6.3 Olivier Berg

I)

$$\cfrac{\cfrac{\cfrac{\forall x(Px \to P_1x)}{Pa \to P_1a}\,\forall Elim \qquad [Pa]}{P_1a}\,\to Elim \qquad \neg P_1a}{\neg Pa}\,\neg Intro$$

II)

$$\cfrac{\cfrac{\cfrac{\forall x\,(Px \to Qx)}{Pa \to Qa}\,\forall Elim \qquad Pa}{Qa}\,\to Elim}{\exists y\, Qy}\,\exists Intro$$

III)

~~∀Intro~~ You are proving the 2nd de-Morgan law here:

$$\neg(P \land Q) \vdash \neg P \lor \neg Q$$

Howeve, this time it has infinite length:

$$\neg(P \land Q \land \ldots) \vdash \neg P \lor \neg Q \lor \neg R \lor \ldots$$

the proof for the 2nd de-Morgan law is:

$$\cfrac{\neg(P \land Q) \qquad \cfrac{\cfrac{\cfrac{[\neg P]}{\neg P \lor \neg Q}\,\lor I \qquad [\neg(\neg P \lor \neg Q)]}{P}\,\neg E \qquad \cfrac{\cfrac{\neg Q}{\neg P \lor \neg Q}\,\lor I \qquad [\neg(\neg P \lor \neg Q)]}{Q}\,\neg E}{P \land Q}}{\neg P \lor \neg Q}\,\neg E$$

the Proof for this exercise is exactly the same

$$\dfrac{\dfrac{\dfrac{\dfrac{[\neg Qa]}{\exists x \neg Qx}\,\exists I \qquad [\neg \exists x \neg Qx]}{Qa}\,\neg E}{\forall x\, Qx}\,\forall I \qquad \neg \forall x\, Qx}{\exists x \neg Qx}\,\neg E$$

IV)

$$\dfrac{\exists x \neg Pxa \qquad \dfrac{\dfrac{[\neg Pba] \qquad \dfrac{[\forall y\, Pya]}{\cancel{\exists}\, Pba}\,\forall E}{\neg \forall y\, Pya}\,\neg E}{\cancel{\exists x \exists \forall}\ \exists z \neg \forall y\, Pyz}\,\exists\ \text{Intro}}{\exists z \neg \forall y\, Pyz}\,\exists\ \text{Elim}$$

Will discuss in class.

7)

$$[\forall z \forall x_1 \, P a b z x_1]$$
$$\forall x_1 \, P a b c x_1 \quad \forall E$$
$$P a b c a_1 \quad \forall E$$
$$\exists y \, P a y c a_1 \quad \exists I$$
$$\forall x_1 \exists y \, P a y c x_1 \quad \forall I$$
$$\exists x \forall x_1 \exists y \, P x y c x_1 \quad \exists I$$
$$\forall z \exists x \forall x_1 \exists y \, P x y z x_1 \quad \forall I$$

this step is valid because c & a do not occur in the undischarged assumption

$$[\exists y \forall z \forall x_1 \, P a y z x_1]$$
$$\forall z \exists x \forall x_1 \exists y \, P x y z x_1 \quad \exists E$$

$$\exists x \exists y \forall z \forall x_1 \, P x y z x_1$$
$$\forall z \exists x \forall x_1 \exists y \, P x y z x_1 \quad \exists E$$

this makes sense, consider the following

$$\forall x \exists y \, P x y \nvdash \exists y \forall x \, P x y$$

this is not valid because you are saying:

~~there is for every one, some one is better than everyone, therefore Everyone there is for~~ For everyone, the is atleast one person who is better, therefore, one person is better than everyone. this is of course not valid.

However, if you go in the opposite direction:

$$\exists x \forall y\, Pxy \vdash \forall y \exists x\, Pxy$$

this is valid: there is one person who is better than everyone (including himself), therefore, for everyone, there is at least one person better.
You are going from more ~~inform~~ information to less information, ~~this~~ instead of from less information to more information.
therefore this is valid.

b)

~~$\forall x$ P: is a Philosopher~~

~~G: has studied Gödel~~

$\cancel{\forall x(Px \to Gx) \therefore \forall x Px \to \forall x(Px \to Gx)}$

~~Proof~~

$\cancel{\forall x(Px \to Gx)}$

$\cancel{\to I}$

$\cancel{\forall x Px \to \forall x(Px \to Gx)}$

[illegible]

