IMPERIAL COLLEGE LONDON

BAYESIAN INFERENCE AND ARGUMENTATION

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Contents

1 INTRODUCTION

Bayesian Network and Argumentation are two commonly used methods in areas like disease diagnosis, forensic scenarios and recommended system, which need to output a decision result and, sometimes, a corresponding explanation.

These techniques can be used separately for different scenarios. For example, we can use extensions in argumentation to determine whether an argument is *acceptable* [1], and we can also use Bayes to get the probability of a determined decision. However, they can also be combined into one approach, to generate a comprehensive result with both statistical perspective and expert domain to help people make accurate decisions. Currently,combinations of Bayesian and Argumentation comprise a set of techniques for different scenarios and different purposes. These techniques are mainly about: (i) Explaining Bayesian Network by Argumentation;(ii) extracting (modify) Bayesian Network based on Argumentation (iii) combining Argumentation and Bayesian Network (BN).

1.1 Explaining Bayesian Network by Argumentation

Because of the unreadability of the BN, it's necessary to extract the explanation from the BN. To extract explanation from a constructed BN, if the BN is structured [2], then we can easily generate the explanation based on some rules [1]. But in most situations, the BNs are not structured. Then based on the relationship between BN and Argumentation, we can use some general methods to extract explanatory arguments [2, 3]. In chapter 3 and 4, we will show 2 concrete cases which use argumentation to explain BN.

1.2 Extracting (modifying) Bayesian Network based on Argumentation

To construct a BN is usually a difficult task, especially when we don't have the data for constructing the BN. Then a possible approach is to first construct the Argumentation framework based on some schemes. Also, to confirm the acceptability of an argument, we need to compute the semantics from Probabilistic Argumentation (PA)[3]. A reduction method for computing Probabilistic Assumption-based Argumentation (PABA) [4] semantics is proposed by [4]. First thing is to construct a PABA framework, then we can use sem-

[1] If an argument is in the extension we choose, say, grounded, then it's acceptable
[2] the BN follows a predefined structure
[3] probabilistic extension(Bayesian Inference in this paper) of Argumentation
[4] An instance of PA

derivation[5] to get semantics, and these derivation can also be applied to other PA models.

1.3 Combining Argumentation and Bayesian Network

To integrate the information of Argumentation frameworks and BN, with the different starting point(i.e start from BN or that combine the advantages of both BN and Argumentation framework, i.e, accuracy and readablity.

1.4 Structure of the report

To summarize, the main goal of combining Argumentation and BN is to combine their advantages and get a more comprehensive result. To achieve this goal, we can either use a new system that inherits the properties of Argumentation system and BN that we need, or transform between Argumentation system and BN based on the connection of BN and Argumentation system. In the following chapters, we illustrate these techniques by examples.

The rest of this report is structured as follows: chapter 2 presents the preliminaries; chapters 3 and 4 present the 2 methods and corresponding case studies for explaining BN by Argumentation; chapter 5 presents the method and corresponding case study for transforming Argumentation to BN; chapters 6 and 7 present 2 new systems that combine some of the properties of BN and Argumentation system and their corresponding examples; chapter 8 is the conclusion part.

[5] sem = credulous, ideal, ground

2 PRELIMINARIES

2.1 Bayesian Network

In this section we briefly review Bayesian Network which usually consists of 2 parts, a graph and tables. The graph shows the dependencies of each variables and tables show the probability of each variable.

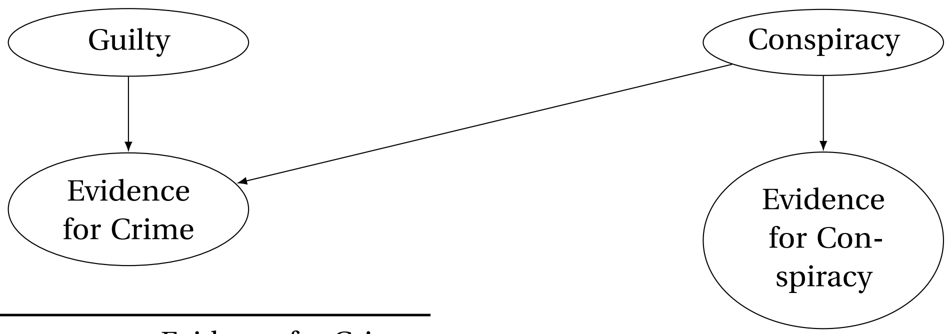
Definition 1 (Bayesian Network)**.** *A BN is a triple B = (V, A, P) where:*

- *G = (V, A) is an acyclic directed graph with nodes V and arcs* $A \subset V \times V (arc(V_i, V_j)$ *is directed from* V_i *to* V_j*);*
- $P = Pr_v \mid v \in V, eachPr_v$ *is the probability of variable v, and these probabilities are typically repreasented as tables.*

Figure 1 is one example of BN [3], which is also used in chapter 3.

Guilty	
T	F
0.1	0.9

Conspiracy	
T	F
0.01	0.99



		Evidence for Crime	
Guilty	Conspiracy	T	F
F	F	0.001	0.999
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.9	0.1

	Evidence for Conspiracy	
Conspiracy	T	F
F	0.9	0.1
T	0.1	0.9

Figure 1: An example of Bayesian Network

In this BN, we can know from the graph that the probability of *Evidence for Crime* depends on *Guilty* and *Conspiracy*, and when both Guilty and Conspiracy are *false*, then the probability of Evidence for Crime being true is 0.001, which means, for example, when the suspect is neither Guilty nor Conspiracy, the probability of finding the Evidence for Crime is very low.

To get the dependencies of each nodes, we use the concept, d-separation. If A and B are independent when we observe C, A and B are d-separated by C. To confirm if A and B are d-separated by C, we need to find all undirected paths between A and B, if all these paths are blocked, then A and B are d-separated by C.

Definition 2. *A path is said to be blocked if there exist a node X, either*

- *X is a head to tail or tail to tail node, and X belongs to C*
- *X is a head to head node, and X and its children don't belong to C*

Based on the tables and dependencies we get from Figure 1, this BN can uniquely define the joint probability distribution Pr(V):

$$Pr(V) = \prod_{v \in V} Pr(v|par(v)) \tag{1}$$

Where par(v) means the parents of this variable v.

2.2 Argumentation

Argumentation can be used to model conflicting or contradictory information by using attack relationship between arguments. And in argumentation, using defeasible rules, which can have exceptions, is one way to deal with uncertainty[6].

There are many different kinds of Argumentation system, and we use Argumentation schemes, ASPIC+ Argumentation, Abastract Argumentation, Assumption-based Argumentation in this report. Following is the brief review of these systems.

2.2.1 Argumentation schemes

An Argumentation scheme usually have 2 parts, the first part is the argumentation scheme itself, one example is *the argument from sign scheme* [5] which is also used in case-4:

- A (a finding) is true in this situation.
- B is generally indicated as true when its sign, A, is true.

