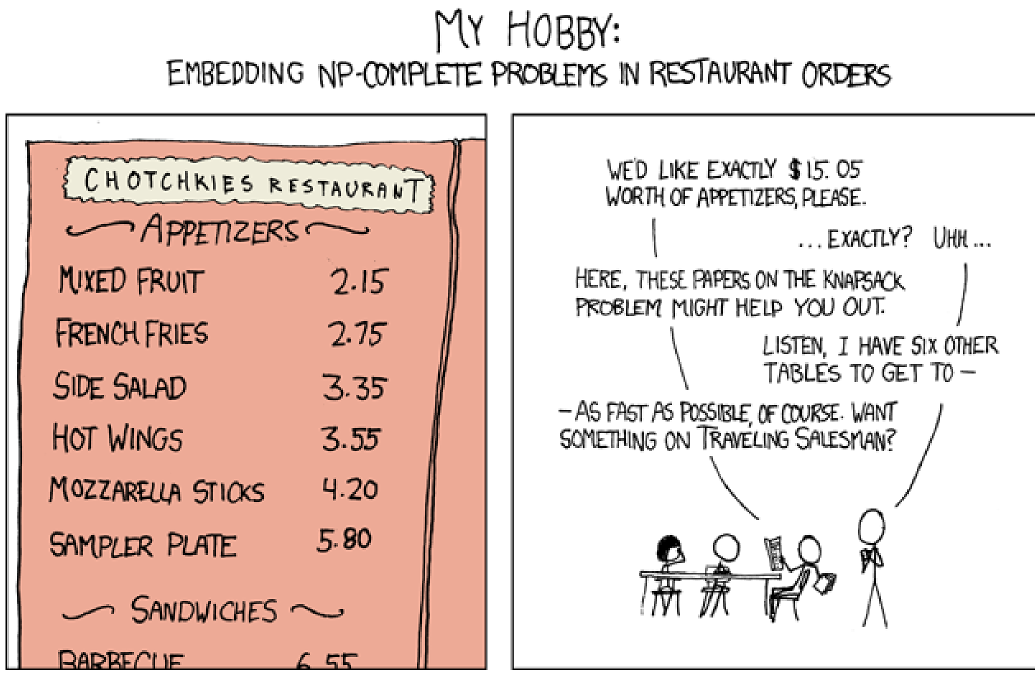
COMP 360 - Fall 2019 - Assignment 5

Due: 11:59pm Dec 6th

General rules: In solving these questions you may consult books or other available notes, but you need to provide citations in that case. You can discuss high level ideas with each other, but each student must find and write his/her own solution. Copying solutions from <u>any source</u>, completely or partially, allowing others to copy your work, will not be tolerated, and will be reported to the disciplinary office.
You should upload the pdf file (either typed, or a <u>clear and readable</u> scan) of your solution to mycourses.
There are in total 22 points, but your grade will be considered out of 20.



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1. (3 points) Either prove that the following problem is NP-complete or show that it belongs to P by giving a polynomial time algorithm:
   - Input: An undirected graph G.
   - Question: Does G have a proper colouring with three colours R, G, B that assigns the colour B to exactly one vertex?

2. (8 points) Consider the triangle elimination problem. We are given an undirected graph $G = (V, E)$, and want to find the smallest possible set of vertices $U \subseteq V$ such that deleting these vertices removes all the triangles (i.e. cycles of length 3) from the graph. For each one of the following algorithms, either show that it is a 3-factor approximation algorithm, or give an example to show that it is not.

   Algorithm I:

- While there is still a triangle C left in G:
- Delete all the three vertices of C from G
- EndWhile
- Output the set of the deleted vertices

Algorithm II:

- While there is still a triangle C left in G:
- Delete one arbitrarily chosen vertex of C from G
- EndWhile
- Output the set of the deleted vertices

Algorithm III:

- While there is still a triangle left in G:
- Delete a vertex that is in the largest number of triangles
- EndWhile
- Output the set of the deleted vertices

Algorithm VI:

- Write an Integer Program for this problem with constraints $x_u + x_v + x_w \geq 1$ for every triangle (u, v, w) in G (and constraints $x_u \in \{0, 1\}$).
- Solve the LP relaxation of this Integer Program (with constraints $0 \leq x_u \leq 1$).
- EndWhile
- Output the set of the vertices u with $x_u \geq \frac{1}{3}$.

3. (4 points) We are given a graph G together with an ordering of the vertices of G such that every vertex v has at most 5 neighbours that appear before v in that order (but v can have many neigbhours appear later in the order).

   - Show that the vertices of G can be properly coloured using 6 colours.
   - Next we want to colour the vertices of G with 5 colours so as to maximize the number of edges that are properly coloured (that is they have different colours on their endpoints). Design a $\frac{14}{15}$-factor approximation algorithm for this problem.

4. Let x be a string of length n of 0's and 1's. Consider the following operations:

   - del(x, i) (for $1 \leq i \leq n$) deletes the i-th bit of the string x, and thus decreases its length to $n - 1$.
   - set(x, i, b) (for $1 \leq i \leq n$ and $b \in \{0, 1\}$) sets the i-th bit of x to the bit b.
   - insert(x, i, b) (for $1 \leq i \leq n+1$ and $b \in \{0, 1\}$) inserts b after the $i-1$-th bit of x, and thus increases the length of x.

   Let a and b be two strings 0's and 1's. Define the distance $d(a, b)$ to be the smallest number of operations required to convert a to b.

   (a) (1 point) Show that $d(a, b) = d(b, a)$.
   (b) (2 point) Explain briefly how $d(a, b)$ can be computed in polynomial time using dynamic programming.
   (c) (4 points) We are given 3 strings a, b, c, and we want to find a fourth string d that minimizes $d(a, d) + d(b, d) + d(c, d)$. Give a 4/3-approximation algorithm for this problem.