

③ a. An allocation is possible where both get  $(\frac{7}{2}, \frac{4}{2})$  where both are better off. ~~By contradiction~~

~~No Pareto dominated~~

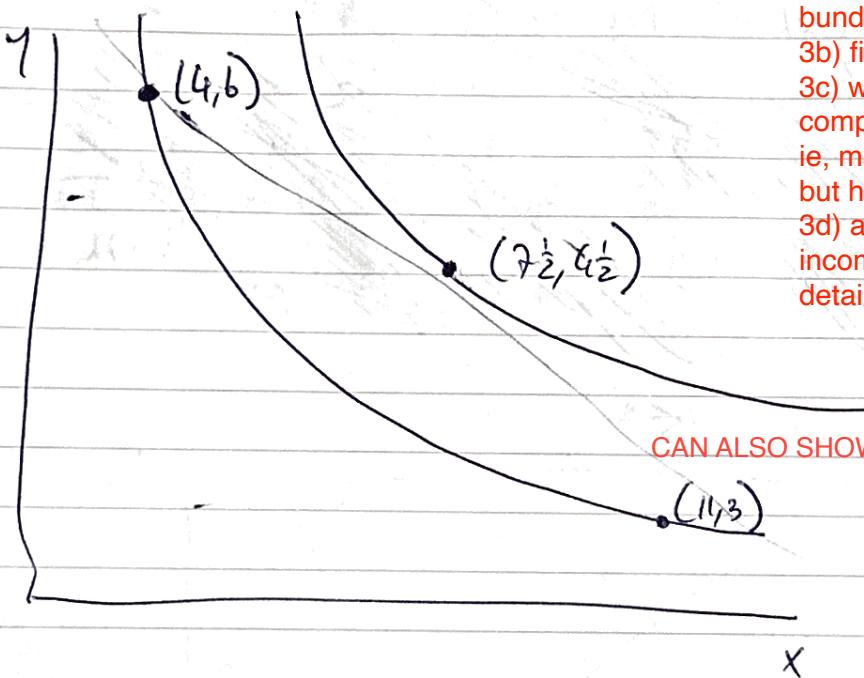
Well-behaved Preferences are continuous, convex, and monotonic. Therefore, ICs are continuous, convex and downward-sloping.

3a) correct, just say that the  $7.5, 4.5$  is the average of two bundles.

3b) fine.

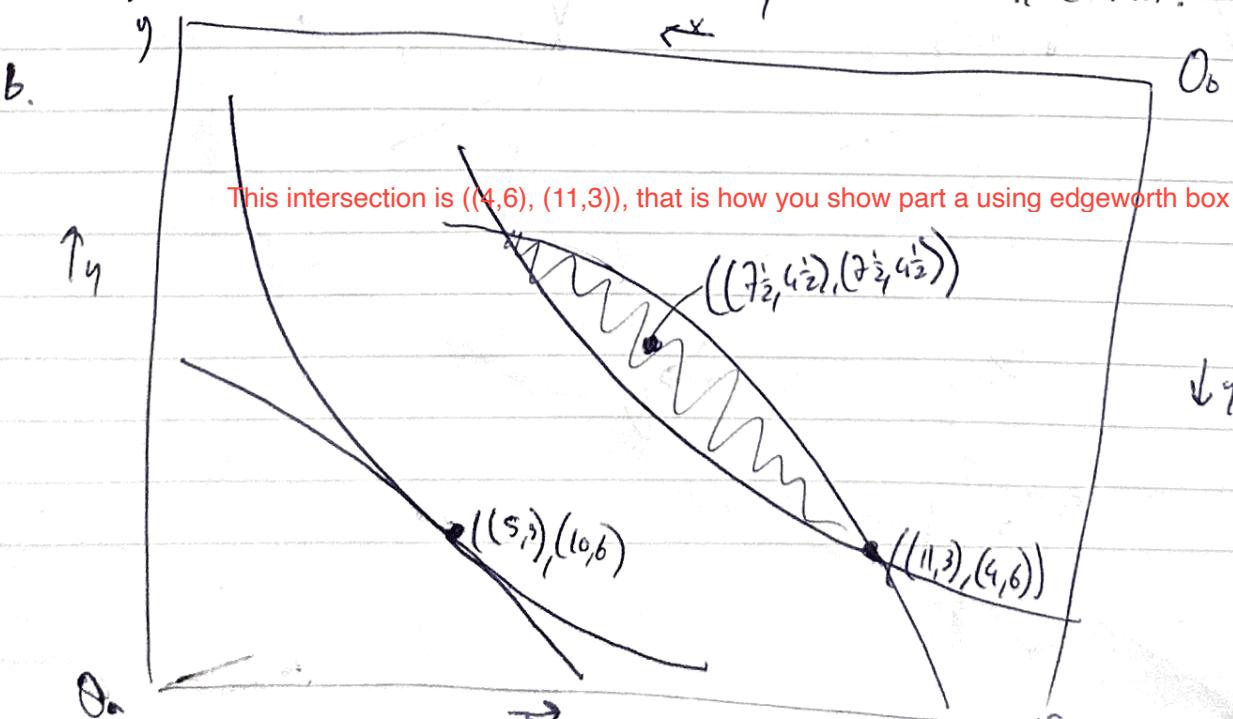
3c) we can't pareto compare these bundles, ie, makes b better off but hurts a.

