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Bayesian Inference and Argumentation

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Bayesian Network (Formal definition)

Definition 1 (Bayesian Network)**.** *A BN is a triple $B = (V, A, P)$ where:*

- $G = (V, A)$ *is an acyclic directed graph with nodes V and arcs $A \subset V \times V$ ($arc(V_i, V_j)$ is directed from V_i to V_j);*
- $P = \{Pr_v \mid v \in V\}$*, each Pr_v is the probability of variable v, and these probabilities are typically represented as tables.*

Bayesian Network

Graph : Dependencies

- Suggested by directed arcs
- These arcs only have means in combination with other arcs : d-separation

Conditional Probability Tables : Distribution

- Joint probability:

$$Pr(V) = \prod_{v \in V} Pr(v|par(v))$$

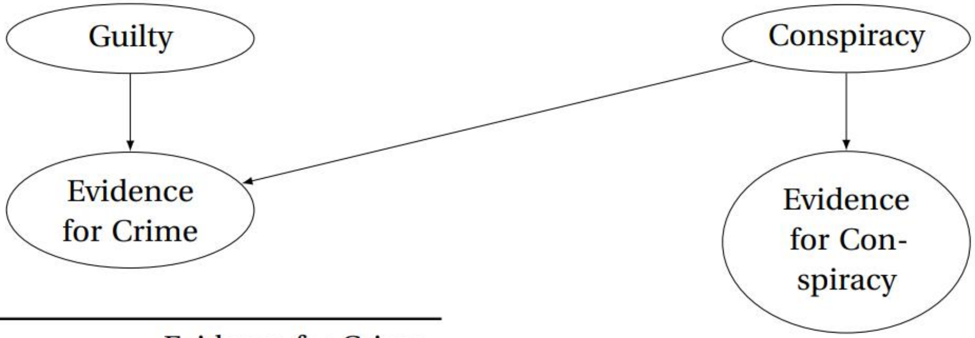
V is a set of variables

par(v) is the parents of variable v which v is dpendent on

Bayesian Network (A simple example)

Guilty T	Guilty F
0.1	0.9

Conspiracy T	Conspiracy F
0.01	0.99



Guilty	Conspiracy	Evidence for Crime T	Evidence for Crime F
F	F	0.001	0.999
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.9	0.1

Conspiracy	Evidence for Conspiracy T	Evidence for Conspiracy F
T	0.9	0.1
F	0.1	0.9

Bayesian Network (d-separation)

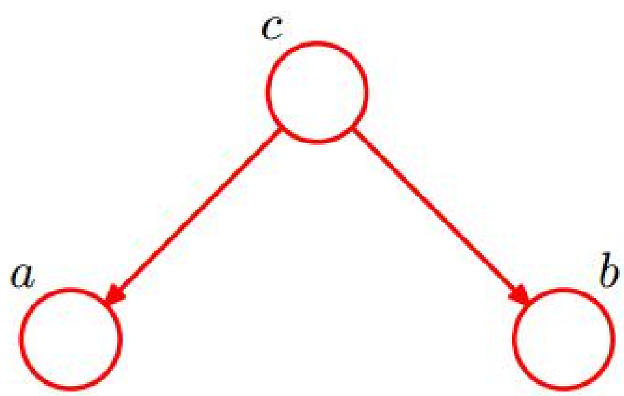
- A undirected path is blocked when there exist a node X in this path, either

    - *X is a head to tail or tail to tail node, and X belongs to C*
    - *X is a head to head node, and X and its children don't belong to C*

- If A and B are d-separated by observed variables C, then all undirected paths between A and B are blocked.

Bayesian Network (d-separation)

- Tail to tail



- If c is not observed

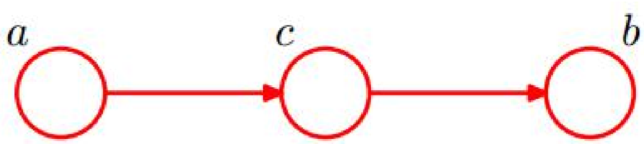
$$p(a,b) = \sum_c p(a|c)p(b|c)p(c).$$

- If c is obeserved, a and b are d-separated by c

$$\begin{aligned} p(a,b|c) &= \frac{p(a,b,c)}{p(c)} \\ &= p(a|c)p(b|c) \end{aligned}$$

Bayesian Network (d-separation)

- Head to tail



- If c is not observed

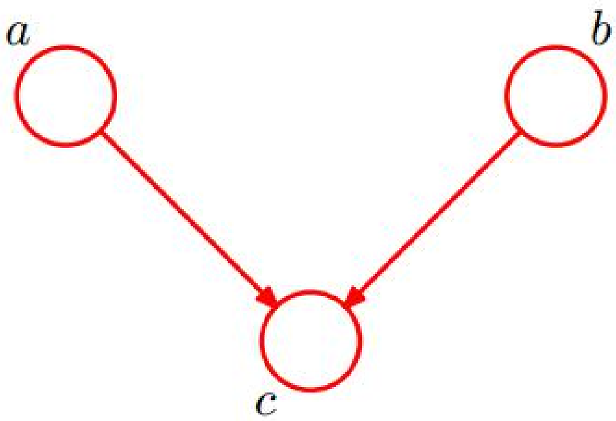
$$p(a,b) = p(a)\sum_c p(c|a)p(b|c) = p(a)p(b|a).$$

- If c is obeserved, a and b are d-separated by c

$$\begin{aligned} p(a,b|c) &= \frac{p(a,b,c)}{p(c)} \\ &= p(a|c)p(b|c) \end{aligned}$$

Bayesian Network (d-separation)

- Head to head



- If c is not observed, and c doesn't have any children, so a and b are d-separated by c

  p(a,b)=p(a)*p(b)

- If c is not observed, then

$$
\begin{aligned}
p(a,b|c) &= \frac{p(a,b,c)}{p(c)} \\
&= \frac{p(a)p(b)p(c|a,b)}{p(c)}
\end{aligned}
$$

Bayesian Network (Conclusion)

- if a and b are independent, then joint probablity of a and b is

$$p(a,b) = p(a) * p(b)$$

- if a and b are d-separated by c, which means a and b are independent, then

$$p(a,b|c)=p(a|c)*p(b|c)$$

Argumentation schemes

Scheme

(the argument from sign scheme)

- A (a finding) is true in this situation.
- B is generally indicated as true when its sign, A, is true.
- B is true in this situation.

Critical Questions

(serves as scheme-specific norms)

- CQ1:What is the strength of the correlation of the sign with the event signified?
- CQ2:Are there other events that would more reliably account for the sign?

Abstract Argumenation(AA) and Assumption-Based Argumenation(ABA)

AA

- A pair (AR, Att)
  - AR : Arguments
  - Att : Attack relations over arguments

ABA

- An instance of AA which shows the internal structure of arguments
- Given a language, an ABA is a triple (R, A, $\bar{\ }$)
  - R : a set of inference rules derived from the language
  - A : a set of assumptions
  - $\bar{\ }$: one-to-one mapping from A into L, where $\overline{x}$ is referred to as the contrary of x.

