

(5)

a. ~~$y = 6K^{\frac{1}{2}}L^{\frac{1}{2}}$~~

$$6(\lambda K)^{\frac{1}{2}} \cdot (\lambda L)^{\frac{1}{2}} = 6 \cdot \lambda^{\frac{1}{2}} \cdot K^{\frac{1}{2}} \cdot \lambda^{\frac{1}{2}} \cdot L^{\frac{1}{2}} \\ = 6\lambda^1 K^{\frac{1}{2}} L^{\frac{1}{2}}$$

So it has a constraint returns to scale

B.

$$y = 6K^{\frac{1}{2}}L^{\frac{1}{2}}$$

$$K^{\frac{1}{2}} = \cancel{M} \frac{y}{6} K^{\frac{1}{2}}$$

$$K = \left(\frac{y}{6}\right)^2 L^{-2}$$

L



$$L^{\frac{1}{2}} = \frac{y}{6} K^{-\frac{1}{2}}$$

$$L = \left(\frac{y}{6}\right)^2 K^{-1}$$

$$MRPS = \frac{dK}{dL} = -\left(\frac{y}{6}\right)^2 L^{-2}$$

$$= -\left(\frac{y}{6}\right)^2 \left(\frac{y}{6} K^{-1}\right)^{-2}$$

$$= \frac{y}{\left(\frac{y}{6}\right)^2 K^2} \times -\frac{1}{\left(\frac{y}{6}\right)^2} \\ = -\left(\frac{y}{6}\right)^2 \frac{1}{\left(\frac{y}{6}\right)^2 K^2} K^2 = -\frac{1}{6} K^2$$