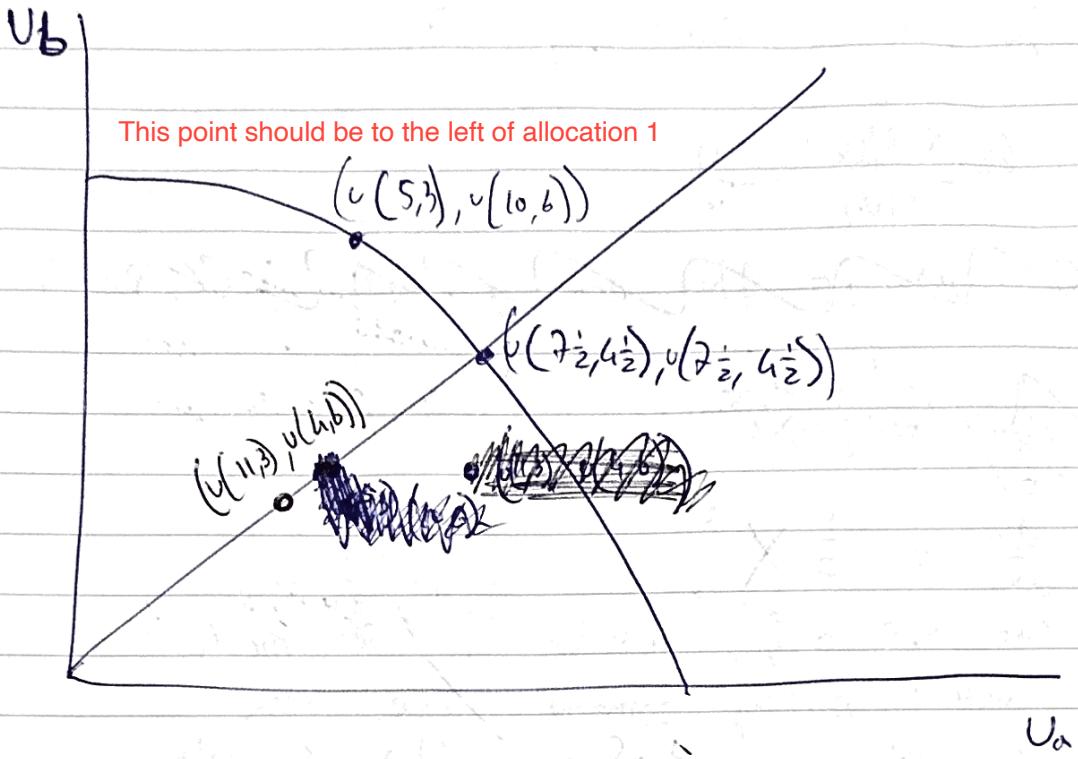
3d) answer is incomplete, not detailed.