ASPIC + Argumentation

Given a Language L and a knowledgebase $K_n \subseteq L$, then an argument can be

- ψ, if $\psi \in K_n$, and we define

$$Premise(A) = \psi$$

$$Conclusion(A) = \psi$$

$$SubArgument(A) = \psi$$

$$TopRule(A)^7 = undefined$$

$$ImmediateSubArgument(A) = \emptyset$$

$$DefeasibleRules(A) = \emptyset$$

ASPIC + Argumentation

Given a Language L and a knowledgebase $K_n \subseteq L$, then an argument can be

- $A_1, ..., A_n \Rightarrow \psi$, *if* $A_1, ..., A_n$ *are arguments such that there is a defeasible rule* $Conclusion(A_1)$, ... , $Conclusion(A_n) \Rightarrow \psi$ *in* R_d, *and we define*

  $Premise(A) = Premise(A_1) \cup ... \cup Premise(A_n)$

  $Conclusion(A) = \psi$

  $SubArgument(A) = SubArgument(A_1) \cup ... \cup SubArgument(A_n) \cup A$

  $TopRule(A) = Conclusion(A_1), ..., Conclusion(A_n) \Rightarrow \psi$

  $ImmediateSubArgument(A) = A_1, ..., A_n$

  $DefeasibleRules(A) = DefeasibleRules(A_1) \cup ... \cup DefeasibleRules(A_n) \cup TopRule(A)$

Extensions

Extensions are important when we want to judge if an argument is acceptable

A set of arguments S is

- **conflict-free** (c-f) iff it does not attack itself;
- **admissible** iff it's c-f and attacks each attacking argument;
- **preferred**(credulous) iff it's maximal admissible;
- **complete** iff it's admissible and contains all arguments it defends(by attacking all attacks against them) ;
- **grounded** iff it is the set inclusion minimal complete extension;
- **stable** iff it is preferred and it defeat any arguments outside S.

Bayesian Inference and Argumentation

Bayesian Network

- Allows for computing probability of interest over its variables
- Implicit causal relationship

Argumentation

- model conflicting or contradictory information by using attack relationship between arguments
- model uncertainty by using defeasible rules (Rules which have exceptions)
- No accurate probability can be derived.

How to combine them?

- Explaining Bayesian Network by Argumentation
- Extracting (modifying) Bayesian Network based on Argumentation
- Combining Argumentation and Bayesian Network to get a new model (Network)

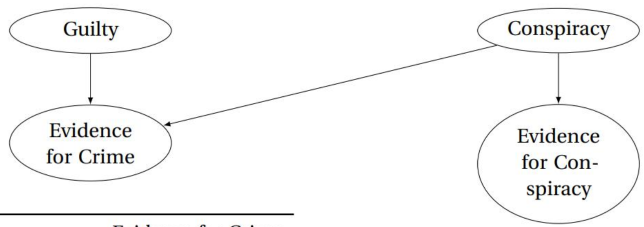
Case-1

Extract explaination from BN using a 2-phase method

- Evidence for Crime(ECr) = true
- Evidence for Conspiracy(ECon) = false
- interested variable = Guilty

Guilty T	Guilty F
0.1	0.9

Conspiracy T	Conspiracy F
0.01	0.99



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T	T	0.9	0.1

Conspiracy	Evidence for Conspiracy T	Evidence for Conspiracy F
T	0.9	0.1
F	0.1	0.9

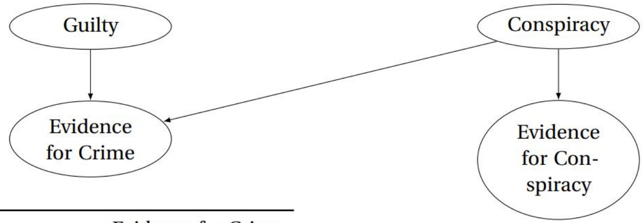
Case-1

Extract explaination from BN using a 2-phase method

- step-1: construct the support graph
- step-2: calculate the strength and find the defeat relations, then keep the undefeated arguments (grounded)

Guilty	
T	F
0.1	0.9

Conspiracy	
T	F
0.01	0.99



		Evidence for Crime	
Guilty	Conspiracy	T	F
F	F	0.001	0.999
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.9	0.1

	Evidence for Conspiracy	
Conspiracy	T	F
T	0.9	0.1
F	0.1	0.9

Support chain

Definition 9. *A Support Chain* of a variable V^* is a set of nodes (chain) such that.

- *there are no repeated nodes*
- *For each immorality*(V_i, V_j, V_k)*(shown in figure 2),we skip* V_j *(i.e,* V_j *is not included in support chain)*
- *ends in* V^*.

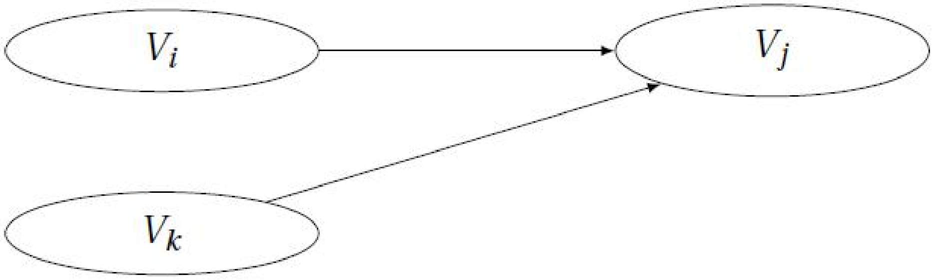
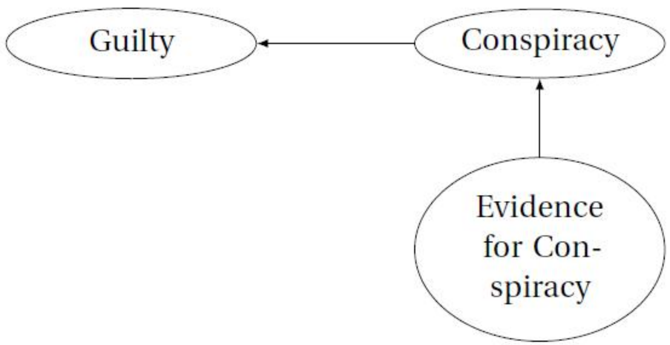


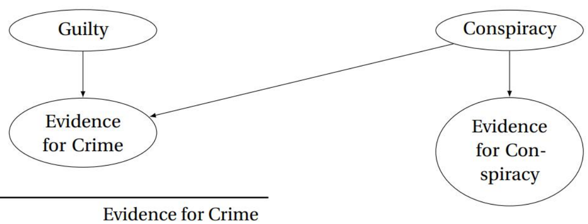
Figure 2: immorality

An example of support chain



Guilty	
T	F
0.1	0.9

Conspiracy	
T	F
0.01	0.99



		Evidence for Crime	
Guilty	Conspiracy	T	F
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	Evidence for Conspiracy	
Conspiracy	T	F
T	0.9	0.1
F	0.1	0.9

