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MICROECONOMICS

COLLECTION PAPER 2026

Time allowed: THREE hours.

There are TEN questions in this paper.

Answer ALL Part A questions and TWO Part B questions.

Part A attracts 40% of the total marks. Each question in Part A has the same weight.
Each question in Part B attracts 30% of the total marks.

For multipart questions, the weight assigned to each part is indicated in brackets.

Candidates may use a calculator as specified by the Department of Economics.

Part A

1. In an exchange economy Andrea and Bee each have the same quasi-linear utility function over goods x and y. Denoting Andrea's consumption bundle by $\{x_a, y_a\}$ and Bee's by $\{x_b, y_b\}$, the utility function is

$$u_i(x_i, y_i) = \ln x_i + y_i \qquad\qquad i = a, b.$$

Andrea is endowed with 4 units of x and 6 units of y. Bee's endowment is zero units of x and 4 units of y.

Let the price of y be 1, so the relative price of x is p.

(a) Compute the demand functions for each of the goods and show that Walras' Law holds in this economy. [50%]

(b) Compute the competitive equilibrium, i.e. the equilibrium value of p and the resulting allocation of each of the goods. Explain why the competitive equilibrium allocation is Pareto efficient. [30%]

(c) The social planner, Professor Rawls, wants Andrea and Bee to have equal utilities and prefers more utility to less utility. He can transfer good y between the agents but not good x, and he is willing to let the agents trade freely with each other once he has determined the new endowments. What would the new endowments be to implement Rawls' plan? [20%]

$m = P_x x + P_y y$

$y = m - \frac{P_x}{P_y} x$

2. Alicia owns a firm that generates pollution in amount x. Her profit is $P(x) = Ax - \frac{x^2}{2}$, where $A > 0$ is a parameter. Alicia's total utility is given by profit plus any other income.

Bob suffers from the pollution. His direct loss from the pollution is $L(x) = Bx + \frac{b}{2}x^2$ where $B > 0$ and $b \geq 0$ are parameters. Bob's total utility is given by income minus the loss from pollution. Assume that $A > B$.

(a) Without computing the optimal amount of pollution, explain first why it is Pareto superior to have a positive amount of pollution rather than zero. [30%]

(b) Compute the Pareto efficient amount of pollution, x^*, as a function of the parameters. Show that the tax that induces this efficient amount of pollution is $t^* = \frac{B+bA}{1+b}$. [50%]

(c) Suppose that the government (i) does not know the value of A in Alicia's profit function, and (ii) knows that $b = 0$ in Bob's loss function. Explain why the tax described in (b) is optimal whatever the value of A. [20%]

$f = e^t - 1 \qquad g = (e^t + 1)^{-1}$

$u = e^t + 1 \qquad y = u^{-1}$

$f' = e^t$

$u' = e^t \qquad y' = -u^{-2}$

$g' = -e^t(e^t + 1)^{-2}$

$$f'g + f \cdot g' = \frac{e^t}{e^t + 1} - \frac{e^t(e^t - 1)}{(e^t + 1)^2}$$

$$= \frac{e^t(e^t + 1)}{(e^t + 1)^2} - \frac{e^t(e^t - 1)}{(e^t + 1)^2}$$

$$= \frac{(e^t + 1 - e^t + 1)e^t}{(e^t + 1)^2}$$

$$= \frac{2e^t}{(e^t + 1)^2}$$

3. Two firms, A and B, sell identical goods in a market. The market demand function is $Q(p) = 10 - p$ when the buying price is $p \leq 10$. Prices are in pounds (£). Denote the announced prices of the firms by p_A and p_B. If a firm sets a lower price than the other then it takes all the demand at its announced price. If the prices are equal then they share equally the demand at the commonly announced price. Each firm has a constant marginal cost of 6 and has no fixed cost. Firms' payoffs equal their profits. The firms choose their prices simultaneously and non-cooperatively.

(a) Suppose that A expects B to set $p_B = 5$. For each of the following prices of A state whether or not it is a best-response to $p_B = 5$ and explain your reasons:

(i) $p_A = 4$;

(ii) $p_A = 5$;

(iii) $p_A = 5.5$;

(iv) $p_A = 6$. [40%]

(b) Show that there is a unique Nash equilibrium in pure strategies, and describe it. [25%]

(c) Suppose now that the firms play the game an infinite number of times. Denote the common discount factor by δ, which satisfies $0 < \delta < 1$; this is the weight that a firm puts on profits one period ahead compared to profit now. Describe a pair of strategies that form a subgame perfect equilibrium with collusion at the monopoly price, as long as the discount factor is large enough, and calculate the critical value of the discount factor. [35%]

4. Consider five used cars: two of these cars are of high quality (H), one is of medium quality (M) and two are of low quality (L). All current owners have a value for these cars equal to $s_H = 60$, $s_M = 30$, and $s_L = 15$.

