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Berg

i)  $\frac{P \wedge (P \rightarrow Q) \rightarrow Q}{t_1 t_2 ? t_5 F_3 F F_1}$

For the sake of completeness  
add columns for P (Q)  
just to be more  
formally accurate

ii)  $\frac{\neg Q \wedge (P \rightarrow Q) \rightarrow \neg P}{t_4 F_5 F_3 t_5 ? t_6 F_3 F F_1 t_2}$

iii)  $\frac{P \vee \neg P}{F_1 F F_2 ?}$

iv)  $\frac{\neg(P \wedge \neg P)}{F t_2 t_1 t_3 ?}$

v)  $\frac{(\neg P \rightarrow P) \rightarrow P}{F_4 ? t_2 F_3 F F_1}$

vi)  $\frac{(P \rightarrow Q) \wedge (\neg P \rightarrow \neg Q) \rightarrow Q}{F_7 t_3 F_6 t_2 F_8 ? t_4 F_5 F F_1}$

vii)  $\frac{\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)}{1 \frac{t_1 ? F_3 t_8 F F_4 t_6 F_2 F_5 t_7}{F_1} 2 \frac{F_1 t_5 t_3 t_4 F F_7 t_6 t_2 t_8 ?}{}}$

VIII)

$$\frac{}{1 \quad \neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)}$$

$$\frac{1 \quad \underline{t_1 F_4 F_3 F_5} \quad F \quad t_2 F_6 \underline{F_2} \quad F_8 ?}{2 \quad \underline{t_1 ? t_3 F_9} \quad F \quad t_4 F_6 \underline{t_2 + t_5} \quad F_7} \quad \checkmark$$

IX)

$$\frac{P \wedge \neg P \rightarrow Q}{t_3 t_2 t_4 ? \quad F \quad F_1} \quad \checkmark$$

(2.6)

#)

$$\frac{P \mid P \wedge P}{\begin{array}{c|ccc} F & F_1 & F & F_2 \\ + & t_1 & + & t_2 \end{array}} \rightarrow \text{Contingency} \quad \checkmark$$

II)

$$\frac{}{P \quad QR \mid ((P \rightarrow Q) \rightarrow R) \leftrightarrow (P \rightarrow (Q \rightarrow R))}$$

$$\frac{\begin{array}{c|cc|cc|cc} & & & & & & \\ T & + & + & F & t_9 & t_u & t_{10} & \underline{F_1 \quad F_3} & + & F_7 & \underline{F_2} & +_8 +_6 F_5 \\ I.2 & & & & & & & & & & & \end{array}}{\begin{array}{c|cc|cc|cc} & & & & & & \\ 2.1 & & & & \bar{F}_8 ? & \cancel{\frac{t_1}{F_7}} & & F & t_3 \underline{F_2} & t_6 F_4 & F_5 \\ 2.21 & & & & t_9 + t_u ? & \cancel{\frac{F_1}{F_3}} & & F & \underline{t_6 + t_2} & F_8 & \underline{t_2 F_5} \\ 2.22 & P P F & & & F_9 + t_4 F_{10} & \cancel{\frac{F_1}{F_3}} & & F & \underline{F_6 + t_2} & F_8 & \underline{t_7 F_5} \end{array}}$$

→ Contingency ✓

III) (I won't do a partial because it'll need to do a lot of assumptions)

$P \otimes R$	$(P \otimes (Q \otimes R)) \leftrightarrow ((P \otimes Q) \otimes R)$							
T T T	F	T	T					
T T F	F	F						
T F T	F	F						
T F F	T	T						
F T T	F	F						
F T F	T	F						
F F T	T	F						
F F F	F	T						