Support Graph

- Definition: A graph contains all support chains
- How to get it:
  - enumerating all support chains
  - using algorithm

Algorithm of deriving Support Graph

- A Forbidden set F is a set of variables that can't be used in other support chains

- Nodes we should add to graph **G** and forbiden set F

  - case I : parents of V_i, shown in figure 4

Figure 4: Parents of V_i

In this case Forbidden set $F_{new} = F \cup \{V_j\}$, and we add V_j to our graph

Algorithm of deriving Support Graph

- A Forbidden set F is a set of variables that can't be used in other support chains
- Nodes we should add to graph **G** and forbiden set F

• case II : Children of V_i, shown in figure 5

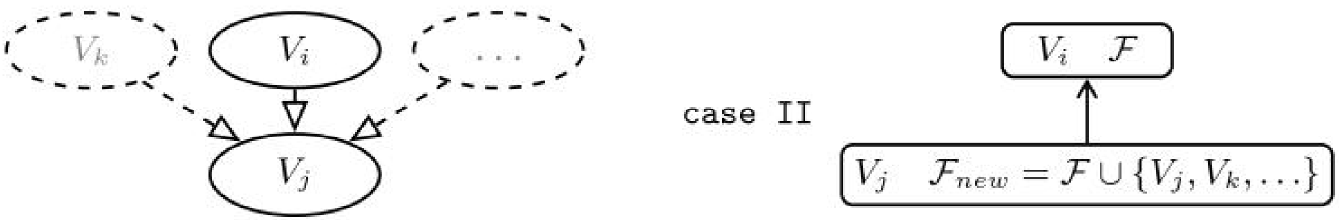


Figure 5: Children of V_i

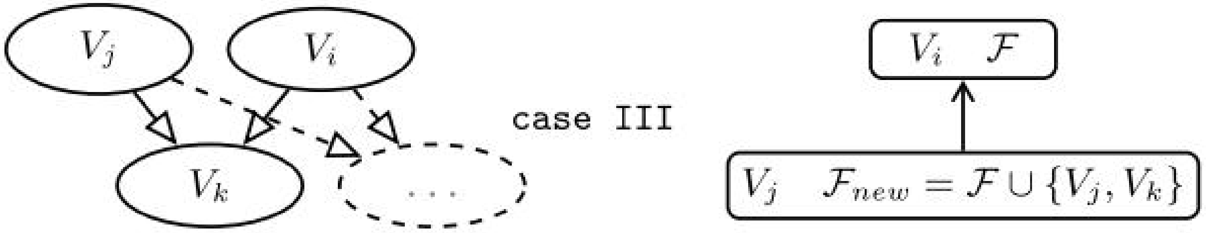
In this case Forbidden set $F_{new} = F \cup \{V_j, V_k\}$,(Here V_k means the parents of V_i's children, if any), and we add V_j to our graph, i.e, we only add one of its children to our graph.

Algorithm of deriving Support Graph

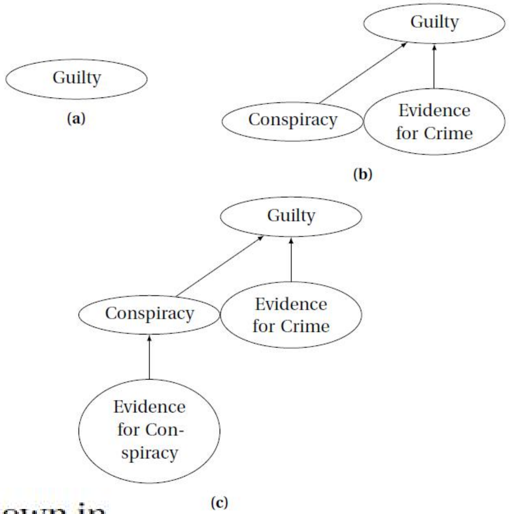
- A Forbidden set F is a set of variables that can't be used in other support chains

- Nodes we should add to graph **G** and forbiden set F
  - case III : Parents of V_i's children, shown in figure 6

    In this case Forbidden set $F_{new} = F \cup \{V_j, V_k\}$,(Here V_k means the common children of V_i and V_j, if any), and we add V_j to our graph, i.e, we only add one of these nodes to our graph.



An example of deriving support graph



- step 1: create a node $N^* = V^* = \mathbf{G}$, $\mathbf{G}$ means the support graph, the result is shown in figure 7a, and now the forbidden set becomes $F_{new} = G$.
- step 2: Because of Case II and Case III, we can add E_{cr} and C to $\mathbf{G}$, the result is shown in figure 7b, and now the forbidden set becomes $F_{new} = G, E_{Cr}, C$
- step 3: repeat step 2, we can add E_{Con} to the graph $\mathbf{G}$, the result is shown in figure 7c, and now the forbidden set becomes $F_{new} = G, E_{Cr}, C, E_{Con}$

Strength calculation

- Likelihood ratio

$$strength = \frac{P(premises(N_i)|(V_i=o) \cap context(N_i))}{P(premises(N_i)|(V_i \neq o) \cap context(N_i))}$$

premises(N_i) means evidences that are N_i's ancestors, and *context*(N_i) means evidences except ancestors of N_j and N_j itself Then we can use LR to calculate the strength of each node.

Strength calculation

- Node Conspiracy (C)
  - if C = true

$$str3 = strength(C = true) = \frac{0.1}{0.9} = \frac{1}{9}$$

  - if C= false, the strength is the reciprocal of the strength of C=true, which is 9
  - we get strength(C=false) > strength(C=true), so argument C=true is defeated by argument C=false.

Strength calculation

- Node Guity (G)
  - if G = true

If G = true, then premise(G) = {Ecr, Econ} and context(G) = {}.

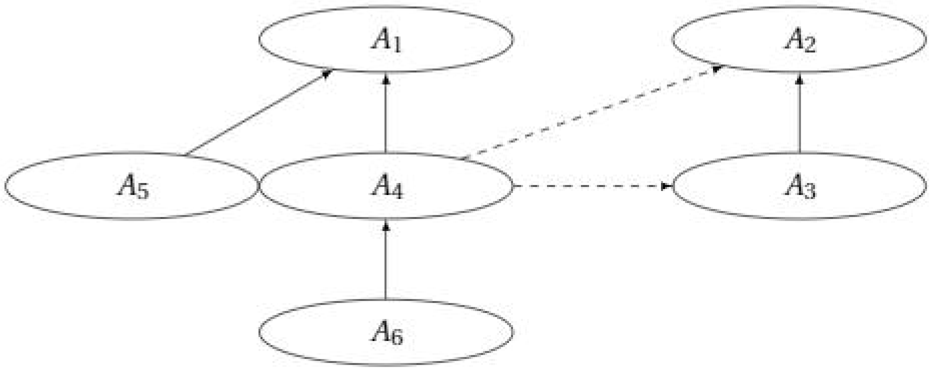
$$\text{strength}(G = \text{true}) = \frac{P(Ecr=true, Econ=false|G=true)}{P(Ecr=true, Econ=false|G\neq true)}$$

  - if G = false, the strength is the reciprocal of the strength of G=true
  - we get strength(G=true) > strength(G=false), so argument G=false is defeated by argument G=true.

