GeolocationSheafEx_OF

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Modeling PNNL+Robinson Geolocation Sheaf:

For specifics and citations, reference https://arxiv.org/abs/1912.05487

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```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --

## v ggplot2 3.3.6     v purrr   0.3.4
## v tibble  3.1.6     v dplyr   1.0.8
## v tidyr   1.2.0     v stringr 1.4.0
## v readr   2.1.2     v forcats 0.5.1

## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

Sensors: [• The GPS reading on the Bear Collar, denoted **G**;

[[• The Radio VHF Device receiver, denoted **R**;] Location Bear

- The Text report, denoted **T**; and
- The Vehicle GPS, denoted **V**.]] Location Researcher

	H =Human position	B =Bear position
V =Vehicle GPS $\langle$lat,long,ft$\rangle$	$\surd$	
T =Text	$\surd$	
R =Receiver $\langle$UTM N,UTM E,m,deg,m$\rangle$	$\surd$	$\surd$
G=GPS on Bear $\langle$UTM N, UTM E,m$\rangle$		$\surd$

Table 1: Sensor matrix for the bear tracking.

Figure 1: Table 1

Vertex	Data Format	Description	Stalk
G =*Bear Collar*	(E, N, m)	Position and elevation of bear from collar	$\mathbb{R}^3$
V=*Vehicle GPS*	(lat, long, ft)	Position and elevation of human from vehicle	$\mathbb{R}^3$
T = *Text*	string	Text description of human's location	set of strings
R =*Radio VHF Device*	(E, N, m, m, deg)	Position and elevation of human and position of bear relative to human	$\mathbb{R}^5$

Table 2: Data feeds for tracking sheaf model

Figure 2: Table 2

U = {V, R, T, G} base sensor set ASC Δ, representing the tracking sensor network, contains the face denoted $H = \{V, R, T\}$

To convert the text descriptions we use Google Maps API [63] and to convert the (lat, long, ft) readings, coming from the vehicle, we use the Python open source package utm 0.4.1 [64].

$$\phi : \mathbb{R}^5 \rightarrow \mathbb{R}^5, \phi((x, y, z, r, \theta)^T) = (x, y, z, rcos(\theta), rsin(\theta))^T$$

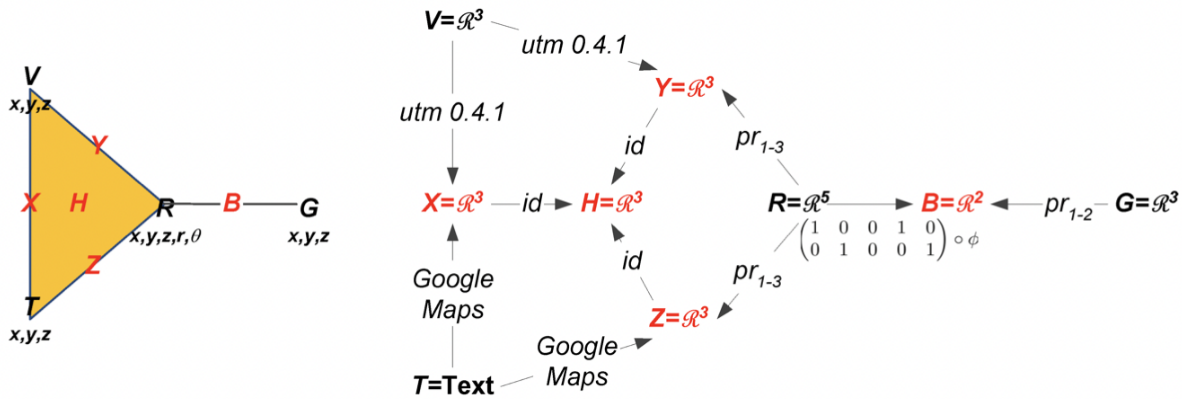

Figure 3: (Left) ASC adorned with stalks on the sensor vertices. (Right) Sheaf model.

Figure 3: Figure 3

Step 1:

Assignment Table?:

Restriction Functions:\

Since the pairwise relations all share UTM coordinates, only id mappings are needed amongst them up to the three-way H face.

utm 0.4.1

Google Maps

id

id

$pr_x - y$: Labels of the form prx-y are projections of the corresponding coordinates (also representable as binary matrices of the appropriate form).

pr_{1-3}

pr_{1-2}

"Finally, the restriction from R up to B is the composition of the polar conversion of the final two components with the projection on the first two to predict the bear position from the radiocollar GPS, bearing, and range."

$$\begin{pmatrix}1 & 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0 & 1\end{pmatrix} \circ \phi : (x, y, z, rcos(\theta), rsin(\theta))^T$$

##Global Sections Definition 3:

Definition 3. *Let $\mathscr{S}$ be a sheaf on an abstract simplicial complex Δ. An* **assignment** $\alpha\colon \Delta \to \prod_{\delta\in\Delta} \mathscr{S}(\delta)$ *provides a value $\alpha(\delta) \in \mathscr{S}(\delta)$ to each face $\delta \in \Delta$. A* **partial assignment**, *β, provides a value for a subset $\Delta' \subset \Delta$ of faces, $\beta\colon \Delta' \to \prod_{\delta\in\Delta'} \mathscr{S}(\delta)$. An assignment s is called a* **global section** *if for each inclusion $\delta \rightsquigarrow \lambda$ of faces, $\mathscr{S}(\delta \rightsquigarrow \lambda)(s(\delta)) = s(\lambda)$.*

$$
\begin{aligned}
&s(\mathit{Text}) = \text{`Intersection of Victoria Rd and Meadow Rd'}\\
&s(\mathit{Vehicle\ GPS}) = \begin{pmatrix} 35.6^\circ \text{ lat} \\ -82.6^\circ \text{ long} \\ 2019 \text{ ft} \end{pmatrix},\\
&s(\mathit{Bear\ Collar}) = \begin{pmatrix} 358391 \text{ E} \\ 3936750 \text{ N} \\ 581 \text{ m} \end{pmatrix},\\
&s(\mathit{Radio\ VHF\ Device}) = \begin{pmatrix} 358943 \text{ E} \\ 3936899 \text{ N} \\ 615.4 \text{ m} \\ 572 \text{ m} \\ 195^\circ \end{pmatrix},\\
&s(\{\mathit{Radio\ VHF\ Device}, \mathit{Bear\ Collar}\}) = \begin{pmatrix} 358391 \text{ E} \\ 3936750 \text{ N} \end{pmatrix}\\
&\text{and } s(\{\mathit{Text}, \mathit{Vehicle\ GPS}\}) = s(\{\mathit{Text}, \mathit{Radio\ VHF\ Device}\})\\
&= s(\{\mathit{Vehicle\ GPS}, \mathit{Radio\ VHF\ Device}\})\\
&= s(\{\mathit{Text}, \mathit{Vehicle\ GPS}, \mathit{Radio\ VHF\ Device}\}) = \begin{pmatrix} 358943 \text{ E} \\ 3936899 \text{ N} \\ 615.4 \text{ m} \end{pmatrix}.
\end{aligned}
$$

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