There is a large number of potential buyers interested in acquiring these cars. All potential buyers have a value for these cars equal to $b_H = 60 + k$, $b_M = 30 + k$, and $b_L = 15 + k$, where k is a strictly positive constant; that is, buyers value the cars more than current owners. When needed, assume that utility is linear in money.

Current owners know the quality of their car, while potential buyers do not (they only know the distribution).

(a) Show that, whenever $k \geq 24$, all cars may be traded in the market. What would be the trading price? [40%]

(b) For the case of $k \geq 24$, is the market outcome efficient? If your answer is positive, explain why this can happen despite the information asymmetry in the market; if your answer is negative, explain the role played by the information asymmetry in this inefficiency. [20%]

(c) What would be the market outcome (cars traded and price) when $k = 20$? And when $k = 5$? Are these outcomes efficient? [40%]

Part B

5. Consider a country in which changes are implemented only whenever there is consensus. It is currently evaluating whether to implement a free-trade policy. Under what conditions would everyone in the economy benefit from free trade? In practice, is it likely that this country would implement the policy?

6. Using a model of public goods, explain why decentralized/private provision would lead to inefficient levels of provision. Select one of the two mechanisms below, explain formally how it may ensure optimal provision, and discuss its practicality:

   (a) The Lindahl scheme of personalized prices

   (b) The Clarke-Groves scheme.

7. Under what conditions would you expect the entry of a new firm in a market to lead to a reduction of the market price (or prices)? Does it follow that mergers between existing firms should be banned?

8. Represent all possible health levels by $[0, 1]$, with 0 representing death (or the worst possible health level), and 1 representing perfect health. Consider a patient that may or may not be an expected utility maximiser. When multiple treatments exist, explain why a doctor should explain the probabilistic consequences of each treatment, instead of implementing what she considers is the best treatment. Would your answer change if you knew that the patient is an expected utility maximiser? And would your answer change if you also knew that there are only two treatments, one of them a modern version of the other leading to first-order stochastic dominance?

9. A large risk-neutral college in Oxford is planning to hire a risk-averse real estate agent to sell one of their properties. Explain why they would agree on a contract in which the real estate agent fee depends, at least partially, on the selling price. After considering an optimal contract for the college in which high effort is planned, and other things being equal, suppose that the reservation utility or the risk aversion level of the real estate agent increase. Discuss the effects on the individual rationality and the incentive compatibility constraints. How should the optimal contract be amended?

(1) ~~...~~ imperfect competition (unilateral effects)
— differentiated goods, diversion effect

(2) if there was collusion, it makes collusion more difficult (coordinated effects)

(3) No should not be banned:

— vertical mergers

— higher efficiency (use graph)

$t = 2e^t \qquad n = (e^t+1)^2$

$t' = 2e^t \qquad n' = 2e^t(e^t+1) = 2e^{2t} + 2e^t$

10. Ace is planning to subscribe to an online tennis channel called Wimbledon Whims. There is a bonus available to those who subscribe: a random number of tickets for a big tournament. The channel offers Ace the following opportunities as a bonus.

- In lottery A, Ace would receive $3, 2, 1$ or 0 tickets, with probabilities $\frac{1}{10}$, $\frac{1}{10}$, $\frac{5}{10}$ and $\frac{3}{10}$, respectively.
- In lottery B, Ace would receive $3, 1$ or 0 tickets, with probabilities $\frac{3}{14}$, $\frac{7}{14}$ and $\frac{4}{14}$, respectively.
- In lottery C, Ace would simply receive 1 ticket with probability 1.

We know that, in deciding over such lotteries, Ace is an expected utility maximiser. He uses the following strictly increasing utility function over the number of tickets t, $u(t) = \frac{2^t-1}{2^t+1}$.

