1 Introduciton

The Consumer Price Index (CPI) is considered one of the most important indicators influencing terms in economics, which simply confirms how prices for goods and services change over time. Meanwhile, the CPI is also a key economic indicator to explain national economic activity widely used by government and central banks that serve as indicators of consumer price change . As a key index in inflation measurement, the CPI is calculated by weighting prices with a basket of consumer goods and services that are commonly demanded among transportation food medicine 'people's livelihood basic consumption style'. As such, it is of the utmost importance to ensure that CPI accurately reviews up-to-date price changes. CPI changes can directly affect the cost of living and purchasing power in household budgets, which makes it a key economic indicator used by central banks to inform inflation (and deflation) trends and also as an essential tool for policy-making purposes. Especially in high price level situations, such a fast increase of CPI will make the residents' living costs more expensive and change people's consumption desire directly due to decreasing purchasing power on money that results from inflation and the business investment demand.

In response to rising inflation, central banks often choose to raise cash interest rates in order to increase borrowing costs and reduce consumer demand in the market (Folger, 2024). By reducing the demand for goods and services, central banks can effectively mitigate inflationary pressures and slow down economic overheating to a certain extent. However, when economic growth slows down and inflation risks are low, the central bank may also choose to cut the cash rate to stimulate consumption and investment and facilitate economic recovery. In view of this, changes and trends in the CPI have far-reaching impacts on policy decisions, corporate strategy formulation and even the financial planning of households. Changes in the CPI affect the central bank's decision-making on the cash rate, and adjustments to the cash rate in turn affect future changes in the CPI. This feedback mechanism is the basis for the central bank to regulate the economy. Accurate forecasting of CPI not only helps the government and enterprises to plan ahead to cope with potential economic uncertainties, but also provides a reference for individuals and families to plan their consumption expenditures.

This report is to develop a multi-step forecasting model based on historical quarterly CPI data for predicting CPI trends in future quarters. By analyzing and modeling a given CPI training dataset (March 1990 to December 2022), we will forecast the CPI for the period March 2023 to June 2024.The results of this forecast will provide data support for future inflation trends and provide important economic information for policy makers, investors and consumers. Meanwhile, we will use the historical cash rate data from the Central Bank for qualitative validation to ensure the reasonableness of the model forecasts.
Through the development and validation of this forecasting model, we can not only provide references for future economic trends, but also gain insights into the applicability of different time series models in CPI forecasting. We expect that this project can provide effective methods and tools for future economic forecasting, as well as practical opportunities for participants' financial data modeling skills.

2 Data description and exploration

The dataset is quarterly CPI (Consumer Price Index) data spanning from the first quarter of 1990 to the fourth quarter of 2022, with 132 quarterly observations. There are no duplicate or missing values in the data set. The dataset provides CPI values for each quarter, which are used to track price levels over time and reflect the level of inflation in an economy. These data will provide the basis for the training and validation of future CPI forecasting models.

We plotted a line graph of the quarterly change in CPI over the period 1990 to 2022 (shown in Figure 2.1). The graph shows a general upward trend in the CPI, especially after the global financial crisis in 2008, when the volatility of the CPI increased significantly. This trend may reflect global economic fluctuations and changes in market prices over the period.From the chart, we can observe that CPI shows peaks at regular intervals. Although not very pronounced, this suggests the presence of seasonality. Additionally, the amplitude of fluctuations remains relatively stable over time, indicating that the seasonality is likely additive.

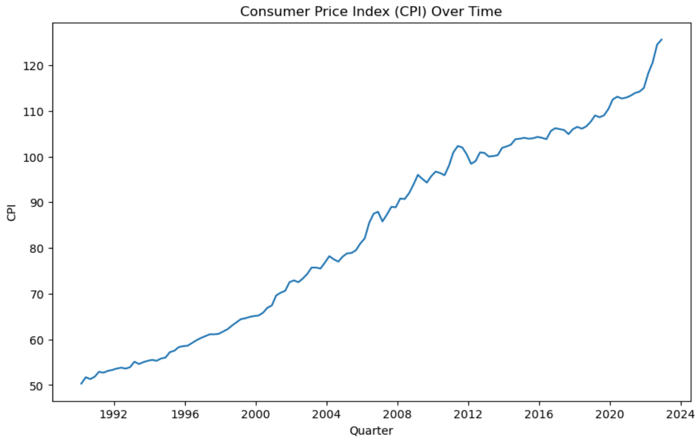

Figure 2.1

The additive decomposition of the CPI data (Figure 2.2) shows that there is significant seasonality in the dataset and that this seasonality is additive. The trend component shows that the CPI has risen consistently since the 1990s, reflecting long-term price growth. The seasonality component, on the other hand, shows that each year the CPI experiences fixed cyclical fluctuations that are stable in magnitude, rise independently of the trend, and do not follow changes in the overall price level. The residual component shows that the remaining fluctuations are more random, indicating that the model successfully captures the main trend and seasonal features. Therefore, it can be concluded that there is a stable additive seasonal pattern in the data.

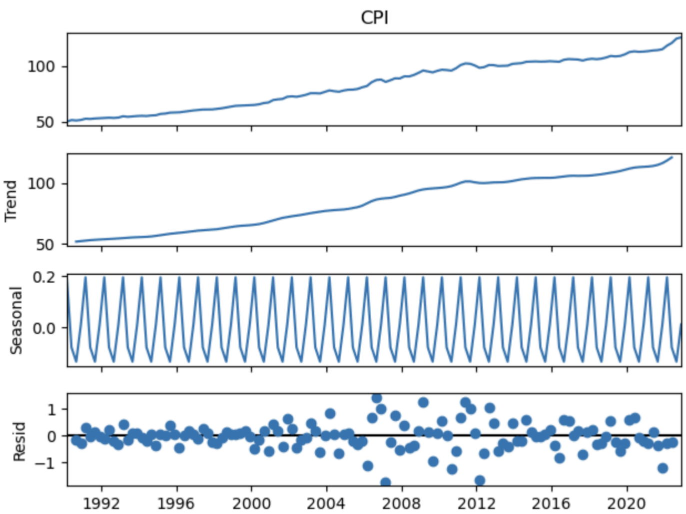

Figure 2.2

The cash rate dataset contains RBA cash rate data for the period 2012 to 2024, divided into historical cash rates and future cash rate forecasts. The data are recorded in months. As can be seen from the cash rate line graph (Figure 2.3), the Australian cash rate has continued to fall between 2012 and 2020, particularly after the global outbreak in 2020, when it fell to a record low of close to 0 per cent. This reflects the loose monetary policy implemented by the central bank to stimulate the economy. However, from 2022 onwards, cash rates rose sharply, suggesting that the central bank adopted a tighter policy in response to rapidly rising inflationary pressures. The data in the outlook section show that the cash rate is expected to remain high for some time to come, implying that inflationary pressures have not yet fully eased and the central bank will continue to curb price rises through its high interest rate policy.

