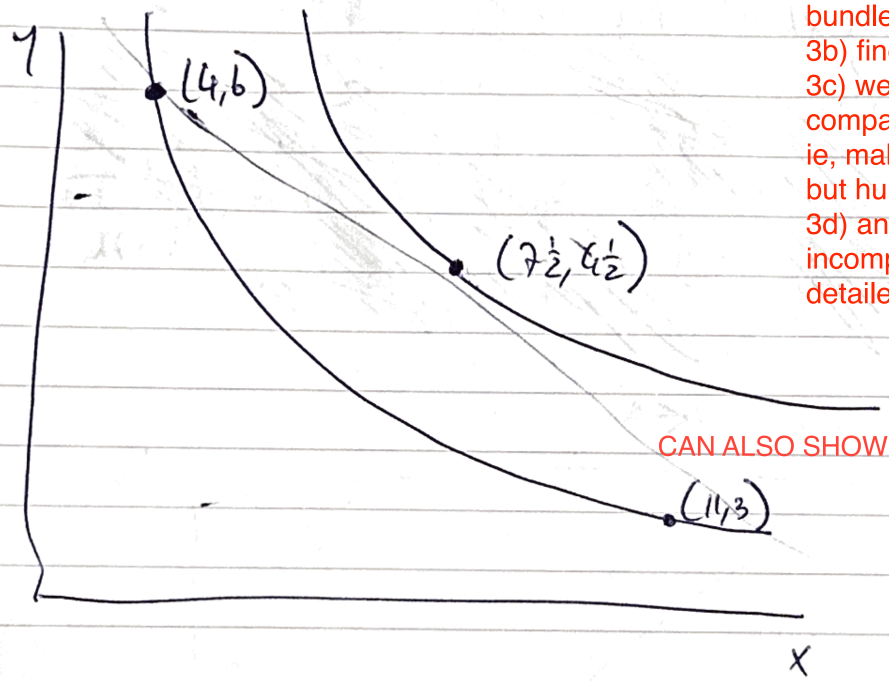
③ a. An allocation is possible where Both get $(7\frac{1}{2}, 4\frac{1}{2})$ where Both are Better off.

Well-Behaved Preferences are Continuous, convex, and monotonic. therefore, ICs are Continuous, convex and downward-sloping.

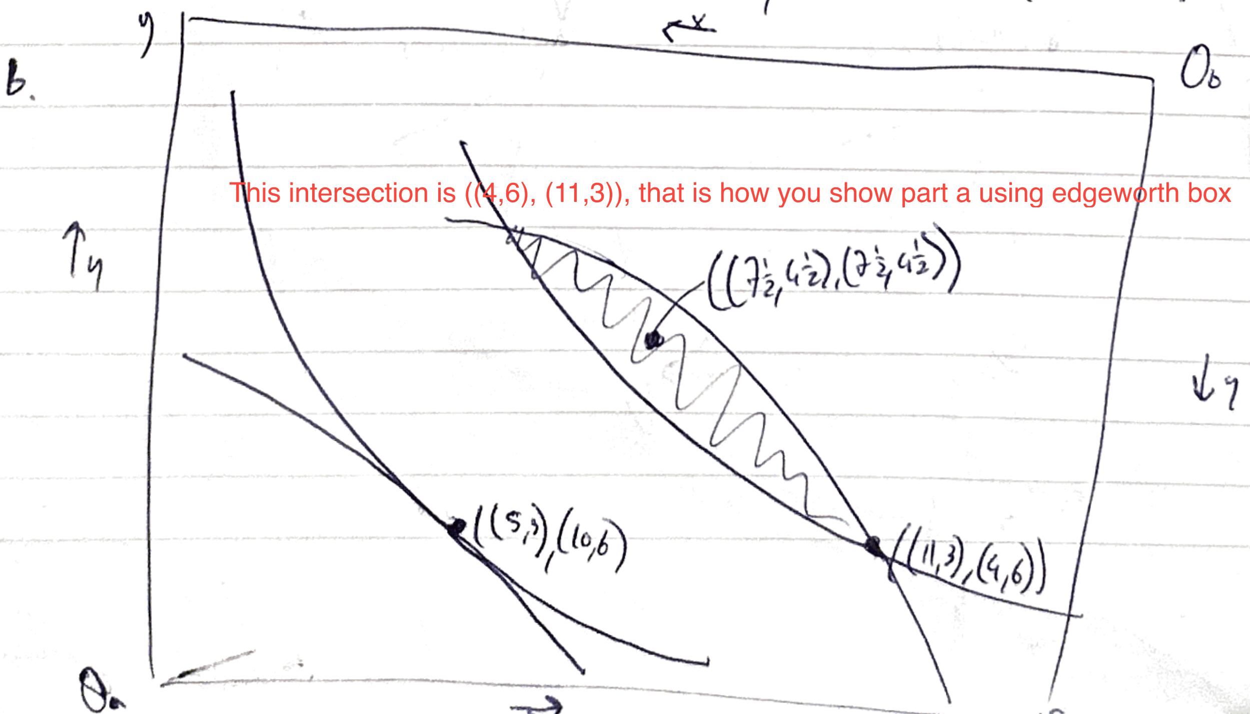
3a) correct, just say that the 7.5, 4.5 is the average of two bundles.
3b) fine.
3c) we can't pareto compare thes bundles, ie, makes b better off but hurts a.
3d) answer is incomplete,not detailed.