$(e^t+1)(e^t-1)$

$e^{2t}-1$

(a) Draw or describe the cumulative distribution functions that correspond to lotteries A and B. Without the need of further computations, do you know whether Ace will prefer any of these two lotteries? (Hint: for your graph or description, simply use the fact that $\frac{4}{14} < \frac{3}{10} < \frac{11}{14} < \frac{8}{10}$). [20%]

(b) Compute Ace's expected utility of lotteries B and C and determine their ranking. What is the certainty equivalent of lottery B? [15%]

Spin is another potential subscriber to the channel. Spin is also an expected utility maximiser with utility function over tickets equal to $v(t) = \frac{e^t-1}{e^t+1}$. In what follows, you can treat t as a continuous variable.

(c) Compute the first and second derivatives of $v(t)$. Is Spin risk averse? Without further computations, do you know Spin's ranking of lotteries A and C? [20%]

(d) Show that Spin's absolute risk aversion coefficient equals $v(t)$. [10%]

(e) How does Spin's absolute risk aversion coefficient change with the number of tickets? Compare Spin's preference to that described by a utility function with Constant Absolute Risk Aversion (CARA) and a utility function with Constant Relative Risk Aversion (CRRA). Suppose that Spin prefers lottery C to lottery B. Consider lotteries $C+$ and $B+$ that are equal to C and B except for a guaranteed additional ticket in all cases. How would Spin rank lotteries $C+$ and $B+$? [20%]

(f) Consider now lottery BD (respectively CD), where there is a fifty-fifty chance of receiving lottery B and lottery D (respectively, a fifty-fifty chance of receiving lottery C and lottery D). Use these lotteries to describe the property of independence. Show that (Ace's, Spin's or any other) expected utility satisfies this property. (Hint: You should mention the reduction property in your discussion.) [15%]

$t = e^t - 1 \qquad n = e^t + 1 \qquad \frac{t' \cdot n - n' \cdot t}{n^2} = \frac{e^{2t} + e^t - e^{2t} + e^t}{(e^t+1)^2}$

$t' = e^t \qquad n' = e^t$

1/5

MERTON COLLEGE

COLLECTION PAPER

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HILLARY	MICROECONOMICS

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TAHEYA TARANNUN

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1) a. $u_i = \ln(x_i) + y_i$ $\quad W_a = \{4, 6\}$ $m_a = 4P + 6$

$W_b = \{0, 4\}$ $m_b = 4$

Both are solving the following problem:

$$\max_{x_i, y_i} \; u_i = \ln(x_i) + y_i \quad \text{s.t. } Px_i + y_i \leq m_i$$

$$\mathcal{L} = \ln(x_i) + y_i - \lambda(Px_i + y_i - m)$$

FOC:

$$\frac{\partial \mathcal{L}}{\partial y_i} = 1 - \lambda = 0 \implies \lambda = 1 \quad (1)$$

So by complementary slackness condition, the BC is binding

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{1}{x_i} - \lambda p = 0$$ By (1) $\lambda = 1$

$$\frac{1}{x_i} = p$$

$$x_i = \frac{1}{p}$$

Plugging into the BC (which ~~Hold~~ Binds as I said)

$$p \cdot \frac{1}{p} + y_i = m_i$$

$$y_i = m_i - 1$$

But if ~~~~ $m_i < 1$, $y_i = 0$ and $x_i = \frac{m_i}{p}$

So for them individually:

$m_a = 4p + 6$ $\quad m_b = 4$ $\quad$ (Both are > 1 given $p \geq 0$)

$$x_a = \frac{1}{p} \qquad y_a = 4p + 6 - 1 = 4p + 5$$

$$x_b = \frac{1}{p} \qquad y_b = 4 - 1 = 3$$

aggregate excess demands are:

$$z_x = \frac{1}{p} + \frac{1}{p} - 4 - 0 = \frac{2}{p} - 4$$

$$z_y = 4p + 5 + 3 - 6 - 4 = 4p - 2$$

Walras' Law states that the value of the aggregate excess demand is zero for all p.

$$p z_x + 1 z_y \equiv 0$$

~~Prew~~

$p\left(\frac{2}{p}-4\right) + 4p-2$

$= 2 - \cancel{4p} + \cancel{4p} - \cancel{2} = 0$

So it holds

B. In competitive equilibrium, aggregate excess demand are zero:

$z_x = \frac{2}{p} - 4 = 0$

$\frac{p}{2} = \frac{1}{4}$

$p = \frac{1}{2}$

By walras law, z_y is now also zero

allocations:

$x_a = \frac{1}{1/2} = 2$ $\qquad y_a = 4 \cdot \frac{1}{2} + 5 = 7$

$x_b = 2$ $\qquad y_b = 3$

By the First welfare theorem, any competitive equilibrium is Pareto-efficient.