[6]s opposed to a strict, or deductive, inference rule

- B is true in this situation.

And an argumentation scheme always have a corresponding part, called critical questions, and the critical questions of the above scheme is

- CQ1:What is the strength of the correlation of the sign with the event signified?
- CQ2:Are there other events that would more reliably account for the sign?

2.2.2 Abstract Argumentation(AA) and Assumption-based Argumentation(ABA)

An AA framework [6] is a pair (AR, Att), in which AR means arguments and $Att \subseteq AR \times AR$ is a binary relation over Args, and (A,B) ∈ Att means A attack B. And its extension will be shown in section 2.2.4.

ABA is an instance of AA which shows the internal structure of arguments. It defines arguments by deductive proofs based on assumptions and inference rules. Given a language L consists of several sentences, an ABA [7] is a triple $(R, A, ^{-})$, in which R is a set of inference rules in the form $\sigma_0 \leftarrow \sigma_1, ..., \sigma_i$ where σ_0 is called head and $\sigma_i \in L$, A is a set of assumptions, and $^{-}$ is a one-to-one mapping from A into L, where $\bar{x}$ is referred to as the contrary of x.

In this paper, we only use flat framework, and contraries of assumptions are not asusumptions.

2.2.3 ASPIC + Argumentation

In this section we describe a simplified version of ASPIC+ because strict rules, presumed knowledge are not used in this report. In this report we mainly use three concepts of ASPIC+ Argumentation, which are *argument, attack, defeat.*

Definition 3 (Argumentation system [8])**.** *An Argumentation system(AS) is a tuple AS = (L,* $^{-}, R_d) where:$

L is a logical language;

$^{-}$ *is the negation function*

R_d *is set of defeasible rules of the form* $_1, ..., \phi_n \Rightarrow \phi$ *(where* ϕ, ϕ_i *are met-variables ranging over wff in L)*

Note that in the definition, ⇒ means the defeasible rules and → means the stric rules which is not used in this report.

To reason with the language and the rules, a knowledge base is required.

Definition 4 (Knowledge base, [8])**.** *In an argumentation system AS, a knowledge base is set* $K_n \subseteq L$.

In this report, we don't use presumed knowledge, so K_n means axiomatic knowledge. An argumentation theory (AT), which can be used to build an argument graph, can be formed by the combination of an AS and a K_n, i.e, AT = (AS, K_n).

Definition 5 (Argument,[8])**.** *An argument A can be one of the following:*

- ψ*, if* $\psi \in K_n$*, and we define*

  Premise(A) = ψ

  Conclusion(A) = ψ

  SubArgument(A) = ψ

  TopRule(A)[7] = *undefined*

  ImmediateSubArgument(A) = $\emptyset$

  DefeasibleRules(A) = $\emptyset$

- $A_1, \ldots, A_n \Rightarrow \psi$*, if* $A_1, \ldots, A_n$ *are arguments such that there is a defeasible rule* $Conclusion(A_1), \ldots, Concl$
  ψ *in* R_d*, and we define*

  Premise(A) = *Premise*(A_1) $\cup \ldots \cup$ *Premise*(A_n)

  Conclusion(A) = ψ

  SubArgument(A) = *SubArgument*(A_1) $\cup \ldots \cup$ *SubArgument*(A_n) $\cup$ *A*

  TopRule(A) = $Conclusion(A_1), \ldots, Conclusion(A_n) \Rightarrow \psi$

  ImmediateSubArgument(A) = $A_1, \ldots, A_n$

  DefeasibleRules(A) = *DefeasibleRules*(A_1) $\cup \ldots \cup$ *DefeasibleRules*(A_n) $\cup$ *TopRule(A)*

Definition 6 (Attack)**.** *Argument A attacks argument B iff A rebuts B. Argument A rebuts argument B iff conclusion of A is the contrary of conclusion of B.*

Definition 7 (Argument defeat)**.** *Given an order* $\preceq$ *for the arguments, then argument A defeats argument B iff A rebuts B and A B*

[7] The last applied rule

2.2.4 Extensions of Argumentation

Extensions are important when we want to judge if an argument is acceptable, and here we only introduce some commonly used extensions, which are also used in this report.

A set of arguments S is

- **conflict-free** (c-f) iff it does not attack itself;
- **admissible** iff it's c-f and attacks each attacking argument;
- **preferred**(credulous) iff it's maximal admissible;
- **complete** iff it's admissible and contains all arguments it defends(by attacking all attacks against them) ;
- **grounded** iff it is the set inclusion minimal complete extension;
- **stable** iff it is preferred and it defeat any arguments ouside S.

These extensions can be used for all three Argumentation we mentioned in this section.

3 CASE-1: EXTRACT EXPLANATION FROM BN

In this case a two-phase method [2], which can help us extract explanations from BN, is introduced. Though we don't really extract all arguments from the BN, but we can extract extensions which can help us to make the judgement. There are basically 2 steps to construct the extensions

- step-1: construct the support graph
- step-2: calculate the strength and find the defeat relations, then keep the undefeated arguments

In the following parts of this section, we use the BN in [3] and use the two-phase method to construct the ground extension of this BN [8].

3.1 Bayesian Network

In this section we use the Bayesian Network shown in Figure 1. For convenience, we represent Node Guilty as G, Node Conspiracy as C, Node Evidence for Crime as E_{Cr}, Node Evidence for Conspiracy as E_{Con}. In this BN, the observed evidences are Evidence for Crime(E_{Cr}) = true and Evidence for Conspiracy(E_{Con}) = false. And we want to know which arguments can be acceptable with these evidences.

3.2 Support Graph

3.2.1 Some basic definitions

Given a BN, we first review some basic definitions before we constructing the support graph.

Definition 8. *A Markov blanket* of a variable is its parents, children and parents of children.

For example, the *Markov blanket* of node G is E_{Cr} and C.

Definition 9. *Support Chain* of a variable V^* is a set of nodes (chain) such that.

- *there are no repeated nodes*
- *For each immorality(V_i, V_j, V_k)(shown in figure 2),we skip V_j (i.e, V_j is not included in support chain)*

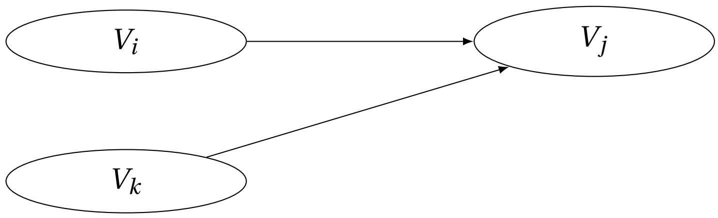


Figure 2: immorality

- *ends in* V^*.

A *support graph* is a graph that contains all support chains in a BN for our interested variable V^*, i.e, support graph captures all chains end with V^*.

3.2.2 An example of deriving support chain

If we take node *Guilty* as interested variable, then from the BN above, we can get the following chain.

