② a. firm **2** is solving the following problem:

$$\max_{q_2} \pi_2 = (a - q_1 - q_2)\, q_2 - c_2 q_2$$

$$= aq_2 - q_1 q_2 - q_2^2 - c_2 q_2$$

FOC:

$$\frac{d\pi}{dq_2} = a - q_1 - 2q_2 - c_2 = 0$$

$$2q_2 = a - q_1 - c_2$$

$$q_2^* = \frac{1}{2}(a - c_2 - q_1)$$

firm 1 solves the same problem so:

$$q_1^* = \frac{1}{2}(a - c_1 - q_2)$$

these are the best response functions so the Nash-equilibrium is where they intersect.

$$q_2 = \frac{1}{2}\left(a - c_2 - \frac{1}{2}(a - c_1 - q_2)\right)$$

$$= \frac{1}{2}a - \frac{1}{2}c_2 - \frac{1}{4}a + \frac{1}{4}c_1 + \frac{1}{4}q_2$$

$$\frac{3}{4}q_2 = \frac{1}{4}a - \frac{1}{2}c_2 + \frac{1}{4}c_1$$

$$q_2^c = \frac{1}{3}a - \frac{2}{3}c_2 + \frac{1}{3}c_1 = \frac{1}{3}(a - 2c_2 + c_1)$$

$$q_1^c = \frac{1}{2}\left(a - c_1 - q_2^c\right)$$

$$= \frac{1}{2}\left(a - c_1 - \frac{1}{3}\left(a - 2c_2 + c_1\right)\right)$$

$$= \frac{1}{2}a - \frac{1}{2}c_1 - \frac{1}{6}a + \frac{1}{3}c_2 - \frac{1}{6}c_1$$

$$= \frac{1}{3}a - \frac{2}{3}c_1 + \frac{1}{3}c_2$$

$$q_1^c = \frac{1}{3}\left(a - 2c_1 + c_2\right)$$

$$\pi_1^c = \left(a - q_1^c - q_2^c\right)q_1^c - c_1 q_1^c$$

$$= \left(a - \frac{1}{3}(a - 2c_1 + c_2) - \frac{1}{3}(a - 2c_2 + c_1)\right)\frac{1}{3}(a - 2c_1 + c_2) - c_1 \cdot \frac{1}{3}(a - 2c_1 + c_2)$$

$$= \left(a - \frac{1}{3}a + \frac{2}{3}c_1 - \frac{1}{3}c_2 - \frac{1}{3}a + \frac{2}{3}c_2 - \frac{1}{3}c_1\right)\left(\frac{1}{3}a - \frac{2}{3}c_1 + \frac{1}{3}c_2\right) - c_1\left(\frac{1}{3}a - \frac{2}{3}c_1 + \frac{1}{3}c_2\right)$$

$$= \left(\frac{1}{3}a + \frac{1}{3}c_1 + \frac{1}{3}c_2\right)\left(\frac{1}{3}a - \frac{2}{3}c_1 + \frac{1}{3}c_2\right) - \frac{1}{3}c_1 a + \frac{2}{3}c_1^2 - \frac{1}{3}c_1 c_2$$

$$= \frac{1}{9}a^2 - \frac{2}{9}ac_1 + \frac{1}{9}ac_2 + \frac{1}{9}ac_1 - \frac{2}{9}c_1^2 + \frac{1}{9}c_1c_2 + \frac{1}{9}ac_2 - \frac{2}{9}c_1c_2 + \frac{1}{9}c_2^2$$

$$- \frac{1}{3}c_1 a + \frac{2}{3}c_1^2 - \frac{1}{3}c_1c_2$$

$$= \frac{1}{9}a^2$$

8. $q_1^c = \frac{1}{3}(a - 2c_1 + c_2)$ $\quad q_2^c = \frac{1}{3}(a - 2c_2 + c_1)$

$\pi_1^c = (a - q_1^c - q_2^c) q_1^c - c_1 q_1^c = \underline{(a - q_1^c - q_2^c - c_1) q_1^c}$

Effect of change in c_1:

Using multi-variable chain rule:

$$\frac{d\pi_1^c}{dc_1} = \underbrace{\frac{\partial \pi_1}{\partial c_1}}_{\text{I) direct}} + \underbrace{\frac{\partial \pi_1}{\partial q_1} \frac{dq_1^c}{dc_1}}_{\text{II) via own } q_1} + \underbrace{\frac{\partial \pi_1}{\partial q_2} \frac{dq_2^c}{dc_1}}_{\text{III) via other } q_2}$$

$$\frac{\partial \pi_1}{\partial c_1} = -q_1^c$$

$$\frac{\partial \pi_1}{\partial q_1} = a - 2q_1^c - q_2^c - c_1$$

However, this is 0 By envelope theorem.

$$\frac{\partial \pi_1}{\partial q_2} = -q_1^c$$

So: $\dfrac{d\pi_1}{dc_1} = -q_1^c + (-q_1^c) \cdot \dfrac{dq_2^c}{dc_1}$

$\dfrac{dq_2^c}{dc_1} = \dfrac{1}{3}$ so $\dfrac{d\pi_1}{dc_1} = -\dfrac{4}{3} q_1^c$

effect of change in c_2: $\frac{d\pi_1^c}{dc_2} = \frac{\partial \pi_1}{\partial c_2} + \frac{\partial \pi_1}{\partial q_1}\frac{dq_1^c}{dc_1} + \frac{\partial \pi_1}{\partial q_2}\frac{dq_2^c}{dc_2}$

there is no direct effect: $\frac{\partial \pi_1}{\partial c_2} = 0$

own-quantity channel is also 0 by envelope theorem: $\frac{\partial \pi_1}{\partial q_1} = 0$ at the optimum

other firm channel: $\frac{\partial \pi_1}{\partial q_2} = -q_1^c$ and $\frac{dq_2^c}{dc_2} = -\frac{2}{3}$

so:

$$\frac{d\pi_1^c}{dc_2} = \frac{2}{3} q_1^c$$

$\frac{d\pi_1^c}{dc_1} = -\frac{4}{3} q_1$ $\qquad$ $\frac{d\pi_1^c}{dc_2} = \frac{2}{3} q_1^c$

so the firm's profit changes more when its own cost changes.

C. If firm 1 becomes the Stackelberg leader, it will set output equal to monopoly level, which will be more than in Cournot:

Best response for firm 2 is still:

$\cancel{q_2^S = \frac{1}{8}(a - 2c_1 + c_2)}$

$q_2^S = \frac{1}{2}(a - c_2 - q_1^S)$

given that, firm 1 will optimize:

$$\max_{q_1} \pi_1^S = \left(a - q_1 - \frac{1}{2}(a - c_2 - q_1)\right)q_1 - c_1 q_1$$
$$= \left(\frac{1}{2}a - \frac{1}{2}q_1 + \frac{1}{2}c_2\right)q_1 - c_1 q_1$$
$$= \frac{1}{2}aq_1 - \frac{1}{2}q_1^2 + \frac{1}{2}c_2 q_1 - c_1 q_1$$

FOC:

$$\frac{d\pi_1^S}{dq_1} = \frac{1}{2}a - q_1 + \frac{1}{2}c_2 - c_1 = 0$$

$$q_1^S = \frac{1}{2}a + \frac{1}{2}c_2 - c_1 = \frac{1}{2}(a - 2c_1 + c_2)$$
$$> q_1^C$$

and
$$q_2^S = \frac{1}{2}\left(a - c_2 - \frac{1}{2}(a - 2c_1 + c_2)\right)$$

Using a graph to show which one is bigger is more helpful

$$= \frac{1}{2}\left(\frac{1}{2}a - \frac{3}{2}c_2 + c_1\right) = \frac{1}{4}(a + 2c_1 - 3c_2)$$
$$< q_2^C$$

Using a graph to show which one is bigger is more helpful
use the isoprofit lines, like in the game theory slides

d. ~~Let's~~

$$q^S = q_1^S + q_2^S = \frac{1}{2}(a - 2c_1 + c_2) + \frac{1}{4}(a + 2c_1 - 3c_2)$$
$$= \frac{3}{4}a - \frac{1}{2}c_1 - \frac{1}{4}c_2$$

So total output gets smaller rather with a higher C_1 than higher C_2. So they want C_2 to be bigger than C_1.
So the one with the lower cost should move first, as total output and therefore consumer surplus would be higher.