MRS's are equal (= can't make anyone better off (w/o hurting others) with trade) + resources are all used up

C. ~~Since this is quasilinear utility~~

Since this is quasilinear utility, both Andrea and Bea will want to consume 2 units of x, regardless of their consumption of y. Then to maximize the equal utilities, $y_a = y_b = \frac{10}{2} = 5$

~~So from the allocation to~~

~~For~~ to get to that from the original allocation he needs to get to a point on the budget line which is tangent to the MRSs of Andrea and Bea:

$$MRS_a = \frac{\frac{\partial U_a}{\partial x_a}}{\partial U_a / \partial y_a} = -\frac{\left(\frac{1}{x_a}\right)}{1} = -\frac{1}{x_a} = -\frac{1}{2}$$

So the slope of the Budget line

$$-\frac{P_x}{P_y} = -p = -\frac{1}{2} \quad \text{so} \quad p = \frac{1}{2}$$

So the Budget line is

$$m = px + y = \frac{1}{2} \cdot 2 + 5 = 6$$

So $m_a = m_b = 6$

$$m_a = 4 \cdot \frac{1}{2} + w_a^y = 6$$

$$2 + w_a^y = 6$$

$$w_a^y = 4$$

So he needs to redistribute 2 units of y from Andrea to Bea.

New allocations are $(4, 4)$

$(0, 6)$

2/5

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~~[illegible]~~	

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2) a. $P(x) = Ax - \frac{x^2}{2}$ $L(x) = Bx + \frac{b}{2}x^2$

~~If then~~

$\frac{dP}{dx} = A - x$ $\frac{dL}{dx} = B + bx$

When pollution is $x = 0$ $\frac{dP}{dx} = A$ and $\frac{dL}{dx} = B$

Since $A > B$, going from zero to some pollution is superior as the profits outweigh the costs, now Bob is worse off so for it to be Pareto superior Alicia could just give Bob some money and they would both be better off.

B. to find the Pareto-efficient amount of Pollution, we can merge the agents to internalize the externality:

$$\max_{x} U(x) = Ax - \frac{x^2}{2} - Bx - \frac{b}{2}x^2$$

FOC:

$$\frac{dU}{dx} = A - x - B - bx = 0$$

$$x + bx = A - B$$

$$(1+b)x = A - B$$

$$x^* = \frac{A-B}{1+b}$$

Now we want Alicia to induce this amount of Pollution By taxing her, She is doing the following Problem

$$\max_{x} P(x) = Ax - \frac{x^2}{2} - tx$$

FOC

$$\frac{dP}{dx} = A - x - t = 0 \qquad x^* = \frac{A-B}{1+b}$$

$$t^* = A - \frac{A-B}{1+b}$$

$$t^* = \frac{A+Ab}{1+b} - \frac{A-B}{1+b} = \frac{B+bA}{1+b}$$

C. Since b is 0, the optimal tax is no longer dependent on the value of A:

$$t^* = \frac{B+0A}{1+0} = \frac{B}{1} = B$$

the optimal tax is only dependent on B, So whatever the value of A, the tax is

3/5

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3 a. I) not a best response, although A would get all the demand, $P_A < MC$ so it would make a ~~the~~ loss.

II) not a best response, now they would share demand equally, But $P_A < MC$ so A would make a loss.

III) it is a best response, A would get no demand and would get profit $\pi = 0$

III) same, But now $P_A = MC$ ~~[illegible]~~ But still no demand

B. A Nash equilibrium is where no actor could become better off By unilaterally deviating. In this case they would set prices

$$P_A = P_B = MC = 6$$

- if firm A or B would unilaterally deviate and lower its price $P < MC$ so they would make a loss and

Be worse off, even though they would get all the demand.
- if one of them would unilaterally deviate and increase Prices they would get no demand and Be just as well off.

