1 a. the consumers are solving the following function:

$$\max_{x_1, x_2} U(x_1, x_2) \quad \text{s.t.} \quad P_1 x_1 + P_2 x_2 = P_1 \omega_1 + P_2 \omega_2$$

their utility function are Cobb-Douglas so the solution will be

$$x_1 = \frac{\alpha}{\alpha+\beta} \frac{m}{P_1}$$ where α & β are from $\alpha \ln(x_1) + \beta \ln(x_2)$

and $m = P_1 \omega_1 + P_2 \omega_2$, and $x_2 = \frac{\beta}{\alpha+\beta} \frac{m}{P_2}$

Consumer a:

$m_a = 20 P_1 \qquad \alpha = 2 \qquad \beta = 3$

$$x_1^a = \frac{2}{2+3} \cdot \frac{20 \cancel{P_1}}{\cancel{P_1}} = 8 \qquad x_2^a = \frac{3}{2+3} \cdot \frac{20 P_1}{P_2} = 12P$$

Consumer b:

$m_b = 12 P_2 \qquad \alpha = 2 \qquad \beta = 1$

$$x_1^b = \frac{2}{2+1} \frac{12 P_2}{P_1} = \frac{8}{P} \qquad x_2^b = \frac{1}{2+1} \frac{12 P_2}{P_2} = 4$$

b. $e_1^a = x_1^a - \omega_1^a = 8 - 20 = -12$

$e_1^b = x_1^b - \omega_1^b = \frac{8}{P} - 0 = \frac{8}{P}$

$z_1 = e_1^a + e_1^b = \frac{8}{P} - 12$

$e_2^a = x_2^a - \omega_2^a = 12P - 0 = 12P$

$e_2^b = x_2^b - \omega_2^b = 4 - 12 = -8$

$z_2 = e_2^a + e_2^b = 12P - 8$

c. Walras' Law is:

$$p_1 z_1 + p_2 z_2 \equiv 0$$

$$p_1 z_1 + p_2 z_2 = p_1\left(8 \cdot \frac{p_2}{p_1} - 12\right) + p_2\left(12 \frac{p_1}{p_2} - 8\right)$$

$$= 8p_2 - 12p_1 + 12p_1 - 8p_2 = 0$$

d. We are in a Walrasian equilibrium if the aggregate excess demands are 0:

$$z_1 = \frac{8}{p} - 12 = 0$$

$$p = \frac{2}{3}$$

By Walras' Law, if aggregate excess demand for good 1 is 0, the same is true for good 2 (assuming $p_2 > 0$). e.g.: $z_2 = 0$

Allocation:

$x_1^a = 8$ $\quad x_2^a = 12p = 8$

$x_1^b = \frac{8}{p} = 12$ $\quad x_2^b = 4$

③ a Both Consumers are solving the Problem

$$\max_{x,y} \; x + \ln(y) \quad \text{s.t.} \quad P_x x + P_y y = P_x \omega_x + P_y \omega_y$$

$P_x = 1$ and $P_y = P$ so the Budget constraint is :

$$x + Py = \omega_x + P\omega_y = m$$

~~Both~~

$$\mathcal{L}(x, y, \lambda) = x + \ln(y) - \lambda(x + Py - m)$$

FOC :

$$\frac{\partial \mathcal{L}}{\partial x} = 1 - \lambda = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{y} - \lambda P = 0 \qquad (2)$$

using (1) & (2) :

$$y = \frac{1}{P}$$

Substituting that into the Budget constraint :

$$x + P \cdot \frac{1}{P} = m$$

$$x = m - 1$$

as it is quasilinear utility, Corner solutions are possible when $x = 0$ so when $m < 1$.

so:

$$X = \begin{cases} m - 1 & \text{if } m \geq 1 \\ 0 & \text{if } m < 1 \end{cases}$$

$$y = \begin{cases} \frac{1}{p} & \text{if } m \geq 1 \\ \frac{m}{p} & \text{if } m < 1 \end{cases}$$

$m_a = 4$ so $x_a = 3$ and $y_a = \frac{1}{p}$

$m_b = 4p$

so $$x_b = \begin{cases} 4p - 1 & \text{if } p \geq \frac{1}{4} \\ 0 & \text{if } p < \frac{1}{4} \end{cases}$$

$$y_b = \begin{cases} \frac{1}{p} & \text{if } p \geq \frac{1}{4} \\ 4 & \text{if } p < \frac{1}{4} \end{cases}$$

First let's assume interior solution so $p \geq \frac{1}{4}$

$$z_x = e_x^a + e_x^b = x_a - \omega_x^a + x_b - \omega_x^b$$

$$= 3 - 4 + 4p - 1 - 0$$

$$= 4p - 2$$

in Walrasian equilibrium, $z_x = 0$ and $z_y = 0$ so

~~$4p - \frac{1}{p} - 2 = 0$~~

~~$4p^2 - 2p - 1 = 0$~~

$4p - 2 = 0$

$p = \frac{1}{2}$

By Walras' law z_y is also 0.