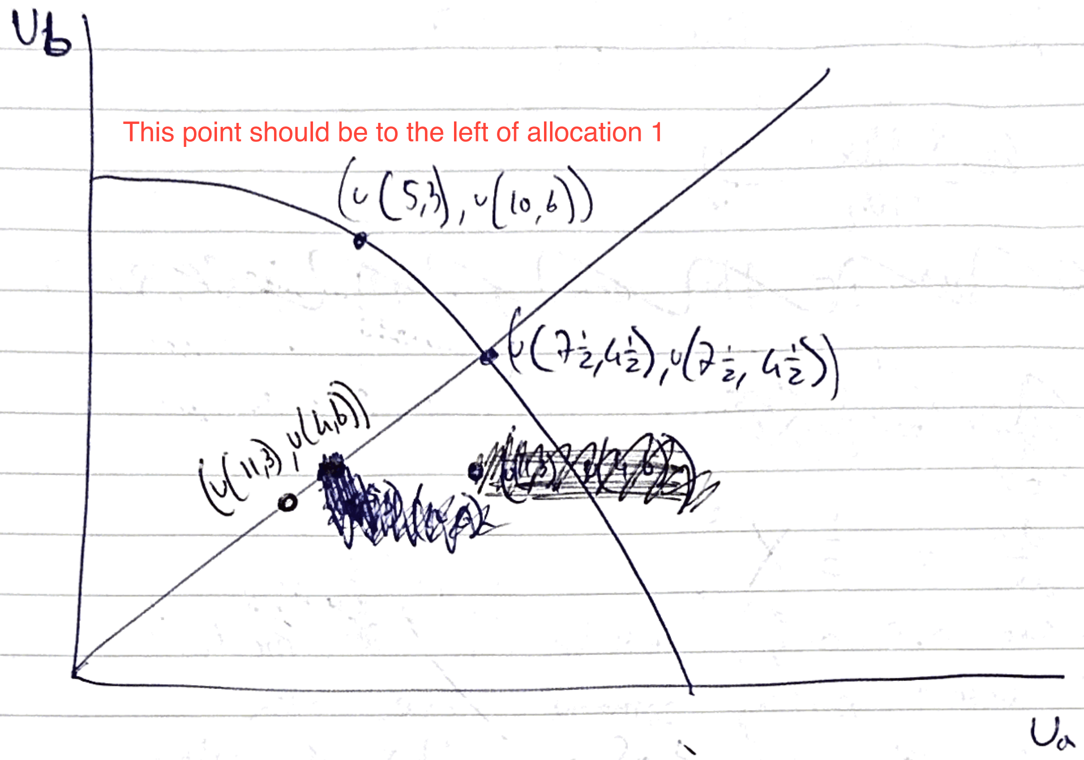


CAN ALSO SHOW THIS USING EDGEWORTH BOX

therefore, Both are Better off with $(7\frac{1}{2}, 4\frac{1}{2})$ so allocation 1 is not Pareto-efficient as it gets Pareto-dominated by this allocation.

b.





The UPF is downward sloping ~~and~~ ~~going~~ ~~from~~ because it is the set of Pareto-efficient allocations and an increase in a's utility must lead to a decrease in b's utility.

c. I) allocation 2 is Pareto-efficient and allocation 1 is not.