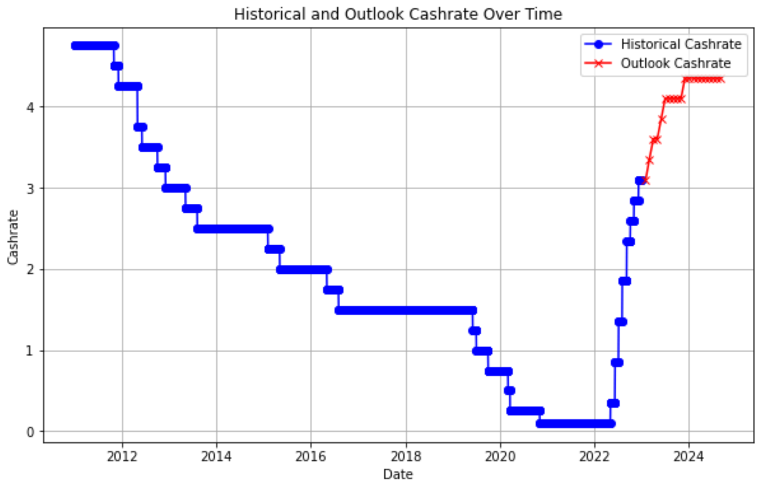

Figure 2.3

3 Methods

Based on EDA, we observed that the data exhibits both trend and seasonality. Therefore, we considered models that can handle both, selecting Decomposition, Holt-Winters, and SARIMA as candidate models.The following section provides a detailed introduction to the three models and the reasons for their selection.

3.1 Decomposition

Decomposition Forecasting Model Introduction

Decomposition forecasting is a statistical method used in time series analysis that decomposes complex time series data into multiple simpler, easier to understand and analyze components. Raw time series data are decomposed into three main components, trend, seasonality and irregular components, which better understand the structure and regularity of the data (Mbuli, Mathonsi, Seitshiro, & Pretorius, 2020).

The trend component reflects the long-term direction of time series data over time, which is usually a function of sustained increase or decrease (Chhetri, Lumpe, Vo, & Kowalczyk, 2017). This can reveal the long-term development trend of the CPI. The seasonal component represents the cyclical fluctuations. Seasonal fluctuations have a fixed cycle length, such as four quarters of a year. In addition, each cycle has the same direction and magnitude (Chhetri, Lumpe, Vo, & Kowalczyk, 2017). This is very important for forecasting future seasonal variations. Irregular component is the irregular fluctuation in the time series. This component cannot be explained by trend or seasonality in a time series due to external short-term events or random variations (Mbuli, Mathonsi, Seitshiro, & Pretorius, 2020). The decomposition model can analyze these three main components to gain a deeper understanding of the impact of each part on the time series. It forecasts future time points by recombining the individual components.

Reasons for Choosing the Decomposition Model

The greatest advantage of the decomposition model is intuitive interpretability. It can analyze trends, seasonality and irregular components to gain a deeper understanding of the impact of each component on the overall data. Each component exists independently, simplifies the process of analyzing the time series and makes it clearer through data visualization.Moreover, after decomposition, it enhances regularity of the data,making the model perform more consistently in long-term forecasting. Through removing complex random fluctuations and noise data, the model can capture the critical trend and seasonal components of the data accurately, which improves the prediction result and interpretability(Chen, Hu, & Zhang, 2022).

Model Principle

Additive model and multiplicative model are commonly used decomposition methods. In the additive model, the time series can be expressed as the sum of the components

$$Y(t) \ = \ T(t) \ + \ S(t) \ + \ R(t)$$

where Y(t) is the time series data, T(t) is the trend component, S(t) is the seasonal component, and R(t) is the remainder component. At the same time, the additive model is appropriate for time series in that seasonal fluctuation is relatively stable in magnitude and does not change significantly over time (Permanasari, Rambli, & Dominic, 2010). However, in multiplicative model, time series could be represented as the product of the individual components (Chhetri, Lumpe, Vo, & Kowalczyk, 2017),

$$Y(t) \ = \ T(t) \times S(t) \times R(t)$$

and it’s more appropriate to use when seasonal fluctuations show an increase or decrease in time series data (Permanasari, Rambli, & Dominic, 2010).

3.2 Holt-Winters

Introduction and Reasons for Choosing the Holt-Winters Model

The Holt-Winters model can handle both trend and seasonality in time series data and can model various types of time series patterns. The mathematical and computational implementation of Holt-Winters is relatively simple, making it easy to understand and use. Additionally, the model's parameters (such as level, trend, and seasonality) have clear physical meanings, which allows for a better interpretation of the model's behavior and results.

Model Principle

The Exponential Smoothing method predicts by assigning weighted averages to past observations, where more recent data gets higher weight. Holt-Winters, or triple exponential smoothing, extends this method for data with trend and seasonality.

The Holt-Winters method includes a forecast equation and three smoothing, one for level l_t, one for the trend b_t and one for the seasonal component s_t, with parameters α , β^*, and γ (Hyndman & Athanasopoulos, 2018).

Holt-Winters has two forms: additive, used when the trend and seasonality are stable, and multiplicative, for when these components grow or shrink over time. In the additive method, the seasonal component is represented as an absolute value on the scale of the observed series, and the sequence is adjusted by subtracting the seasonal component. The seasonal elements typically sum to about zero each year. In the multiplicative method, the seasonal component is expressed as a relative value (percentage), and the sequence is adjusted by dividing by the seasonal component. Each year, the seasonal elements add up to approximately m(frequency of the seasonality) (Hyndman & Athanasopoulos, 2018).

Additive method

The component form for the additive method is:

$$
\begin{aligned}
\hat{y}_{t+h|t} &= l_t + hb_t + s_{t+h-m(k+1)} \\
l_t &= \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1}) \\
b_t &= \beta^*(l_t - l_{t-1}) + (1-\beta^*)b_{t-1} \\
s_t &= \gamma(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m}
\end{aligned}
$$

Multiplicative method

The component form for the multiplicative method is:

$$
\begin{aligned}
\hat{y}_{t+h|t} &= (l_t + hb_t)s_{t+h-m(k+1)} \\
l_t &= \alpha\frac{y_t}{s_{t-m}} + (1-\alpha)(l_{t-1} + b_{t-1}) \\
b_t &= \beta^*(l_t - l_{t-1}) + (1-\beta^*)b_{t-1} \\
s_t &= \gamma\frac{y_t}{(l_{t-1}+b_{t-1})} + (1-\gamma)s_{t-m}
\end{aligned}
$$

Holt-Winters' damped method

The Damped Trend Method in the Holt-Winters model adds an adjustment to control the influence of trends. In a regular Holt-Winters model, trends are assumed to continue growing or declining indefinitely. However, in many real-world scenarios, trends eventually slow down or stop. By adding a damping factor (ϕ) to the trend, the model suppresses the continuation of long-term trends, making predictions more realistic. A method that often provides accurate and robust forecasts for seasonal data is the Holt-Winters method with a damped trend and multiplicative seasonality:

$$
\begin{aligned}
\hat{y}_{t+h|t} &= [l_t + (\phi + \phi^2 + \dots + \phi^h)b_t]s_{t+h-m(k+1)} \\
l_t &= \alpha\frac{y_t}{s_{t-m}} + (1-\alpha)(l_{t-1} + \phi b_{t-1}) \\
b_t &= \beta^*(l_t - l_{t-1}) + (1-\beta^*)\phi b_{t-1}
\end{aligned}
$$

$$s_t = \gamma \frac{y_t}{(l_{t-1}+\phi b_{t-1})} + (1-\gamma)s_{t-m}$$

3.3 SARIMA

Reasons for Choosing the SARIMA Model

Based on the EDA stage above , it is clear to show that the CPI value have a clear increasing trend over the time and it is trend to be linear and it is calculated by quarterly.Therefore,additive SARIMA model is an appropriate model, because it supports to observe both of long-term trends and seasonal components. Furthermore, it assumes that seasonal and trend components are linear combinations and have not experienced any scaling, which is matching with the stable fluctuations that are observed from data. Therefore, the additive SARIMA model is a suitable and appropriate approach to solve the prediction problem.