$p \geq \frac{1}{4}$ so it is indeed an interior solution.

a's consumption : $(3, 2)$

b's consumption : $(1, 2)$

b. ~~same as it is the same because both consumers~~

it is opposite as both consumers have identical preferences :

b : $(3,2)$

a : $(1,2)$

c. ~~assuming~~ $m_a = p$ $m_b = 4 + 3p$

assuming interior :

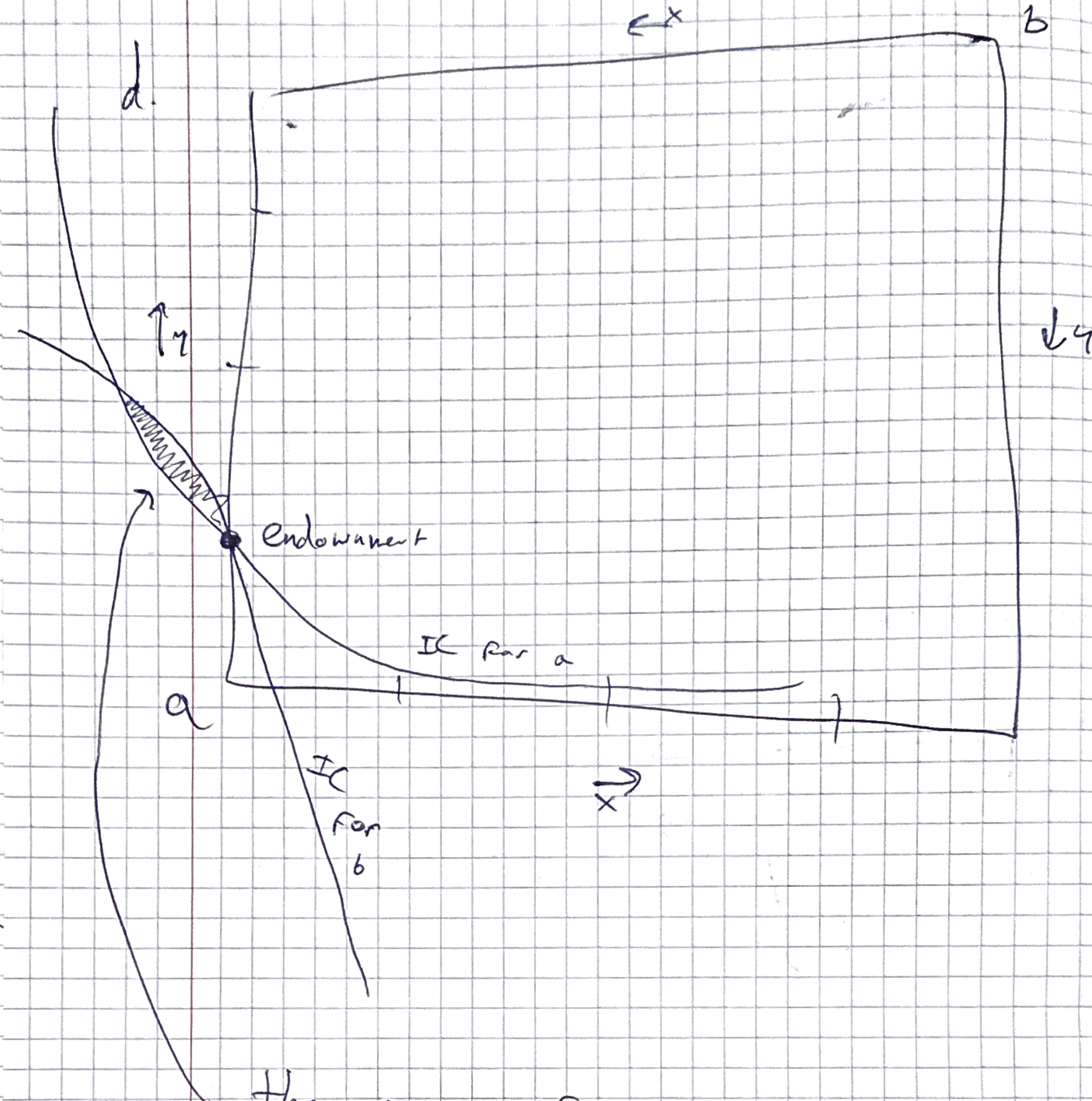
$$z_x = e_x^a + e_x^b = x_a - \omega_x^a + x_b - \omega_x^b$$