Final arguments

$A_1 : G = true$ $\quad$ $A_3 : C = true$ $\quad$ $A_5 : E_{Cr} = true$

$A_2 : G = false$ $\quad$ $A_4 : C = false$ $\quad$ $A_6 : E_{Con} = false$



Case-2

Extract explaination from BN using a brute search method

- Step 1: Extract candidate rules
- Step 2: Caluculate the strength of each rule
- Step 3: Find attack(undercutting) using strength

Motive T	Motive F
0.05	0.95

Motive	Psych_report T	Psych_report F
F	0.6	0.1
T	0.4	0.9

Motive	Crime T	Crime F
F	0.5	0.01
T	0.5	0.99

Twin T	Twin F
0.01	0.99

Crime	Twin	DNA_match T	DNA_match F
F	F	10^{-6}	$1-10^{-6}$
F	T	1.0	0.0
T	F	1.0	0.0
T	T	1.0	0.0

Rules extraction

- Extract candidate rules by enumerating the parents, children and parents of children of every consequent (variables), premise can be any subsets of these nodes.
- Calculate the strength of each rule

this rule. And the strength of the candidate rule $p_1, \ldots, p_n \Rightarrow c$ is defined as:

$$strength(p_1, \ldots, p_n \Rightarrow c|\epsilon) = \frac{Pr(c|p_1, \ldots, p_n \cap \epsilon)}{Pr(c|\epsilon)}$$

Premises $p_1, \ldots, p_n$: parents, parents of children, children

Evidential context ϵ: evidences except $p_1, \ldots, p_n$

- Keep the rules with strength bigger than 1
- Strength can also help us to explain the BN

Attack

Attack(undercut): And an *undercutter* to a candidate rule is a assignment to a varialbe u such that $strength(p_1, ..., p_n \Rightarrow c|\epsilon, u) \leq 1$.

An example of extracting arguments

- Enter the evidence {Psy_report = true, DNA_match = true}

- Step 1: Extract candidate rules. Here we extract two simple rules

$$DNA\_match \Rightarrow Crime$$
$$DNA\_match \Rightarrow Twin$$

- Step 2: Caluculate the strength of each rule, and keep the rules we need. Here we can get the strength of first rule is 1.2, and the strength of second rule is 1.2.

An example of extracting arguments

- Step 3: Find attack(undercutting) using strength.

Motive	
T	F
0.05	0.95

	Psych_report	
Motive	T	F
F	0.6	0.1
T	0.4	0.9

	Crime	
Motive	T	F
F	0.5	0.01
T	0.5	0.99

Twin	
T	F
0.01	0.99

		DNA_match	
Crime	Twin	T	F
F	F	$10^{-}6$	$1-10^{-}6$
F	T	1.0	0.0
T	F	1.0	0.0
T	T	1.0	0.0

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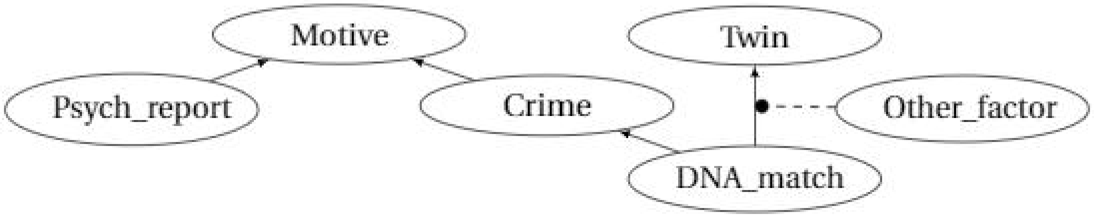
Case-3



Extracting (modifying) Bayesian Network based on Argumentation

Goal : construct a Bayesian Network that can contain the information shown in this Argumentation System

Constraints during construction (For Graph)

Constraint 5.1 (Nodes and Values). *For every atomic proposition in a structured Argumentation Framework(SAF), there exists a corresponding node in BN.*

Constraint 5.2 (Inference chains). *For every rules in a SAF, there exist active chains between all nodes appearing in that rule, given the observed nodes.*

Constraint 5.3 (Attack chains). *For every attack relation (A,B), then there exist active chains between nodes related to the attack(undercut or contrary).*

Constraints during construction (For Tables)

Constraint 5.4 (Inference probabilities). *To get the probability of inferences in the Argumentation system*

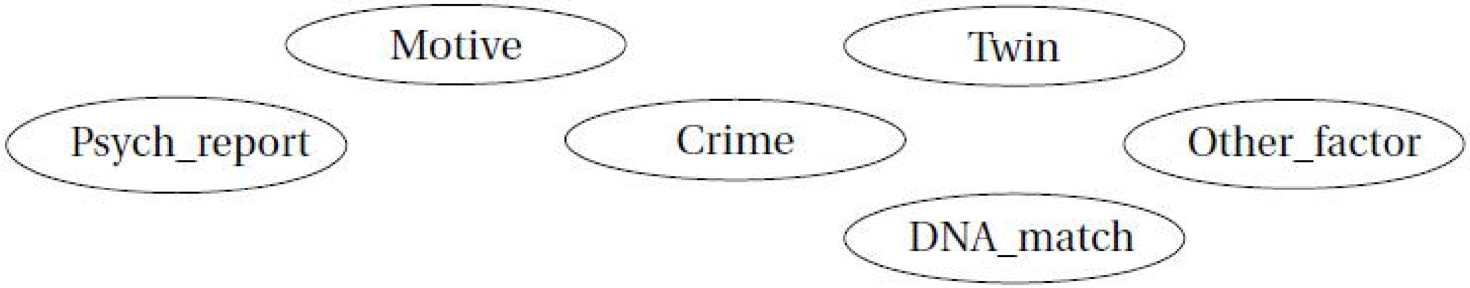
- *For every strict rule* $r : \phi_1, ..., \phi_n \rightarrow \psi$*, we have* $P(\psi|\phi_1, ..., \phi_n) = 1$*;*
- *For every defeasible rule* $r : \phi_1, ..., \phi_n \Rightarrow \psi$*, we have* $P(\psi|\phi_1, ..., \phi_n) > 0$*.*

Constraint 5.5 (Attack probabilities). *To get the probability of attacks in the Argumentation system*

- *If A attack B, then* $P(B|A)=0$*;*
- *If A undercuts B, then* $P(B) < P(A|B)$*.*

