8.1

$$\forall x \forall y \forall z (((Px \wedge Py) \wedge Pz) \rightarrow ((x=y \vee y=z) \vee x=z))$$

8.2

I) $D_{\mathcal{A}} = \{1, 2\}$

$|Q^2|_{\mathcal{A}} = \{\langle 1,2\rangle, \langle 2,1\rangle\}$

$|a|_{\mathcal{A}} = 1$

$|b|_{\mathcal{A}} = 2$

II) $D_{\mathcal{B}} = \{1\}$

$|P^1|_{\mathcal{B}} = \emptyset$

8.3 No, from (I), you can do to (II):

$$\dfrac{\dfrac{[a=a]^{=} \qquad Pa}{a=a \wedge Pa}\wedge I}{\exists x(x=a \wedge Px)}\exists I$$

Very good. For logical equivalence you should also show the other direction:

$$\dfrac{\exists x(x=a \wedge Px) \qquad \dfrac{\dfrac{[b=a \wedge Pb]}{Pb} \qquad \dfrac{[b=a \wedge Pb]}{b=a}}{Pa}{=}\mathrm{Elim}}{Pa}\exists\mathrm{Elim}$$

In case it's of interest to you, Halbach further remarks: "I prefer (i) because it is much simpler. In fact, one could propose to formalise 'x is a philosopher' using an additional existential quantifier."

8.4 I)

$$\dfrac{[a = a]^{=}}{\exists y\; a = y}\ \exists I$$

II)

$$\dfrac{[Pa] \quad \wedge \quad [a = b]}{Pb}\ {=}E$$

$$\dfrac{Pb \qquad [\neg Pb]}{\neg a = b}\ \neg I$$

$$\dfrac{\neg a = b}{\exists y\; \neg a = y}\ \exists I$$

$$\dfrac{\exists y\; \neg a = y}{\exists x \exists y\; \neg x = y}\ \exists I$$

$$\dfrac{\exists x\, \neg Px \qquad \exists x \exists y\; \neg x = y}{\exists x \exists y\; \neg x = y}\ \exists E$$

$$\dfrac{\exists x\, Px \qquad \exists x \exists y\; \neg x = y}{}\ \exists E$$

8.5 I)

I)

$$\dfrac{[\forall y(Py \to a=y) \wedge Pa]}{Pa}\ \wedge E \qquad [a=b]$$

$$Pb$$

$$\dfrac{[\forall y(Py \to a=y) \wedge Pa]}{\forall y(Py \to a=y)}\ \wedge E$$

$$\dfrac{\forall y(Py \to a=y)}{Pb \to a=b}\ \forall E \qquad [Pb]$$

$$\dfrac{Pb \to a=b \qquad [Pb]}{a=b}\ \to E$$

$$\dfrac{Pb \qquad a=b}{Pb \leftrightarrow a=b}\ \leftrightarrow I$$

$$\dfrac{Pb \leftrightarrow a=b}{\forall y(Py \leftrightarrow a=y)}\ \forall I$$

$$\dfrac{\forall y(Py \leftrightarrow a=y)}{\exists x \forall y(Py \leftrightarrow x=y)}\ \exists I$$

$$\dfrac{\exists x(\forall y(Py \to x=y) \wedge Px) \qquad \exists x \forall y(Py \leftrightarrow x=y)}{\exists x \forall y(Py \leftrightarrow x=y)}\ \exists E$$

II)

$$\dfrac{[\forall y(Py \leftrightarrow a=y)]}{Pb \leftrightarrow a=b}\ \forall E \qquad [Pb]$$

$$\dfrac{Pb \leftrightarrow a=b \qquad [Pb]}{a=b}\ \leftrightarrow E$$

$$\dfrac{a=b}{Pb \to a=b}$$

$$\dfrac{Pb \to a=b}{\forall y(Py \to a=y)}\ \forall I$$

$$\dfrac{[\forall y(Py \leftrightarrow a=y)]}{Pa \leftrightarrow a=a}\ \forall E \qquad [a=a]$$

$$\dfrac{Pa \leftrightarrow a=a \qquad [a=a]}{Pa}\ \leftrightarrow E$$

$$\dfrac{\forall y(Py \to a=y) \qquad Pa}{\forall y(Py \to a=y) \wedge Pa}\ \wedge I$$

$$\dfrac{\forall y(Py \to a=y) \wedge Pa}{\exists x(\forall y(Py \to x=y) \wedge Px)}\ \exists I$$

$$\dfrac{\exists x \forall y(Py \leftrightarrow x=y) \qquad \exists x(\forall y(Py \to x=y) \wedge Px)}{\exists x(\forall y(Py \to x=y) \wedge Px)}\ \exists E$$

8.7

I) $\cancel{\exists x \exists y (Q_1 x \land Q_1 y \land \neg y = x \land \forall z (Q_1 z \to (x = z \lor y = z)))}$

$$\exists x \exists y (Q_1 x \land Q_1 y \land \neg x = y \land \forall z (Q_1 z \to (x = z \lor y = z)))$$

Good. Halbach splits his existential and universal quantifiers:
∃x ∃y (Q1x ∧ Q1 y ∧ ¬x = y) ∧ ∀x∀y∀z((Q1x ∧ Q1 y ∧ Q1z) → (x = y ∨ y = z ∨ x = z))

II)

$$\exists x (Q_x \land P_x \land \forall y ((Q_y \land P_y) \to x = y) \land \forall z (Q_1 z \to R_{xz}))$$

Again, good. Halbach uses an ↔ instead of → for
the clever tutor expression: ∃x(∀y(Py∧Qy↔x=y)∧∀z(Q1z→Rxz))

III)

$$\exists x (Q_1 x \land \forall y (Q_y \to R_{xy}) \land \forall z ((Q_1 z \land \forall y (Q_y \to R_{zy})) \to x = z) \land P_x)$$

Careful using universal quantifiers and ∧. Here is Halbach:
∃y∀x(((∀z(Qz→Rxz)∧Q1x)↔x=y)∧Py)

The TA is wrong here, my answer is also correct (note the double open brackets).

IV $\forall x \forall y \forall z ((Qx \land Qy \land Qz) \to (x = y \lor x = z \lor y = z))$