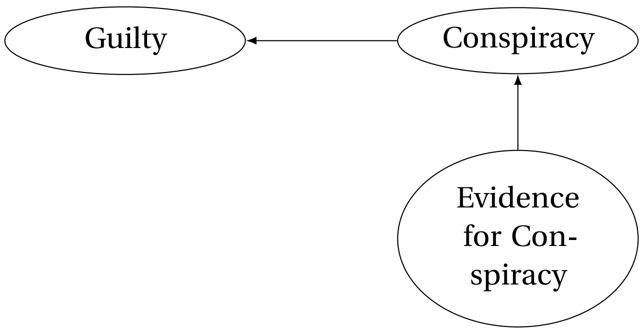


Figure 3: One support chain of BN in figure 1

This support chain start from node E_{Con}, pass nodes C and E_{Cr}, and end in G, but because of the immorality, we skip the node E_{Cr}, then we get the finally support chain.

3.2.3 Construct the support graph

We can simply enumerate all support chain in a BN to get the support graph if it's doable(i.e the number of support chains is not too large). But it's not always a feasible way, then we can use a algorithm to construct a more concise support graph in which we merge

[8] Here we construct arguments based on ASPIC+ Argumentation

the support chains with same prefixes.
Here we first introduce a concept

Definition 10. *Forbidden set F* is a set of variables that can't be used in other support chains

Then we can recursively enumerate the following cases until we can't find a variable not contained in the forbidden set(our interested variable is V_i). First we let the initial forbidden set F = ∅

- case I : parents of V_i, shown in figure 4

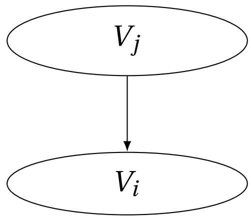


Figure 4: Parents of V_i

In this case Forbidden set $F_{new} = F \cup \{V_j\}$, and we add V_j to our graph

- case II : Children of V_i, shown in figure 5

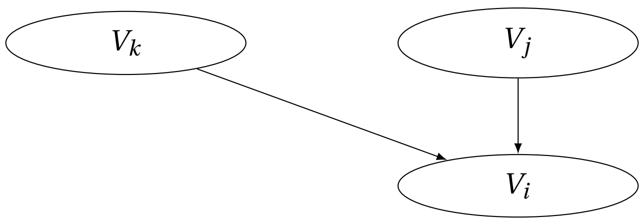


Figure 5: Children of V_i

In this case Forbidden set $F_{new} = F \cup \{V_j, V_k\}$,(Here V_k means the parents of V_i's children, if any), and we add V_j to our graph, i.e, we only add one of its children to our graph.

- case III : Parents of V_i's children, shown in figure 6

  In this case Forbidden set $F_{new} = F \cup \{V_j, V_k\}$,(Here V_k means the common children of V_i and V_j, if any), and we add V_j to our graph, i.e, we only add one of these nodes to our graph.

Based on this algorithm, we can construct the support graph.

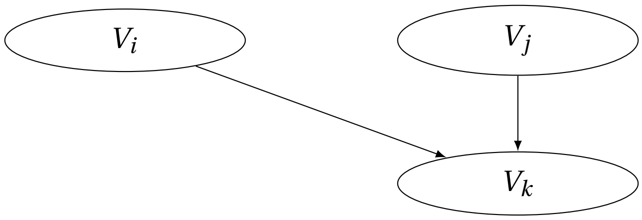


Figure 6: Parents of children

- step 1: create a node $N^* = V^* = \mathbf{G}$, **G** means the support graph, the result is shown in figure 7a, and now the forbidden set becomes $F_{new} = G$.
- step 2: Because of Case II and Case III, we can add E_{cr} and C to **G**, the result is shown in figure 7b, and now the forbidden set becomes $F_{new} = G, E_{Cr}, C$
- step 3: repeat step 2, we can add E_{Con} to the graph **G**, the result is shown in figure 7c, and now the forbidden set becomes $F_{new} = G, E_{Cr}, C, E_{Con}$

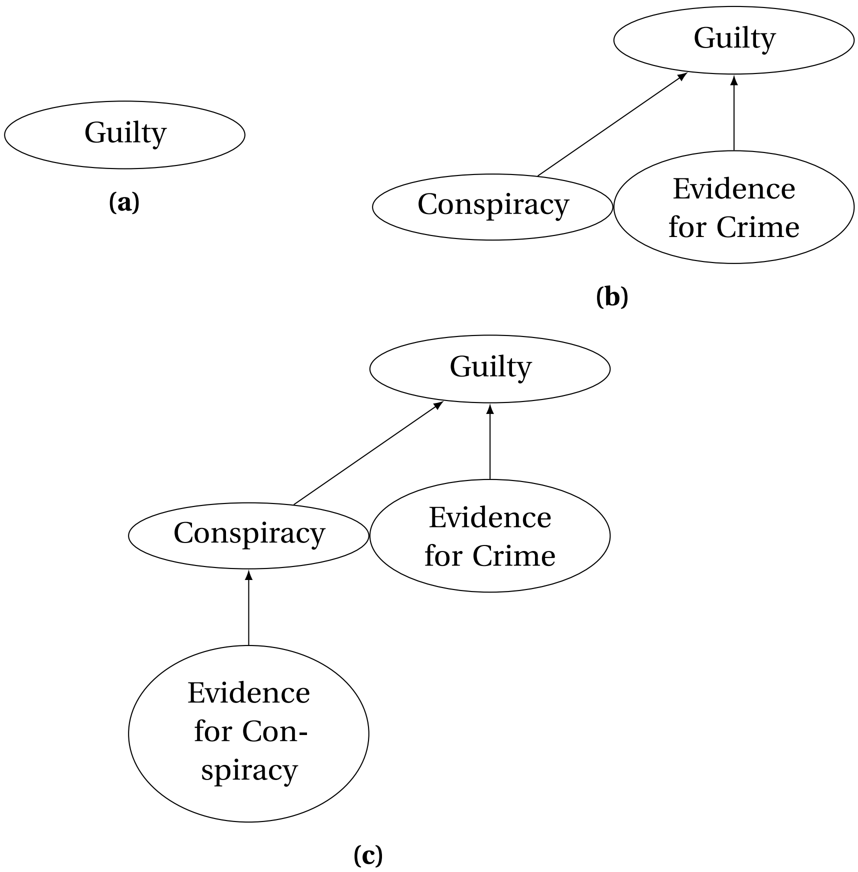


Figure 7: A runing example of deriving the support graph

3.3 Argumentation Construction

In this section we'll construct a ground extension, instead of a conventional Argumentation System, based on the support graph. First we prune the support graph to get the chains start with evidence and end with V^*, these chains can be directly used to generate the defeasible rules, and these arguments (chains) become the ground extension. In this case, we let the evidence e = E_{Cr}=true, E_{Con} = false

3.3.1 pruned support graph

We can prune the support graph by repeatdedly removing the nodes N in G which either:

- N is the ancestor of evidence
- N doesn't belong to evidence, and has no unpruned parents

Based on these rules, we can find that no node satisfied 2 conditions, so after pruning, our graph doesn't change.