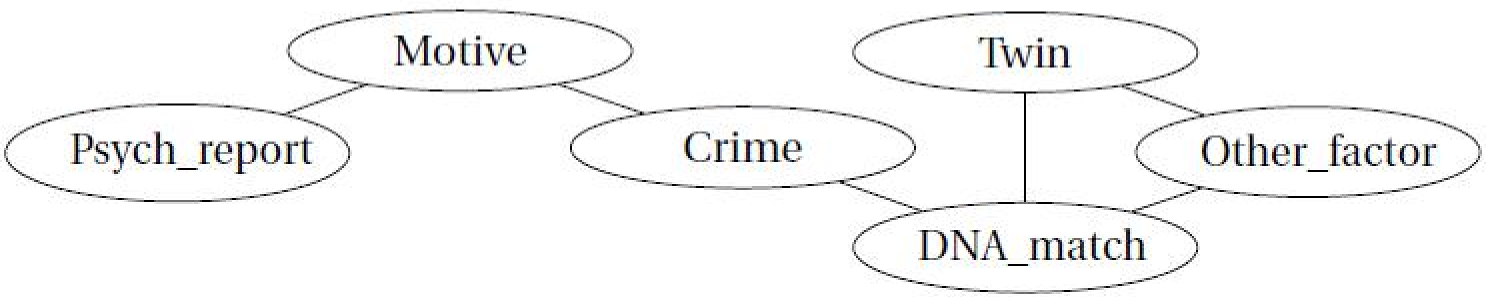
An example of conversion

- Step 1: Construct nodes based on Constraint 5.1



An example of conversion

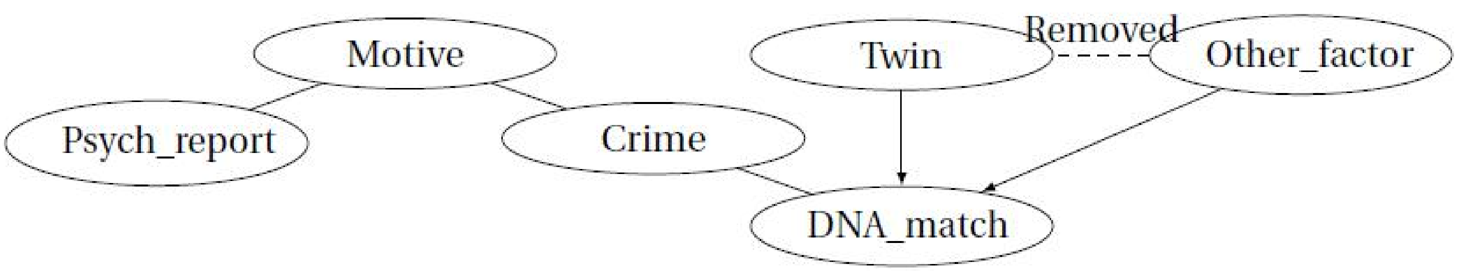
- Step 2: Based on constraint 5.2 and 5.3, we get undirected graph



An example of conversion

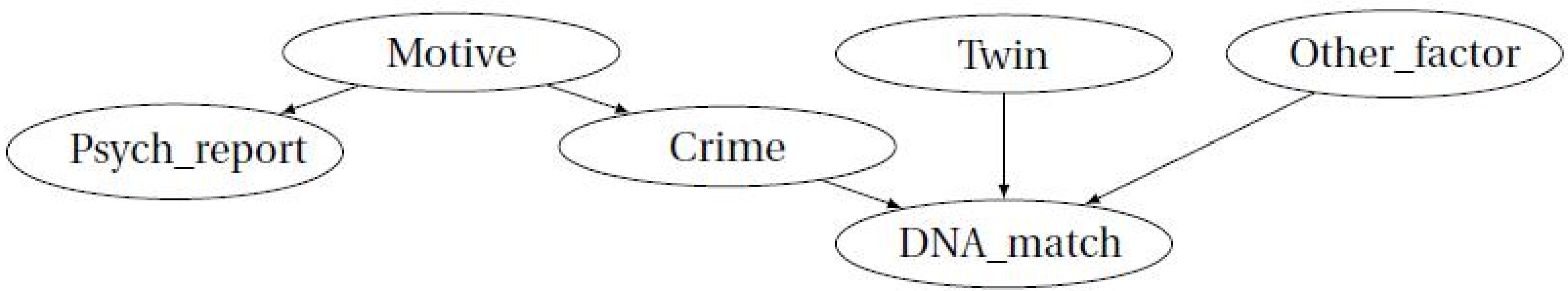
- Step 3: Remove edges:

* Step3-Remove edges:For all undercutting attacks, if a rule $r : \phi_1, ..., \phi_n \Rightarrow \psi$ is undercut by $\chi \in \overline{n(r)}$, then remove edge(χ, ψ)(or (ψ, χ)) and turn the other edges involved in to the following directed ones:(χ, ϕ_i) and (ψ, ϕ_i) for all ϕ_i.



An example of conversion

- Step 4: Step4-Choose causal direction:If there exist causal interpretation for an edge in the graph, then choose a direction that fit the causal interpretation, if not, then we choose an arbitrary direction.



- step 5: Verify d-separation

An example of conversion

- Step 6: assign the probability based on constraint 5.4 and 5.5, then we can know
  - $P(Psy|motive) > 0$, $P(Cr|motive) > 0$, $P(DNA|Cr, Twin) > 0$
  - $P(DNA|Twin) = 1$
  - $P(DNA|Cr, Twin) < Pr(DNA|\, Cr, Twin, Others)$

Case-4

Probabilistic ABA (PABA) and Schemes

- Convert schemes to ABA and PABA
- Extract semantics from PABA

PABA

- A triple $\{A_p, R_p, F\}$
  - A_p means probabilistic assumptions, it represent the uncertain belief about your assumptions.
  - R_p means rules containing A_p
  - F means ABA, which is $F = (R, A, ^-)$.

Argumentation scheme

- Scheme

  S1: Everybody (in a particular reference group) accepts that A. Therefore, A is true (or you should accept A).

  S2: Everybody (in a particular reference group) rejects A. Therefore, A is false (or you should reject A).

- Critical questions

  CQ1:Does a large majority of the cited reference group accept A s true?

  CQ2:Is there other relevant evidence available that would support the assumption that A is not true?

  CQ3:What reason is there for thinking that the view of this large majority is likely to be right?

  (Position to know ad popular argument is one of the answer of this question)

- Position to know

  Everybody in this group G accepts A.

  This group is in a special position to know that A is true.

  Therefore, A is (plausibly) true.

