COMP 360 - Fall 2019 - Assignment 2

Due: 11:59pm Oct 10th.

General rules: In solving these questions you may consult books or other available notes, but you need to provide citations in that case. You can discuss high level ideas with each other, but each student must find and write his/her own solution. Copying solutions from <u>any source</u>, completely or partially, allowing others to copy your work, will not be tolerated, and will be reported to the disciplinary office.
You should upload the pdf file (either typed, or a <u>clear and readable</u> scan) of your solution to mycourses.
There are in total 25 points, but your grade will be considered out of 20, however no partial marks are given for the last bonus question

"Is this going to be on the midterm?"

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1. (3 points) Prove or disprove: Let G be a flow network where all the capacities are 1. If G has at least two different integer-valued maximum flows f and f', then there are at least two different minimum cuts (A, B) and (A', B').

2. (3 points) We are given an $n \times n$ matrix A with integer entries. We want to select the smallest number of rows and columns (in total) so that after removing these rows and columns all the remaining entries are strictly positive. Show that this problem be solved efficiently.

3. (3 points) Give an efficient algorithm based on network flow techniques to solve the following problem. Justify that your algorithm is correct. The input is an undirected bipartite graph $G = (V, E)$ with parts X and Y, where some of the edges are coloured red. We would like to find out whether we can colour more edges red (if necessary) so that every vertex is incident to exactly 10 red edges.

4. (3 points) We are given an $n \times n$ matrix A with ± 1 entries. We want to rearrange the rows of A and then the columns of A to obtain a new matrix B such that the sum of the <u>diagonal</u> entries of B is maximized. That is $\sum_{i=1}^{n} B_{ii}$ is maximized. Show that this problem be solved efficiently.

5. (3 points) Give an efficient algorithm based on network flow techniques to solve the following problem. Justify that your algorithm is correct.

   We are given as input, positive integers $a_1, \ldots, a_k$ and $b_1, \ldots, b_t$ such that $a_1 + \ldots + a_k = b_1 + \ldots + b_t = n$. Let G be the graph on n vertices that is formed by taking the disjoint union of k cliques, on $a_1, \dots, a_k$ vertices, respectively. We want to know if we can colour the vertices of G with t colours such that the first colour is used b_1 times, the second colour b_2 times, etc, so that no two adjacent vertices are coloured with the same colour.

6. (3 points) Give an efficient algorithm based on network flow techniques to solve the following problem. Justify that your algorithm is correct. In particular, explain how backup sets and flows correspond to each other.

Input: The coordinates of n antennas on the plane, where the coordinate of the i-th antenna is of the form (x_i, y_i) where x_i and y_i are integers.

Output: For each antenna i, select a backup set B_i of five other antennas such that the antennas in B_i are all within distances of at most 100 from the antenna i, and moreover in total no antenna belongs to more than 10 backup sets.

7. (3 points) Consider a graph where some of the edges have "directions", but some are undirected edges. We want to assign direction to all the undirected edges such that in the resulting directed graph, the incoming degree of every vertex is equal to its outgoing degree. Note that we are not allowed to change the direction of the edges that have directions in the beginning.

   Give an efficient algorithm based on network flow techniques to solve this problem. Justify that your algorithm is correct.

8. (Bonus problem: 4 points) Show that for every network flow, there is always a sequence of augmenting paths that leads to maximum flow, where none of these paths decreases the flow on any of the edges.