CAN ALSO SHOW THIS USING EDGEWORTH BOX

Therefore, both are better off with  $(\frac{7}{2}, \frac{4}{2})$  so allocation 1 is not Pareto-efficient as it gets Pareto-dominated by this allocation.





the UPF is downward sloping and convex because it is the set of Pareto-efficient allocations and an increase in a's utility must lead to a decrease in b's utility.

c. i) allocation 2 is Pareto-efficient and allocation 7 is  
Cannot Pareto compare them, allocation 2 is not a pareto improvement, its a Kaldor Hicks improvement not.

ii) the minimum utility in allocation 2 is  $U(5,3)$ ,  
in allocation 7 the utilities are equal  $U(11,3) = U(4,6)$ .  
By monotonicity:  $U(11,3) > U(5,3)$  so allocation 7 would be better.

iii) For the Utilitarian way, you need cardinal information about the preferences. So this is impossible to compare.

d. None objects to a Pareto-improvement, but it does not capture other ideas such as inequality, even making some potentially worse off to make Does not account for externalities  
Not all bundles can be pareto-compared, there are too many pareto-efficient allocations Rawlsian: you need minimum Utilitarian: you need cardinal Kaldor-Hicks  
Society as a whole better off (Kaldor-Hicks)

(4)

$$a. MU_x = \frac{\partial U}{\partial x} = \frac{1}{x} \quad MU_y = \frac{\partial U}{\partial y} = \frac{1}{y}$$

$$MRS = -\frac{MU_x}{MU_y} = \frac{y}{x}$$

~~the PPF of the economy is  $y = 48 - x^{\frac{1}{2}}$~~

~~$MRT = \frac{dy}{dx}$~~

$$MRT = \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

Define: MRT = Marginal Rate of Transformation = Slope of PPF

- b. At the Pareto-efficiency, nobody can be made better off without making someone worse off.  
As there is only one consumer, that is where their utility is maximized which is where  $MRT = MRS$

$$\frac{1}{2}x^{-\frac{1}{2}} = -\frac{1}{x} \quad (1)$$

~~We also know the budget constraint~~

~~$MRT = \frac{dy}{dx} = 48 - x^{\frac{1}{2}}$~~

$$y = (48 - x)^{\frac{1}{2}}$$

Substituting that into (1):

$$\frac{1}{2}x^{-\frac{1}{2}} = -\frac{1}{(48 - x)^{\frac{1}{2}}}$$

$$\frac{1}{2}x = 48 - x$$

$$x =$$

We also know that

$$X = 48 - \frac{1}{8}Y \quad \text{By } MRS = MRT$$

So Substituting that in:

$$\frac{1}{2}(48-X)^{-\frac{1}{2}} = -\frac{1}{8} \frac{Y}{X}$$

$$4(48-X) = \cancel{MRS}$$

Plugging in  $X=32$  &  $Y=4$  works so it is.  
Efficient according to the tangency condition  
of  $MPS = MRT$

### c. the Budget Line is Tangent

The Pareto-efficient allocation is part of a competitive equilibrium when the budget line is tangent to the MRS/MRT.

for  $X=32, Y=4$  that means that  
the slope of the budget line is:

$$-\frac{P_X}{P_Y} = \cancel{MRS} = -\frac{1}{32}$$

$$\frac{P_X}{P_Y} = \frac{1}{32}$$

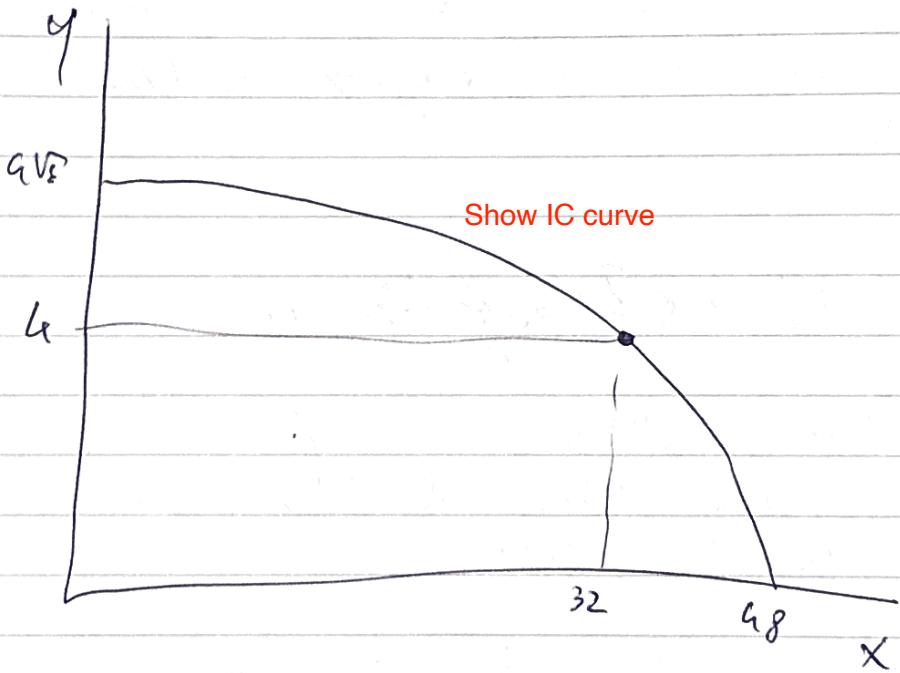
the profits are: (If we set  $P_x = 7$ )  
and  $P_y = 8$

$$\Pi = 8y - 7x(48 - \frac{x}{7}) = 8y + 560 - 16x = 32 - 16 = 16$$

The consumer budget constraint is satisfied:

$$48 \cdot 1 + 16 = P_x x + P_y y = 7 \cdot 32 + 8 \cdot 4 = 64$$

d. PPF:  $y = (48-x)^{\frac{1}{2}}$



(5)

a. the set of all Pareto-efficient allocations is ~~where~~ where the ~~Max~~ MRSs are equal: (Contract-curve)

$$MRS_a = MRS_b \quad (1)$$

$$MRS_a = -\frac{MU_1^a}{MU_2^a} = -\frac{\partial U/\partial x_1^a}{\partial U/\partial x_2^a} = -\frac{2/x_1^a}{3/x_2^a} = -\frac{2x_2^a}{3x_1^a}$$

$$MRS_b = -\frac{MU_1^b}{MU_2^b} = -\frac{\partial U/\partial x_1^b}{\partial U/\partial x_2^b} = -\frac{2/x_1^b}{1/x_2^b} = -\frac{2x_2^b}{x_1^b}$$

plugging this into (1):

$$-\frac{2}{3} \frac{x_2^a}{x_1^a} = -\frac{2x_2^b}{x_1^b} \quad (2)$$

we also know that  $x_1^a + x_1^b = 20 \Rightarrow x_1^b = 20 - x_1^a$   
 and  $x_2^a + x_2^b = 12 \Rightarrow x_2^b = 12 - x_2^a$

Substituting these in (2):