3.3.2 strength calculation

In this report we use Likelihood ratio to calculate the strength. The LR strength of an assignment $V_i = o$ for a given support graph node N_i with $V(N_i) = V_i$ is

$$strength = \frac{P(premises(N_i)|(V_i=o) \cap context(N_i))}{P(premises(N_i)|(V_i \neq o) \cap context(N_i))}$$

premises(N_i) means evidences that are N_i's ancestors, and *context(N_i)* means evidences except ancestors of N_j and N_j itself Then we can use LR to calculate the strength of each node.

1. Node G

   If G = true, then premise(G) = {E_{Cr}, E_{Con}} and context(G) = {}.

   strength(G = true) = P(E$_{Cr}$= true, E_{Con} = false|G=true)$\frac{}{P(}$E$_{Cr}$=true,E_{Con}=false|G$\neq true$)

- if C = true

$$str1 = \frac{0.9 \times 0.1 \times 0.01}{0.8 \times 0.1 \times 0.01} = \frac{9}{8}$$

- if C = flase

$$str1' = \frac{0.9}{0.001} = 90$$

If G = false, the strength is the reciprocal of the strength of G=true, so we can get str2 = $\frac{8}{9}$(C = true) and str2' = $\frac{1}{90}$(C = false). And we can easily find that str2 > str1 and str2' > str2, which means no matter which value the node C choose, G = false is always defeated by G = true

Node C
If C = true, then premise(C) = {E_{Con} = false}, context(C)= {E_{Cr} = true}

strength(C = true) = P(E$_{Con}$ = false|E_{Cr} = true, C = true)$\frac{}{P(}$E$_{Con}$ = false|E_{Cr} = true, C = false)
Then we can get

$$\text{str3} = \text{strength(C = true)} = \frac{0.1}{0.9} = \frac{1}{9}$$

Similarly, we can get str4 = strength(C = false) = $\frac{1}{str3}$ = 9, then wen can know cons = true is defeated by cons = false.

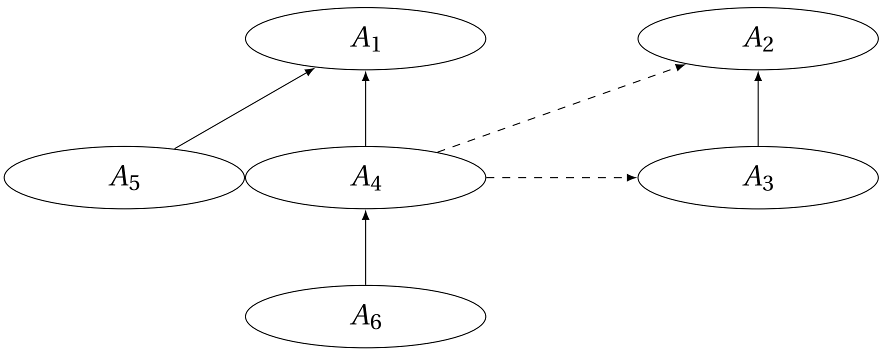


Figure 8: Final result of case 1

3.3.3 Final result

Firstly, based on the evidence and the support graph we can get the following arguments

$$A_1 : G = true \qquad A_3 : C = true \qquad A_5 : E_{Cr} = true \tag{2}$$

$$A_2 : G = false \qquad A_4 : C = false \qquad A_6 : E_{Con} = false \tag{3}$$

Then we can get the final arguments(Figure 8, dashed arrows mean defeat), and the arguments we keep are A_1, A_4, A_5, A_6.

4 CASE-2: EXTRACT EXPLANATION FROM BN(2)

In case-1, we use an efficient way, by using support graph, to derive the ground extension from BN. Here if we know the connection between the BN and Argumentation system, then we can use brute search to get the Arguments of a BN [3], which may be inefficient if the BN is complicated.

In the following parts of this section, we use the BN in [2] and use the brute search method to construct 2 arguments based on this BN.[9]

4.1 Bayesian Network

Motive T	Motive F
0.05	0.95

Motive	Psych_report T	Psych_report F
F	0.6	0.1
T	0.4	0.9

Motive	Crime T	Crime F
F	0.5	0.01
T	0.5	0.99

Twin T	Twin F
0.01	0.99

Crime	Twin	DNA_match T	DNA_match F
F	F	$10^{-}6$	$1-10^{-}6$
F	T	1.0	0.0
T	F	1.0	0.0
T	T	1.0	0.0

Figure 9: Bayesian Network of case 2

4.2 Some basic definitions

- **Explaining away**: In this case, we use *explaining away* as attacking in Argumentation system, which can be interpreted as an inferences where the second conclusion attack the first conclusion by offering another explanation for the evidence.

[9]Here we construct arguments based on ASPIC+ Argumentation

If an evidence has two explanations, say, A and B. Either A and B could bring about the evidence. Then if we observe cause A, then the belief of B drops. We can say that A explain away B, vice versa. And explaining away can be modelled as undercutting(attacking) in an Argumentation system.

- **Rule extraction**: First we find all candidate rules by simply enumerating the parents, children and parents of children of every consequent. And because of the dynamic of the BN, we need to calculate the strength of each rule to determine whether we need this rule. And the strength of the candidate rule $p_1,...,p_n \Rightarrow c$ is defined as:

$$strength(p_1,...,p_n \Rightarrow c|\epsilon) = \frac{Pr(c|p_1,...,p_n \cap \epsilon)}{Pr(c|\epsilon)}$$

  Premises $p_1,...,p_n$: parents, parents of children, children
  Evidential context ϵ: evidences except $p_1,...,p_n$

  And an *undercutter* to a candidate rule is a assignment to a varialbe u such that $strength(p_1,...,p_n \Rightarrow c|\epsilon, u) \leq 1$.

  Finally, we only keep the rules with strength bigger than 1.

4.3 Arguments derivation

If we enter the evidence {Psy_report = true, DNA_match = true}, then from the background knowledge of last section, we can derive the arguments and attack relations from BN by following steps:

- Step_1: Extract all candidate rules. Here we extract two simple rules

$$DNA\_match \Rightarrow Crime$$
$$DNA\_match \Rightarrow Twin$$

- Step_2: Caluculate the strength of each rule, and keep the rules we need. Here we can get the strength of first rule is 1.2, and the strength of second rule is 1.2.
  These two rules have same strength, because from the original BN we can know that the probabilty of finding the evidence given that there is a crime and there are twins are same.

- Step_3: Find attack(undercutting) using strength

5 CASE-3: TRANSFORM ARGUMENTATION FRAMEWORK TO BAYESIAN NETWORK

In this section, we adapt an BN in [2] to an Argumentation Framework, and then use a heuristic method [9] to convert it to a corresponding BN which holds the same information but with different properties.

