Olivia Woodhouse, 260734701 Assignment #1

(1) Number Representation, 2s Comlement, and Floating Point

1. Convert 0.101011_2 from binary → decimal

$$0.101011_2 = (0\times2^0)+(1\times2^{-1})+(0\times2^{-2})+(1\times2^{-3})+(0\times2^{-4})+(1\times2^{-5})+(1\times2^{-6})$$
$$= 0+\frac{1}{2}+0+\frac{1}{8}+0+\frac{1}{32}+\frac{1}{64}$$
$$= \frac{32}{64}+\frac{8}{64}+\frac{2}{64}+\frac{1}{64} = \frac{43}{64} = 0.671875$$

2. Convert 6.75_{10} decimal → binary AND hexadecimal

i. binary:

$2\overline{)6_{10}}$
$2\overline{)3}$ + 0
$2\overline{)1}$ + 1
0 + 1 $= 110_2$

$0.75_{10}\times2 = 0.5 + 1$
$0.5\times2 = 0.0 + 1$
$0.0\times2 = 0.0 + 0$ $= 0.110_2$

$\therefore 6.75_{10} = 110.110_2$

ii. hexadecimal:

$16\overline{)6_{10}}$
0 + 6 $= 6_{16}$

$0.75_{10}\times16 = 0.0 + (12 = C)$
$0.0\times16 = 0.0 + 0$ $= 0.C0_{16}$

$\therefore 6.75_{10} = 6.C0_{16}$

3. Convert $1FA.U06G_{32}$ base 32 → binary → hexadecimal

i. decimal (to use for binary and hexadecimal):

F = 15
A = 10
U = 30
G = 16

$$1FA.U06G_{32} = (1\times32^2)+(15\times32^1)+(10\times32^0)+(30\times32^{-1})+(0\times32^{-2})$$
$$+(6\times32^{-3})+(16\times32^{-4}) = 1024+480+10+\frac{30}{32}+0$$
$$+\frac{6}{32768}+\frac{16}{1048576} = 1514+\frac{983040}{1048576}+\frac{192}{1048576}+\frac{16}{1048576}$$
$$= 1514+\frac{983248}{1048576} = 1514+\frac{491624}{524288} = 1514+\frac{245812}{262144}$$
$$= 1514+\frac{61453}{65536} = 1514.937698364_{10}$$

ii. binary:

1514.937698364_{10}

$2\overline{)1514_{10}}$
$2\overline{)757}$ + 0
$2\overline{)378}$ + 1
$2\overline{)189}$ + 0
$2\overline{)94}$ + 1
$2\overline{)47}$ + 0
$2\overline{)23}$ + 1
$2\overline{)11}$ + 1
$2\overline{)5}$ + 1
$2\overline{)2}$ + 1
$2\overline{)1}$ + 0
0 + 1 $= 10111101010_2$

$0.937698364_{10}\times2 = 0.875396728 + 1$
$0.875396728\times2 = 0.750793456 + 1$
$0.750793456\times2 = 0.501586912 + 1$
$0.501586912\times2 = 0.003173824 + 1$
$0.003173824\times2 = 0.006347648 + 0$
$0.006347648\times2 = 0.012695296 + 0$
$0.012695296\times2 = 0.025390592 + 0$
$0.025390592\times2 = 0.050781184 + 0$
$0.050781184\times2 = 0.101562368 + 0$
$0.101562368\times2 = 0.203124736 + 0$
$0.203124736\times2 = 0.406249472 + 0$

...TBC

$0.406249472 \times 2 = 0.812498944 + 0$

$0.812498944 \times 2 = 0.624997888 + 1$

$0.624997888 \times 2 = 0.249995776 + 1$

$0.24999577 \times 2 = 0.499991552 + 0$

$0.499991552 \times 2 = 0.999983104 + 0$

$0.999983104 \times 2 = 0.999966208 + 1$

$0.999966208 \times 2 = 0.999932416 + 1$

$0.999932416 \times 2 = 0.999864832 + 1$

$0.999864832 \times 2 = 0.999729664 + 1$

$\vdots$

$= 0.11110000000011001111\ldots_2$

$\therefore 1FA.U066_{32} = 1514.937698364_{10} = 10111101010.11110000000011001111\ldots_2$

iii. hexadecimal:

