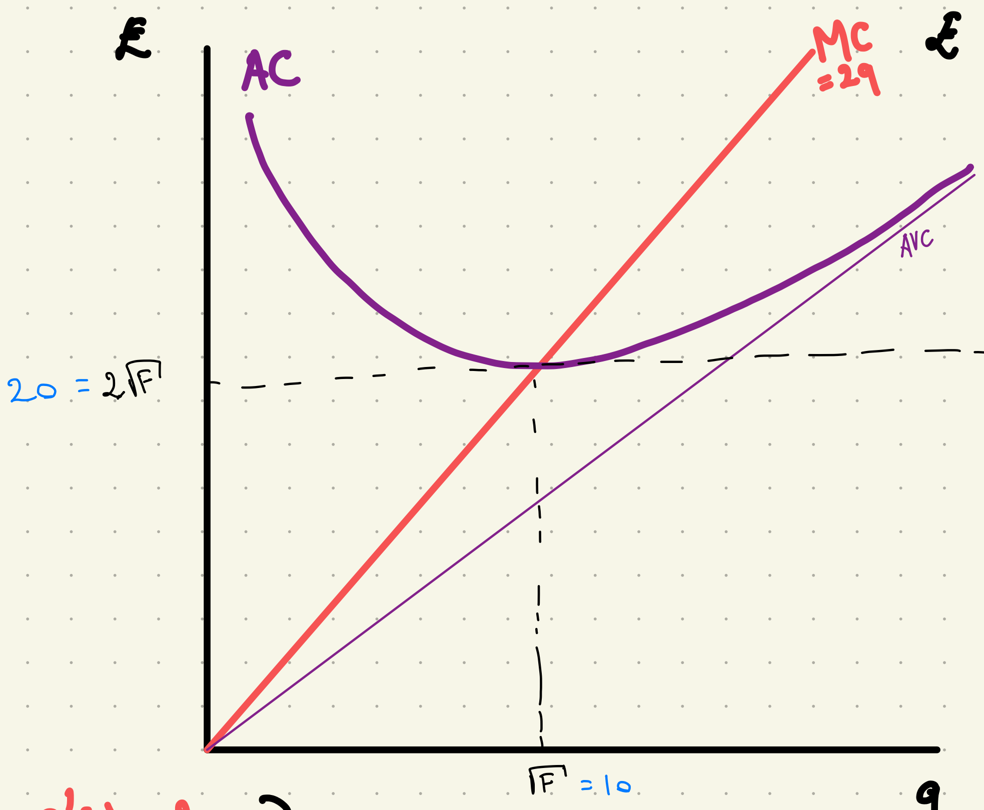
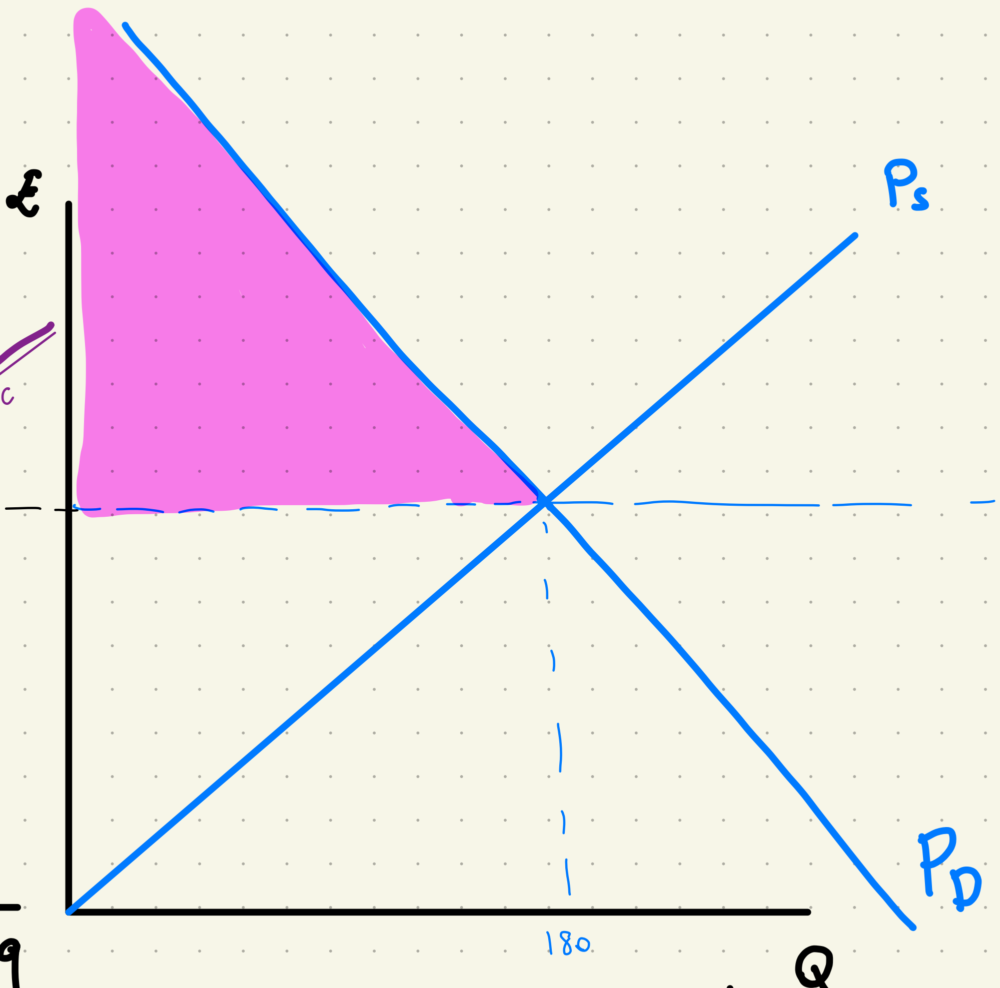
Q1

