4) I)

$$C = Y = AL^{\alpha} \qquad U = \ln(C) - 2L^2$$

Substitute it in:

$$\max_{L} \; U = \ln(AL^{\alpha}) - 2L^2$$

$$\frac{\partial U}{\partial L} = \frac{\alpha A L^{\alpha - 1}}{AL^{\alpha}} - 4L = \frac{\alpha}{L} - 4L = 0$$ (set to 0 to maximize)

$$\alpha - 4L^2 = 0$$

$$4L^2 = \alpha$$

$$L^2 = \frac{1}{4}\alpha$$

$$L = \sqrt{\frac{1}{4}\alpha} = \frac{1}{2}\sqrt{\alpha}$$

L is not dependent on A, so if productivity changes, the labour supply will stay the same.

~~Therefore if productivity increases labour supply increases proportional to the square root of the productivity increase.~~

II)

$$C = wL + \pi \qquad U = \ln(C) - 2L^2$$

Substitute it in:

$$\max_{L} \; U = \ln(wL + \pi) - 2L^2$$

$$\frac{\partial U}{\partial L} = \frac{w}{wL + \pi} - 4L = 0$$ (set to 0 to optimize)

$$\frac{w}{C} - 4L = 0$$

$$4L = \frac{w}{C}$$

$$L = \frac{w}{4C}$$

III) $\max_N \pi = AN^\alpha - wN$

$$\frac{\partial \pi}{\partial N} = \alpha A N^{\alpha-1} - w = 0$$ (set to 0 to maximize)

(1) $\alpha A N^{\alpha-1} = w$

$$N^{\alpha-1} = \frac{w}{\alpha A}$$

$$N = \left(\frac{w}{\alpha A}\right)^{\frac{1}{\alpha-1}}$$

~~MPL = ∂~~

$$y = AN^\alpha$$

$$MPL = \frac{\partial y}{\partial N} = \alpha A N^{\alpha-1}$$

From (1) you see that

$MPL = w$

IV) $U = \ln(C) - 2L^2$ $\quad C = nAN^\alpha$ $\quad L = nN$

$w = \alpha A N^{\alpha-1}$

$L = \frac{w}{4C}$

$$L = \frac{\alpha A N^{\alpha-1}}{4nAN^\alpha}$$

$$L = \frac{\alpha}{4n \cdot N}$$

$$L = \frac{\alpha}{4L}$$

$L = \frac{1}{2}\sqrt{\alpha}$

$L = \frac{\alpha}{4L}$

$4L^2 = \alpha$

$L = \frac{\sqrt{\alpha}}{2}$

So, again, it is not dependent on A.

V) $w = \alpha A N^{\alpha-1}$ So w not depends on n Because N stays the same Because each individual firm will stay with the same N.

$L = nN\uparrow$

wage drops

Output per head $= nY = nAN^{\alpha}$ so if n falls, output per head falls proportionally.

For output $= c$ so the same.

VI $\pi = AN^{\alpha} - wN$ $\qquad N = \frac{L}{n}$

$\pi = A\left(\frac{L}{n}\right)^{\alpha} - w\frac{L}{n}$ $\qquad w = \alpha A N^{\alpha-1}$

$\pi = A\left(\frac{L}{n}\right)^{\alpha} - \alpha A \left(\frac{L}{n}\right)^{\alpha-1} \cdot \frac{L}{n}$

$= A\left(\frac{L}{n}\right)^{\alpha} - \alpha A\left(\frac{L}{n}\right)^{\alpha}$

$\pi = (A - \alpha A)\left(\frac{L}{n}\right)^{\alpha}$

~~So π changes proportional to $\frac{1}{n^{\alpha}}$~~

~~In the long run, profits will be zero~~

So π is proportional to $\frac{1}{n^{\alpha}}$.

basically, if there are profits, firms will enter, $n\uparrow \to w\uparrow \to$ then profits reduced $\to$ back to original level!