→ tautology ✓

IV

$P Q$	$\neg(P \rightarrow Q) \leftrightarrow (P \wedge \neg Q)$							
1.1 + F	<u>t<sub>1</sub></u>	<u>t<sub>5</sub></u>	<u>F<sub>3</sub></u>	<u>F<sub>4</sub></u>	+		<u>t<sub>6</sub></u>	<u>t<sub>2</sub></u>
1.2							<u>t<sub>7</sub></u>	<u>F<sub>8</sub></u>
2.1	<u>t<sub>1</sub></u>	<u>t<sub>5</sub></u>	<u>F<sub>3</sub></u>	<u>F<sub>4</sub></u>	F		<u>t<sub>6</sub></u>	<u>F<sub>2</sub></u>
2.2	<u>F<sub>1</sub></u>	?	<u>t<sub>3</sub></u>	<u>F<sub>2</sub></u>	F		<u>t<sub>6</sub></u>	<u>F<sub>7</sub></u> ?
							<u>t<sub>6</sub></u>	<u>t<sub>2</sub></u>
							<u>t<sub>6</sub></u>	<u>t<sub>5</sub></u>

→ tautology ✓

(2.7)

III)  $| \phi \vee \psi |_A = F$  iFF  $|\phi|_A = F$  and  $|\psi|_A = F$

IV)  $| \phi \rightarrow \psi |_A = F$  iFF  $|\phi|_A = T$  and  $|\psi|_A = F$

V)  $| \phi \leftrightarrow \psi |_A = F$  iFF  $|\phi|_A \neq |\psi|_A$

(2.8)

According to definition 2.9  $I \models \phi$  iFF

there is no  $L_1$ -structure in which all sentences ~~over true~~ in  $I$  are true and  $\phi$  is false.

According to definition 2.11 a set of sentences is consistent iFF there is an  $L_1$ -structure under which all sentences in the structure are true.

IE so, if a set is inconsistent there is no  $L_1$ -structure under which all sentences are true.

If the set containing all sentences in  $I$  and  $\neg\phi$  is inconsistent then there is no counterexample as in definition 2.10. therefore, the argument must be valid.

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Answer:  $\Gamma \vdash \phi$  iFF there is no  $L_1$ -structure where  $\Gamma$  all true and  $\phi$  false

iFF ~~that~~ there is no  $L_1$ -structure in which  $\neg\phi$  and all  $\Gamma$  are true

iFF the set containing  $\neg\phi$  and all in  $\Gamma$  ~~are~~ is inconsistent

- ⑦
- A. Jones arrives at the airport after the scheduled departure time
  - B. the Plane will wait for Jones
  - C. Nobody noticed that
- The argument being made is:

$$\frac{A \rightarrow w \therefore (A \wedge N) \rightarrow w}{? + F_5 \quad t_3, t_7, t_4 \models F_2} \checkmark$$

There is no counterexample so the argument seems valid. However, you could argue that the plane will of course only wait for Jones (it can inflict a loss) if they ~~do~~ know that he is late, so you could argue that the argument should actually be formalised the following way:

$$\frac{(A \wedge N) \rightarrow w \therefore (A \wedge N) \rightarrow w}{t_5 \models F_2, t_6 \models F_9 \quad t_3, t_7, t_4 \models F_2}$$

So in this case there is a counterexample and so the argument is not valid.

This, however, does not have to be the mechanism, there could be some other mechanism that will cause the plane to wait which does not require anyone to know that Jones is late. So, in my opinion we should apply the principle of charity and assume the latter.

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I) Robin believes that A

A	Robin believes that A
T	?
F	?



II) Robin knows that A

A	Robin knows that A
T	?
F	F → however, this depends on your definition of what it means to know something



III) Robin knows that A, But it's not true that A

A	Robin knows that A, But it's not true that A
T	<del>?</del> F → this is false Because we just said that if something is not true, one cannot know it
F	<del>?</del> F



Could be done as

A	(Robin knows that A $\wedge \neg A$ )
T	?
F	+ <del>?F</del> F +

~~?F~~ F +

~~That would give different results.~~

~~That is because in this case,  
Robin would never~~

~~You would argue here that there is  
not be a question mark because in  
the case of A being true there is  
a contradiction because~~

~~It is a contradiction, Robin would never~~

~~believe A if A is false so both statements  
can't be true at the same time.~~

III)

A	the infallible Clairvoyant believes that A
T	?
F	✓

This depends on the definition of "Believing".