NB:- If ~~....~~ they Both set a higher Price $P_A = P_B > 6$, one of them would undercut and get all profit.
- if they would Both set a lower Price:

$P_A = P_B < 6$ ~~they both~~ one of them could Be better off By increasing the Price.

c. If they collude, to maximize profits they would set the monopoly Price:

$$\max_{P} \pi_M = P \cdot Q(P) - 6\,Q(P) = (P-6)\,Q(P)$$

$$= (P-6)(10-P)$$

$$= 10P - P^2 - 60 + 6P$$

$$= 16P - P^2 - 60$$

FOC:

$$\frac{d\pi_M}{dP} = 16 - 2P = 0$$

$$P_M = 8$$

$$\pi_M = (8-6)(10-8) = 4$$

they would have to Share the monopoly Profit:

$$\frac{\pi_M}{2} = 2$$

A Pair of Strategies ~~for~~ for SPE with collusive Would be:

Start ~~by~~ By colluding ($P = P_M$) and if any Firm deviates ($P < P_M$) Punish them ($P = c$) Forever.

Profits from collusion would be:

$$\frac{\pi_M}{2} + \frac{\pi_M}{2}\delta + \frac{\pi_M}{2}\delta^2 + \overset{=2}{\frac{\pi_M}{2}}\delta^3 = \frac{2}{1-\delta}$$

Profits from defection would be (Punishment Profits are 0 since $P = MC$)

$$\pi_M + 0 + 0 + 0 + 0 = 4$$

So the critical value of the discount factor is:

$$\frac{2}{1-\delta} = 4$$

$$2 = 4 - 4\delta$$

$$\delta \geq \frac{1}{2}$$

~~Strategy~~? (I missed it above)

4/5

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4 a. the expected value of the cars for the Buyers is:

~~E(Car)=~~ $E(b) = \frac{2}{5}(60+k) + \frac{1}{5}(30+k) + \frac{2}{5}(15+k)$

$= 24+6+6+k = 36+k$

if $k \geq 24$ $E(b) \geq 60$, ~~so every seller would sell~~ so ~~the~~ the Buyers would Be willing to Pay more than 60 so every seller would sell their Car, so ~~the Buyers could just offer 60 so this would be the trading price~~ the trading Price would Be Between 60 and 60+k depending on the Specific circumstances.

Expected value is 36+K

B. Yes the market outcome is efficient, the Buyers get all the cars, which they value more highly than the sellers. the reason for this is that the Buyers value the cars so much more that even though they do not ~~fat~~ know the exact value of the cars, they value the average car higher or equal to the value that the sellers give to the highest quality car.

C. ~~at k~~ $k=20$:

$E(b) = 36 + k = 56$, & at that price, the sellers would not sell so only medium and low quality cars would be left so:

$E(b) = \frac{1}{3} \cdot 30 + \frac{2}{3} \cdot 18 + \cancel{k} = 40$

at that price the sellers would sell their medium and low quality cars and the price would be somewhere between 30 and 40.

$k=5$:

$E(b) = 36 + k = 41$ so the sellers would not sell high quality so:

$E(b) = 20 + k = 25$ But then the sellers would not sell medium quality cars so only the low quality cars would remain:

$E(b) = 15 + k = 20$ so low quality cars would be traded at a price between 15 and ~~#~~ 20

these outcomes are not efficient since

Both Buyer and seller would be better off if all cars were traded.

essay

7) Under what conditions would you expect the entry of a new firm in a market to lead to a reduction of the market price (or prices)? Does it follow that mergers between existing firms should be banned?

Plan:

- imperfect competition (unilateral effects)
  ↳ differentiated goods → diversion effect
- ~~...~~ entry makes collusion more difficult (coordinated effect) (Christie & Sotheby, White Salt)
- mergers should not be banned
  ↳ vertical mergers, eliminate double margins (AT&T & time warner)
  ↳ higher efficiency (graph, Exxon Mobil)

Essay:

In this essay, I argue that under two conditions, prior imperfect competition and prior collusion, entry of a new firm in a market will lead to a fall in prices. I will use a Cournot model. ~~and~~ then I argue that this does not imply that mergers should be banned as this should be a tradeoff between ~~...~~ market power and efficiency gains for horizontal mergers, and can eliminate double margins for vertical mergers.

If there is prior imperfect competition, the entry of a new firm ~~...~~ can increase competition and lower prices. To show this, I ~~...~~ use a Cournot model with ~~...~~ n initial firms (duopoly) and I will briefly comment on the case with goods that are imperfect substitutes.