$$-\frac{2}{3} \frac{x_2^a}{x_1^a} = -\frac{2(12-x_2^a)}{20-x_1^a} =$$

$$\frac{2x_2^a}{3x_1^a} = \frac{24 - 2x_2^a}{20 - x_1^a}$$

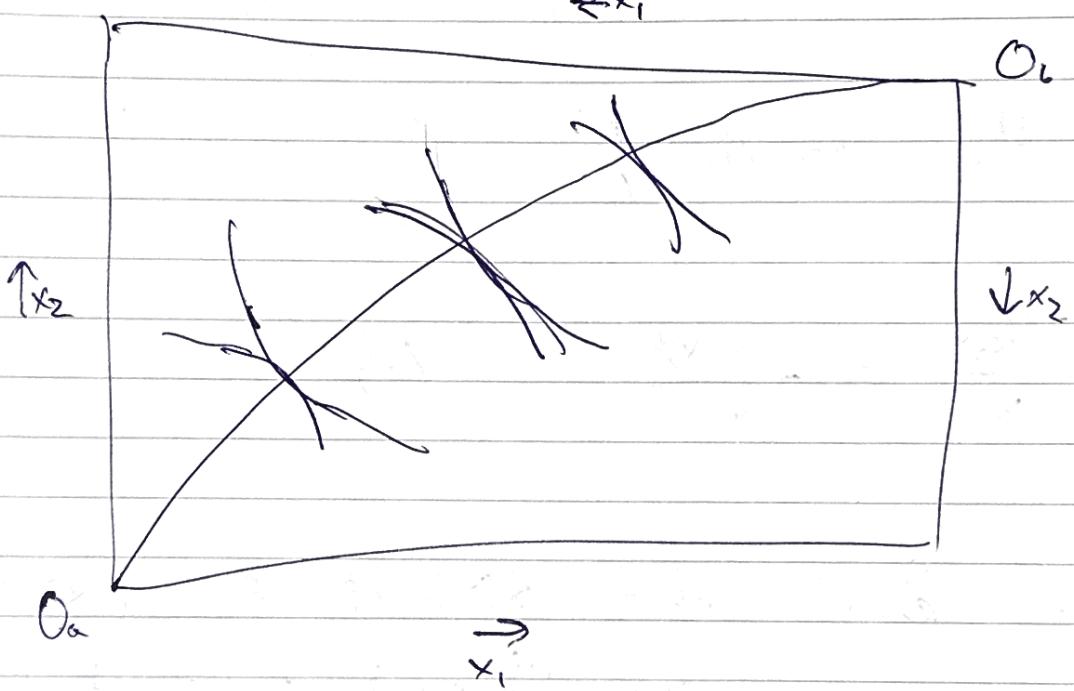
$$2x_2^a(20 - x_1^a) = 3x_1^a(24 - 2x_2^a)$$

$$40x_2^a - 2x_2^a x_1^a = 72x_1^a - 6x_2^a x_1^a$$

~~$$40x_2^a + 4x_2^a x_1^a = 72x_1^a$$~~

$$x_2^a(40 + 4x_1^a) = 72x_1^a$$

$$x_2^a = \frac{72x_1^a}{40 + 4x_1^a} = \frac{18x_1^a}{x_1^a + 10}$$



b. Consumer a: (Cardinal just used Cobb-Douglas Standard)  
 (normalizing  $P_2 = 1$ )

$$\max U = 2 \ln(x_1^a) + 3 \ln(x_2^a) \quad \text{S.t. } P_1 x_1^a + x_2^a \leq 20 P_1$$

$$L = 2 \ln(x_1^a) + 3 \ln(x_2^a) - \lambda(P_1 x_1^a + x_2^a - 20P_1)$$

$$\frac{\partial L}{\partial x_1^a} = \frac{2}{x_1^a} - \lambda P_1 = 0 \quad (1) \Rightarrow \lambda = \frac{2}{x_1^a P_1} \quad \text{so } \lambda > 0 \text{ if } x_1^a > 0 \text{ and } P_1 > 0$$

$$\frac{\partial L}{\partial x_2^a} = \frac{3}{x_2^a} - \lambda = 0 \quad (2) \quad \text{so } \frac{\partial L}{\partial \lambda} = 0 \quad (3)$$

Using (1) & (2):

$$\frac{2x_2^a}{3x_1^a} = P_1 \Rightarrow 2x_2^a = 3P_1 x_1^a \quad (4)$$

Using (3):

$$\frac{\partial L}{\partial \lambda} = -P_1 x_1^a - x_2^a + 20P_1 = 0$$

$$P_1 x_1^a = 20P_1 - x_2^a$$

~~Eqn 1 & Eqn 2~~ Substituting (4):

$$2x_2^a = 3(20P_1 - x_2^a)$$

$$2x_2^a = 60P_1 - 3x_2^a$$

$$5x_2^a = 60P_1$$

$$x_2^a = 12P_1$$

and substituting into (1):