So, if K increases, the marginal rate

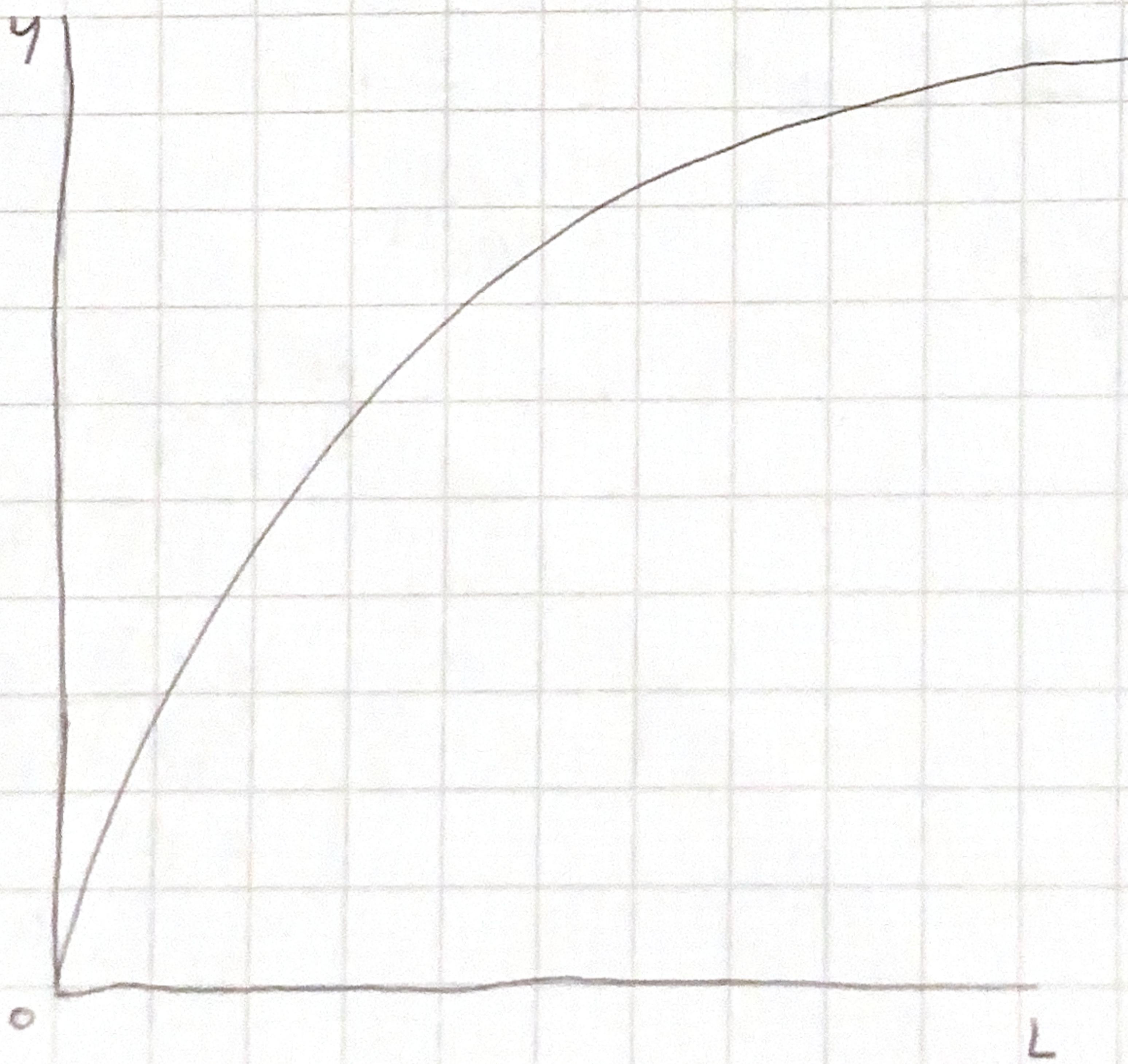
of technical substitution

decreases, increases

this makes sense, if K ^{is larger} increases, then more and more Capital is needed ~~to~~ to substitute ~~the~~ ~~for~~ a unit of Labour IF you look at the isoquant, ~~isn't~~ if K is smaller, you need ~~more~~ ~~more~~ ~~you~~ can substitute ~~for~~ a unit of Labour with much less Capital than if ~~tasked~~ K is Bigger.