$10111101010.11110000000011001111\ldots_2 = 1514.937698364_{10}$

$16 \overline{)1514_{10}}$

$16 \overline{)94} \quad + (10 = A)$

$16 \overline{)5} \quad + (14 = E)$

$0 \quad + 5$

$= 5EA_{16}$

$0.937698364_{10} \times 16 = 0.00317382 + (15 = F)$

$0.0031782 \times 16 = 0.050781184 + 0$

$0.050781184 \times 16 = 0.812498944 + 0$

$0.812498944 \times 16 = 0.999983104 + (12 = C)$

$0.999983104 \times 16 = 0.999729664 + (15 = F)$

$0.999729664 \times 16 = 0.995674624 + (15 = F)$

$0.995674624 \times 16 = 0.930793984 + (15 = F)$

$0.930793984 \times 16 = 0.892703744 + (14 = E)$

$0.892703744 \times 16 = 0.283259904 + (14 = E)$

$0.283259904 \times 16 = 0.532158464 + 4$

$0.532158464 \times 16 = 0.514535424 + 8$

$0.514535424 \times 16 = 0.232566784 + 8$

$0.232566784 \times 16 = 0.721068544 + 3$

$0.721068544 \times 16 = 0.537096704 + (11 = B)$

$0.537096704 \times 16 = 0.593547264 + 8$

$0.593547264 \times 16 = 0.496756224 + 9$

$0.496756224 \times 16 = 0.948099584 + 7$

$0.948099584 \times 16 = 0.169593344 + (15 = F)$

$\vdots$

$= 0.F00CFFFEE4883B897F\ldots_{16}$

$\therefore 1FA.U066_{32} = 1514.937698364_{10}$

$= 5EA.F00CFFFEE4883B897F\ldots_{16}$

4. Convert 3031004_5 to hexadecimal

i. decimal: $(3\times5^6)+(0\times5^5)+(3\times5^4)+(1\times5^3)+(0\times5^2)+(0\times5^1)+(4\times5^0)$

$= 46875+0+1875+125+0+0+4 = 48879_{10}$

ii. hexadecimal: $3031004_5 = 48879_{10}$

$16 \overline{)48879_{10}}$		
$16 \overline{)3054}$	+	$(15=F)$
$16 \overline{)190}$	+	$(14=E)$
$16 \overline{)11}$	+	$(14=E)$
0	+	$(11=B)$

$= BEEF_{16}$

5. Convert -4128786_{10} to a 24 bit signed binary number using 2's complement (and then to hexadecimal in 6 symbols)

i. $|-4128786_{10}|$ to binary:

$2 \overline{)4128786_{10}}$		
$2 \overline{)2064393}$	+	0
$2 \overline{)1032196}$	+	1
$2 \overline{)516098}$	+	0
$2 \overline{)258049}$	+	0
$2 \overline{)129024}$	+	1
$2 \overline{)64512}$	+	0
$2 \overline{)32256}$	+	0
$2 \overline{)16128}$	+	0
$2 \overline{)8064}$	+	0
$2 \overline{)4032}$	+	0
$2 \overline{)2016}$	+	0
$2 \overline{)1008}$	+	0
$2 \overline{)504}$	+	0
$2 \overline{)252}$	+	0
$2 \overline{)126}$	+	0
$2 \overline{)63}$	+	0
$2 \overline{)31}$	+	1
$2 \overline{)15}$	+	1
$2 \overline{)7}$	+	1
$2 \overline{)3}$	+	1
$2 \overline{)1}$	+	1
0	+	1

$= 001111110000000000010010_2$

ii. Two's Complement:

$= 110000001111111111101101_2 + 1_2$

$= 110000001111111111101110_2$

iii. hexadecimal:

$\underbrace{1100}_{C}\ \underbrace{0000}_{0}\ \underbrace{1111}_{F}\ \underbrace{1111}_{F}\ \underbrace{1110}_{E}\ \underbrace{1110}_{E}$

$= C0FFEE_{16}$

6. -2.625_{10} as an IEEE single precision floating number (in both binary and hex)... is this representation exact?

$|-2.625_{10}| = 2\overline{)2}_{10}$

$2\overline{)1} + 0$

$0 + 1 \uparrow = 10_2$

$0.625 \times 2 = 0.25 + 1$

$0.25 \times 2 = 0.5 + 0$

$0.5 \times 2 = 0.0 + 1$

$0.0 \times 2 = 0.0 \quad 0 \downarrow = 0.1010_2$

$|-2.625_{10}| = 10.101_2 \xrightarrow{\text{normalize}} 1.0101_2 \times 2^1$

$\text{exponent} = 1 \xrightarrow{\text{bias notation}} 127 + 1 = 128$

$2\overline{)128}_{10}$

$2\overline{)64} + 0$

$2\overline{)32} + 0$

$2\overline{)16} + 0$

$2\overline{)8} + 0$

$2\overline{)4} + 0$

$2\overline{)2} + 0$

$2\overline{)1} + 0$

$0 + 1 \uparrow = 1000\,0000_2$

normalized mantissa: 01010000...