Model Principle

The SARIMA model is followed by the following mathematical function:

$$\phi_p(B^s)\phi_p(B)(1-B)^d(1-B^s)^D y_t = \theta q(B^s)\theta q(B)\varepsilon_t$$

Auto-regressive (AR) Terms

Auto-regressiveis used to detect influence of previous value, and can be represented by following formula:

$$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p Bp$$

Where parameter p is the number of lagged values, and Φ are the coefficients, on behalf of the influence of lagged values to current.

Seasonal auto-regressive terms

To detect seasonal change, the mathematical formula of seasonal component can be represented as below:

$$\phi_p(B^s) = 1 - \phi_1 B^s - \phi_2 B^{2s} - \cdots - \phi_p B^{ps}$$

The B^s is refer to the backshift operator applied with a seasonal lag.

Differencing Terms

The differencing components help to convert a non-stationary time series into a stationary one by removing trends and seasonality. The two types of differencing are:

Non-seasonal difference: $(1-B)^d$ represents differential operations to help eliminate linear trends.

Seasonal difference: $(1-B^s)^D$ represents seasonal differences to help eliminate cyclical fluctuations

Moving average (MA) component

θ_q(B)and $\theta(B^s)$is to predict past errors to smooth the current predicted value, and treat non-seasonal and seasonal errors separately.

$$\theta_q(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q$$

Similarly, the seasonal moving average component accounts for seasonal error terms:

$$\theta_Q(B^s) = 1 + \theta_1 B^S + \theta_2 B^{2s} + \cdots + \theta_Q B^{Qs}$$

Based on the formula above, the SARIMA model combines the three mechanisms of AR, differencing, and MA, and processes the seasonal period in the data through the parameter “seasonal order”, so that it can deal with time series data with trends and seasonal components. Those mathematical formulas and parameters allow the SARIMA model to capture complex structures in time series, including short-term dependencies, long-term trends, and seasonal changes, to generate reasonable predictions.

Model initialization

When initializing the additive SARIMA model, it has 2 key parameters, which are “order” and “seasonal order” ; they can observe the non-seasonal and seasonal components in the data respectively. At the same time, “Order” parameters have three parameters (p, d, q),including auto-regressive relationship (p), moving average (q), and elimination of non-stationarity through integration (d). By contrast, “seasonal order” parameter has four parameters (P, D, Q, s), and are all used to capture period cycles. Through combining them together, the model could detect short-term fluctuations and long-term period cycles efficiently. Moreover, with the help of “enforce stationarity” and “enforce invertibility” parameters, the model could adapt non-stationarity and randomness quickly.

Advantages

The SARIMA model could address non-seasonal and seasonal components, especially for time series data with notable cyclical patterns, such as quarterly or annual data. By combining seasonal order parameters, the model can detect seasonal fluctuations quickly, for analyzing data with recurring trends. At the same time,the model is adept at handling time series with trends and random variations, because it could capture short-term dependencies and trends. The order parameters (p, d, q) address trend and noise components through AR, MA, and differencing processes, allowing the model to effectively manage trends and random fluctuations. In summary, these parameter combinations (p, d, q and P, D, Q) provide strong interpretability of the model, revealing important insights into trends, seasonality, and other essential patterns within the time series.

Disadvantages

Firstly, the complexity of the SARIMA model requires a large range of selection of parameters, including seasonal and non-seasonal parameters. This makes the model's tuning process long, and when the data scale is large, a lot of cross-validation is needed to implement, to find the best combination of parameters. Meanwhile, differential processing is needed when dealing with non-stationary data. If there are significant structural changes in the data or the influence of external elements, the model does not capture these changes efficiently. In addition, the model does not include the ability to handle external variables. In cases where time series trends and multiple external factors need to be considered at the same time, the model may perform poorly. Finally, the model generally performs well in short-term forecasting, but struggles for long-term forecasting, because it relies only on past trends and seasonality, the model's performance will become unreliable. When there are significant long-term changes or mutations in the time series, its predictive ability is relatively limited.

4 Results and Discussion

4.1 Testing Set Configuration

In this task, the data is divided into training sets and validation sets, using the simple chronological split method. The training set is composed of all data points except the last six quarters, and the validation set is composed of the last six quarters.The reason for doing this is that our final prediction is for 6 data points. By aligning the validation set with the forecast horizon, we can directly assess the model's ability to predict future data, providing a clearer evaluation of its performance for the actual forecasting task.

There is a key point is that the 'Decomposition model' requires the training data to have a length that is a multiple of 4. The reason is that the 'Decomposition model' needs to identify seasonal components, trends, and residuals within the time series, so there can be inconsistencies or reduced effectiveness in capturing the seasonal patterns if the data length is not a multiple of the seasonal cycle.

4.2 Description of Metrics Used

1) Quantitative Metrics

In the time series forecasting project, the core of evaluating model performance is to measure the difference between the predicted value and the true value. In this project, aiming to quantify the performance of the predictive models, Mean Squared Error (MSE) evaluation metrics are employed. MSE are used to evaluate the error between the predicted value and the actual value. The smaller the better.Since both our validation and test sets have 6 data points, the MSE is calculated by dividing the sum of squared errors between each predicted and actual value by 6 to obtain the mean squared error.

$$test\_error = \frac{1}{6}\sum_{h=1}^{6}\left(\widehat{y}_{T+h|1:T} - y_{T+h}\right)^2$$

2) Qualitative Metrics

In addition to quantitative analysis, qualitative evaluation is important to further understand the model's performance. We use a series of metrics to analyze, such as time series plot, residual analysis, comparison with cash rate data.

- **Time series plot:** By plotting a time series graph of actual values and predicted values, we can visually observe whether the model's predictions closely follow the fluctuations and trends of the actual data.
- **Residual analysis:** Including the Residual Plot, Residual Histogram, Autocorrelation Function (ACF) plot, and Partial Autocorrelation Function (PACF) plot.
- **Residual Plot**: Used to check if residuals are randomly and evenly distributed, and if heteroscedasticity exists. If the plot shows a random pattern and residuals are evenly distributed around 0, it indicates a good model fit. Otherwise, it may suggest the model hasn't fully captured the data features and needs adjustment.

- **Residual Histogram**: Checks if the residuals follow a normal distribution. If they do, it indicates that model errors are random and the model fit is good. If not, model parameters may need adjustment, or a different model may be required.
- **ACF and PACF Plots**: Used to check for autocorrelation in the residuals. If most lags in the plot fall within the confidence interval, it indicates residuals are random with no significant autocorrelation. If certain lags show significant correlation, the model may not fully capture the time dependencies and may need adjustment.
- **Combined with cash rate analysis:** A rise in CPI typically signals inflation, and central banks often raise the cash rate to curb spending. Conversely, when CPI falls, indicating deflation, banks usually lower the cash rate to stimulate consumption. Therefore, observing cash rate changes can help assess the validity of our predictions.