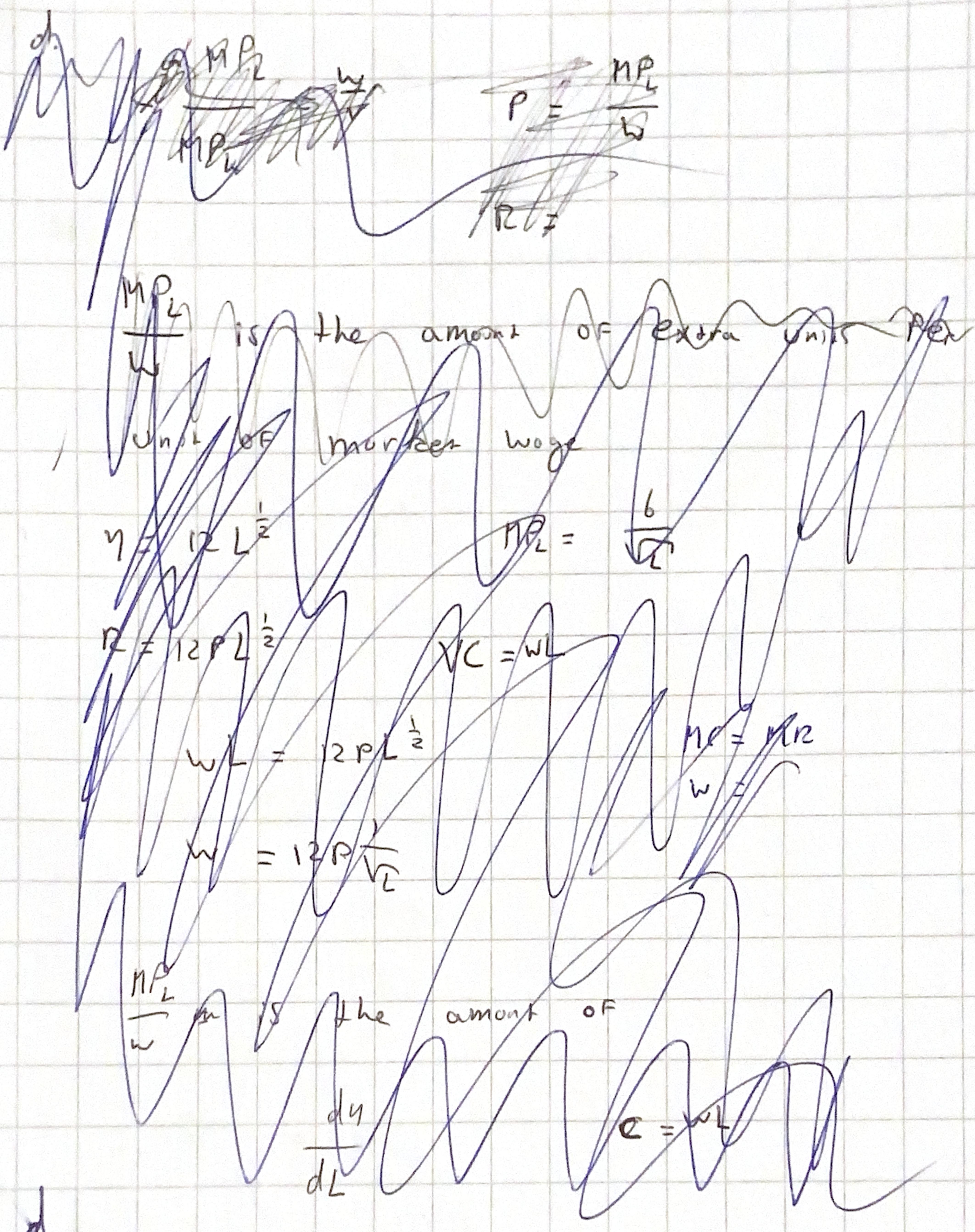
$$C. \bar{K} = 4$$

$$y = 6\sqrt{4} L^{\frac{1}{2}} = 12L^{\frac{1}{2}}$$



$$MP_L = \frac{\partial y}{\partial L} = 6 L^{\frac{2}{2}-\frac{1}{2}}$$

$$\frac{dMP_L}{dL} = -3g L^{-\frac{3}{2}} < 0 \text{ so diminishing Product of Labour}$$



d.

$$y = 12\sqrt{L}$$

$$r = p \cdot \cancel{y} - (wL + gr)$$

$$\text{max if } \frac{dr}{dL} = 0 = p \cdot \cancel{\frac{dy}{dL}} - w \rightarrow MP_L$$

$$w = p \cdot MP_L$$

$$MP_L = \frac{6}{\sqrt{L}} \quad 2 = 7 \cdot \frac{6}{\sqrt{L}} \Rightarrow L = 9$$

(4)

$$a. F(\lambda K, \lambda L) = \sqrt{\lambda} V_K + \sqrt{\lambda} V_L$$

$$= \lambda^{\frac{1}{2}} (\sqrt{K} + \sqrt{L})$$

So decreasing returns to scale

$$B. MP_L = \frac{\partial F}{\partial L} = \frac{1}{2} L^{-\frac{1}{2}}$$

$$MP_K = \frac{\partial F}{\partial K} = \frac{1}{2} K^{-\frac{1}{2}}$$

$$MRTS = - \frac{MP_L}{MP_K} = - \frac{\frac{1}{2} L^{-\frac{1}{2}}}{\frac{1}{2} K^{-\frac{1}{2}}} = - \frac{\sqrt{L}}{\sqrt{K}}$$

$$C. \text{Cost } C = wR + rwt$$