5.1 Argumentation system

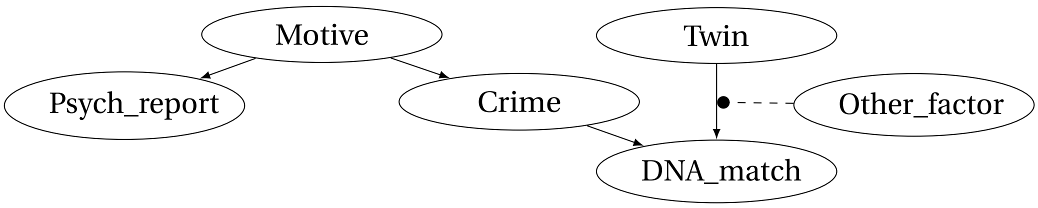


Figure 10: Argumentation system

5.2 Conversion Rules

Constraint 5.1 (Nodes and Values)**.** *For every atomic proposition in a structured Argumentation Framework(SAF),there exists a corresponding node in BN.*

Constraint 5.2 (Inference chains)**.** *For every rules in a SAF, there exist active chains between all nodes appearing in that rule, given the observed nodes.*

Constraint 5.3 (Attack chains)**.** *For every attack relation (A,B), then there exist active chains between nodes related to the attack(undercut or contrary).*

Constraint 5.4 (Inference probabilities)**.** *To get the probability of inferences in the Argumentation system*

- *For every strict rule* $r : \phi_1, ..., \phi_n \rightarrow \psi$, *we have* $P(\psi|\phi_1, ..., \phi_n) = 1$;
- *For every defeasible rule* $r : \phi_1, ..., \phi_n \Rightarrow \psi$, *we have* $P(\psi|\phi_1, ..., \phi_n) > 0$.

Constraint 5.5 (Attack probabilities)**.** *To get the probability of attacks in the Argumentation system*

- *If A attack B, then P(B|A)=0;*
- *If A undercuts B, then P(B) < P(A|B).*

5.3 Conversion Procedures

* Step1-Construct nodes based on Constraint 5.1

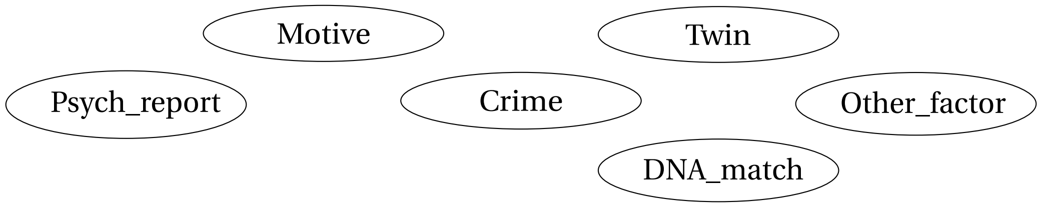


Figure 11: Step1 - Get Nodes

* Step2-Based on constraint 5.2 and 5.3, we get undirected graph

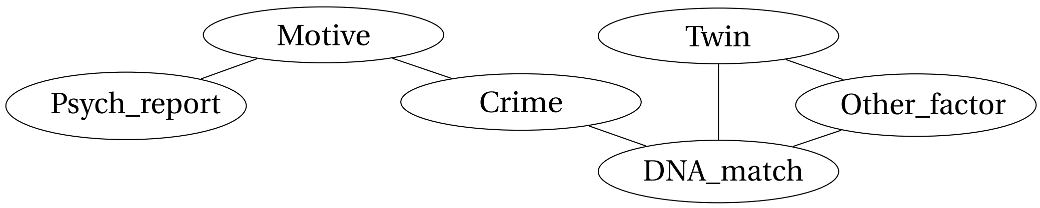


Figure 12: Step2 - Undirected Graph

* Step3-Remove edges:For all undercutting attacks, if a rule $r : \phi_1, ..., \phi_n \Rightarrow \psi$ is undercut by $\chi \in \overline{n(r)}$, then remove edge(χ, ψ)(or (ψ, χ)) and turn the other edges involved in to the following directed ones:(χ, ϕ_i) and (ψ, ϕ_i) for all ϕ_i.

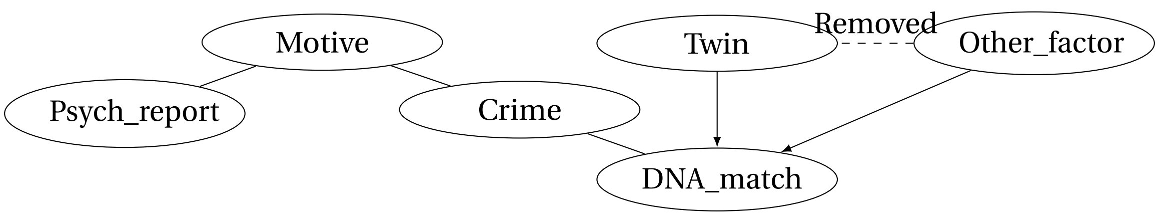


Figure 13: Step4 - Pruned Graph

* Step4-Choose causal direction:If there exist causal interpretation for a edge in the graph, then choose a direction that fit the causal interpretation, if not, then we choose an arbitrary direction.

* step5-Verify d-separation

* Step6-assign the probability based on constraint 5.4 and 5.5, then we can know

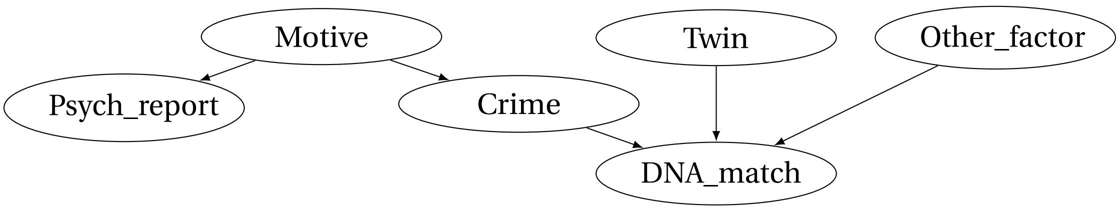


Figure 14: Step6 - Graph of Bayesian Network

- P(Psy|motive) > 0, P(Cr|motive) > 0, P(DNA|Cr,Twin) > 0
- P(DNA|Twin) = 1
- P(DNA|Cr,Twin) < Pr(DNA| Cr, Twin, Others)

Based on these constraints, we get

Motive T	Motive F
0.05	0.95

Others T	Others F
0.01	0.99

Motive	Psych_report T	Psych_report F
F	0.6	0.1
T	0.4	0.9

Motive	Crime T	Crime F
F	0.5	0.01
T	0.5	0.99

Twin T	Twin F
0.01	0.99

Crime	Twin	DNA_match T	DNA_match F
F	F	10¯6	1 − 10¯6
F	T	1.0	0.0
T	F	1.0	0.0
T	T	1.0	0.0

Figure 15: Step6 : Tables of Bayesian Network

6 Case-4: Combination of Bayesian Network and Argumentation(Probabilistic ABA)

In this section, we first introduce the definition of PABA, and then convert some common schemes[5] to PABA and ABA, then we use inference procedures(derivation)[4] to get the semantics of one of these schemes.

