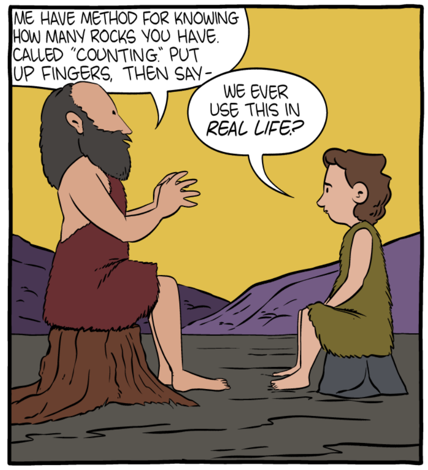
COMP 360 - Fall 2019 - Assignment 4

Due: 11:59pm Nov 21st.

General rules: In solving these questions you may consult books or other available notes, but you need to provide citations in that case. You can discuss high level ideas with each other, but each student must find and write his/her own solution. Copying solutions from any source, completely or partially, allowing others to copy your work, will not be tolerated, and will be reported to the disciplinary office.
You should upload the pdf file (either typed, or a clear and readable scan) of your solution to mycourses.
There are in total 22 points, but your grade will be considered out of 20.



The first math class.

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1. (4 points) Consider the following problem: We are given a number r and a maximization linear program LP (say in the standard form) as follows:

$$\begin{array}{lll} \max & c^T \cdot \vec{x} & \\ \text{s.t.} & A\vec{x} & \leq \vec{b} \\ & \vec{x} & \geq 0 \end{array}$$

   That is the input is $[A, c, b, r]$, where A is an $m \times n$ matrix, b is an m-dimensional vector, and c is an n-dimensional vector, and r is a number, and we want to know if $\text{Opt}(LP) \geq r$?

   Without using the fact that Linear Programming can be solved efficiently using the ellipsoid method show that this problem belongs to both NP and CoNP.

2. We are given the coordinates of n points $p_1, \dots, p_n$ on the plane. We want to find non-overlapping disks centred on these points such that the sum of their radii is maximized.

   (a) (2 points) Show that this problem can be solved as a linear program.

   (b) (4 points) Consider a complete directed graph on the points $p_1, \dots, p_n$ where for every i and j, there is an edge from p_i to p_j, and one edge from p_j to p_i, both with a costs equal to $d(p_i, p_j)$, the distance between the two points. We want to cover the vertices of this graph with cycles (that is every vertex belongs to at least one cycle) so that the total sum of the cost of the edges that participate in these cycles is minimized (Note that we also allow cycles with two edges).

   Use the linear programming duality to prove that the solution to this problem is at least twice the solution to the non-overlapping disk problem.

3. (4 points) We are given an $n \times m$ table. The rows correspond to n students and the columns correspond to their favourite things (For example, food, book, sport, composer, movie, etc). For example if the entry in the food column of the i-th row is Samosa, it means that Samosa is the favourite food of the i-th student. We are also given a number k, and our goal is to select k students so that every two of them share at least one favourite thing in life. Prove that this problem is NP-complete.

4. (4 points) We are given n numbers (each in decimal representation), and a number k. We want to see if it is possible to select k of these numbers so that if we add them up, the digit 2 will not appear in the decimal representation of the sum. Prove that this problem is NP-complete.

5. (4 points) Prove that the following problem is NP-complete.

   - Input: A CNF ϕ.
   - Question: Does ϕ have a truth assignment that assigns True to exactly half the <u>terms</u> in each clause?

   (For example $x_1 = T, x_2 = F, x_3 = F, x_4 = F$ assigns True to half the terms in each clause of $\phi = (x_1 \vee \overline{x_2} \vee x_3 \vee x_4) \wedge (x_1 \vee x_2)$.)