n identical firms.
Perfectly competitive ; i.e. price takers.
$c(q) = q^2 + F$
Total supply : $Q = n.q$
Demand : $Q = D - p$; $D > 0$ is a parameter.

(a) Show that in eqm: $q^* = \sqrt{F}$ and $p^* = 2\sqrt{F}$.
[15] Explain why long-run p^* does not depend on D.





$MC(q) = c'(q) = 2q$
$AVC(q) = \frac{q^2}{q} = q$
$AC(q) = q + \frac{F}{q}$

In equilibrium, each $\underbrace{\text{firms maximizes}}_{(1)}$ profits and $\underbrace{\text{supply = demand}}_{(2)}$.

(1) $\max_q \pi = p.q - c(q) \Rightarrow$ FOC : $p - 2q = 0 \Rightarrow q = \frac{1}{2}p$.

(2) For supply = demand in LR, we need firms to make $\pi = 0$. otherwise there would either be entry or exit.
But $\pi = 0 \Leftrightarrow q\left[p - AC(q)\right] = 0 \Rightarrow p = AC(q) = q + \frac{F}{q}$.

From (1) & (2) we therefore have $2q^* = q^* + \frac{F}{q^*} \Rightarrow q^* = \sqrt{F}$

Hence $p^* = 2q^* = 2\sqrt{F}$.

Long run p does not depend on D : long run demand is essentially flat at $p = \min_q AC(q)$. Whatever D may be, free entry + exit ensures that $p = \min_q AC(q)$, which depends on firms' cost structure but not on D.

(b) [15]

$\boxed{\text{Suppose } F = 100. \text{ What are } p^* \text{ and } q^*? \text{ If, additionally, } D = 200, \text{ what is } n? \text{ What is PED? What is CS?}}$

- Clearly, $q^* = \sqrt{100} = 10$ and $p^* = 2\sqrt{100} = 20$.
- At the industry level, we need supply = demand, i.e. $\overbrace{n\, q^*}^{\text{supply}} = \overbrace{200 - p^*}^{\text{demand}}$