6.1 Probabilistic ABA

In the concrete cases in our world, we don't always have certain beliefs about some assumptions, so we add a new type of assumption, probabilistic assumption A_p, to normal ABA. Here we define assumptions as sentences that are open to challenge, and the assumptions become a probabilistic one if the belief(strength) of this assumption is uncertain. Then a PABA framework P can be defined as $\{A_p, R_p, F\}$, where

- A_p means probabilistic assumptions, it represent the uncertain belief about your assumptions.
- R_p means rules containing A_p
- F means ABA, which is $F = (R, A, ^-)$.

6.1.1 Basic definitions in PABA

Definition 11 (frame , [4])**.** *A frame is a set of partial world. A partial world is a set that contains few possible probabilistic assumptions which is possible.*

In this case, our final goal is to get a cr-frame. [10]

6.2 Convert schemes to ABA and PABA

To convert schemes to ABA and PABA, there are 2 basic steps

- Step-1 : Transform critical questions to assumptions.
- Step-2 : Extract rules from the scheme.

[10]i.e credulous frame, a frame that is credulous in Argumentation

In ABA, we can simply count all CQs as assumptions, but in PABA, we should first confirm that if this critical question is statistical, if it's statistical, then it can be transformed into probabilistic assumption, if not, then it belongs to normal assumptions. For example, in scheme-3:

- Question *How credible is E as an expert source* is a statistical question, because the credibility can be represented as probability
- Question *Is E an expert in the field that A is in* is a non-statistical question, because this question can be answered by simply observing it.

6.2.1 Scheme-1: Argument from sign

- Scheme

  A (a finding) is true in this situation.

  B is generally indicated as true when its sign, A, is true.

  B is true in this situation.

- Critical questions

  CQ1:What is the strength of the correlation of the sign with the event signified?

  CQ2:Are there other events that would more reliably account for the sign?

- ABA

  - A

    sign(A,B) - A is a sign of B

    strength(A) - correlation between A and B is strong

  - R

    $A \leftarrow \qquad B \leftarrow A, sign(A,B), strength(B)$

- PABA

  - A_p

    $P_{cor}(A)$: correlation between A and B

    $P_{rel}(other)$: there exist other signs that are more reliable than A

- R_p

  $[P_{cor}(A){:}0.7] \leftarrow \neg P_{rel}(other)$ $[P_{cor}(A){:}0.3] \leftarrow \mathrm{P}_{rel}(other)$

  $[P_{rel}(other){:}0.5] \leftarrow$

- $A' = A$
- R'

  $A \leftarrow;$ $B \leftarrow A, sign(A, B), P_{cor}(A)$

6.2.2 Scheme-2: Argument from evidence to a hypothesis

- Scheme

  Major premise If A (a hypothesis) is true, then B (a proposiion reporting an event) will be observed to be true.

  Minor premise B has been observed to be true, in a given instance.

  Conclusion Therefore, A is true

- Critical questions

  CQ1:Is it the case that if A is true, then B is true?

  CQ2:Has B been observed to be true?

  CQ3:Could there be some reason why B is true, other than its being because of A being true? (other factors)

- ABA

  - A

    observed(B), correlated(B), correlated(others)

  - R

    $correlated(B) \leftarrow \neg correlated(others)$ $B \leftarrow observed(B)$

    $A \leftarrow true(B), correlated(B)$

- PABA

  - A_p

    P_{cor_B} : correlation between A and B

    P_{cor_oth} : there exist other reasons

– R_p

$[P_{cor\_oth} : 0.9] \leftarrow \neg P_{cor\_oth}$ $[P_{cor\_B}:0.1] \leftarrow \mathrm{P}_{cor\_oth}$

(These two rules can be explained as correlation between A and B is affected by other reliable factors)

$[P_{cor\_oth}:0.3] \leftarrow$

– A'

observed(B)

– R'

$B \leftarrow$ observed(B) $\mathrm{A} \leftarrow B, P_{cor\_oth}$

6.2.3 Scheme-3: Argument from Expert opinion

- Scheme

  Source E is an expert in subject domain S containing proposition A.

  E asserts that proposition A (in domain S) is true(false)

  A may plausibly be taken to be true(false)

- Critical questions

  CQ1:How credible is E as an expert source?

  CQ2:Is E an expert in the field that A is in?

  CQ3:What did E assert that implies A?

  CQ4:Is E personally reliable as a source?

  CQ5:Is A consistent with what other experts assert?

  CQ6:Is E's assertion based on evidence?

  (There are also some sub-CQs, and they can also be treated as assumptions in ABA framework)

- ABA

  – A

  credible(expert(E)), expert(E), assert(A),
  credible(E), consistent(A), evidence(E)

- R

  A ← assert(A), credible(expert(E)), credible(E)

  credible(expert(E)) ← expert(E)

  credible(E) ← consistent(A), evidence(E)

- PABA

  - A_p

    P_{cre} : credibility of E as an expert

    P_{rel} : reliability of E as a person

  - R_p

    [P_{cre}:0.8] ← expert(E) [P_{cre} : 0.1] ← ¬ expert(E)

    [P_{rel}:0.7] ← consistent(A) [P_{rel}:0.7] ← ¬ consistent(A)

  - A'

    expert(E), assert(A), consistent(A), assert(A)

  - R'

    A ← assert(A), P_{cre}, P_{rel}

6.2.4 Scheme-4: The appeal to popular opinion

- Scheme

  S1: Everybody (in a particular reference group) accepts that A. Therefore, A is true (or you should accept A).

  S2: Everybody (in a particular reference group) rejects A. Therefore, A is false (or you should reject A).

- Critical questions

  CQ1:Does a large majority of the cited reference group accept A s true?

  CQ2:Is there other relevant evidence available that would support the assumption that A is not true?

  CQ3:What reason is there for thinking that the view of this large majority is likely to be right?

  (Position to know ad popular argument is one of the answer of this question)

- Position to know Everybody in this group G accepts A.

  This group is in a special position to know that A is true.

  Therefore, A is (plausibly) true.