$$-\frac{w}{r} = MRTS = -\frac{\sqrt{K}}{\sqrt{L}}$$

$$\frac{w}{r} = \frac{\sqrt{K}}{\sqrt{L}}$$

$$y = \sqrt{K} + \sqrt{L}$$

$$\sqrt{K} = y - \sqrt{L}$$

$$\frac{w}{r} = \frac{y - \sqrt{L}}{\sqrt{L}} = \frac{y}{\sqrt{L}} - 1$$

$$\frac{y}{\sqrt{L}} = \frac{w}{r} + 1$$

$$\frac{\sqrt{L}}{y} = \frac{1}{\frac{w}{r} + 1}$$

$$\sqrt{L} = \frac{y}{\frac{w}{r} + 1}$$

$$L = \left(\frac{y}{\frac{w}{r} + 1} \right)^2$$

and $\sqrt{K} = y - \frac{y}{\frac{w}{r} + 1}$

$$K = \left(y - \frac{y}{\frac{w}{r} + 1} \right)^2$$

$$d. C = rk + wl$$

$$= r \left(y - \frac{y}{\frac{w}{r} + 1} \right)^2 + w \left(\frac{y}{\frac{w}{r} + 1} \right)^2$$

So, if move out for it produced
costs

$$= r \left(y^2 - 2 \cdot \frac{y^2}{\frac{w}{r} + 1} + \frac{y^2}{\left(\frac{w}{r} + 1 \right)^2} \right) + w \cdot \frac{y^2}{\left(\frac{w}{r} + 1 \right)^2}$$

$$= y^2 \left(r - \frac{2r}{\frac{w}{r} + 1} + \frac{r}{\left(\frac{w}{r} + 1 \right)^2} + \frac{w}{\left(\frac{w}{r} + 1 \right)^2} \right)$$

So if y goes up, costs go up
exponentially.