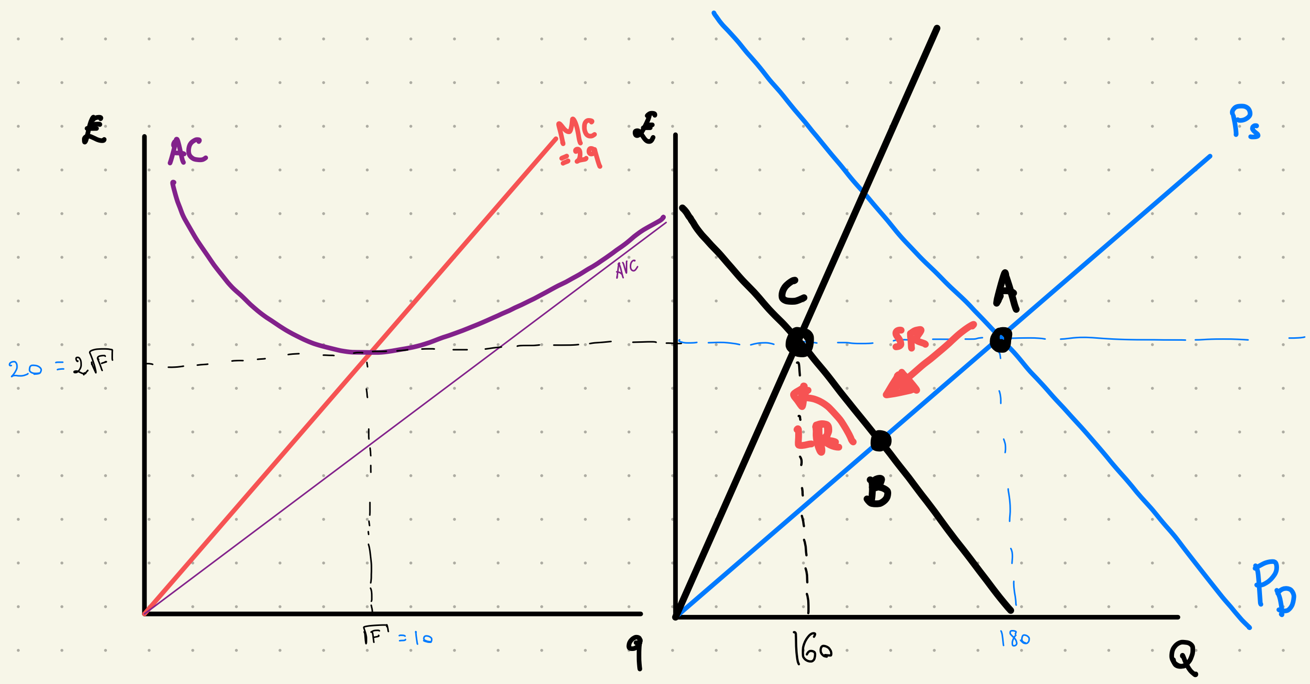
  Therefore $n = \dfrac{200-20}{10} = 18$, and $Q^* = n \cdot q^* = 18 \cdot 10 = 180$
- $\text{PED} \overset{\text{def}}{=} \dfrac{\partial Q}{\partial P} \cdot \dfrac{P}{Q} = -1 \cdot \dfrac{P^*}{Q^*} = -1 \cdot \dfrac{20}{180} = -\dfrac{1}{9}$.
- $\text{CS} \overset{\text{def}}{=} (\text{WTP} - p)$ aggregated over all units traded.

  = shaded area above in pink.

  $= (200-20) \cdot \dfrac{Q^*}{2} = \dfrac{180^2}{2} = 16200$ £

(c) [10]

$\boxed{\text{What are the effects of a demand shock } D = 200 \text{ to } D = 180 \text{ on } n \text{ and PED?}}$



In the short run, demand shifts inwards (A to B).

Firms make a loss and exit the market, so in the long run, supply swivels inwards as shown (B to C).

The long run price remains at $p^* = 20$.