$$= p - 1 - 0 + 4 + 3p - 1 - 4$$

$$= 4p - 2$$

in Walrasian equilibrium, $z_x = 0$ so $4p - 2 = 0$

$p = \frac{1}{2}$



the gains from trade are worth negative x. ~~there are no~~

⑤

$$U = \ln(t) + \ln(1-\ell)$$

$$T = L^{\frac{1}{2}} F^{\frac{1}{2}}$$

$$P_t = 1$$

a. $P_t t \leq w\ell + rF$ ← supply of fields

$F = 1$ and $P_t = 1$ so:

$$t \leq w\ell + r$$

B. $\max_{t,\ell} \quad U = \ln(t) + \ln(1-\ell) \quad$ s.t. $\quad t \leq w\ell + r$

$$\mathcal{L} = \ln(t) + \ln(1-\ell) - \lambda(t - w\ell - r)$$

Kkt -conditions:

$$\frac{\partial \mathcal{L}}{\partial t} = \frac{1}{t} - \lambda = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial \ell} = -\frac{1}{1-\ell} + w\lambda = 0 \quad (2)$$

By (1) $\lambda > 0$ $(t > 0)$ So By the Complementary Slackness condition the Budget constraint Binds.

using (1) & (2):

$$\frac{1}{t} \cdot \frac{1-\ell}{1} = \frac{1}{w}$$

$$\frac{t}{1-\ell} = w$$

$$t = w - w\ell$$

using the BC

$$w - wf \cancel{wf} \cancel{2w} = wf + r$$

$$\cancel{}\ 2wf = w - r$$

$$f = \frac{w-r}{2w}$$

so $t = w - w\frac{w-r}{2w} = \frac{2w}{2} - \frac{w-r}{2} = \frac{w+r}{2}$

~~But~~ if $r > w$ $f = 0$ and $t = r$

c. $C(L, F) = wL + rF$

d.

$$\min_{L,F} C = wL + rF \quad \text{s.t.} \quad L^{\frac{1}{2}}F^{\frac{1}{2}} = T$$

$$\mathcal{L} = wL + rF - \lambda\left(L^{\frac{1}{2}}F^{\frac{1}{2}} - T\right)$$

FOC:

$$\frac{\partial \mathcal{L}}{\partial L} = w - \frac{1}{2}\lambda L^{-\frac{1}{2}}F^{\frac{1}{2}} = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial F} = r - \frac{1}{2}\lambda L^{\frac{1}{2}}F^{-\frac{1}{2}} = 0 \quad (2)$$

so :

$$\frac{w}{r} = \frac{F}{L}$$

$$F = \frac{wL}{r}$$

Using the BC

$$L^{\frac{1}{2}} \left(\frac{w}{r} L \right)^{\frac{1}{2}} = t$$

$$L \sqrt{\frac{w}{r}} = t$$

$$L = t \sqrt{\frac{r}{w}}$$

$$F = \frac{w}{r} t \sqrt{\frac{r}{w}} = t \sqrt{\frac{w}{r}}$$

By ~~E~~t:

$$\frac{dC^*}{dt} = \lambda^*$$

So lambda is the marginal cost of production.

Since

$$C^*(F^*, L^*) = wL^* + vF^* = w t \sqrt{\frac{r}{w}} + r t \sqrt{\frac{w}{r}}$$

$$= t\sqrt{rw} + t\sqrt{rw}$$

$$= 2t\sqrt{rw}$$

So $$\frac{dC^*}{dt} = 2\sqrt{rw} = \lambda$$

All markets clear

$F = 1$ (1) $\quad L = \ell$ (2) $\quad t = t$ (3)

using (1)

$$F = t\sqrt{\frac{w}{r}} = 1$$

$$t = \sqrt{\frac{r}{w}} \quad (4)$$ so this is turnip supply

$$L = t\sqrt{\frac{r}{w}}$$ Since we know turnip supply (4):

$$L = \sqrt{\frac{r}{w}}\sqrt{\frac{r}{w}} = \frac{r}{w} \quad (5)$$

now using (2) and (5)

$$L = \ell$$

$$\frac{r}{2w} = \frac{w-r}{2w}$$

$$r = \frac{w-r}{2}$$

$$1\tfrac{1}{2}r = \tfrac{1}{2}w$$

$$\frac{r}{w} = \frac{1}{3} \quad (6)$$

also recall that if $r > w$, $\ell = 0$ (from Part B) but $w > r$ so clearly an interior solution.

~~Since two out of three markets are now cleared, the turnip market will also clear~~

~~that~~ from (4) and (6) we know that

$r = \sqrt{\frac{1}{3}}$ and using (3) $\cancel{t}$ $t = \sqrt{\frac{2}{3}}$

and from (5):

$L = \frac{r}{w} = \frac{1}{3} = \rho$ (By (2))

$\longrightarrow$ $\rho = \frac{1}{3}$ & $t = \frac{1}{3}\sqrt{3}$