Argumentation scheme

- ABA
  - *A*

    accept(A)-large majority of the group accept A as true(false)

    others(A)-other factors

    position(A) - position to know
  - *R*

    $A \leftarrow accept(A), right(maj)$

    $right(maj) \leftarrow \neg\, others(A), position(A)$

Argumentation scheme

- PABA
  - A_p

    P_{oth} : other factors

    P_{right} : the view of majority is right
  - R_p

    $[P_{right}{:}0.8] \leftarrow \neg P_{oth}, \text{position(A)}$

    $[P_{right}{:}0.8] \leftarrow \neg P_{oth}, \neg \text{position(A)}$

    $[P_{right}{:}0.2] \leftarrow P_{oth}, \neg \text{position(A)}$

    $[P_{right}{:}0.5] \leftarrow \neg P_{oth}, \text{position(A)}$

    $[P_{oth}{:}0.3] \leftarrow$
  - A'

    accept(A), position(A)
  - R'

    $\text{A} \leftarrow \text{accept(A)}, P_{right}$

PABA (frame)

Definition 11 (frame , [4]). *A frame is a set of partial world. A partial world is a set that contains few possible probabilistic assumptions which is possible.*

PABA (well-formed)

Definition 2. A PABA framework $\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ is well-formed if it satisfies the four constraints below.

1. For each $\alpha \in \mathcal{A}_p \cup \neg\mathcal{A}_p$, α does not occur in $\mathcal{A}$ or in the head of a rule in $\mathcal{R}$, and $[\alpha : x]$ does not occur in the body of a rule in $\mathcal{R} \cup \mathcal{R}_p$.
2. If $\mathcal{R}_p$ contains $[\alpha : x] \leftarrow \beta_1, \ldots, \beta_n$, then it also contains a complementary rule $[\neg\alpha : 1 - x] \leftarrow \beta_1, \ldots, \beta_n$.[9]
3. For each $\alpha \in \mathcal{A}_p$, there exists $Pa_\alpha \subseteq \mathcal{A}_p$ s.t. for each maximal consistent subset $\{\beta_1, \ldots, \beta_m\}$ of $Pa_\alpha \cup \neg Pa_\alpha$, $\mathcal{R}_p$ contains a rule $[\alpha : x] \leftarrow \beta_1, \ldots, \beta_m$ (and the complementary rule $[\neg\alpha : 1 - x] \leftarrow \beta_1, \ldots, \beta_m$).
4. If $\mathcal{R}_p$ contains two rules r_1, r_2 with heads $[\alpha : x]$ and $[\alpha : y]$, $x \neq y$, then either of the conditions below holds.
   (a) $body(r_1) \subset body(r_2)$ or $body(r_2) \subset body(r_1)$.
   (b) $\theta \in body(r_1)$ and $\neg\theta \in body(r_2)$ for some $\theta \in \mathcal{A}_p$.

PABA (adapted PABA)

- A

  a(A), p(G)

- A_p

  $P_1(P_{oth}), P_2(P_{right})$

- R_p

  $[P_2{:}0.8] \leftarrow P_1$　　$[P_1{:}0.3] \leftarrow$

  $[P_2{:}0.8] \leftarrow \neg P_1$

- R

  $A \leftarrow a(A), p2$　　$\neg a(A) \leftarrow \neg p1$

	P_{right}	
P_{oth}	T	F
F	0.8	0.2
T	0.2	0.8

P_{oth}	
T	F
0.3	0.7

Figure 16: Bayesian Network of PABA above

cr-frame derivation (algorithm)

1. $\mathcal{P}_i$ is a multi-set of propositions, $\mathcal{O}_i$ is a multi-set of finite multi-sets of propositions, A_i, C_i are sets of assumptions.
2. For each step $i \geq 0$, selection strategy sl selects a sentence $\sigma \in \mathcal{P}_i$ or $\sigma \in S$ for some $S \in \mathcal{O}_i$, and
   (a) If $\sigma \in \mathcal{P}_i$ is selected then
      i. if $\sigma \in \mathcal{A}$ then $\mathcal{P}_{i+1} = \mathcal{P}_i \setminus \{\sigma\}$ and $\mathcal{O}_{i+1} = \mathcal{O}_i \cup \{\{\overline{\sigma}\}\}$[11]
      ii. otherwise, there exists some rule $\sigma \leftarrow Bd \in \mathcal{R}$ such that $C_i \cap Bd = \emptyset$ and $\mathcal{P}_{i+1} = \mathcal{P}_i \setminus \{\sigma\} \cup (Bd \setminus A_i)$ and $A_{i+1} = A_i \cup (\mathcal{A} \cap Bd)$
   (b) If $\sigma \in S$ for some $S \in \mathcal{O}_i$ is selected, then
      i. If $\sigma \in \mathcal{A}$, then one of the cases below hold.
         A. $\mathcal{O}_{i+1} = \mathcal{O}_i \setminus \{S\} \cup \{(S \setminus \{\sigma\})\}$
         B. $\sigma \notin A_i$ and $\sigma \in C_i$ and $\mathcal{O}_{i+1} = \mathcal{O}_i \setminus \{S\}$
         C. $\sigma \notin A_i$ and $\sigma \notin C_i$ and $\mathcal{P}_{i+1} = \mathcal{P}_i \cup \{\overline{\sigma}\}$ and $\mathcal{O}_{i+1} = \mathcal{O} \setminus \{S\}$ and $C_{i+1} = C_i \cup \{\sigma\}$
      ii. Otherwise, $\mathcal{O}_{i+1} = \mathcal{O}_i \setminus \{S\} \cup \{S \setminus \{\sigma\} \cup Bd \mid \sigma \leftarrow Bd \in \mathcal{R}$ and $Bd \cap C_i = \emptyset\}$

cr-frame derivation (algorithm)

Definition 15. A **cr-frame derivation** in a PABA framework $\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ using a selection strategy *sl* is a possibly infinite sequence $\mathcal{T}_0, \mathcal{T}_1, \ldots, \mathcal{T}_i, \ldots$ where

1. $\mathcal{T}_i$ is a set of pairs of the form (t, ω) where t is a tuple $\langle \mathcal{P}, \mathcal{O}, A, C \rangle$ as defined in Definition 8 and ω is a partial world.
2. For each i, the following conditions hold.
   (a) *sl* selects a pair (t, ω) from $\mathcal{T}_i$, and selects a sentence σ or an empty set from either the $\mathcal{P}$ or $\mathcal{O}$ component of t.
   (b) $\mathcal{T}_{i+1} = \mathcal{T}_i \setminus \{(t, \omega)\} \cup \Delta\mathcal{T}$ where
      i. $\Delta\mathcal{T} = \{(t, \omega \cup \{\sigma\}), (t, \omega \cup \{\neg\sigma\})\}$ if *sl* selects a sentence σ and σ is a probabilistic assumption σ not occurring in ω,[16]
      ii. $\Delta\mathcal{T} = \{(t', \omega) \mid t' \in FollowAB_{\mathcal{F}_\omega}(t, sl)\}$ otherwise.

cr-frame derivation (example)

- A

  a(A), p(G)

