I) F_2 is how much the individual saves for period 2.

$$C_1 + f_2 = wL$$

$$C_2 = (1+R) f_2$$

$$C_1 + \frac{C_2}{1+R} = wL$$

II) $$\max_{C_1, C_2, L} \quad U = \log(C_1) + \log(C_2) + \log(1-L)$$

$$\text{s.t.} \quad C_1 + \frac{C_2}{1+R} = wL$$

$$\mathcal{L}(C_1, C_2, L, \lambda) = \log(C_1) + \log(C_2) + \log(1-L) - \lambda\left(C_1 + \frac{C_2}{1+R} - wL\right)$$

$$\frac{\partial \mathcal{L}}{\partial C_1} = \frac{1}{C_1} - \lambda = 0 \Rightarrow \lambda = \frac{1}{C_1}$$

$$C_1 = \frac{1}{\lambda}$$

$$\frac{\partial \mathcal{L}}{\partial C_2} = \frac{1}{C_2} - \frac{\lambda}{1+R} = 0 \Rightarrow C_2 = \frac{1+R}{\lambda}$$

$$\frac{\partial \mathcal{L}}{\partial L} = -\frac{1}{1-L} + w\lambda = 0 \Rightarrow 1-L = \frac{1}{w\lambda}$$

$$L = 1 - \frac{1}{w\lambda}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -C_1 - \frac{C_2}{1+R} + wL = 0$$

$$C_1 + \frac{C_2}{1+R} = wL$$

$$\frac{\partial G}{\partial C_1} = \frac{1}{C_1} - \lambda = 0 \Rightarrow \lambda = \frac{1}{C_1} \qquad (1)$$

$$\frac{\partial G}{\partial C_2} = \frac{1}{C_2} - \frac{\lambda}{1+R} \Rightarrow \lambda = \frac{1+R}{C_2} \qquad (2)$$

$$\frac{\partial G}{\partial L} = \frac{-1}{1-L} + \lambda w = 0 \Rightarrow \lambda = \frac{1}{w(1-L)} \qquad (3)$$

From (1) and (2):

$$\frac{1}{C_1} = \frac{1+R}{C_2} \Rightarrow C_2 = (1+R)\,C_1 \qquad (4)$$

From (1) and (3):

$$\frac{1}{C_1} = \frac{1}{w(1-L)} \Rightarrow C_1 = w(1-L) \qquad (5)$$

Using the budget constraint:

Using (4):

$$C_1 + C_1 = wL \Rightarrow 2C_1 = wL$$

Using (5):

$$2w(1-L) = wL$$

$$2(1-L) = L$$

$$2-2L = L$$

$$3L = 2$$

$$L^* = \frac{2}{3}$$

this makes sense, utility is a third leisure and two thirds consumption.

income effect cancels out substitution effect (so if wages rise)

Consumption today and in future will be the same in their current value:

$$c_1^* = w(1-L) = w\left(1-\frac{2}{3}\right) = \frac{1}{3}w$$

$$c_2^* = c_1(1+R) = \frac{1}{3}w(1+R)$$

III) $$c_1 + F_2 = wL - t$$ (where t is the tax ~~and F_2 =~~ and P is the pension)

$$c_2 = (1+R)(F_2+P)$$

~~But the intertemporal budget constraint stays the same!~~

$$\cancel{c_1 + \frac{c_2}{1+R} = wL}$$

~~the outcomes are exactly the same but in this case the saving is done for the individual by the government. So unless the government taxes more than the individual would have saved there is no effect.~~

should be P/(1+R)? But if you say P = value of pension in current value, it is still true
But it makes sense conceptually, if you assume that the pension fund will grow over time (if you understand pensions like that)

$$c_1 + \frac{c_2}{1+R} = wL - t + P$$

$$\cancel{c_1 \ldots F_2 = (1-t)wL}$$

$$c_2 = (1+R)\left(\cancel{F_2 + twL}\right)$$

If I do the lagrangian again:

$$\max_{c_1, c_2, L} u = \log(c_1) + \log(c_2) + \log(1-L)$$

$$\text{s.t.} \quad c_1 + \frac{c_2}{1+R} = wL - t + P$$

$$\mathcal{L} = \log(c_1) + \log(c_2) + \log(1-L) - \lambda\left(c_1 + \frac{c_2}{1+R} - wL + t - P\right)$$

$$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{1}{c_1} - \lambda = 0 \Rightarrow \lambda = \frac{1}{c_1} \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = \frac{1}{c_2} - \frac{\lambda}{1+R} = 0 \Rightarrow \lambda = \frac{1+R}{c_2} \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial L} = \frac{-1}{1-L} + \lambda w = 0 \Rightarrow \lambda = \frac{1}{w(1-L)}$$

From (1) and (2):

$$\frac{1}{c_1} = \frac{1+R}{c_2} \Rightarrow c_2 = (1+R)c_1 \qquad (4)$$

From (1) & (3):

$$\frac{1}{c_1} = \frac{1}{w(1-L)} \Rightarrow c_1 = w(1-L) \qquad (5)$$

with the Budget constraint and (4):

$$C_1 + C_1 = 2C_1 = wL - t + p$$

and using (5)

$$2w(1-L) = wL - t + p$$

$$2w - 2wL = wL - t + p$$

$$2w = 3wL - t + p$$

$$3wL = 2w + t - p$$

$$L^* = \frac{2w + t - p}{3w}$$

$$C_1^* = w\left(1 - \frac{2w + t - p}{3w}\right) \Rightarrow \frac{w - t + p}{3}$$

$$C_2^* = \frac{(1+R)(w - t + p)}{3}$$

So, ~~it~~ with a large lump-sum tax, people will start working more because it is just an income effect so ~~the~~ leisure will reduce together with consumption at C_1 and C_2. the opposite is true for the pension.

IV) this will also cause a substitution effect because consuming will become proportionally more expensive than leisure.

$$C_1 + f_2 = \cancel{\ } (1-t)wL$$

$$C_2 = f_2 + p \qquad (R=0 \Rightarrow 1+R=1)$$

$$C_1 + C_2 = (1-t)wL + p$$

$$\max_{C_1, C_2, L} \quad U = \log(C_1) + \log(C_2) + \log(1-L)$$

$$\text{s.t.} \quad C_1 + C_2 = (1-t)wL + p$$

$$\mathcal{L} = \log(C_1) + \log(C_2) + \log(1-L) - \lambda(C_1 + C_2 - (1-t)wL - p)$$

$$\frac{\partial \mathcal{L}}{\partial C_1} = \frac{1}{C_1} - \lambda \Rightarrow \lambda = \frac{1}{C_1} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial C_2} = \frac{1}{C_2} - \lambda \Rightarrow \lambda = \frac{1}{C_2} \quad (2)$$

From (1) and (2):

(4) $C_1 = C_2$ this is because the interest rate $R=0$ so consumption today is exactly equal to consumption in the future

$$\frac{\partial \mathcal{L}}{\partial L} = \frac{-1}{1-L} + \lambda (1-t) w = 0$$

$$\lambda = \frac{1}{w(1-t)(1-L)}$$

From (1) & (3):

$$C_1 = w(1-t)(1-L) \quad (5)$$

with the constraint and (4)

$$2C_1 = (1-t) wL + P$$

and with (5):

$$2w(1-t)(1-L) = (1-t) wL + P$$

$$2w(1-t) - 2wL(1-t) = (1-t) wL + P$$

$$3wL(1-t) = 2w(1-t) - P$$

$$L^* = \frac{2w(1-t) - P}{3w(1-t)}$$

~~So~~ and $C_1^* = C_2^* = \frac{(1-t)w + P}{3}$

So as taxes increase, L will ~~de~~ decrease so leisure increases. This is because there is also a substitution effect now as consumption becomes more expensive in terms of labor.