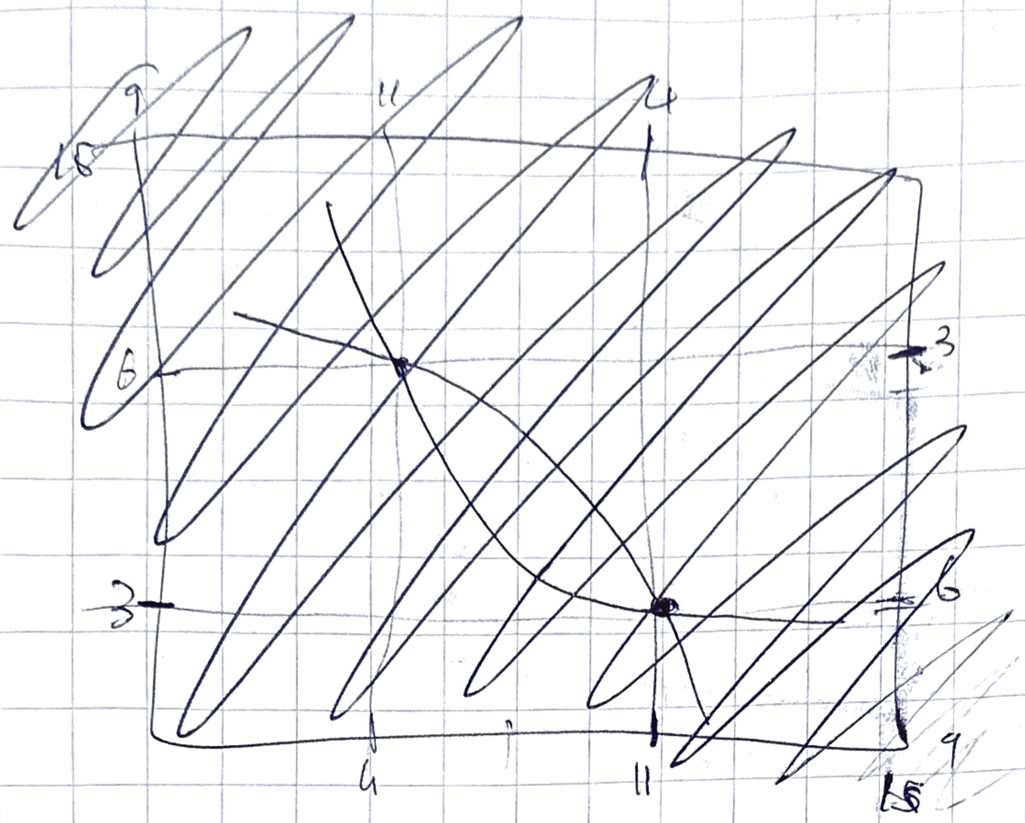
Cannot Pareto compare them, allocation 2 is not a pareto improvement, its a kaldor hicks improvement

II) the minimum utility in allocation 2 is $u(5,3)$, in allocation 1 the utilities are equal $u(11,3) = u(4,6)$. By monotonicity: $u(11,3) > u(5,3)$ so allocation 1 would be better.

III) For the Utilitarian way, you need cardinal information about the preferences. So this is impossible to compare.

d. Noone objects to a Pareto-improvement, but it does not capture other ideas such as inequality, even making someone potentially worse off to make society as a whole better off (Kaldor-Hicks).

Does not account for externalities
Not all bundles can be pareto-compared, there are too many pareto-efficient allocations
Rawlsian: you need minimum
Utilitarian: you need cardinal
Kaldor-Hicks



④ $u = \ln(x) + \ln(y)$ (1)

$Y = X^{1/2}$ (2)

$MRS = -\dfrac{\partial U/\partial x}{\partial U/\partial y} = -\dfrac{y}{x}$

MRT is the slope of the PPF.

$X = 48 - x$ (good x used for production is 48 minus the consumption of good x)

$Y = y$ (the amount of good y that is produced is also consumed)

so using (2):

PPF: $y = (48-x)^{1/2}$

$MRT = \dfrac{dy}{dx} = -\dfrac{1}{2}(48-x)^{-\frac{1}{2}}$

B. if it's ~~effe~~ efficient then

MRT = MRS

$$-\frac{1}{2}(48-x)^{-\frac{1}{2}} = -\frac{y}{x}$$

Plugging in $x=32$ and $y=4$ gives

$-\frac{1}{8}$ on both sides

so that is efficient.