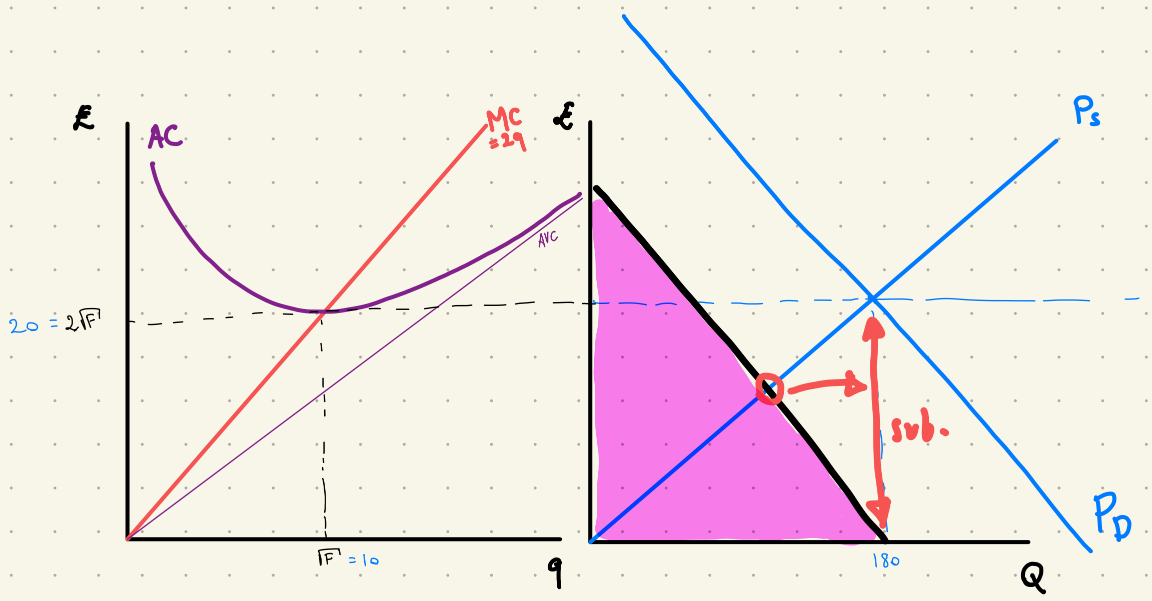
At the new long run equ of C, $Q^* = 180 - p^* = 160$.

So $n^* = \dfrac{Q^*}{q^*} = \dfrac{160}{10} = 16$, and $\text{PED} = \dfrac{\partial Q}{\partial P} \cdot \dfrac{P^*}{Q^*} = -1 \cdot \dfrac{20}{160} = -\dfrac{1}{8}$.

$|\text{PED}|$ is larger here because price is unchanged but quantity is lower.

(d) [20]

What if govt gives consumers a per-unit subsidy so that Q bought is the same as before the shock?

