

(6.7)

$$\text{I) } \frac{\frac{[P]}{R \vee P} \vee \text{Intro 2}}{P \rightarrow (R \vee P)} \rightarrow \text{Intro} \quad \checkmark$$

$$\text{II) } \frac{\frac{R \wedge Q}{Q \wedge R} \wedge \text{Elim 2} \quad \frac{R \wedge Q}{R \wedge Q} \wedge \text{Elim 1}}{Q \wedge R} \wedge \text{Intro} \quad \checkmark$$

$$\text{III) } \frac{P \rightarrow Q \quad [P]}{Q} \rightarrow \text{Elim} \quad \frac{Q \quad [\neg Q]}{\neg P} \rightarrow \text{Intro} \quad \frac{\neg P}{\neg Q \rightarrow \neg P} \rightarrow \text{Intro} \quad \checkmark$$

$$\text{IV) } \frac{[P \rightarrow \neg P] \quad [P]}{\neg P} \rightarrow \text{Elim} \quad \frac{\neg P \quad [P]}{\neg P} \rightarrow \text{Intro} \quad \frac{\neg P}{(P \rightarrow \neg P) \rightarrow \neg P} \rightarrow \text{Intro} \quad \checkmark$$

$$\text{V) } \frac{P \leftrightarrow Q \quad [P]}{Q} \leftrightarrow \text{Elim 1} \quad \frac{Q \quad [\neg Q]}{\neg P} \rightarrow \text{Intro} \quad \checkmark$$

VI)

$$\begin{array}{c}
 \frac{[P] \quad [Q]}{P \wedge Q} \quad \wedge \text{Intro} \\
 \frac{P \wedge Q}{(P \wedge Q) \rightarrow R} \rightarrow \text{Elim} \\
 \hline
 R \\
 \hline
 Q \rightarrow R \quad \rightarrow \text{Intro} \\
 \hline
 P \rightarrow (Q \rightarrow R) \quad \rightarrow \text{Intro}
 \end{array}$$

VII)

$$\begin{array}{c}
 \frac{[P]}{\neg Q \rightarrow \neg P} \rightarrow \text{Intro} \\
 \hline
 \neg Q \rightarrow \neg P \quad [\neg Q] \quad \rightarrow \text{Elim} \\
 \hline
 \neg P \quad [P] \quad \neg \text{Elim} \\
 \hline
 Q \quad \rightarrow \text{Intro} \\
 \hline
 P \rightarrow Q \quad \neg(P \rightarrow Q) \quad \neg \text{Elim} \\
 \hline
 P
 \end{array}$$

A faster way would be:

$$\begin{array}{c}
 \frac{[P] \quad [P]}{Q} \quad \neg \text{Elim} \\
 \hline
 Q \quad \rightarrow \text{Intro} \\
 \hline
 P \rightarrow Q \quad \neg(P \rightarrow Q) \quad \neg \text{Elim} \\
 \hline
 P
 \end{array}$$

↑
this is how
Holt does it

Handwritten proof for $\vdash (P \rightarrow (Q \vee R)) \wedge \neg(Q \wedge R) \vdash \neg(Q \vee R)$ using natural deduction rules:

$$\begin{array}{c}
 \frac{P \rightarrow (Q \vee R) \wedge \neg(Q \wedge R) \quad P \rightarrow P \quad \rightarrow\text{Elim}}{P \rightarrow (Q \vee R) \wedge \neg(Q \wedge R)} \rightarrow\text{Elim} \\
 \frac{(Q \vee R) \wedge \neg(Q \wedge R)}{Q \vee R} \wedge\text{Elim1} \\
 \frac{P \rightarrow (Q \vee R) \wedge \neg(Q \wedge R) \quad Q \vee R}{\neg(Q \vee R)} \rightarrow\text{Elim}
 \end{array}$$

Q -Elim -R

Q \wedge -R Intro

↓

$$Q \wedge R$$

12