If you say that it is impossible to believe something without having thoughts about it, the first row is a "?".

Clairvoyant could be a name

II)

AB	A, But B
TT	T
TF	F
FT	F
FF	F

Same as  $\Lambda$

AB | Suppose A; then B

+	+	+
+	F	F
F	+	?
F	F	?

It behaves the same way as an if - statement (that is not the same as the truth table for ~~the same~~  $\rightarrow$ ).

Just live with the if, & it is not truth functional in the case that the antecedent is false (counterfactual).

two examples:

- Suppose Giovanni did not go to London, then he would not have gotten ill.

I

imagine the case that Giovanni did go to London ( $\neg L$ ), but did not get sick

(I) (this is now F+) it is ambiguous whether the sentence is true or false.

- 1. Suppose Kamala Harris would have won the election, then she would have been president

2. Suppose Kamala Harris would have won the election, then aliens would have attacked the earth.

Both of these are now & FF Chandra  
did not win and the latter did not  
happen, it is ambiguous whether the 2nd  
sentence would be true ~~or~~ or false.

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Also "Suppose ..." is an imperative  
sentence, and not declarative, so one  
could argue that all the values  
in the truth table are "?".<sup>5</sup>

3.6

S : Many Students will be in Schopenhauer's lectures

Good.

H :  $\neg \exists$  " Hegel's "

will prove  
this is clear  
in down  
then

+ : they are scheduled at the same time

$E_1$  : Hegel's lectures are entertaining

$E_2$  :  $\neg$  in Schopenhauer's "

Initially you'd think it is Formulated like this:

$$+ \rightarrow (H \vee S), +, E_1 \rightarrow H, E_2 \rightarrow E_1,$$
$$S \rightarrow E_2 \quad \therefore H \wedge \neg S$$

$\neg H \wedge E_1, E_2$	$+ \rightarrow (H \vee S)$	$+   E_1 \rightarrow H   E_2 \rightarrow E_1   S \rightarrow E_2   H \wedge \neg S$
$t_{11} + t_{12} + t_{13} + t_{14}$	$t_1 + \underline{t_3 + t_2 + t_4}$	$t_{13} + t_5   t_{11} + t_{12}   t_6 + t_7   t_8 F F_{10} + t_9$

So there is a counterexample,

then

However, "many students come to Schopenhauer's lectures or Hegel's lectures" does ~~not~~ ~~mean~~ could be interpreted as an exclusive or, ~~is~~ ~~you~~ ~~should~~ ~~suggest~~ that other students that there could not be many students ~~are~~ in both lectures.

	$t \rightarrow ((H \vee S) \wedge \neg(H \wedge S))$	$E_1 \rightarrow H$	$E_2 \rightarrow E_1$	$S \rightarrow E_2 \wedge \neg E_1$
1	$t_1 + \underline{t_6 + t_3 F_3} + t_2 + t_4 + \underline{t_8 F_5 F_9}$	$t + t_{10}$	$F_{11} + F_{10}$	$F_{11} + t_2 F F_{13}$
2	$t_1 + \underline{F_6 + t_3 + t_7 + t_2 + t_4} + F_9 F_5 + t_9$	$2$	$F_{11} + F_{10} F_{13} + F_{12} ? + F_{13} F$	

You only have to check these two because there are only two ways for an XOR to be true. So there are no counterexamples, so the assumption makes the argument valid.

→ truth table:

AQ	A XOR B
+ +	F
+ F	F
F +	F
F F	F

→ So only the way for it to be true.