3.1)

Olivier Berg

I) $1 \notin \{2\}$

$|a|_s \notin |P^1|_s$

$|Pa|_s = F$ ✓

II) $\langle 1,3\rangle \in \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$

$\langle |a|_s, |b|_s\rangle \in |R^2|_s$

$|Rab|_s = T$ ✓

III) $\langle 3,1\rangle \notin \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$

$\langle |b|_s, |a|_s\rangle \notin |R^2|_s$

$|Rba|_s = F$ ✓

IV see II & III

$|Rab \leftrightarrow Rba|_s = F$ ✓

V) $\langle 3,3\rangle \notin \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$

$\langle |b|_s, |b|_s\rangle \notin |R^2|_s$

$|Rbb|_s = F$

$1 \notin \{2\}$

$|a|_s \notin |P^1|_s$

$|Pa|_s = F$ $\Big\} = I$

$|\neg Pa|_s = T$

$\langle 1,1\rangle \notin \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$

$\langle |a|_s, |a|_s\rangle \notin |R^2|_s$

$|Raa|_s = F$

$|\neg Raa|_s = T$

$|\neg Pa \wedge \neg Raa|_s = T$

$|Rbb \vee (\neg Pa \wedge \neg Raa)|_s = T$ ✓

VI) Let α be the variable assignment over S where ~~x agents the value~~ $|x|_S^\alpha = 2$

$$\exists \langle |a|_S^{\alpha}, |x|_S^\alpha \rangle \in |R^2|_S^{\alpha}$$
$$\langle 1,2 \rangle \in \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$$

$$|Rax|_S^\alpha = t$$

$$|\exists x Rax|_S = t$$ ✓

VII)

$$\langle 1,2 \rangle \in \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$$
$$\langle |a|_S^{\alpha}, |x|_S^\alpha \rangle \in |R^2|_S^{\alpha}$$

$$|Rax|_S^\alpha = t$$

VI

$$\langle 2,3 \rangle \in \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$$
$$\langle |x|_S^\alpha, |b|_S^{\alpha} \rangle \in |R^2|_S^{\alpha}$$

$$|Rxb|_S^\alpha = t$$

$$|Rax \wedge Rxb|_S^\alpha = t$$

$$|\exists x (Rax \wedge Rxb)|_S = t$$ ✓

VIII)

$$3 \notin \{2\}$$

$$|b|_S \notin |P^1|_S$$

$$|Pb|_S = F$$

Let β be a variable assignment over S

where $|x|_S^\beta = 1$ ~~is either 1, 2 or 3~~

γ the ~~a~~ one where $|x|_S^\gamma = 2$

δ the one where $|x|_S^\delta = 3$

$\langle 1,1\rangle \notin |R^2|_S\ \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$

$\langle 2,2\rangle \notin |R^2|_S\ \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$

$\langle 3,3\rangle \notin |R^2|_S\ \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$

$\langle |x|_S^\beta, |x|_S^\beta\rangle \notin |R^2|_S$

$\langle |x|_S^\gamma, |x|_S^\gamma\rangle \notin |R^2|_S$

$\langle |x|_S^\delta, |x|_S^\delta\rangle \notin |R^2|_S$

$|Rxx|_S^\beta = F$

$|Rxx|_S^\gamma = F$

$|Rxx|_S^\delta = F$

$|\exists x\, Rxx|_S = F$

$|P_1 \lor \exists x\, Rxx|_S = F$

* What is a better way to do this? Or should I just do it in english?

Halbach literally just wrote "This sentence is false in S as both disjuncts are"

IX) Let ε be a variable assignment over S so that $|x|_S^\varepsilon = 3$

~~Let θ be a variable assignment over S~~

? the $|y|_S^\varepsilon$ is 1 or 2 or 3.

Neither $\langle 3,1\rangle$ nor $\langle 3,2\rangle$ nor $\langle 3,3\rangle$ is in $|R|_S$, that is, in $\{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$

∗ Is this the only way to do this?

therefore:

$$\langle |x|_S^\varepsilon, |y|_S^\varepsilon\rangle \notin |R^2|_S$$

$$|Rxy|_S^\varepsilon = F$$

$$|\exists y Rxy|_S^\varepsilon = F$$

$$|\forall x \exists y Rxy|_S = F$$

X) Let θ be the variable assignment over S where $|x|_S^\theta = 2$

$$2 \in \{2\}$$

$$|x|_S^\theta \in |P^1|_S$$

$$|Px|_S^\theta = T$$

Let in variable assignment θ, ~~either 1 or 2 or 3~~ in the y the part $\exists y Ryx$ is ~~either~~ $|y|_S^\theta = 1$

$$\langle 1,2\rangle \in \{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$$

$|R$ ~~...~~

$\langle |y|_s^\theta , |x|_s^\theta \rangle \in |R^2|_s$

$|Ryx|_s^\theta = t$

$|\exists y Ryx|_s^\theta = t$

Let in variable θ, the y-variable

in Part Part $\exists y Rxy$, Be $|y|_s^\theta = 3$

How do I deal with the same variable home in a different scope?

$\langle 2,3 \rangle \in \{\langle 1,2 \rangle, \langle 2,3 \rangle, \langle 1,3 \rangle\}$

$\langle |x|_s^\theta , |y|_s^\theta \rangle \in |R^2|_s$

$|Rxy|_s^\theta = t$

I can't use multiple different variable assignments, should I just substitute in y_1 and then y_2?