C. ~~Competitive equilibrium requires that~~ ~~the MRS on the contract curve~~ ~~At that point the~~

The Pareto-optimal allocation is part of a competitive equilibrium when the slope of the Budget line is ~~tangent~~ equal to the MRS/~~MRT~~.

$$MRS = MRT = -\frac{1}{8} \quad \text{for } x = 32 \quad y = 4$$

The slope of the Budget line is

$$-\frac{P_x}{P_y} = -\frac{1}{8}$$

$$\frac{P_x}{P_y} = \frac{1}{8}$$

the profits are: (If we set $P_x = 1$ and $P_y = 8$)

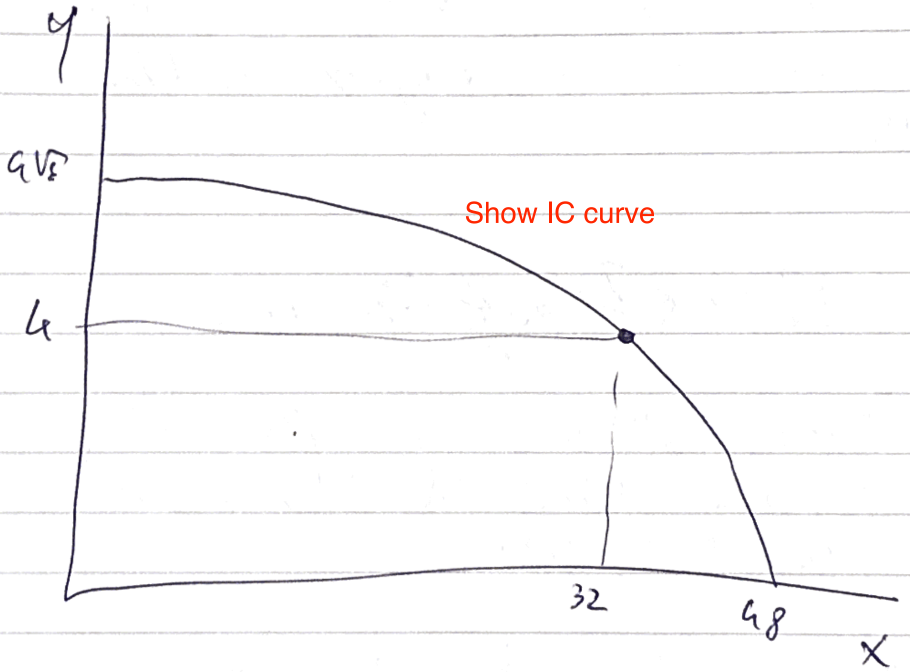
$\pi = 8y - 1(48 - x) = 32 - 16 = 16$

exactly, pi=8Y - 1X = 8y - 1(48-x) since Y=y and X = 48-x
Important distinction!

the consumer budget constraint is satisfied:

$48 \cdot 1 + 16 = P_x X + P_y Y = 1 \cdot 32 + 8 \cdot 4 = 64$

d. PPF: $y = (48-x)^{\frac{1}{2}}$



Show IC curve

⑤

a. the set of all Pareto-efficient allocations (contract-curve) is ~~where~~ where the ~~max~~ MRSs are equal:

$$MRS_a = MRS_b \qquad (1)$$

$$MRS_a = -\frac{MU_1^a}{MU_2^a} = -\frac{\partial u^a/\partial x_1^a}{\partial u^a/\partial x_2^a} = -\frac{2/x_1^a}{3/x_2^a} = -\frac{2x_2^a}{3x_1^a}$$

$$MRS_b = -\frac{MU_1^b}{MU_2^b} = -\frac{\partial u^b/\partial x_1^b}{\partial u^b/\partial x_2^b} = -\frac{2/x_1^b}{1/x_2^b} = -\frac{2X_2^b}{X_1^b}$$

plugging this into (1):

$$-\frac{2}{3}\frac{x_2^a}{x_1^a} = -\frac{2x_2^b}{x_1^b} \qquad (2)$$

we also know that $x_1^a + x_1^b = 20 \Rightarrow x_1^b = 20 - x_1^a$

and $x_2^a + x_2^b = 12 \Rightarrow x_2^b = 12 - x_2^a$

substituting those in (2):