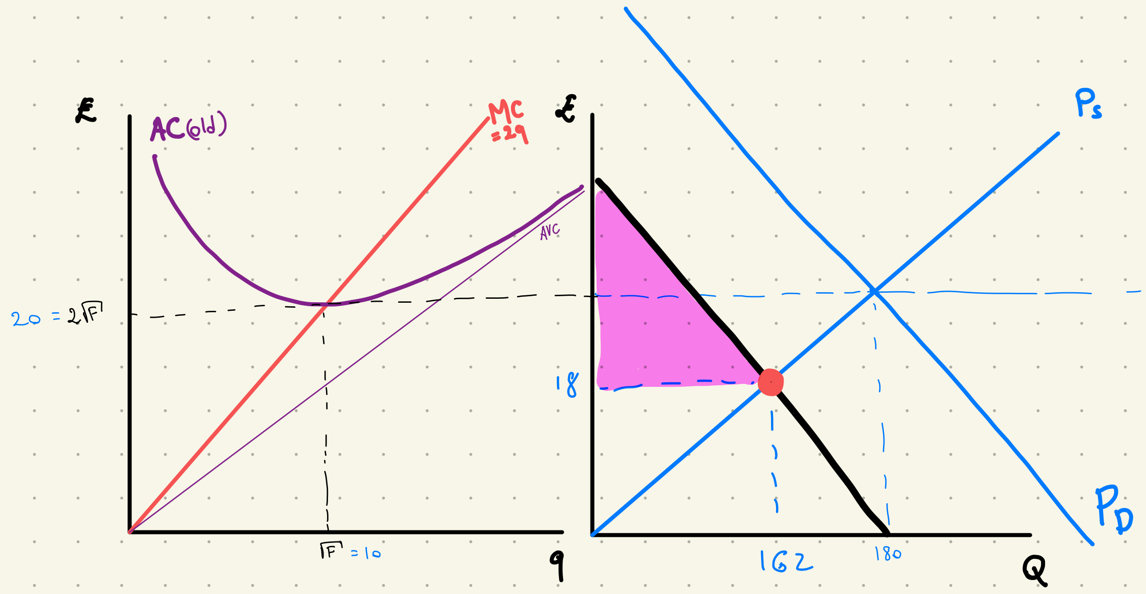


From the diagram, one can see that the subsidy per unit must be 20.
i.e. the govt simply pays for each unit entirely on behalf of the consumers.
Consumers pay 0, producers get 20 per unit.
The CS is shaded in pink : $\frac{(180-0)\cdot 180}{2} = \frac{180^2}{2} = 16200$ – same as before.

The cost of the subsidy is $\underbrace{20}_{\text{sub. per unit.}} \times \underbrace{180}_{\text{units traded}} = 3600$ £.

(e) [20]

What if govt provides a lump-sum subsidy to firms to cover firms' fixed costs.



We are holding n = 18.
New equm at ●, i.e. $180 - p^* = 18 \cdot q^*$. Moreover each firm optimizes via $p = MC$ i.e. $p^* = 2q^*$.
Solving yields $180 - 2q^* = 18 \cdot q^* \Rightarrow q^* = \frac{180}{20} = 9$, and $p^* = 18$.
Total output is $Q^* = n \cdot q^* = 18 \cdot 9 = 162$.
Finally CS is shaded in pink: $\frac{(180-18)\cdot 162}{2} = \frac{162^2}{2} = 13122$ £.
Point of the subsidy is to ensure that each firm makes $\pi = 0$.
But $\pi = p^* \cdot q^* - c(q^*) = 18 \cdot 9 - (9^2 + 100) = -19$.
So govt must pay each firm 19 £ $\Rightarrow$ Total cost of subsidy $= 19 \cdot 18 = 342$ £

(f) $\boxed{\text{Which policy is better?}}$
[20]

Price subsidy (i) vs. lump sum subsidy (ii).

Either way, firms make zero profits. So all we really need to look at is CS + cost of subsidy. (i) yields higher CS, but is more expensive to run: 16200 - 3600 = 12600.
(ii) yields CS - cost of subsidy of: 13122 - 342 = 12780.

So, <u>net</u>, there is a DWL of 180 in (i) compared to (ii).

Discuss any other reasons to prefer one to the other.

Q2

$$u(x_i, g) = 2\log(x_i) + \log(g)$$

private good   public good $g = g_1 + g_2$

Income $I_i = 100$.
Prices are $P_x = P_g = 1$ per unit.

(a) [30] **Derive the best-response functions for public good contributions**

$$\max_{x_i, g_i} \; 2\log(x_i) + \log(g_i + g_j) \quad \text{s.t. } x_i + g_i = 100$$

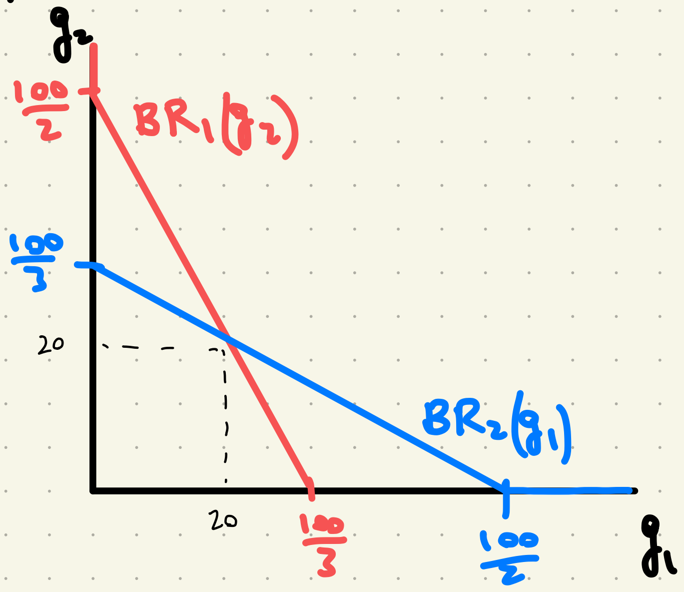
$$\Leftrightarrow \max_{g_i} \; 2\log(100 - g_i) + \log(g_i + g_j)$$