4.3 Results of Model

1) Decomposition model

The Decomposition model decomposes CPI data into three parts: trend, seasonality and residual. The results show that the trend component captures the long-term upward trend of CPI and reflects the impact of economic events. The 'detrended series' highlights the variations remaining after the trend has been removed. While the seasonal component shows cyclical changes within the quarter. The normalized seasonal indices further confirm the stability of these quarterly cycles. However, the decomposition model only focuses on trend and seasonality, so the treatment of residuals is relatively limited, there is still a certain error in predicting future data. The mean square error (MSE) on the test set is 66.8346.

Step 1: Trend estimation can reveal long-term trends in time series analysis, so the data needs to be smoothed. The moving average method can effectively remove noise. This is quarterly data, so a window size of 4 is appropriate. Calculating the mean by moving the window makes the trend line smoother and clearly reflects the initial trend estimation of the time series (Figure 4.1). This helps to understand the general trend of the CPI independent of short-term fluctuations.

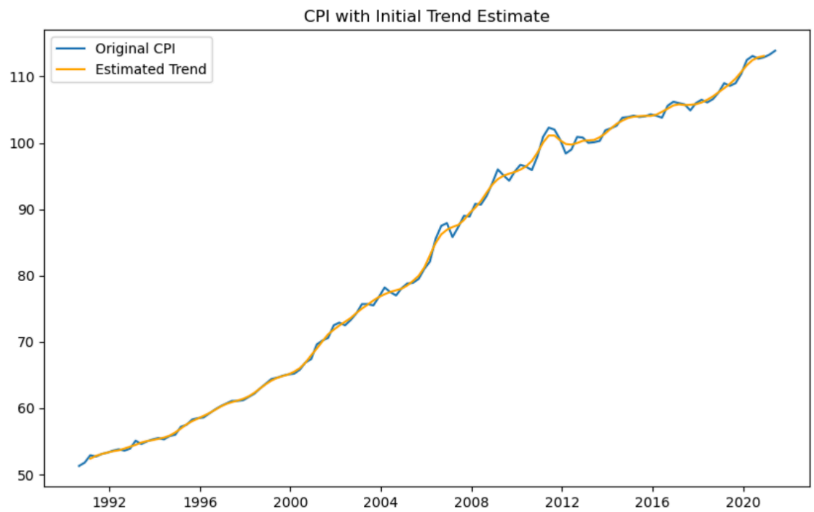

Figure 4.1: CPI with Initial Trend Estimate

Step 2: De-trending is an important step in the decomposition model. According to Figure 4.1 it can be seen that the variance is relatively stable and there is no obvious trend of increasing over time.

Therefore, it is more appropriate to use an additive model for trend decomposition. The trend component is subtracted from the time series, only keeping seasonality and remainder to better observe the cyclical fluctuations. Figure 4.2 shows that CPI has volatility and cyclicality in different time periods. The fluctuations from 2008 to 2012 were large, which may be related to the changes in the global economic environment at that time.

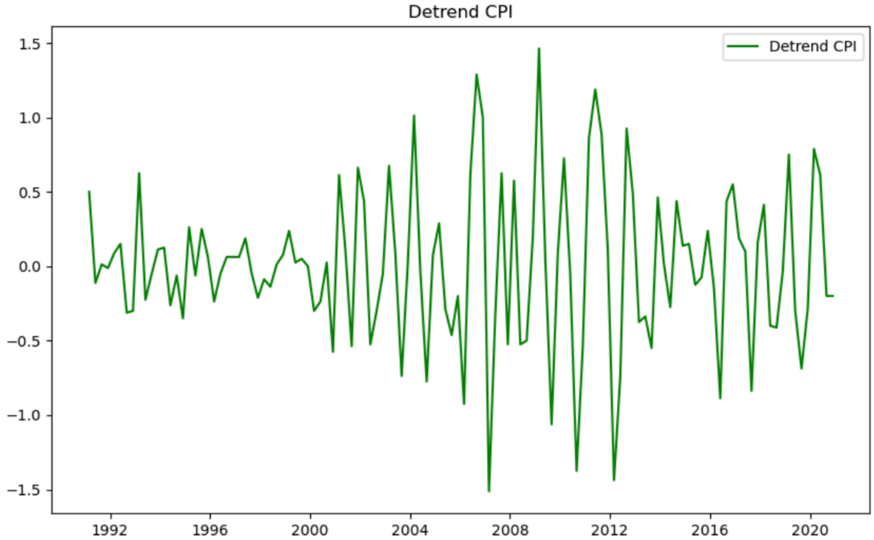

Figure 4.2: Detrend CPI

Step 3: Group the detrended data by quarter and calculate the mean value of each quarter to obtain the seasonal index. The seasonal index is normalized to ensure that its mean value is zero, which makes the prediction more accurate. Figure 4.3 shows that the seasonal index has obvious cyclical variations, and its fluctuations are mainly concentrated in the range of -0.1 to 0.15, which indicates that the data have stable seasonal characteristics.

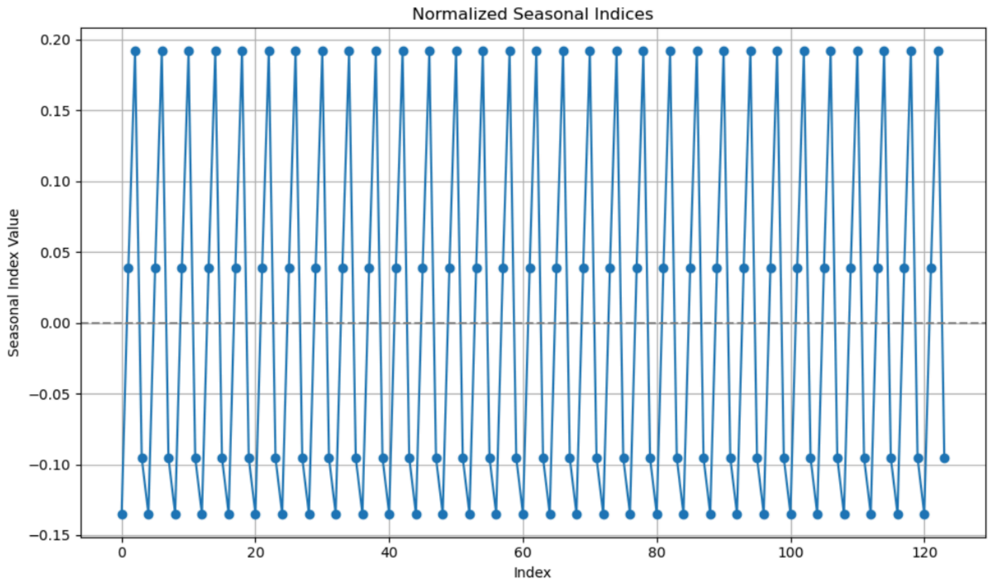

Figure 4.3: Normalized Seasonal Indices

Step 4: Subtract the corresponding seasonal index from the raw data, which can eliminate the influence of cyclical fluctuations on the forecast. According to Figure 4.4, after reducing the seasonal disturbances the changes in CPI are more stable and the CPI values are generally on an upward trend.

In addition, the fluctuations between 2008 and 2012 may reflect the impact of some non-seasonal economic factors and events during this period.