$$-\frac{2}{3}\frac{x_2^a}{x_1^a} = -\frac{2(12-x_2^a)}{20-X_1^a} =$$

$$\frac{2}{3}\frac{x_2^a}{x_1^a} = \frac{24-2x_2^a}{20-x_1^a}$$

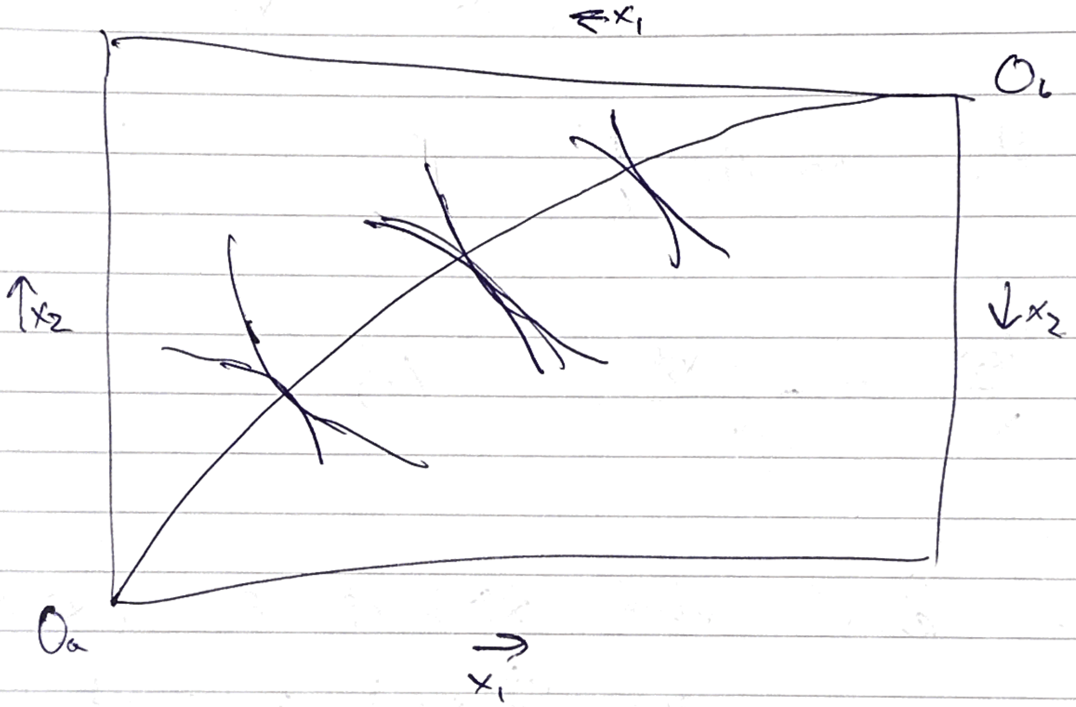
$$2x_2^a(20-x_1^a) = 3x_1^a(24-2x_2^a)$$

$$40x_2^a - 2x_2^a x_1^a = 72x_1^a - 6x_2^a x_1^a$$

$$40x_2^a + 4x_2^a x_1^a = 72x_1^a$$

$$x_2^a(40+4x_1^a) = 72x_1^a$$

$$x_2^a = \frac{72x_1^a}{40+4x_1^a} = \frac{18x_1^a}{x_1^a+10}$$



(Could've just used Cobb-Douglass Standard!)

b. Consumer a: (normalising $P_2 = 1$)

$$\max U = 2\ln(x_1^a) + 3\ln(x_2^a) \quad \text{s.t.} \quad P_1 x_1^a + x_2^a \leq 20P_1$$

$$\mathcal{L} = 2\ln(x_1^a) + 3\ln(x_2^a) - \lambda(P_1 x_1^a + x_2^a - 20P_1)$$

$$\frac{\partial \mathcal{L}}{\partial x_1^a} = \frac{2}{x_1^a} - \lambda P_1 = 0 \quad (1) \quad \Rightarrow \lambda = \frac{2}{x_1^a P_1}$$

So $\lambda > 0$ if $x_1^a > 0$ and $P_1 > 0$

so $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$ (3)

$$\frac{\partial \mathcal{L}}{\partial x_2^a} = \frac{3}{x_2^a} - \lambda = 0 \quad (2)$$