- A_p

  $P_1(P_{oth}), P_2(P_{right})$

- R_p

  $[P_2{:}0.8] \leftarrow P_1$ $\quad$ $[P_1{:}0.3] \leftarrow$

  $[P_2{:}0.8] \leftarrow \neg P_1$

- R

  $A \leftarrow a(A), p2$ $\quad$ $\neg a(A) \leftarrow \neg p1$

	P	O	A	C	w
T_1	A	ϕ	ϕ	ϕ	ϕ
T_2	$\underline{a(A)}, P_2$	ϕ	ϕ	ϕ	ϕ
T_3	P_2	$\underline{\neg P_1}$	{a(A)}	ϕ	ϕ
T_4	P_2	$\underline{\neg P_1}$	{a(A)}	ϕ	$\{\neg P_1\}$
	P_2	$\underline{\neg P_1}$	{a(A)}	ϕ	$\{P_1\}$
T_5	$\underline{P_2}$	ϕ	{a(A)}	ϕ	$\{P_1\}$
T_6	$\underline{P_2}$	ϕ	{a(A)}	ϕ	$\{P_1, P_2\}$
	$\underline{P_2}$	ϕ	{a(A)}	ϕ	$\{P_1, \neg P_2\}$
T_7	ϕ	ϕ	{a(A)}	ϕ	$\underline{\{P_1, P_2\}}$

Table 1: Derivation of complete admissible frame

Compute probability

the probability of a set of partial world $S = \{s_1, ..., s_n\}$ is $\vee_{i=1}^{n} \wedge_{j=1}^{s_i} p_{ij}$

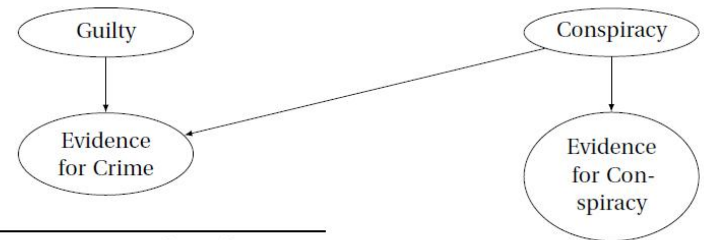
$$p_1 \times p_2 = 0.2 \times 0.3 = 0.06$$

Case-5

Bayesian Argumentation Network(BAN)

Guilty T	Guilty F
0.1	0.9

Conspiracy T	Conspiracy F
0.01	0.99



Guilty	Conspiracy	Evidence for Crime T	Evidence for Crime F
F	F	0.001	0.999
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.9	0.1

Conspiracy	Evidence for Conspiracy T	Evidence for Conspiracy F
F	0.9	0.1
T	0.1	0.9

- Construct the BAN of Evidence for Crime
- BAN of other nodes can be constructed by same method

BAN

Definition 12 (Bayesian Argumentation Network, [10]). *A BAN is a triple* $\{S, R, e\}$, *where S is a set of arguments, R is the attack relation between non-empty subsets of S and an element of S, e is a transmission function giving each h in the pair (H,x) a value* $\in [0,1]$

In the following graph, we can see H = $h_1, \dots, h_k$, and e(H,x, h_k) = e_k

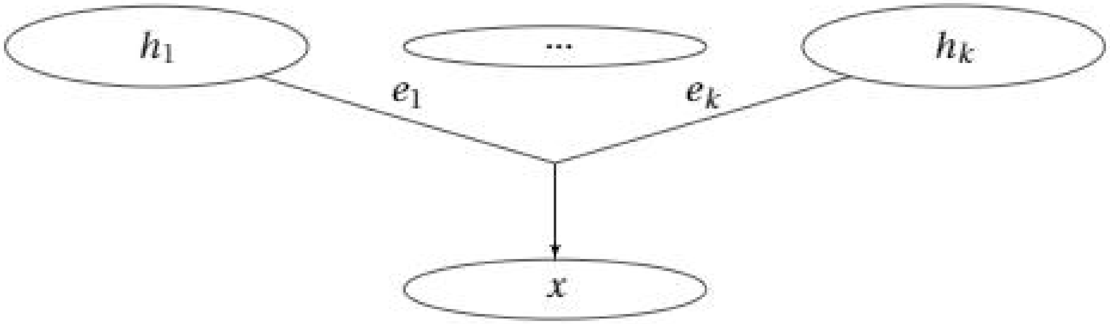


Figure 17: Transmission function

Example of constructing BAN

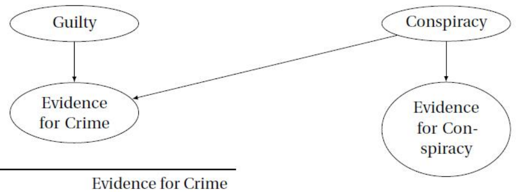
We first add 2 nodes $\neg G$ and $\neg C$, and add point c_{00}, c_{01}, c_{10} and c_{11}

$$c_{00} = \neg G \wedge \neg C$$
$$c_{01} = \neg G \wedge C$$
$$c_{10} = G \wedge \neg C$$
$$c_{11} = G \wedge C$$

$$P(c_{11}) = (1 - P(C)) \cdot (1 - P(G)) = 0.9 \times 0.99 = 0.891$$
$$P(c_{10}) = (1 - P(G)) \cdot (1 - (1 - P(C))) = (1 - P(G)) \cdot P(C) = 0.9 \times 0.01 = 0.009$$
$$P(c_{10}) = (1 - P(C)) \cdot (1 - (1 - P(G))) = (1 - P(C)) \cdot P(G) = 0.1 \times 0.99 = 0.099$$
$$P(c_{10}) = (1 - (1 - P(G))) \cdot (1 - (1 - P(G))) = P(G) \cdot P(C) = 0.1 \times 0.01 = 0.001$$

Guilty	
T	F
0.1	0.9

Conspiracy	
T	F
0.01	0.99



Guilty	Conspiracy	Evidence for Crime T	Evidence for Crime F
F	F	0.001	0.999
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.9	0.1

Conspiracy	Evidence for Conspiracy T	Evidence for Conspiracy F
F	0.9	0.1
T	0.1	0.9