~~With initially two identical firms with ...~~
~~the firms are optimizing:~~

~~$\max_{q_i} \pi_i = P(q_i, Q_{-i}) q_i - 0 = q_i A - q_i^2 - q_i Q_{-i}$~~ ~~$Q_{-i}$ is the output of all other firms~~

~~FOC:~~

~~$$\frac{d\pi_i}{dq_i} = A - 2q_i - Q_{-i} = 0$$~~

~~$$q_i = \frac{A - Q_{-i}}{2}$$~~

~~$$P = A - \left(\frac{A - Q_{-i}}{2}\right) - Q_{-i}$$~~

No time to prove in model but with increasing n, price falls and so the entry of a new firm decreases the price.
For imperfect substitute goods, because of the opposite diversion effect (when firm A increases their price, firm B will gain some of the lost demand), the prices will also fall if another firm enters. (unilateral effects)

If there is already collusion, an additional firm will mean that the discount factor will have to be higher $\delta = 1 - \frac{1}{n}$ (no time to prove in Bertrand model) so it makes it more difficult to sustain collusion. For instance, in the Christie and Sotheby case it was very easy to sustain collusion because there were only 2 firms.
Moreover, other factors can also make collusion more challenging. For instance, the new firm might not be perfectly symmetric, and as we saw in the White Salt case, that makes it harder to sustain collusion.

Because collusion is no longer possible, prices will fall.

~~__~~ If one sees ~~the~~ a merger simply as the exit of a firm, then this ~~__~~ story is opposite to what I just outlined: it reduces competition (unilateral) and makes collusion easier (coordinated effect).

— No time for efficiency and vertical merger points.