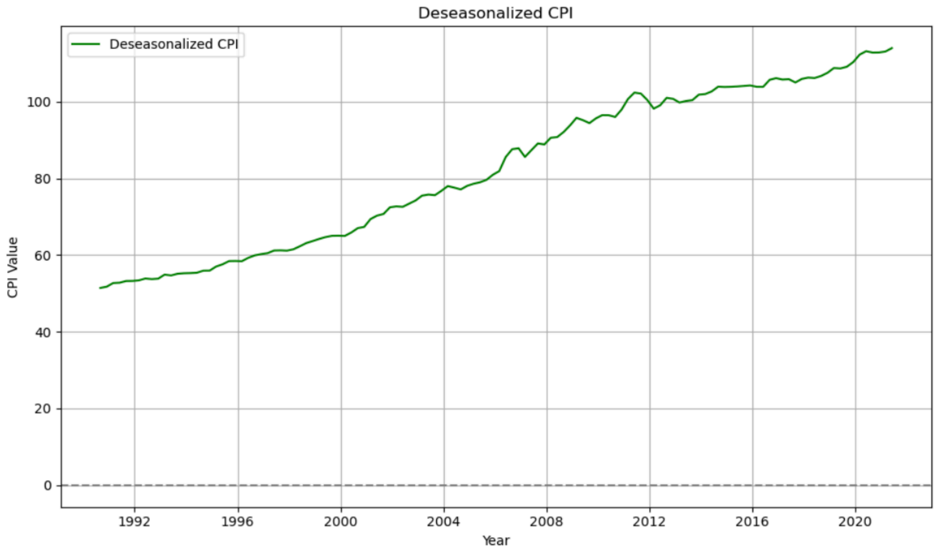

Figure 4.4: Deseasonalized CPI

Step 5: Perform moving average processing on the deseasonalized data again to obtain a more accurate trend. As in Figure 4.5, the trend component becomes smoother, it is helpful for subsequent forecasting.

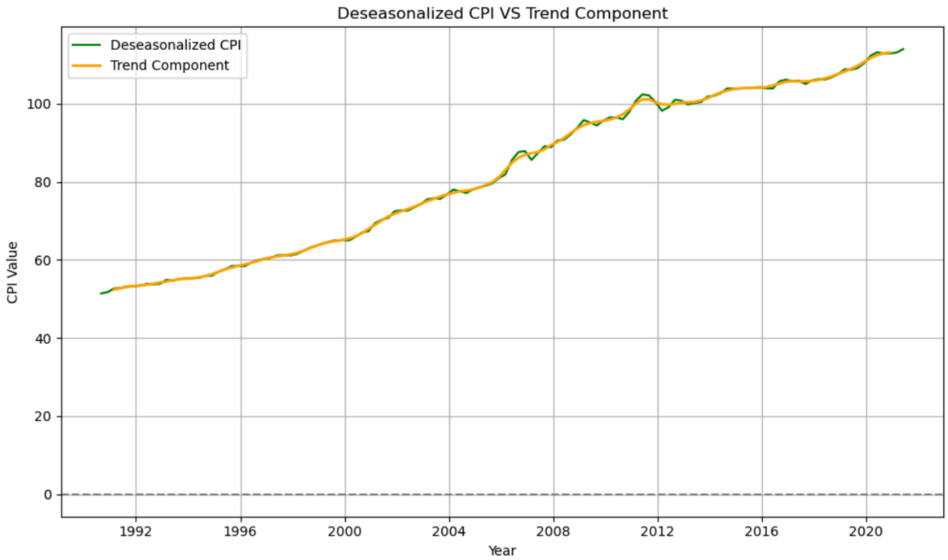

Figure 4.5: Deseasonalized CPI VS Trend Component

Step 6: Forecast the CPI data for the next 6 quarters based on the trend and seasonal components decomposed previously.

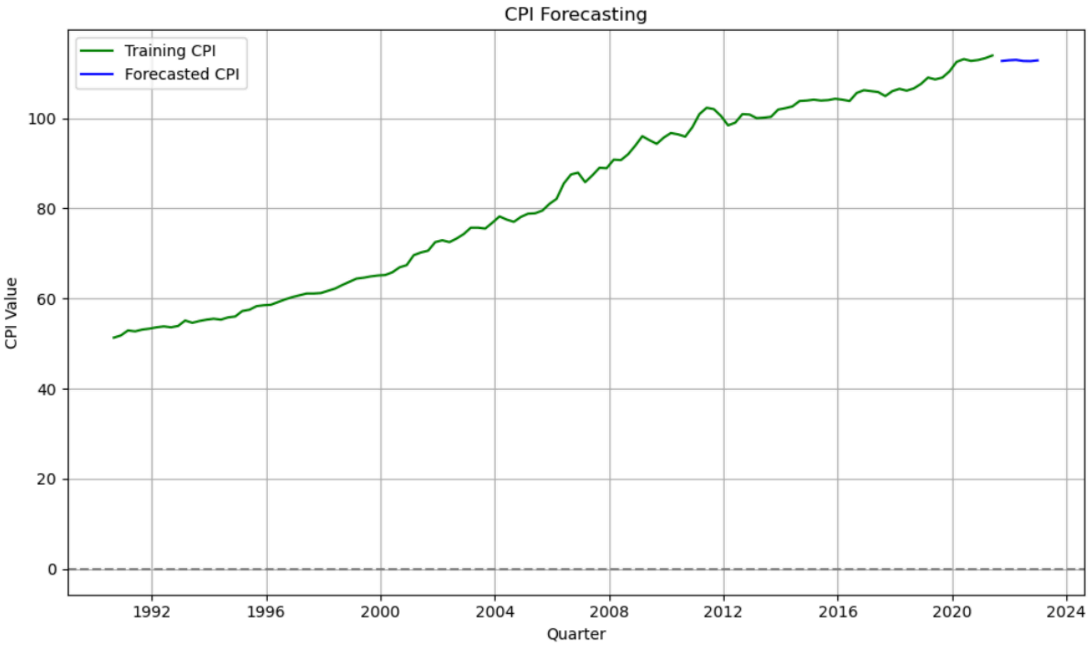

Figure 4.6: CPI Forecasting

The residual diagnostics are shown in Figure 4.7, Figure 4.8 and Figure 4.9.From the chart, we can see that the residuals are randomly and normally distributed, mostly around 0, and the autocorrelation values fall largely within the confidence interval. This indicates that the model's fit is reasonably good.

