varphi|^2 \rangle$ with $\|\varphi\|^2 = 1$

$$J[\varphi] = \langle |v^k, \varphi|^2 \rangle - \lambda(\|\varphi\|^2 - 1) \Rightarrow$$
$$\int_{\Omega} \langle v^k(x) v^k(x') \rangle \varphi(x') dx' = \lambda \varphi(x)$$

Bullet Points

- In practice, the data is discretized in space and time, with n observations and m -dimensional vector, a $(m \times n)$ matrix is obtained. Then use matrix decomposition to obtain basis functions.
- Obtaining dominant POMs(modes, basis functions) can construct low-order dynamical models for system. These low-dimensional models can be effective in capturing the global dynamics of a nonlinear system, which can save a lot of computational effort for dynamical studies of structural modifications or for control system design.
- Since POMs are computed at discrete points in a domain, interpolations can be performed to obtain continuous POMs valid over the entire domain of the computation.(B-spline interpolation functions of 6th order in paper's example)

- POVs give clear indications about the participation of the corresponding POMs(i.e. the energy percentage of each POMs).
- The spatial and time information is explicitly separated. Insight into the POMs and time coefficient can be more efficient.
- "A safe heaven in the intimidating world of nonlinearity; although it may not do the physical violence of linearization methods". Although POD is linear methods, the POMs and the portion of energy captured by each of the leading (dominant) modes are valuable tools for studying the strength of nonlinear effects in these systems.

Further thoughts

- In practice computation, as for the decomposition of matrix. The main advantage in considering SVD instead of eigenvalue decomposition is that additional information is obtained through matrix V . From the definition of the SVD, it follows that each column of U corresponds to a POM mode φ_k , while each line of V contains the time coefficients of the corresponding POM. [Lenaerts V., Kerschen, G., and Golinval J., 2003]
- POD cannot provide an efficient description of a dataset where nonlinear dependencies exist. (updated NLPCA).
- Due to the computation methods of POD, it is only sensitive to second-order statistics. Thus, if the assumption that the variables are Gaussian is violated, POD does not yield statistical independence. (independent component analysis)