Example of constructing BAN

Then we add attacks from G and C to all c_{ij} and get

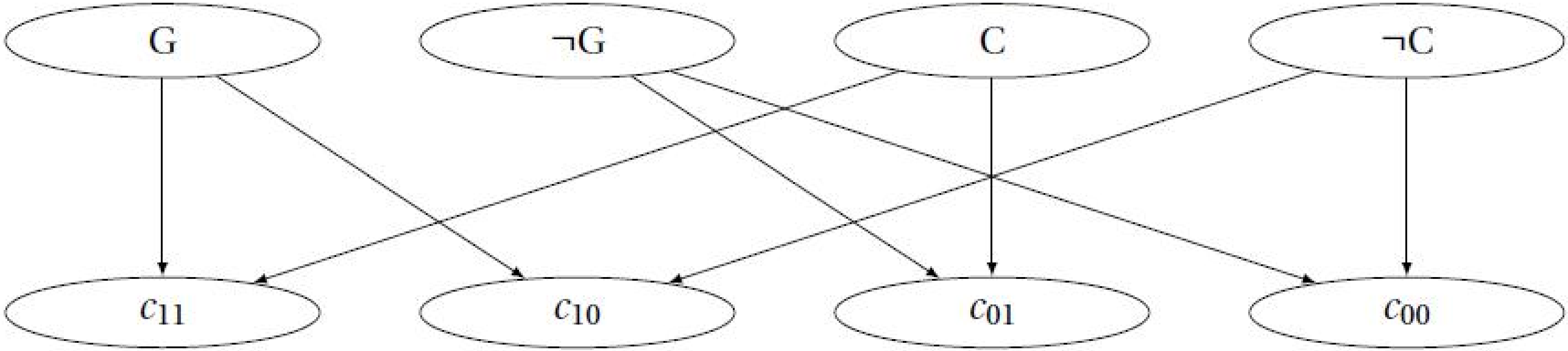


Figure 18: After adding attacks

Example of constructing BAN

Then we add an additional node x_{ij} for each c_{ij} to incorporate the transmission factors e_{ij}, now we get

And $e_{xij|\phi ij}$ can be find in the tables of original BN, we can get that

$$\mathrm{e}_{x11|\phi 11} = 0.9, P(X_{11}) = 1 - e_{x11|\phi 11} \cdot c_{11} = 0.1981$$

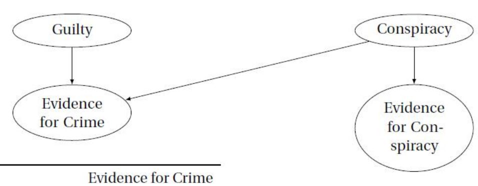
$$\mathrm{e}_{x10|\phi 10} = 0.9, P(X_{11}) = 1 - e_{x10|\phi 10} \cdot c_{10} = 0.9919$$

$$\mathrm{e}_{x01|\phi 01} = 0.8, P(X_{11}) = 1 - e_{x01|\phi 01} \cdot c_{01} = 0.9992$$

$$\mathrm{e}_{x00|\phi 00} = 0.001, P(X_{11}) = 1 - e_{x00|\phi 00} \cdot c_{00} = 0.999999$$

Guilty	
T	F
0.1	0.9

Conspiracy	
T	F
0.01	0.99

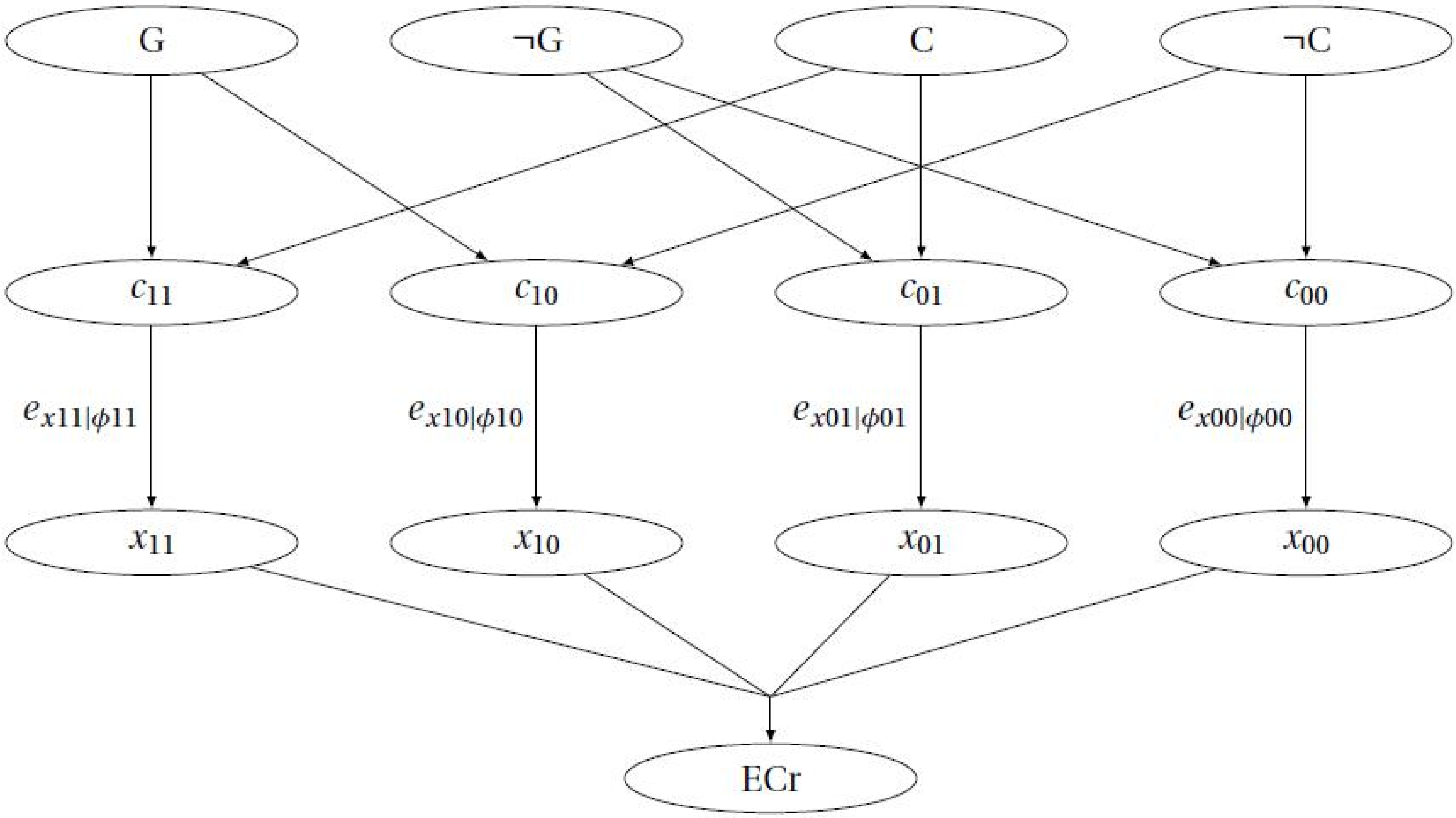


		Evidence for Crime	
Guilty	Conspiracy	T	F
F	F	0.001	0.999
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.9	0.1

	Evidence for Conspiracy	
Conspiracy	T	F
F	0.9	0.1
T	0.1	0.9

All x_{ij} jointly attack ECr,

Example of constructing BAN



Example of constructing BAN

And Finally we can get the probability of ECr

$$= min\{1, \sum(1 - P(x_{ij}))\} \tag{5}$$

$$= min\{1, 0.810801\} \tag{6}$$

$$= 0.810801 \tag{7}$$

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Thank you