5/5

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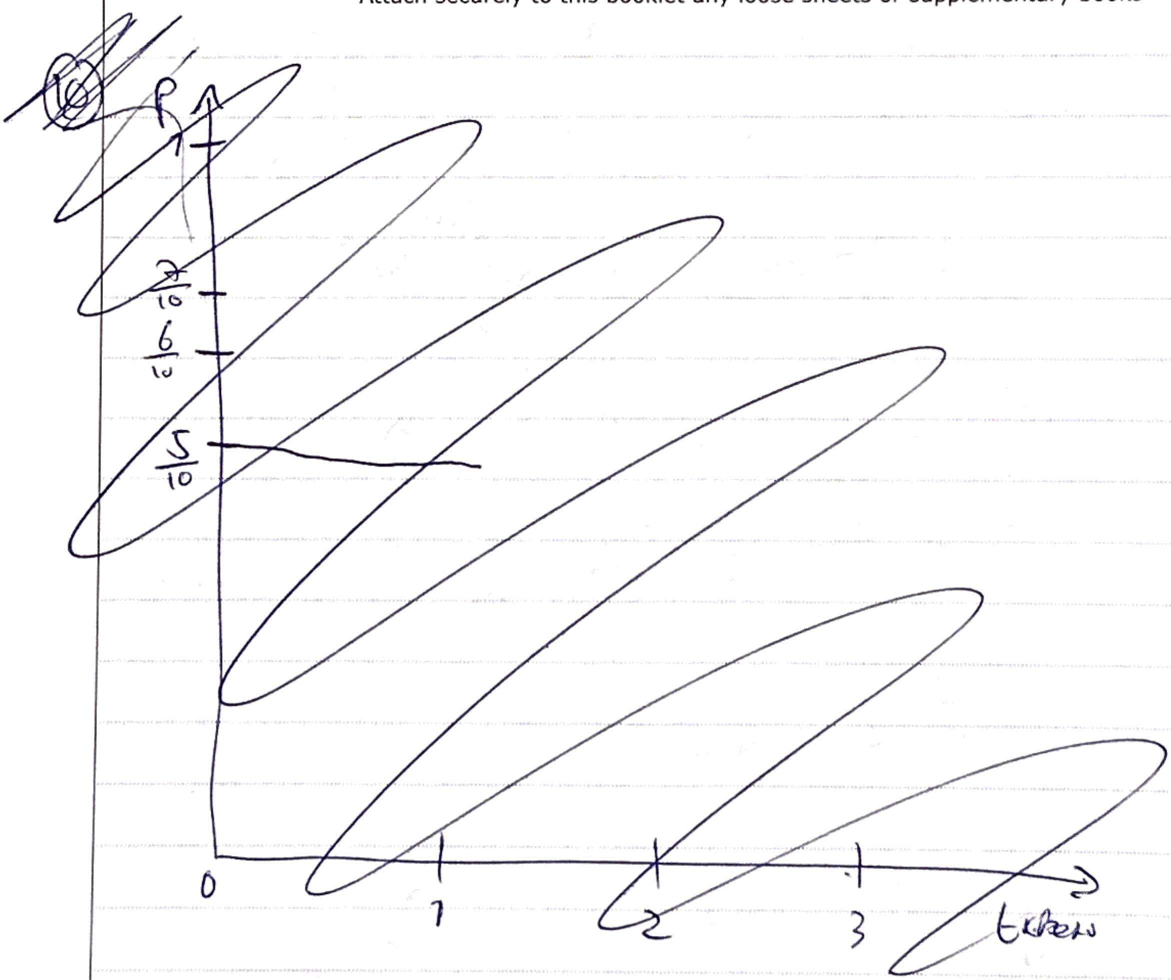
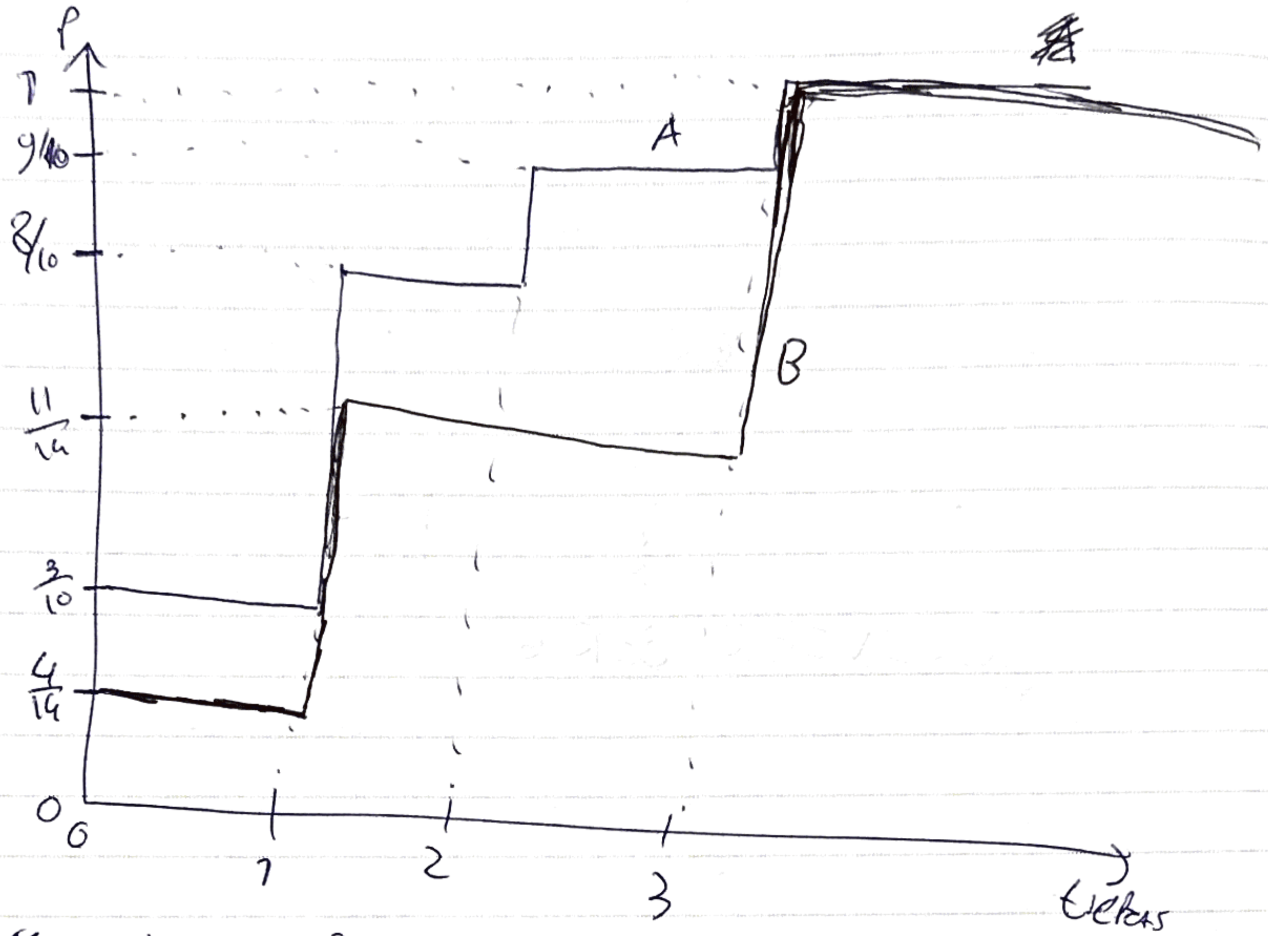


(10) a



~~Ace~~ Lottery B is every where to the right ~~of~~ ~~B~~ and below lottery A, so Ace prefer B over A by FOSD

$B \succ_{FOSD} A$

meaning?