FOC: $-\dfrac{2}{100 - g_i} + \dfrac{1}{g_i + g_j} \overset{\text{set}}{=} 0$

$\Rightarrow \; 100 - g_i = 2(g_i + g_j) \quad \cdots (*)$

$\Rightarrow \; \boxed{g_i = \dfrac{100 - 2g_j}{3}}$

So $BR_i(g_j) = \dfrac{100 - 2g_j}{3}$



(b) [10] **Solve for the Nash equilibrium**

At a Nash equilibrium, $g_1 = BR_1(g_2)$
& $g_2 = BR_2(g_1)$

From the FOC $(*)$ we know that $g_i = 100 - 2(g_i + g_j)$. This implies that $g_i = g_j$.

Hence the NE is found by solving $g_i = 100 - 4g_i$, so $g_i^* = 20$ for $i \in \{1, 2\}$.

(c) [20] **What is the socially optimal level of g?
How does it compare with Nash, and why does it differ?**

- Either use the Samuelson condition ($\Sigma MRS = MRT$) directly, or derive it:

We can find a Pareto optimal allocation (where no one can be made better off without making someone worse off) by looking for an allocation that maximizes the sum of utilities subject to the economy's constraints:

$$\max_{x_1, x_2, g} \; 2\log(x_1) + 2\log(x_2) + 2\log(g) \quad \text{s.t. } x_1 + x_2 + g = 200$$

$$\Leftrightarrow \max_{x_1, g} \; 2\log(x_1) + 2\log(200 - g - x_1) + 2\log(g)$$

$$\left.\begin{aligned} FOC_1:&\ \frac{2}{x_1} - \frac{2}{200-g-x_1} \overset{set}{=} 0 \\ FOC_g:&\ \frac{2}{g} - \frac{2}{200-g-x_1} \overset{set}{=} 0 \end{aligned}\right\} \Rightarrow x_1 = g \text{ and } x_1 = 200-g-x_1 \text{ so } g = \frac{200}{3}$$

- NB. Alternative: $MRS_1 = \frac{2g}{x_1}$, $MRS_2 = \frac{2g}{x_2}$, $MRT = 1$ ← price ratio of goods is 1.

  Now solve via Samuelson: $\underbrace{MRS_1 + MRS_2 = MRT}_{2g = x_1+x_2}$ and $\underbrace{x_1+x_2+g = 200}_{3g=200 \text{ so } g = \frac{200}{3}}$. $\xrightarrow{sub.}$

- The optimal g is higher than Nash. This is because g is a "positive externality". We can see this in the model because g_i enters into u_j directly. People can benefit without providing it, so there is too little provision compared to what is optimal.

(d) [20] $\boxed{\text{A tax of 5 is imposed on each person to finance the public good. What happens to } g\text{?}}$

The optimization problem for i becomes:

$$\max_{x_i, g_i} \ 2\log(x_i) + \log(g_i + g_j \underbrace{+10}_{\text{gov't provision of } g \text{ via collecting 5 from each person.}}) \quad \text{s.t. } x_i + g_i = 100 \underbrace{-5}_{\text{tax}}$$

$$\Leftrightarrow \max_{g_i} \ 2\log(95-g_i) + \log(g_i+g_j+10)$$

$$FOC: -\frac{2}{95-g_i} + \frac{1}{g_i+g_j+10} \overset{set}{=} 0$$

$$\Leftrightarrow 95 - g_i = 2(g_i+g_j+10) \quad \cdots (\#)$$

$$\Leftrightarrow \frac{75-2g_j}{3} = g_i \quad \leftarrow \text{Best-response function.}$$

From (#) we conclude that $g_i = g_j$ so in the new equm, $95 - g_i = 4g_i + 20$

$$\Rightarrow g_i = g_j = \frac{75}{5} = 15.$$

- There is a "crowding out" effect: each person reduces their contribution in a manner that exactly offsets the tax.

(e) General conclusions?

[20]

- Public good; positive externality. Underprovided.
- 'Partial' solution to finance public good via taxation doesn't solve the problem. People re-adjust their contribution downwards.
- What would be needed is f<u>ull</u> provision of the socially optimal level of g by the govt.