- ABA
  - A

    accept(A)-large majority of the group accept A as true(false)

    others(A)-other factors

    position(A) - position to know
  - R

    A ← accept(A), right(maj)

    right(maj) ← ¬ others(A), position(A)

- PABA
  - A_p

    P_{oth} : other factors

    P_{right} : the view of majority is right
  - R_p

    $[P_{right}:0.8] \leftarrow \neg P_{oth}$, position(A) $[P_{right}:0.8] \leftarrow \neg P_{oth}$, ¬ position(A)

    $[P_{right}:0.2] \leftarrow P_{oth}$, ¬ position(A) $[P_{right}:0.5] \leftarrow \neg P_{oth}$, position(A)

    $[P_{oth}:0.3] \leftarrow$
  - A'

    accept(A), position(A)
  - R'

    A ← accept(A), P_{right}

6.3 Extract semantics from PABA

6.3.1 PABA framework

We adapt the PABA framework of scheme-4 to make it well-formed, then the PABA framework become:

- A

  a(A), p(G)

- A$_p$

  $P_1(P_{oth})$,$P_2(P_{right})$

- R$_p$

  $[P_2{:}0.8] \leftarrow P_1$ $[P_1{:}0.3] \leftarrow$

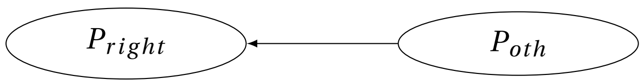
  $[P_2{:}0.8] \leftarrow \neg\, P_1$

- R

  A ← a(A), p2 ¬ a(A) ← ¬ p1

6.3.2 Bayesian Network constructed from PABA

P_{oth}	P_{right} T	P_{right} F
F	0.8	0.2
T	0.2	0.8



P_{oth} T	P_{oth} F
0.3	0.7

Figure 16: Bayesian Network of PABA above

6.3.3 Extract complete(credulous) admissible frames

	P	O	A	C	w
T_1	A	ϕ	ϕ	ϕ	ϕ
T_2	$\underline{\text{a(A)}}$,P_2	ϕ	ϕ	ϕ	ϕ
T_3	P_2	$\underline{\neg P_1}$	{a(A)}	ϕ	ϕ
T_4	P_2	$\underline{\neg P_1}$	{a(A)}	ϕ	$\{\neg P_1\}$
	P_2	$\underline{\neg P_1}$	{a(A)}	ϕ	$\{P_1\}$
T_5	$\underline{P_2}$	ϕ	{a(A)}	ϕ	$\{P_1\}$
T_6	$\underline{P_2}$	ϕ	{a(A)}	ϕ	$\{P_1, P_2\}$
	$\underline{P_2}$	ϕ	{a(A)}	ϕ	$\{P_1, \neg P_2\}$
T_7	ϕ	ϕ	{a(A)}	ϕ	$\underline{\{P_1, P_2\}}$

Table 1: Derivation of complete admissible frame

Finally we know the complete admissible frame of this PABA is $\{P_1, P_2\}$

6.4 Compute probability

To compute the probability of a frame, we first simplify the frame, then compute the probability.

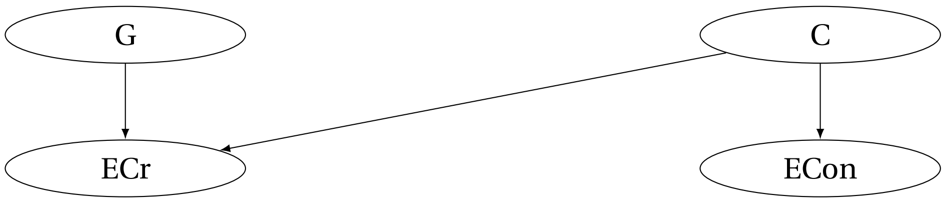
7 Case-5: Combination of Bayesian Network and Argumentation Argumentation network)

In this case, we use a method to convert a BN in [3] to a new kind of network, named Bayesian Argumentation Network [10].

7.1 Bayesian Network

Guilty	
T	F
0.1	0.9

Conspiracy	
T	F
0.01	0.99



		Evidence for Crime	
Guilty	Conspiracy	T	F
F	F	0.001	0.999
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.9	0.1

	Evidence for Conspiracy	
Conspiracy	T	F
F	0.9	0.1
T	0.1	0.9

7.2 Bayesian Argumentation Network

In this part we will introduce the definition of Bayesian Argumentation Networks (BANs).

Definition 12 (Bayesian Argumentation Network, [10]). *A BAN is a triple S, R, e, where S is a set of arguments, R is the attack relation between non-empty subsets of S and an element of S, e is a transmission function giving each h in the pair (H,x) a value* $\in [0,1]$

In the following graph, we can see H = $h_1, ..., h_k$, and e(H,x, h_k) = e_k

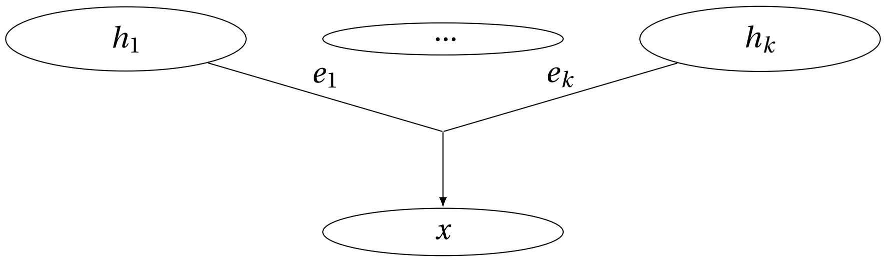


Figure 17: Transmission function

7.3 Translation from Bayesian Network to Bayesian Argument Network

In order to get the new Network, we first get the corresponding BAN of ECr, so we add 2 nodes $\neg G$ and $\neg C$, and add point c_{00}, c_{01}, c_{10} and c_{11} which means

$$\begin{aligned}
\mathrm{c}_{00} &= \neg G \wedge \neg C \\
\mathrm{c}_{01} &= \neg G \wedge C \\
\mathrm{c}_{10} &= G \wedge \neg C \\
\mathrm{c}_{11} &= G \wedge C
\end{aligned}$$

Then we add attacks from G and C to all c_{ij} and get

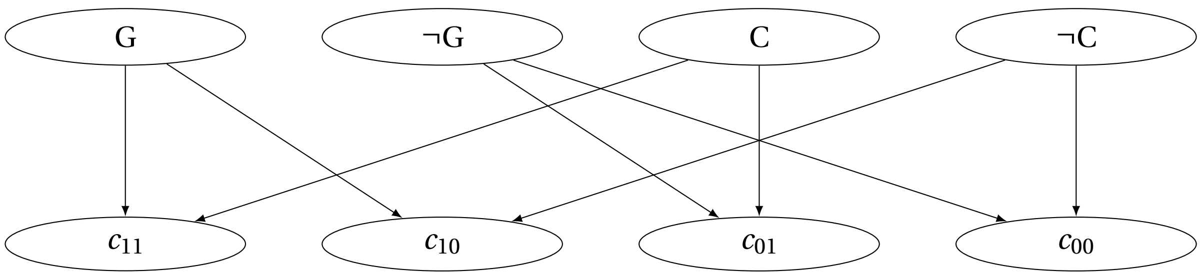


Figure 18: After adding attacks

And now we can compute the probabilities of c_{ij}

$$\begin{aligned}
\mathrm{P}(\mathrm{c}_{11}) &= (1 - P(C)) \cdot (1 - P(G)) = 0.9 \times 0.99 = 0.891 \\
\mathrm{P}(\mathrm{c}_{10}) &= (1 - P(G)) \cdot (1 - (1 - P(C))) = (1 - P(G)) \cdot P(C) = 0.9 \times 0.01 = 0.009
\end{aligned}$$