$\underbrace{1}_{\text{sign}} \; \underbrace{1000\,0000}_{\text{exponent}} \; \underbrace{01010000000000000000000}_{\text{fraction}}$

IEEE single precision = 11000000 010100000000000000000000

$\rightarrow$ hexadecimal: $\underbrace{1100}_{C}\,\underbrace{0000}_{0}\,\underbrace{0010}_{2}\,\underbrace{1000}_{8}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}\,\underbrace{0000}_{0}{}_2$

$= C0280000_{16}$

2 Seven Segment Decoder

1. Truth Table

A_3	A_2	A_1	A_0	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7
0	0	0	0	1	1	1	1	1	0	1	X
0	0	0	1	1	1	0	0	1	1	1	X
0	0	1	0	0	1	1	0	1	1	0	X
0	0	1	1	1	0	0	1	1	1	1	X
0	1	0	0	1	1	1	0	1	1	0	X
0	1	0	1	1	1	1	0	1	0	0	X
0	1	1	0	1	1	1	1	0	1	1	X
0	1	1	1	1	1	0	0	1	0	1	X
1	0	0	0	0	1	0	0	1	0	0	X
1	0	0	1	0	0	0	1	1	1	1	X
X	X	X	X	X	X	X	X	X	X	X	X

Karnaugh Maps: special form of truth table which enables easier pattern recognition

K-maps for the circuit:

* a grouping can only contain 2^k members
* as few groups as possible

S_0:

$\neg A_3 \cdot \neg A_1$ $\quad$ $\neg A_3 \cdot A_0$ $\quad$ A_2

A_3A_2 \ A_1A_0	00	01	11	10
00	1	1	1	0
01	1	1	1	1
11	X	X	X	X
10	0	0	X	X

$$S_0 = A_2 + \neg A_3 \neg A_1 + \neg A_3 A_0$$

S_1:

$\neg A_1 \cdot \neg A_3$ $\quad$ $\neg A_0$ $\quad$ A_2

A_3A_2 \ A_1A_0	00	01	11	10
00	1	1	0	1
01	1	1	1	1
11	X	X	X	X
10	1	0	X	X

$$S_1 = A_2 + \neg A_0 + \neg A_3 \neg A_1$$

$\neg A_0 \cdot \neg A_3$

S_2:

$A_2 \cdot \neg A_1$

A_3A_2 \ A_1A_0	00	01	11	10
00	1	0	0	1
01	1	1	0	1
11	X	X	X	X
10	0	0	X	X

$$S_2 = \neg A_3 \neg A_0 + A_2 \neg A_1$$

S_3:

$\neg A_3 \cdot \neg A_2 \cdot \neg A_1 \cdot \neg A_0$ $\quad$ $\neg A_2 \cdot A_1 \cdot A_0$ $\quad$ $A_2 \cdot A_1 \cdot \neg A_0$ $\quad$ $A_3 \cdot A_0$

A_3A_2 \ A_1A_0	00	01	11	10
00	1	0	1	0
01	0	0	0	1
11	X	X	X	X
10	0	1	X	X

$$S_3 = A_3A_0 + \neg A_2 A_1 A_0 + A_2 A_1 \neg A_0 + \neg A_3 \neg A_2 \neg A_1 \neg A_0$$

S_4:

$\neg A_1$ $\quad$ A_0 $\quad$ $\neg A_2$

A_3A_2 \ A_1A_0	00	01	11	10
00	1	1	1	1
01	1	1	1	0
11	X	X	X	X
10	1	1	X	X

$$S_4 = \neg A_2 + \neg A_1 + A_0$$

S_5:

$\neg A_2 \cdot A_0$ $\quad$ $\neg A_2 \cdot A_1$ $\quad$ $A_2 \cdot \neg A_0$

A_3A_2 \ A_1A_0	00	01	11	10
00	0	1	1	1
01	1	0	0	1
11	X	X	X	X
10	0	1	X	X

$$S_5 = \neg A_2 A_0 + \neg A_2 A_1 + A_2 \neg A_0$$

S_6:

$\neg A_3 \cdot \neg A_2 \cdot \neg A_1$ $\quad$ $\neg A_2 \cdot A_0$ $\quad$ $A_2 \cdot A_1$

A_3A_2 \ A_1A_0	00	01	11	10
00	1	1	1	0
01	0	0	1	1
11	X	X	X	X
10	0	1	X	X

$$S_6 = \neg A_3 \neg A_2 \neg A_1 + \neg A_2 A_0 + A_2 A_1$$