$$2 \cdot 12P_1 = 3P_1 X_1^a$$

$$X_1^a = 8$$

Consider B:

$$\max U = 2\ln(X_1^b) + \ln(X_2^b) \quad \text{s.t. } P_1 X_1^b + X_2^b \leq 12$$

$$L = \ln(X_1^b) + \ln(X_2^b) - \lambda(P_1 X_1^b + X_2^b - 12)$$

$$\frac{\partial L}{\partial X_1^b} = \frac{2}{X_1^b} - \lambda P_1 = 0 \quad (1) \Rightarrow \lambda > 0 \Rightarrow \frac{\partial L}{\partial X_2^b} = 0 \quad (2)$$

$$\frac{\partial L}{\partial X_2^b} = \frac{1}{X_2^b} - \lambda = 0 \quad (2)$$

(1) & (2):

$$\frac{2X_2^b}{X_1^b} = P_1$$

$$2X_2^b = P_1 X_1^b \quad (4)$$

From (3):

$$P_1 X_1^b + X_2^b = 12$$

$$P_1 X_1^b = 12 - X_2^b$$

$$X_2^a = 12 - x$$

$$4P_1 = R(X_2^a - x)^2$$

using (a):

$$2X_2^b = 12 - X_2^a$$

$$3X_2^b = 12$$

$$\textcircled{X_2^b = 4}$$

and using (a):

$$2 \cdot 4 = P_1 X_1^b$$

$$\textcircled{X_1^b = \frac{8}{P_1}}$$

$$Z_1 = X_1^a - W_1^a + X_1^b - W_1^b = 8 - 20 + \frac{8}{P_1} - 0 = \frac{8}{P_1} - 12$$

In Competitive Equilibrium, aggregate ex. excess demand is 0:

$$\frac{8}{P_1} - 12 = 0$$

$$\textcircled{P_1 = \frac{2}{3}}$$

$$\text{Allocation a: } (8, 8)$$

$$\text{ " b: } (12, 4)$$

Thus allocation lies on the contract curve!

$$x_2^a = \frac{18x_1^a}{x_1^a + 10}$$

$$8 = \frac{18 \cdot 8}{8+10} = 8$$

So it is Pareto Efficient

as  $MRS_{a,b} = MRS_{b,a}$  so there are no gains from trade.

However, By welfare theorem 1, every competitive equilibrium is P-efficient  
(do you want me to give the proof?)

c.

$$x_2^a = \frac{18x_1^a}{x_1^a + 10}$$

$$6 = \frac{18 \cdot 5}{5+10} = 6 \quad \text{So P.S. on the Contract curve.}$$

By welfare there 2 it can be, the slope of the budget line is equal to the  $MRS_{a,b}$ :

WRONG way around  
should be  $-p_1/p_2$

$$-\frac{P_2}{P_1} = MRS_{a,b} = -\frac{2x_2^a}{3x_1^a} = -\frac{2}{3} \cdot \frac{6}{5} = -\frac{4}{5}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{5}{4}$$

Plugged in the wrong allocation here and got p1 and p2 wrong way around

d. The transfer should be to a point on the budget line ~~for a~~ more is  
(with  $p_1 = 5$  &  $p_2 = 4$ )

$$x_2^a = \frac{20 - 5}{4} x_1^a = \frac{20}{4} - \frac{5}{4} x_1^a$$

so She should transfer good 1 until ~~she~~

~~part of the budget~~ She is on that line

$(20, 0)$  is already on that line.

Right answer:

Budget line  $p_1 = 4$   $p_2 = 5$

$$m = 5 * p_1 + 6 * p_2 = 50$$

$$50 = p_1 x_1 + p_2 x_2$$

$$p_2 x_2 = 50 - p_1 x_1$$

$$x_2 = 10 - \frac{4}{5} x_1$$

for consumer a:

plug in  $x_2 = 0$  and then you find  $x_1 = 12.5$

so you redistribute 7.5 units of  $x_1$

Also look at question 6!