B. $EU(B) = \frac{4}{14} u(0) + \frac{7}{14} u(1) + \frac{3}{14} u(3)$ $\quad u(t) = \frac{2^t - 1}{2^t + 1}$

$= \frac{4}{14} \cdot \frac{2^0 - 1}{2^0 + 1} + \frac{7}{14} \cdot \frac{2^1 - 1}{2^1 + 1} + \frac{3}{14} \cdot \frac{2^3 - 1}{2^3 + 1}$

$= \cancel{\frac{4}{14} \cdot \frac{0}{2}} + \frac{7}{14} \cdot \frac{1}{3} + \frac{3}{14} \cdot \frac{7}{9} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$EU(C) = 1 \cdot \frac{2^1 - 1}{2^1 + 1} = \frac{1}{3}$

So she is indifferent between the lotteries.

the CE is therefore 1:

$$U(CE) = EU(B) = \frac{1}{3} \quad \text{so} \quad U(CE) = EU(C)$$

so $CE = 1$

or more formally:

$$\frac{2^{CE}-1}{2^{CE}+1} = \frac{1}{3}$$

$$CE = 1$$

4 c. $$V(t) = \frac{e^t - 1}{e^t + 1}$$

$$V'(t) = 2\frac{e^t}{(e^t+1)^2}$$

$$V''(t) = \frac{2e^t(e^t+1)^2 - 2e^t(2e^{2t} + 2e^t)}{(e^t+1)^4}$$

$$= \frac{(\cancel{e^{2t}} + \cancel{2e^t} + 1 - 2e^{2t} \cancel{-2e^t})2e^t}{(e^t+1)^4}$$

$$= \frac{2e^t - 2e^{3t}}{(e^t+1)^4}$$

For any positive t, this is smaller than zero so ~~SB~~ his utility function is concave so By Jensen's inequality he prefers the expected value of a lottery over the lottery itself so he is risk averse. Without further computations we know that he prefers

d. $A(t) = -\frac{v'(t)}{v''(t)} = -\frac{2e^t}{(e^t+1)^2} \cdot \frac{(e^t+1)^4}{2e^t - 2e^{3t}}$

$$= -\frac{2e^t(e^t+1)^2}{2e^t - 2e^{3t}} = -\frac{(e^t+1)^2}{1-e^{2t}}$$

$$= \frac{(e^t+1)^2}{e^{2t}-1} = \frac{(e^t+1)^2}{(e^t+1)(e^t-1)}$$

$$= \frac{e^t+1}{e^t-1}$$

Oh I must have mixed them up somewhere, no time to fix

e. As the number of tickets go up, the absolute risk aversion coefficient gets closer and closer to 1:

$$\lim_{t \to \infty} A(t) = 1$$

So as the number of tickets go up it ~~starts to be~~ his preferences start behaving more like a utility function with CARA(1) but it is not similar to ~~a~~ CRRA preferences as his relative risk aversion would in the limit be t.

If we are saying that he will act pretty much like he has ~~a~~ CARA(1) preferences he prefers C^+ to B^+ since his risk averseness does not change with an increasing t (in reality it is even increasing in t, getting closer to 1 so he would definitely prefer C^+ as he becomes more risk averse with an increasing t and C^+ offers lower risk).

f. If two lotteries B and C are independent, then if $C \succeq B$, the also then ~~[illegible]~~ also $DC \succeq DB$ when thes are the lotteries where you receive lottery D or C & B respectively with a 50/50 chance.

The reduction property is the idea that a rational expected utility maximizer will reduce a complicated lottery (e.g. consisting of multiple lotteries) into one simplified lottery.

Ace's expected utility satisfies this property, first she reduces the lottery into the simpler form and how if

$$EU(C) > EU(B)$$

then

$$EU(CD) > E(BD)$$

since she reduces the lottery this is equivalent to:

$$\frac{1}{2}EU(C) + \cancel{\frac{1}{2}EU(D)} > \frac{1}{2}EU(B) + \cancel{\frac{1}{2}EU(D)}$$

Cancel out so indeed it satisfies it.