(4) a.

$$\max_{q_2} \pi_2 = (40 - q_1 - q_2)q_2 - (0 + 4q_2)$$

$$= 40q_2 - q_1q_2 - q_2^2 - 4q_2 = 36q_2 - q_1q_2 - q_2^2$$

FOC:

$$\frac{d\pi_2}{dq_2} = 36 - q_1 - 2q_2 = 0$$

$$2q_2 = 36 - q_1$$

$$q_2^a = \frac{36 - q_1}{2}$$

Give this, firm 1 will optimize:

$$\max_{q_1} \pi_1^a = (40 - q_1 - (18 - \tfrac{1}{2}q_1))q_1 - 2q_1$$

$$= 22q_1 - \tfrac{1}{2}q_1^2 - 2q_1 = 20q_1 - \tfrac{1}{2}q_1^2$$

FOC:

$$\frac{d\pi_1}{dq_1} = 20 - q_1 = 0$$

$$q_1^a = 20$$

$$q_2^a = \frac{36-20}{2} = 8$$

$$P_a = 40 - (20 + 8) = 12$$

$$\pi_1^a = 12 \cdot 20 - 2 \cdot 20 = 200 \qquad \pi_2^a = 12 \cdot 8 - 4 \cdot 8 = 64$$

B. $\max_{q_2} \pi_2^b = 40q_2 - q_1q_2 - q_2^2 - (4+4q_2)$

$= 36q_2 - q_1q_2 - q_2^2$

Best response function stays the same

It will not enter if profits are not positive.

$\pi_2^b = (40 - q_1 - 18 + \frac{1}{2}q_1)(18 - \frac{1}{2}q_1) - F - 4(18 - \frac{1}{2}q_1)$

$(22 - \frac{1}{2}q_1)(18 - \frac{1}{2}q_1) - F - (72 - 2q_1) = 0$

$396 - 20q_1 + \frac{1}{4}q_1^2 - 4 - 72 + 2q_1 = 0$

$320 - 18q_1 + \frac{1}{4}q_1^2 = 0$

$\pi_2^b = (40 - q_1 - q_2)q_2 - F - 4q_2 = (36 - q_1 - q_2)q_2 - F$

But $q_2 = \frac{36 - q_1}{2}$

So $\pi_2^b = \left((36 - q_1) - \frac{36 - q_1}{2}\right)\frac{36 - q_1}{2} - F$

$= \frac{(36 - q_1)^2}{4} - F = 0$

So for $F = 4$

$\frac{(36 - q_1)^2}{4} = 4$

$36 - q_1 = \pm 16^{\frac{1}{2}} = \pm 4$

$q_1 = 32$ or $q_1 = 40$

↑ irrelevant as it's already achieved $\pi_2 = 0$ with $q_1 = 32$

$\pi_2 = 0$ at $\pi_2 = 0$

$\pi_1^b = (40 - 32)32 - 2 \cdot 32 = 192 < \pi_1^a$

So deterrence is worse for firm 1 than the Stackelberg solution so it will do the same as in a.

c. $\frac{(36 - q_1)^2}{4} = 36$

$36 - q_1 = \pm\sqrt{144} = \pm 12$

$q_1 = 28$

$\pi_1^c = (40 - 28)28 - 2 \cdot 28 = 280 > \pi_1^a$

So firm 1 will choose deterrence and set $q_1 = 28$

d. $\frac{(36 - q_1)^2}{4} = 160$

$36 - q_1 = \pm 12$

$q_1 = 16$

Now firm 2 is already deterred by $q_1 > 16$ so firm 1 will just set the ~~Stackelberg~~ monopoly output of ~~$q_1 = 20$~~

Monopoly output, and if firm 2 is a monopolist, ~~that~~ it optimizes:

$$\max_{q_1} \pi_1^m = (40-q_1)q_1 - 2q_1$$

$$= 38q_1 - q_1^2$$

FOC:

$$\frac{d\pi_1^m}{dq_1} = 38 - 2q_1 = 0$$

$$q_1 = 19 \quad > 16$$

So it will set output to 19

(this is also lower than for c so that is correct).

$$\pi_1^m = (40-19)19 - 2\cdot 19 = 361$$

e. - In part B, deterrence is too costly so it will keep doing the same as in part a.

- in part C, deterrence is profitable

- But in part d it is so easy that it can set output at the monopoly level and still deter firm 2.