$|\exists y Rxy|_s^\theta = t$

Very good, will cover in class

$|\exists y Ryx \wedge \exists y Rxy|_s^\theta = t$

$|Px \to (\exists y Ryx \wedge \exists y Rxy)|_s^\theta = t$

Let η be the variable assignment over S where $|x|_s^\eta$ is either 1 or 3. Neither 1, or 3 is in $|P^1|_s$.

therefore:

$|x|_s^\eta \notin |P^1|_s$

$|Px|_s^\eta = F$

$|Px \to (\exists y Ryx \wedge \exists y Rxy)|_s^\eta = t$

So: $|\forall x (Px \to (\exists y Ryx \wedge \exists y Rxy))|_s = t$

Because ~~th~~ we have considered every variable assignment for x.

XI) ~~Let λ be the variable assignment over S where $|x|^{\lambda}_{S}$ is either 1 or 3. Neither~~

Let λ be the ~~the~~ variable assignment over S where $|x|^{\lambda}_{S} = 2$.

$$2 \in \{2\}$$

$$|x|^{\lambda}_{S} \in |P^1|_S$$

$$|P^1|^{\lambda}_{S} = T$$

Let in variable assignment λ

$|y|^{\lambda}_{S}$ be either 1 or 2 or 3.

$\langle 1,2\rangle$ is in $|R^2|_S$, that is, $\{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$.

But $\langle 2,2\rangle$ and $\langle 3,2\rangle$ are not.

So $|Ryx|^{\lambda}_{S}$ is true iff $|y|^{\lambda}_{S} = 1$.

However, $\langle 2,1\rangle$ is not in $|R^2|_S$, that is,

$\{\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle\}$. therefore

$|Ryx|^{\lambda}_{S}$ and $|Rxy|^{\lambda}_{S}$ cannot both be true. therefore:

$$|Ryx \wedge Rxy|^{\lambda}_{S} = F$$

$$|\exists y(Ryx \wedge Rxy)|^{\lambda}_{S} = F$$

$|Px \rightarrow \exists y (Ryx \wedge Rxy)|_s^{\lambda} = F$

$|\forall x (Px \rightarrow \exists y (Ryx \wedge Rxy))|_s = F$

Because there is ~~a~~ at least one case in which it is false, that is under variable assignment λ. Good - will cover in class.

5.2

I) G_2-Structure $\mathcal{H}$:

$D_{\mathcal{H}} = \{1, 2\}$

$|a|_{\mathcal{H}} = 1$

$|P^1|_{\mathcal{H}} = \{1\}$

$|Q^1|_{\mathcal{H}} = \{2\}$

II) G_2-Structure $\mathcal{X}$:

$D_{\mathcal{X}} = \{1, 2\}$

$|P^1|_{\mathcal{X}} = \{1, 2\}$

$|R^2|_{\mathcal{X}} = \{\langle 1,2\rangle, \langle 2,1\rangle\}$

III) G_2-Structure $\mathcal{Y}$:

$D_{\mathcal{Y}} = \{1, 2\}$

$|R^2|_{\mathcal{Y}} = \{\langle 1,1\rangle, \langle 2,2\rangle\}$

$|a|_{\mathcal{Y}} = 1$

$\Gamma \models \varphi$

(5.3) ~~$\Gamma \models \varphi$~~ - iff there is not a $\mathcal{L}_2$-Structure where Γ are all true and φ is false.

~~$\Gamma \models \varphi$~~ - iff there is no $\mathcal{L}_2$-Structure in which $\neg\varphi$ and all of Γ are true.

- iff the set containing $\neg\varphi$ and all the ~~set~~ sentences in Γ is inconsistent.

(5.4) * (I have added a few unnecessary ones in my ~~[illegible]~~ answers)

I) $\exists x \exists y \exists z (Px \wedge \neg Py \wedge \neg Pz \wedge \neg Qx \wedge Qy \wedge \neg Qz$
$\wedge \neg Rx \wedge \neg Ry \wedge Rz)$

Some here are also unnecessary I think.

(they have to all be different)

II) $\exists x \exists y (Rxy \wedge \neg Rxx \wedge \neg Ryy)$ unnecessary

III) $\exists x \exists y \exists z ($~~$Rxyz \wedge \neg Rx$~~ $Rxy \wedge Rxz \wedge Ryz \wedge$
$\neg Rxx \wedge \neg Ryy \wedge \neg Rzz)$

IV) $\forall x \exists y (Rxy \wedge \neg Ryx \wedge \neg Rxx \wedge \neg Ryy)$

So there must always be a new value.

You could see this as x is larger than y. If x is larger than y, x is not larger than x and y is not larger than y and y is not larger than x

* You could also use $\forall$ i.o. $\exists$. However, I don't like it because it would not work in natural language. There there is no restriction for the empty set as the domain

7.5 Depends on the interpretation of the 2nd sentence

Either:

D: everything

P2: ..., ~~is something~~ has ... 2

Q1: ..., is a cause

$$\forall x \exists y (Q^*y \land Pxy) \vDash \forall x \exists y (Qy \land Pxy)$$

Here the two sentences are identical and therefore logically equivalent. Therefore there will never be a L_2-structure where the premiss is true and the conclusion false because it is impossible for them to have a different truth-value.

Or:

$$\forall x \exists y (Qy \land Pxy) \nvDash \exists y \forall x (Qy \land Pxy)$$

Counterexample: ~~L_2-structure~~ there does not exist a case which is the cause of everything therefore this ~~sentence~~ argument is obviously invalid.

Good, will discuss