$\mathrm{P}(\mathrm{c}_{10}) = (1 - P(C)) \cdot (1 - (1 - P(G))) = (1 - P(C)) \cdot P(G) = 0.1 \times 0.99 = 0.099$
$\mathrm{P}(\mathrm{c}_{10}) = (1 - (1 - P(G))) \cdot (1 - (1 - P(G))) = P(G) \cdot P(C) = 0.1 \times 0.01 = 0.001$

Then we add an additional node x_{ij} for each c_{ij} to incorporate the transmission factors e_{ij}, now we get

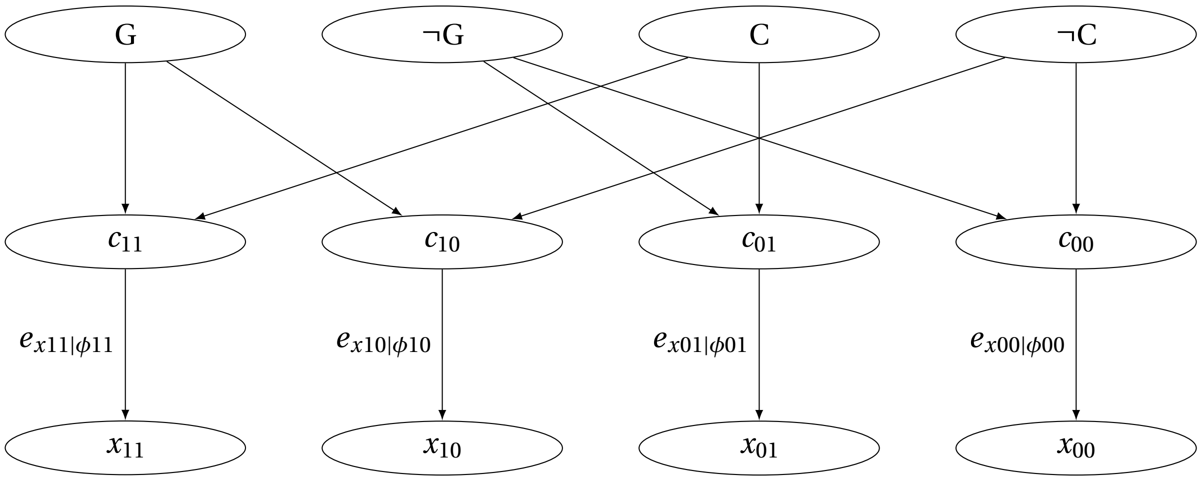


Figure 19: Adding additional nodes

And $e_{xij|\phi ij}$ can be find in the tables of original BN, we can get that

$\mathrm{e}_{x11|\phi11} = 0.9, P(X_{11}) = 1 - e_{x11|\phi11} \cdot c_{11} = 0.1981$
$\mathrm{e}_{x10|\phi10} = 0.9, P(X_{11}) = 1 - e_{x10|\phi10} \cdot c_{10} = 0.9919$
$\mathrm{e}_{x01|\phi01} = 0.8, P(X_{11}) = 1 - e_{x01|\phi01} \cdot c_{01} = 0.9992$
$\mathrm{e}_{x00|\phi00} = 0.001, P(X_{11}) = 1 - e_{x00|\phi00} \cdot c_{00} = 0.999999$

All x_{ij} jointly attack ECr, and we can do the similar thing to node ECon, so the final graph is
And Finally we can get the probability of ECr

$$= min\{1, \sum(1 - P(x_{ij}))\} \tag{4}$$
$$= min\{1, 0.810801\} \tag{5}$$
$$= 0.810801 \tag{6}$$

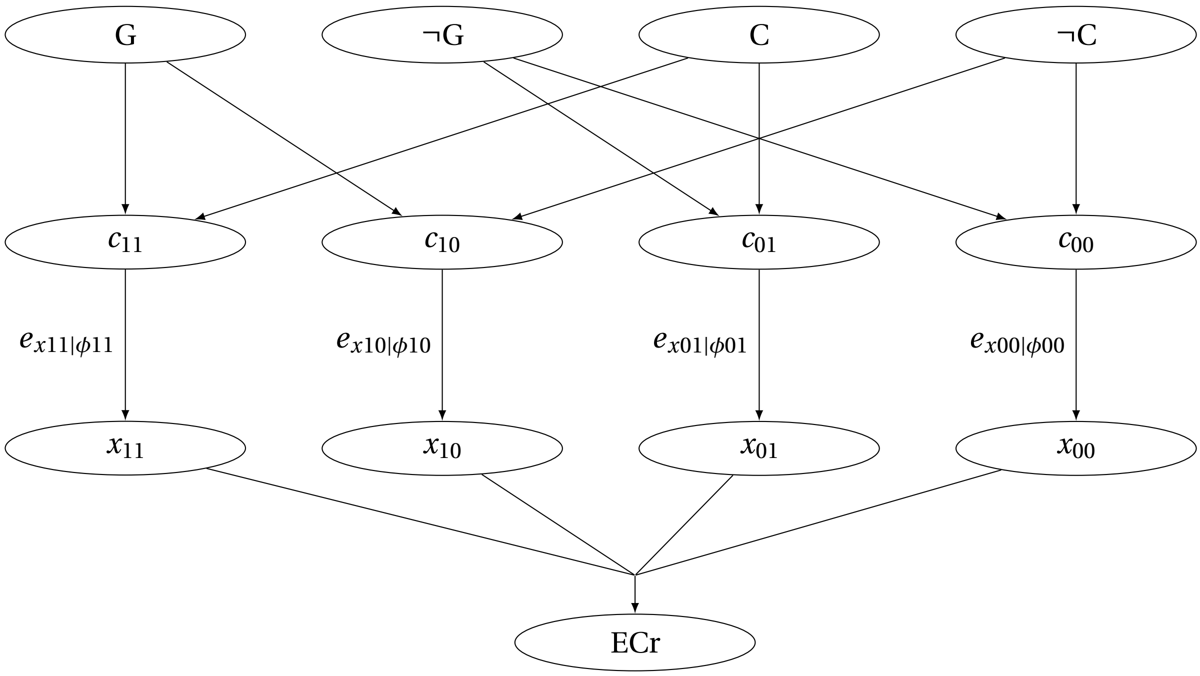


Figure 20: Bayesian Argumentation Network

8 CONCLUSION

Several models or methods have been proposed to combine the strength of Probabilistic reasoning (Bayesian Inference) and Argumentation. And in this report, we basically choose 3 directions, from BN to Argumentation, from Argumentation to BN, from Argumentation or BN to new model, in combining BN and Argumentation, then we choose 3 methods and 2 new models and made comprehensive case-study of them.

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