Using (1) & (2):

$$\frac{2x_2^a}{3x_1^a} = P_1 \quad \Rightarrow \quad 2x_2^a = 3P_1 x_1^a \quad (4)$$

Using (3):

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -P_1 x_1^a - x_2^a + 20P_1 = 0$$

$$P_1 x_1^a = 20P_1 - x_2^a$$

Substituting (4):

$$2x_2^a = 3(20P_1 - x_2^a)$$

$$2x_2^a = 60P_1 - 3x_2^a$$

$$5x_2^a = 60P_1$$

$$x_2^a = 12P_1$$

and substituting into (a):

$$2 \cdot 12 P_1 = 3 P_1 x_1^a$$

$$x_1^a = 8$$

Consumer B:

$$\max \quad U = 2\ln(x_1^b) + \ln(x_2^b) \quad \text{s.t.} \quad P_1 x_1^b + x_2^b \leq 12$$

$$\mathcal{L} = 2\ln(x_1^b) + \ln(x_2^b) - \lambda(P_1 x_1^b + x_2^b - 12)$$

$$\frac{\partial \mathcal{L}}{\partial x_1^b} = \frac{2}{x_1^b} - \lambda P_1 = 0 \quad (1) \quad \Rightarrow \lambda > 0 \Rightarrow \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial x_2^b} = \frac{1}{x_2^b} - \lambda = 0 \quad (2)$$

(1) & (2):

$$\frac{2x_2^b}{x_1^b} = P_1$$

$$2x_2^b = P_1 x_1^b \quad (4)$$

From (3):

$$P_1 x_1^b + x_2^b = 12$$

$$P_1 x_1^b = 12 - x_2^b$$

$\cancel{x^{1/2} = 48 - x}$

$\cancel{y = k(48 - x)^{1/2}}$

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using (a):

$2x_2^b = 12 - x_2^b$

$3x_2^b = 12$

$x_2^b = 4$ and using (a):

$2 \cdot 4 = p_1 x_1^b$

$x_1^b = \frac{8}{p_1}$

$z_1 = x_1^a - w_1^a + x_1^b - w_1^b = 8 - 20 + \frac{8}{p_1} - 0 = \frac{8}{p_1} - 12$

In competitive equilibrium, aggregate excess demand is 0:

$\frac{8}{p_1} - 12 = 0$

$12p_1 = 8$

$p_1 = \frac{2}{3}$ allocation a: $(8, 8)$

" b: $(12, 4)$

This allocation lies on the contract curve!

$$x_2^a = \frac{18x_1^a}{x_1^a+10}$$

$$8 = \frac{18\cdot 8}{8+10} = 8$$

So it is Pareto efficient as $MRS_a = MRS_b$ so there are no gains from trade.

However, by welfare theorem 1, every competitive equilibrium is P-efficient (do you want me to give the proof?)

C.

$$x_2^a = \frac{18\,x_1^a}{x_1^a+10}$$

$$6 = \frac{18\cdot 5}{5+10} = 6$$

so this on the contract curve.

By welfare theorem 2 it can be, the slope of the budget line is equal to the MRSs:

WRONG way around
should be -p1/p2

$$-\frac{P_2}{P_1} = MRS_a = -\frac{2x_2^a}{3x_1^a} = -\frac{2}{3}\,\frac{6}{5} = -\frac{4}{5}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{5}{4}$$

Plugged in the wrong allocation here and got p1 and p2 wrong way around

d. The transfer should be to a point on the budget line. The budget line for a ~~[illegible]~~ is

(with $p_1 = 5$ & $p_2 = 4$)

$$x_2^a = \frac{20 \cdot 5}{4} - \frac{5}{4}x_1^a = \cancel{100}\ 25 - \frac{5}{4}x_1^a$$

so she should transfer good 1 until ~~the~~ ~~point of the goods~~ she is on that line

(20, 0) is already on that line.

Right answer:
Budget line p1 = 4 p2=5

m = 5 * p1 + 6 * p2 = 50

50 = p1x1 + p2x2
p2x2 = 50 - p1x1
x2 = 10 - 4/5 x1

for consumer a:
plug in x2 = 0 and then you find x1 = 12.5

so you redistribute 7.5 units of x1