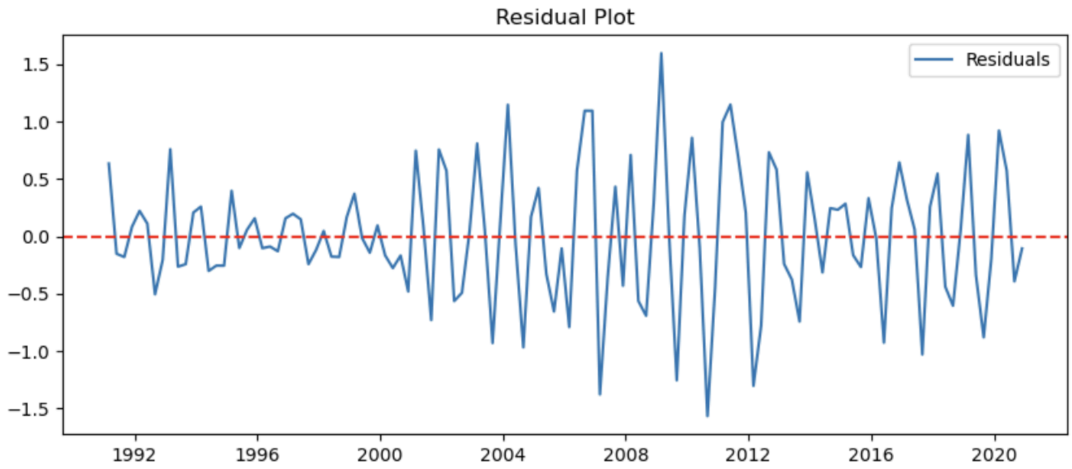

Figure 4.7: Residual Plot of Decomposition model

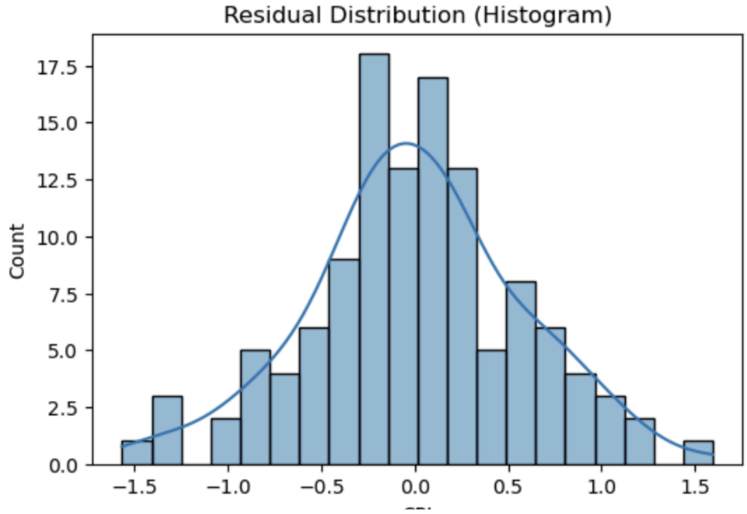

Figure 4.8: Residual Distribution Plot of Decomposition model

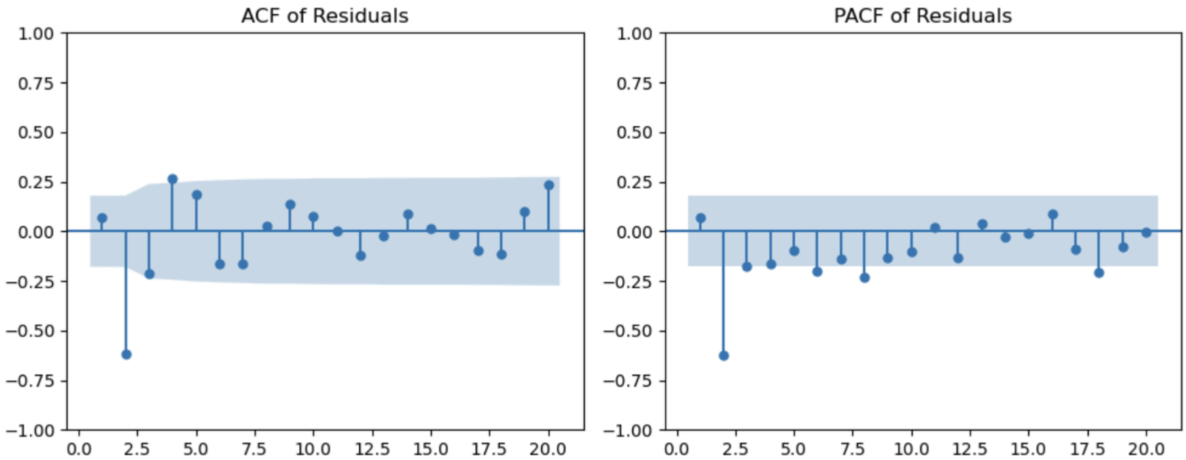

Figure 4.9: ACF & PACF of Decomposition model

2) Holt-Winters

Training the Holt-Winters model requires some settings, including the selection of trend and seasonality, as well as whether to use damping.

- Trend: Additive. The dataset shows a relatively stable linear trend over time, with consistent changes at each point, indicating an additive trend. Therefore, the trend parameter in the model is set to additive.
- Seasonality: Additive. The seasonality also maintains stable variance over time, suggesting additive seasonality, so we use "additive" for the seasonal parameter.
- Damping: Not applied. There is no indication of trend damping, and since we are focusing on short-term forecasting (six future data points), damping is not applied to suppress the trend.

The model uses simple additions for trend and seasonality, making it good for data with steady growth and regular seasonal changes. The forecast value continues to increase and maintain steady growth. The model's MSE on the test data is 27.8363, meaning it does a good job of tracking trends and seasonal changes.

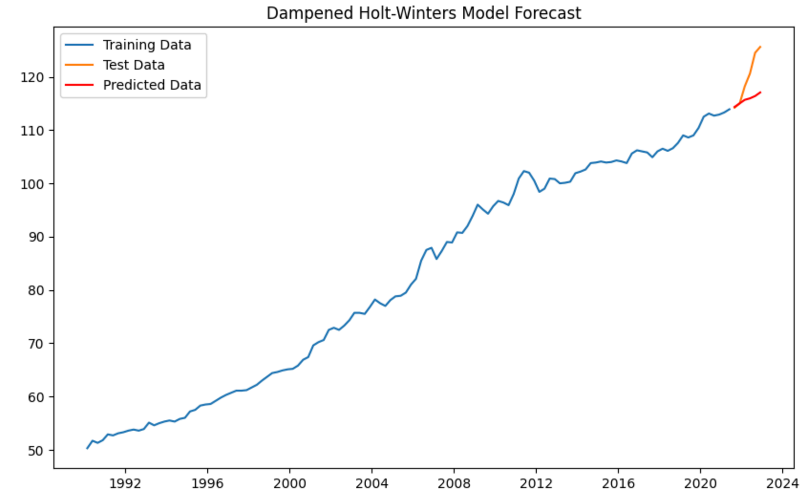

Figure 4.10: Dampened Holt-Winters Model Forecast

After model training, we perform state decomposition on it. In Figure 4.11, the level rises slowly year by year, indicating an upward long-term trend in the time series. Besides, the trend shown in the plot exhibits fluctuations, which shows that there are different growth rates across different time periods, and the seasonal component shows a pattern of periodic fluctuations.