6

a. $C = 16 + 4q^2$

$$MC = \frac{dc}{dq} = 8q$$

$$AVC \text{ or } AC = 4q$$

min of AVC when $AVC = MC$

$$4q = 8q$$

$$q = 0$$

So if $q > 0$ (AVC' is positive)

$$P = 8q \quad q = \frac{1}{8}P$$

~~margin~~

$$q = 24 \cdot \frac{1}{8}P = 3P$$

$$180 - 2P = 3P$$

$$180 = 5P$$

$$P = 36$$

B. In the long run the price is the minimum LR average cost

$$AC = \frac{16}{q} + 4q$$

$$8q = \frac{16}{q} + 4q$$

$$4q = \frac{16}{q}$$

$$4q^2 = 16 \quad q = 2$$

φ_E ~~ρ_{agg}~~ _{tot}

$$y_{\text{agg}} = n \cdot y$$

$$\text{Durch} \rightarrow P = 8y = 8 \cdot 2 = 16$$

$$2n = 180 - 2P$$

$$2n = 180 - 32$$

$$1n = 75 - 16 = 59$$

② College task sheet

a. $MC = 2y^2 + 1$

II) $AC = \frac{2}{3}y^2 + 1 + \frac{36}{y}$

$$2y^2 + 1 = \frac{2}{3}y^2 + 1 + \frac{36}{y}$$

$$\frac{4}{3}y^2 - 1 = \frac{36}{y}$$

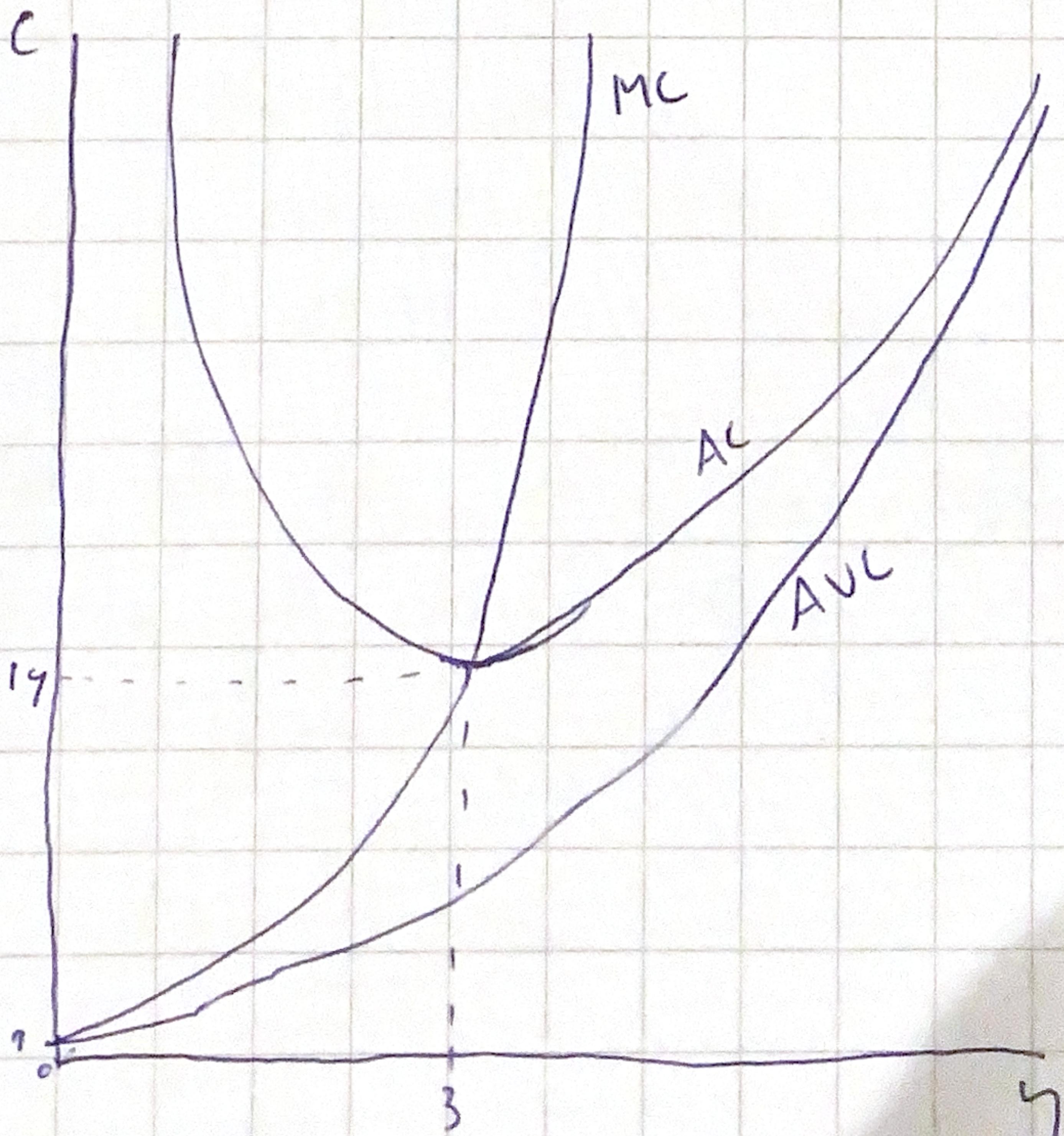
$$\frac{4}{3}y^3 = 36$$

$$y^3 = 27$$

$$y = 3$$

$$MC = 2y^2 + 1 = 2 \cdot 3 + 1 = 19$$

III)



$$\text{IV) } AVC = \frac{2}{3}y^2 + 1 \quad MC = 2y^2 + 1$$

$$\frac{2}{3}y^2 + 1 = 2y^2 + 1$$

$$y = 0$$

so for $y \geq 0$, the supply is
the MC:

$$P = MC = 2y^2 + 1$$

$$y = \sqrt{\frac{1}{2}P - \frac{1}{2}}$$

B.

