CSC 383, S'23 **Homework 3** Due Mar. 3^{rd}

Directions:

Write your solutions using Python, Jupyter, and LaTeX. Then submit the files `hw3.tex`, `hw3.pdf`, and `2p_Nash.ipynb`.

Problem 1

Prove that for any finite game, the set of mixed-strategy profiles is convex. That is, in a game with P players and A_p actions for player p, given any two mixed-strategy profiles $\vec{\sigma}_1, \vec{\sigma}_2 \in \Delta^{A_1} \times \ldots \times \Delta^{A^P}$, and any constant $\alpha \in [0,1]$ the profile $\alpha\vec{\sigma}_1 + (1-\alpha)\vec{\sigma}_2 \in \Delta^{A_1} \times \ldots \times \Delta^{A^P}$.

Since the summation of probabilities in $\sigma_1 = 1$ and $\sigma 2 = 1$, the profile $\alpha\vec{\sigma}_1 + (1-\alpha)\vec{\sigma}_2 = 1$ for all players. We know that this strategy is in the simplex because it sum to 1.

$$\triangle_p = \alpha\vec{\sigma}_1^p + (1-\alpha)\vec{\sigma}_2^p$$

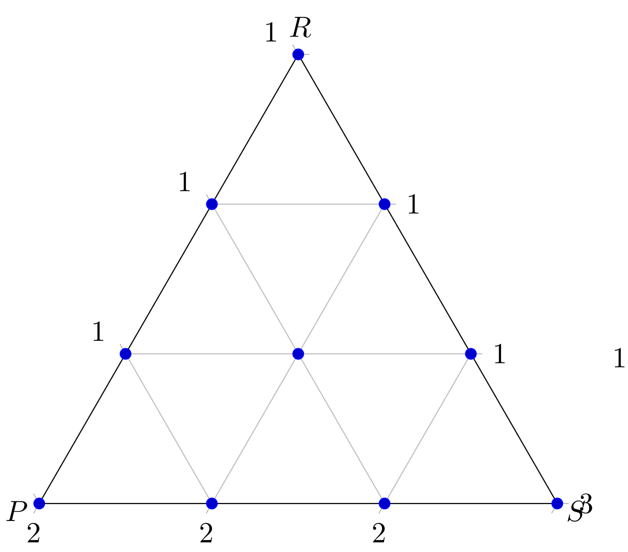
This represents the simplex per player. For each probability in the mixed profile, when multiplied by α and $1-\alpha$, it will still sum to 1. As a result, it will still be considered a simplex in the set of possible simplices for that player.

$$\prod_{n=1}^{\infty} \triangle_n$$

By the definition of a simplitope, the above expression will perform the cartesian product across all simplexes (all players). The resulting cartesian product will be our simplotope.

Problem 2

In the following Rock-Paper-Scissors variant, compute the Brouwer labels for the vertices of a 3-way subdivision of the symmetric mixed-strategy simplex.



	R	P	S
R	$0, 0$	$-1, 1$	$2, -1$
P	$1, -1$	$0, 0$	$-1, 2$
S	$-1, 2$	$1, -1$	$0, 0$

'The middle node of the simplex is labeled as 1 (can be 1 or 2 so we will break the tie arbitrarily).

For the given profile of RPS $\begin{bmatrix} 0 \\ 1/3 \\ 2/3 \end{bmatrix}$ the deviation payoffs are:

R $= \frac{1}{3}(-1) + \frac{2}{3}(2) = 1$

P $= \frac{2}{3}(-1) = -\frac{2}{3}$

S $= \frac{1}{3}(1) = \frac{1}{3}$

The utility of the strategy is:
$u_1 = (\frac{1}{3})(\frac{-2}{3}) + \frac{2}{3}(1) = (\frac{4}{9})$

Therefore the gain is : $1 - \frac{4}{9} = \frac{5}{9}$ and the advantage for S is: $\frac{11/9}{14/9} = 0.78$ and the advantage for P is: $\frac{3/9}{14/9} = 0.22$ Therefore P was most decreased so we label the point 2.

Problem 3

In the Jupyter notebook `2p_Nash.ipynb` implement the function to identify Nash equilibria in 2-player games via linear programming. Note that this version of the notebook contains lots of extra examples; look them over before you get started!