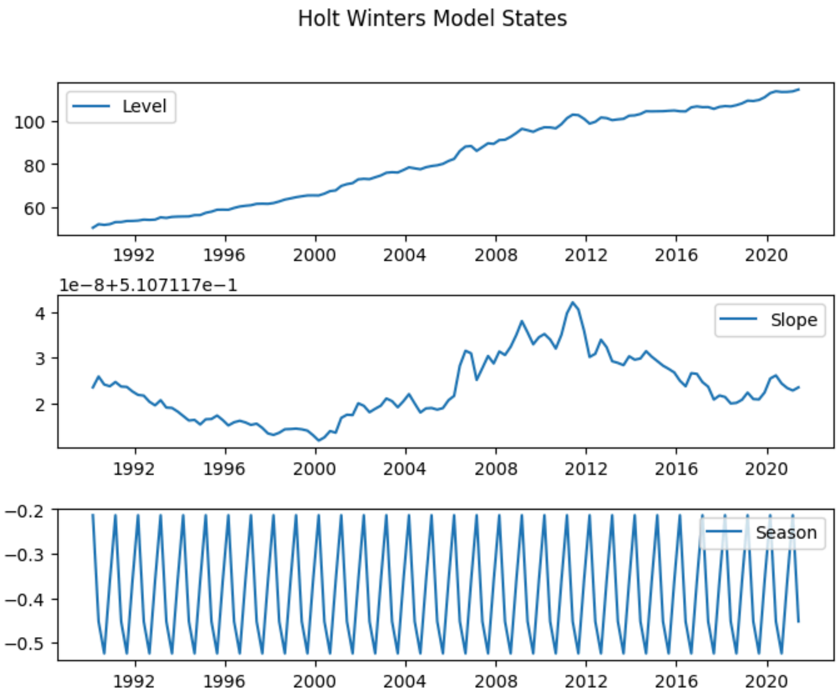

Figure 4.11; ACF & PACF of Decomposition model

To evaluate the model's performance qualitatively, we analyze the residuals (the differences between actual and predicted values) to assess the model's fit and identify potential issues. As shown inFigure 4.12, the plot shows that the residuals are randomly distributed around 0, there are no obvious trends or patterns, showing that the model fits the data well.

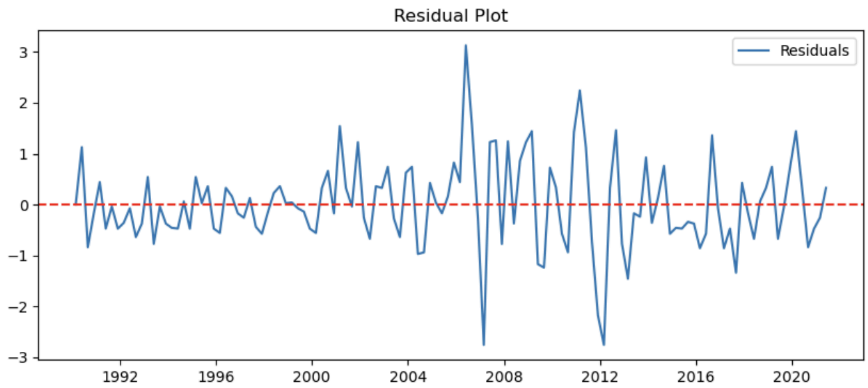

Figure 4.12: Residual Plot of Holt-Winters

An ideal residual distribution should approximate a normal distribution. As shown in Figure 4.13, the residuals are roughly normally distributed, indicating that the model has captured most of the data's characteristics well.

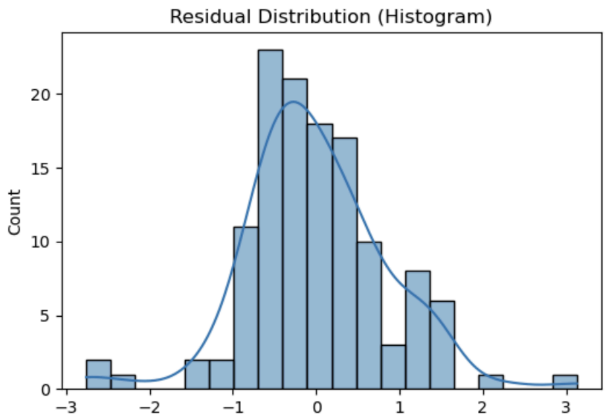

Figure 4.13: Residual Distribution Plot of Holt-Winters

As shown in Figure 4.14, this is the ACF & PACF of Holt-Winters, the residuals' autocorrelation and partial autocorrelation coefficients for most lags are close to 0, indicating that the model performs well in eliminating autocorrelation, leaving no significant autocorrelation.

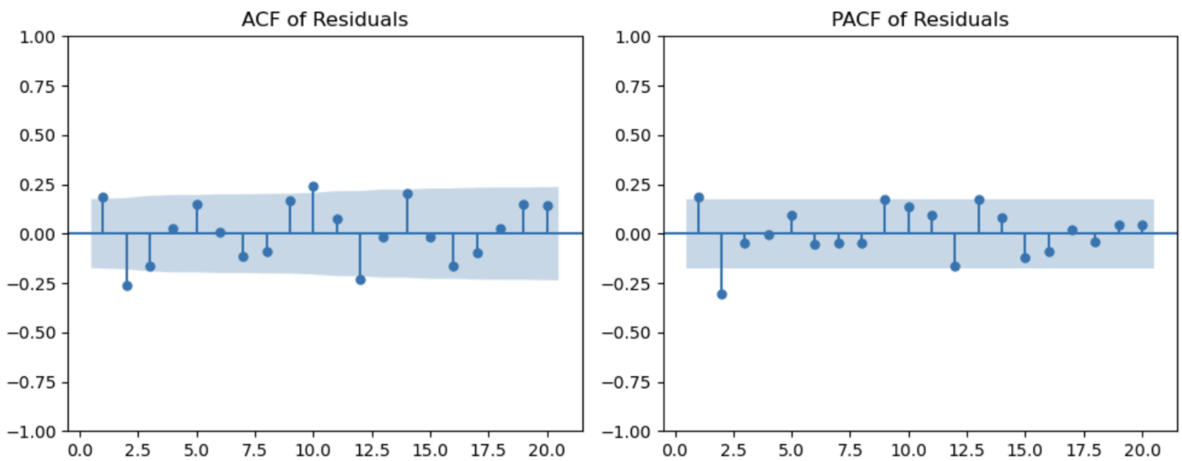

Figure 4.14: ACF & PACF of Holt-Winters

3) SARIMA

To build the basic model of SARIMA, we extracted the CPI data in the training set firstly, and set the order parameter = (1,1,1). The three numbers respectively represent that the model constantly uses the data of the previous quarter as the basis for the subsequent prediction data, and only performs one difference operation. The unstable time series is converted to stationary data, and the previous data prediction error is used to improve the current forecast. At the same time, the seasonal parameter represents the seasonal pattern in the model, and the meaning of the order parameter set before is the same. The CPI value of the previous quarter is used for forecasting and differential operation. The enforce_stationarity and enforce_invertibility parameters simultaneously mean that the model can accept non-stationary data and allow data that does not meet the reversible condition respectively. Furthermore, the model is fitted for the first time and the second fine fitting for the purpose of helping the model find a better initial point. The powell method is used here to improve the convergence and fitting effect of the model, and fit the model again to achieve the better imitative effect.

```
sarima_model = SARIMAX(train_data['CPI'],
                       order=(1, 1, 1),  # (p, d, q): Non-seasonal parameters
                       seasonal_order=(1, 1, 1, 4),  # (P, D, Q, s): Seasonal parameters (s=4 because it is quarterly data)
                       enforce_stationarity=False,  # agree non-stationary data
                       enforce_invertibility=False)  # agree non-invertible models
```

The SARIMA model performs well in capturing the long-term trend and seasonal fluctuations in the CPI data, and the MSE of the testing dataset is 27.0059. The forecast of this model indicates that CPI is on an upward trend in the future.