6.4

P: ... is a Philosopher

R: ... ~~[illegible]~~ has Studied logic

G: ... knows Gödel

$$\forall x((Px \land Rx) \to Gx) \vdash \forall x(Px \to Rx) \to \forall x(Px \to Gx)$$

$$\dfrac{[\forall x(Px \to Rx)]}{Pa \to Ra}\ \forall E$$

$$\dfrac{Pa \to Ra \qquad [Pa]}{Ra}\ \to E$$

$$\dfrac{\forall x((Px \land Rx) \to Gx)}{(Pa \land Ra) \to Ga}\ \forall E \qquad \dfrac{[Pa] \qquad Ra}{Pa \land Ra}\ \land I$$

$$\dfrac{(Pa \land Ra) \to Ga \qquad Pa \land Ra}{Ga}\ \to E$$

$$\dfrac{Ga}{Pa \to Ga}\ \to I$$

$$\dfrac{Pa \to Ga}{\forall x(Px \to Gx)}\ \forall I$$

$$\dfrac{\forall x(Px \to Gx)}{\forall x(Px \to Rx) \to \forall x(Px \to Gx)}\ \to I$$

5.c First, lets do the following one

$$R \leftrightarrow \neg R \vdash P$$

~~to do others~~

$$\dfrac{\dfrac{\dfrac{R\leftrightarrow\neg R \quad [R]}{\neg R}\leftrightarrow E \quad [R]}{\neg R}\neg I \qquad \dfrac{\dfrac{R\leftrightarrow\neg R \quad [\neg R]}{R}\leftrightarrow E \quad [\neg R]}{R}\neg E}{P}\neg E$$

In the same way, one would do this one

$\cancel{[\forall y(Ray \leftrightarrow \neg Ryy)]}$ $\forall E$

$$\cancel{[\forall y(Ray \leftrightarrow \neg Ryy)]}$$

$$\cancel{\dfrac{\dfrac{\dfrac{[\forall y(Ray \leftrightarrow \neg Ryy)]}{Raa \leftrightarrow \neg Raa}\,\forall E \quad [Raa]}{\neg Raa}\,\leftrightarrow E \quad [Raa]}{\neg Raa}\,\neg I}$$

$$\cancel{\dfrac{\dfrac{[\forall y(Ray \leftrightarrow \neg Ryy)]}{Raa \leftrightarrow \neg Raa} \quad [\neg Raa]}{}}$$

$$\cancel{[\forall y(Raa}$$

$$\dfrac{\exists x \forall y(Rxy \leftrightarrow \neg Ryy) \qquad \dfrac{\exists x \forall y(Rxy \leftrightarrow \neg Ryy) \qquad \dfrac{\dfrac{\dfrac{\dfrac{[\forall y(Ray \leftrightarrow \neg Ryy)]}{Raa \leftrightarrow \neg Raa}\,\forall E \quad [Raa]}{\neg Raa}\,\leftrightarrow E \quad [Raa]}{\neg Raa}\,\neg I \qquad \dfrac{\dfrac{\dfrac{[\forall y(Ray \leftrightarrow \neg Ryy)]}{Raa \leftrightarrow \neg Raa}\,\forall E \quad [\neg Raa]}{Raa}\,\leftrightarrow E \quad [\neg Raa]}{Raa}\,\neg E}{\rho}\,\neg E}{\rho}\,\exists E}{\rho}\,\exists E$$

(7.1) I) the Problem is that when the Proof goes from:

$$\dfrac{\exists x\,(Px \land Qx) \qquad \dfrac{\cancel{[Pa \land Qa]}}{Pa}}{Pa}\ \exists\text{Elim}$$

this is not allowed in the $\exists$-Elim rule Because the arbitrary Variable used in the assumption is used in the conclusion.

→ I am not sure if I could follow this, but it's about the incorrect application of $\exists$Elim (I think you are saying the same thing)

it should be

$$\dfrac{\dfrac{\exists x\,(Px \land Qx) \qquad \dfrac{\dfrac{[Pa \land Qa]}{Pa}}{\exists x\, Px}}{\exists x\, Px} \qquad \dfrac{\exists x\,(Px \land Qx) \qquad \dfrac{\dfrac{[Pa \land Qa]}{Qa}}{\exists x\, Qx}}{\exists x\, Qx}}{\exists x\, Px \land \exists x\, Qx}$$

✓

✓ II) As I pointed out in 6.3 V, this is not valid, Let A be the Structure with

$$D_A = \{1, 2\}$$

$$|R^2|_A = \{\langle 1,2\rangle, \langle 2,1\rangle\}$$

the Problem in the Proof is again in

$$\dfrac{\exists y\, Ray \qquad [Rab]}{Rab}$$

the arbitrary variable B occurs in the conclusion

III) the Problem is in the Step

$$\frac{P_a \to Q_a}{\forall x (P_x \to Q_x)}$$

You cannot do this Because a occurs ~~in~~ in undischarged assumptions.

It is also not valid. Let $\mathcal{A}$ be the $\mathcal{L}_2$-structure with:

$$D_{\mathcal{A}} = \{1, 2\}$$

$$|P|_{\mathcal{A}} = \{1, 2\}.$$

$$|Q|_{\mathcal{A}} = \{1\}$$