$$\text{I)} \quad C = Cy + F$$

$$AC = \frac{Cy}{y} + \frac{F}{y} = c + \frac{F}{y}$$

$$\frac{d AC}{dy} = -\frac{F}{y^2} < 0 \text{ so } AC \text{ decreases}$$

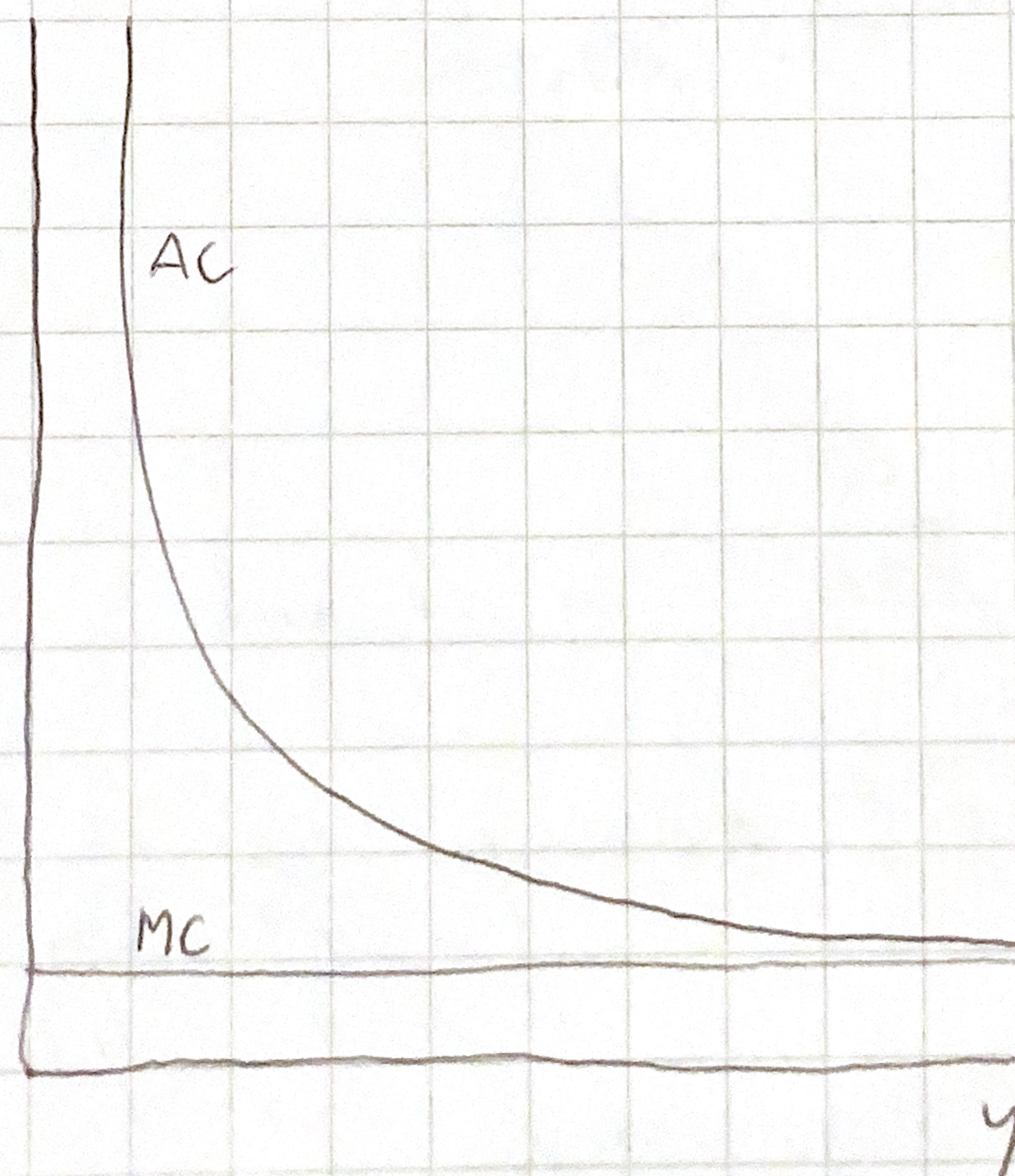
For every $y (> 0)$

II)

$$MC = c$$

$$AC - MC = c + \frac{F}{y} - c = \frac{F}{y} > 0$$

III)



$$c. \quad MC = \frac{dc}{dy}$$

index 201:

$$\frac{dc}{dy} = \frac{c}{y}$$

$$y = c \cdot \left(\frac{dc}{dy}\right)^{-1}$$

$$4c = \frac{c}{y} \cdot c \cdot \frac{1}{y}$$

thus to be:
 $\frac{dMC}{dy} = -\frac{c}{y^2} = 0$

and $\frac{d^2 MC}{dy^2} = 2 \frac{c}{y^3} < 0$

thus to be:

so:

$$\frac{dAC}{dy} = \frac{dc}{dy} \cdot \frac{1}{y} + -c \frac{1}{y^2}$$

$$= \frac{dc}{dy} \frac{1}{y} - \frac{c}{y^2} = 0$$

$$0 = \frac{dc}{dy} \cdot \frac{1}{c} - \frac{c}{c \left(\frac{dc}{dy}\right)^2} \left(c \left(\frac{dc}{dy}\right)^{-1}\right)^2$$

Also

$$\frac{c}{c^2 \left(\frac{dc}{dy}\right)^2} = \frac{\frac{dc}{dy}}{dy} \frac{\frac{dc}{dy}}{c}$$

$$\frac{\left(\frac{dc}{dy}\right)^2}{c} = \frac{\left(\frac{dc}{dy}\right)^2}{c}$$

so it works.

Also: $\frac{d^2 AC}{dy^2}$ ~~to do for it to be~~
~~a minimum~~

$$\frac{d^2 AC}{dy^2} = \frac{dc}{dy^2} \cdot \frac{1}{y} + \frac{dc}{dy} \cdot -\frac{1}{y^2} + \frac{dc}{dy} \cdot \frac{1}{y^2} + c \cdot -2 \frac{1}{y^3}$$

$$= \frac{dc}{dy^2} \cdot \frac{1}{y} + \frac{dc}{dy} \cdot -\frac{1}{y^2} + \frac{dc}{dy} \cdot \frac{1}{y^2} + \frac{2c}{y^3} = \frac{dc}{dy^2} \cdot \frac{1}{y} - 2 \frac{dc}{dy} \frac{1}{y^2} + \frac{2c}{y^3}$$