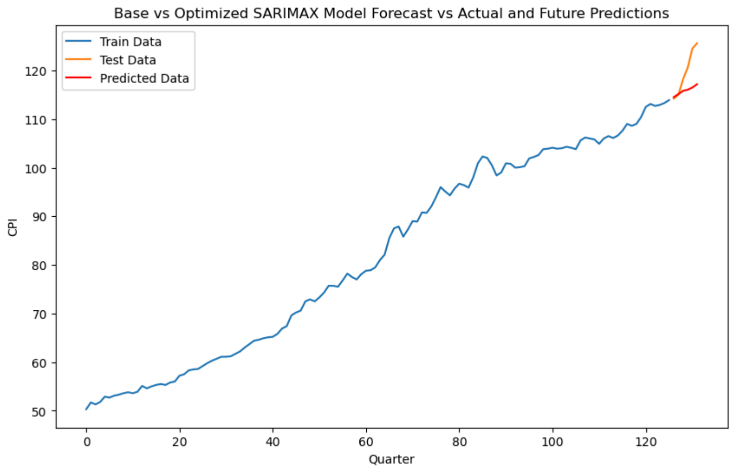

Figure 4.15: SARIMA Model Forecast

The residual diagnostics are shown in Figure 4.16, Figure 4.17 and Figure 4.18.In the residual plot, the large fluctuations before 1992 may indicate poor model fit during the initial phase. After that, the residuals randomly fluctuate around the zero line, suggesting smaller prediction errors and improved fit in the later period. The residual histogram shows that the residuals can be considered normally distributed. The ACF plot indicates that most residuals are random with no significant autocorrelation, meaning the model fits well overall. However, there is notable correlation at lag 4, but this is the best result after multiple optimizations.

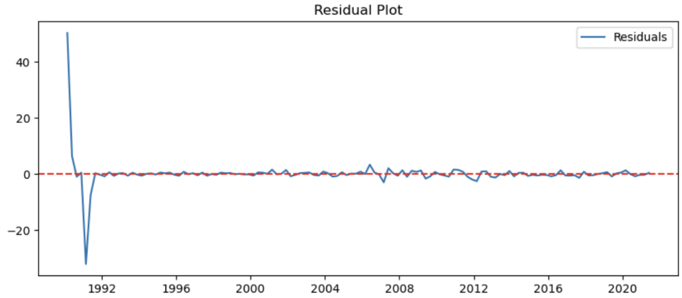

Figure 4.16: Residual Plot of SARIMA

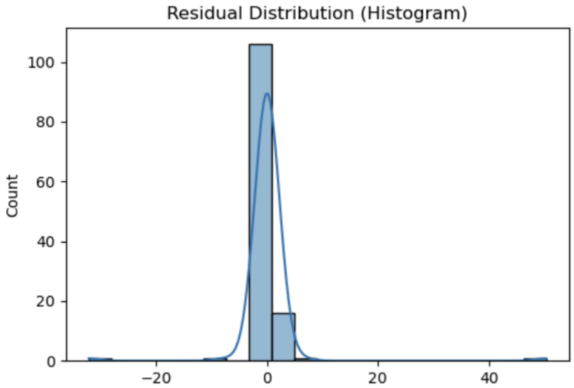

Figure 4.17: Residual Distribution Plot of SARIMA

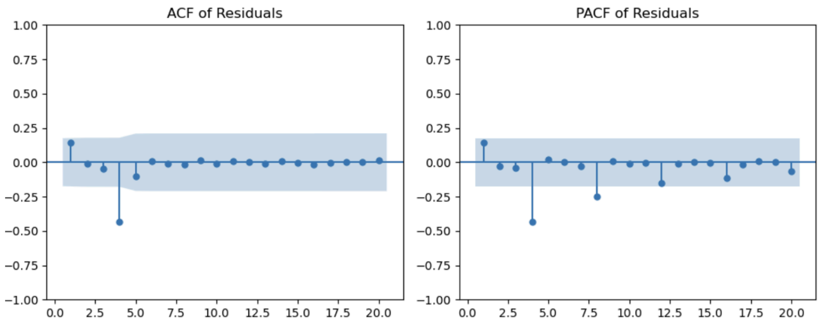

Figure 4.18: ACF & PACF of SARIMA

4.4 Final Model

The MSE of SARIMA model on testing data is the lowest among three models, which shows its prediction accuracy is the highest. In addition, the SARIMA model can capture both seasonality and long-term trends, which is suitable for multi-step forecasting tasks.Therefore, we selected the SARIMA model as the final model for predicting the final results.

Table 4.1

MODEL	MSE
Decomposition	66.8346
Holt-Winters	27.8363
SARIMA	**27.0059**

4.5 Final Results

We retrain the SARIMA model using the entire dataset (including the validation set) and then make the final prediction, to make the model use all the information to make predictions, ensuring that the model can learn all patterns and trends. This results in a better performance in the final 6-step forecast. The final result is as follows:

Table 4.2

Time	Forecasted CPI values	cashrate (%)
2023-03-01	125.9341	3.85
2023-06-01	126.2915	4.10
2023-09-01	126.8277	4.35
2024-12-01	127.5191	4.35
2024-03-01	128.2443	4.35
2024-06-01	128.5419	4.35

The forecast results of the SARIMA model shows that the CPI will keep rising in the next six quarters. This shows that although inflation will continue to rise in the future, the growth rate will gradually slow down than before.

Based on the bank's cash rate forecast, we observe an upward trend from March 2023 to June 2024, increasing from 3.85% to 4.35%. Since the cash rate is generally related to inflation levels, when CPI rises, indicating increased inflation, the bank typically raises the cash rate to curb consumption. Therefore, the bank's decision to raise the cash rate during this period suggests that CPI was increasing, which aligns with our forecast trend, confirming the validity of our prediction.

References

1. Chen, Y., Hu, X., & Zhang, L. (2022). A review of ultra-short-term forecasting of wind power based on data decomposition-forecasting technology combination model. *Energy Reports, 8*, 14200–14219. https://doi.org/10.1016/j.egyr.2022.10.342

2. Chhetri, M. B., Lumpe, M., Vo, Q. B., & Kowalczyk, R. (2017). On forecasting Amazon EC2 spot prices using time-series decomposition with hybrid look-backs. In *2017 IEEE International Conference on Edge Computing (EDGE)* (pp. 158–165). IEEE. https://doi.org/10.1109/ieee.edge.2017.29

3. Gardner, E. S. (n.d.). Exponential smoothing: The state of the art—Part II. *International Journal of Forecasting, 22*(4), 637–666.

4. Hyndman, R. J., & Athanasopoulos, G. (2018). *Forecasting: Principles and practice*. OTexts.

5. Mbuli, N., Mathonsi, M., Seitshiro, M., & Pretorius, J.-H. C. (2020). Decomposition forecasting methods: A review of applications in power systems. *Energy Reports, 6*, 298–306. https://doi.org/10.1016/j.egyr.2020.11.238

6. Permanasari, A. E., Rambli, D. R. A. R., & Dominic, P. D. D. (2010). Forecasting method selection using ANOVA and Duncan multiple range tests on time series dataset. In *2010 International Conference on Intelligent and Advanced Systems (ICIAS)* (pp. 941–945). IEEE. https://doi.org/10.1109/itsim.2010.5561535

7. Shang, Y., Zhu, C., & Li, S. (2018). Estimation and explanation of time-varying weights in Chinese CPI. *Emerging Markets Finance and Trade, 54*(15), 3426–3437. https://doi.org/10.1080/1540496X.2018.1485095

8. Folger, J. (2024, June 27). What is the relationship between inflation and interest rates? *Investopedia*. https://www.investopedia.com/ask/answers/12/inflation-interest-rate-relationship.asp