② a. type X would get low and type Y would get medium and high

B. Same as in a

c. expected value for type X is 80, for Y its $\frac{110+85+60}{3} = 85$ so type Y will Buy all the Bikes.

d. they know the expected value is 85, But for that price the high quality sellers would not sell, So then the expected value would B ~~...~~ $72.5 \leq 88$ so ~~they~~ there would be no trade.

c is not efficient

e. a, b, c are efficient, But because of incomplete information, d is not efficient.

⑤ a. the high productivity workers would like to signal their type to get a higher wage equal to their productivity. they can make it credible ~~by~~ By giving a signal costly enough that L-type workers would find it too costly.

B. With competitive, risk-neutral firms, the wage must be equal to the expected productivity of the worker:

$$w = \frac{1}{4} \cdot 100 + \frac{3}{4} \cdot 80 = 85$$

c. education is an observable signal. A separating equilibrium exists if H-types had to take the education to increase their wage an L-types do not want to mimic.

if a worker gets education, $w = 100$
Otherwise $w = 80$.

So for H-types, their payoff is

$100 - 12 = 88 > 80$ so they will get an education.

L-types:

$100 - 22 = 78 < 80$

So they won't educate.

$\rightarrow$ there is a unique separating equilibrium.

d. if the wage from education would be 100, they would both get an education, as it is more than 80. But then firms can't differentiate anymore so the wage will just be the same as in part b and nobody will get education.

$\longrightarrow$ Not credible signal

c. Yes, total payoff with education is lower. In this model, education is merely a cost.

(6) a. ~~Wage~~ Wage will equal IR

$$\sqrt{w_e} - e = 8$$

$$w_e = (8+e)^2$$

$w_0 = (8+0)^2 = 64$ $\quad w_1 = (8+1)^2 = 81$

b. Payoff from high-~~effort~~ effort:

$$\frac{3}{5} \cdot 195 + \frac{2}{5} \cdot 60 - 81 = 60$$

Payoff from low effort:

$$\frac{2}{5} \cdot 195 + \frac{3}{5} \cdot 60 - 64 = 50$$

$60 > 50$ so high effort is preferred so

$\{1, w_1\}$

Same: $\frac{3}{5} \cdot 165 + \frac{2}{5} \cdot 60 - 81 = 42 > \frac{2}{5} \cdot 165 + \frac{3}{5} \cdot 60 - 64 = 38$

$\{0, w_0\}$:

$$\frac{3}{5} \cdot 135 + \frac{2}{5} \cdot 60 - 81 = 24 < \frac{2}{5} \cdot 135 + \frac{3}{5} \cdot 60 - 64 = 26$$

B. If effort is unobservable, the Principal must give the agent a wage that makes him just willing to accept when exerting $e=0$.

IR for $e=0$: $\sqrt{w} = 8 \Rightarrow w = 64$

Utility under $e=1$:

$$U_1 = \frac{3}{5}\sqrt{w(\pi_H)} + \frac{2}{5}\sqrt{w(\pi_L)} - 1$$

Under $e=0$:

$$U_0 = \frac{2}{5}\sqrt{w(\pi_H)} + \frac{3}{5}\sqrt{w(\pi_L)}$$

IC for $e=1$ requires $U_1 \geq U_0$:

$$\frac{3}{5}\sqrt{w(\pi_H)} + \frac{2}{5}\sqrt{w(\pi_L)} - 1 > \frac{2}{5}\sqrt{w(\pi_H)} + \frac{3}{5}\sqrt{w(\pi_L)}$$

$$\sqrt{w(\pi_H)} - \sqrt{w(\pi_L)} \geq 5$$

IR for $e=1$ requires $U_1 \geq 8$:

$$\frac{3}{5}\sqrt{w(\pi_H)} + \frac{2}{5}\sqrt{w(\pi_L)} \geq 9$$

~~Solving~~ ~~together~~ gives:

$$\frac{3}{5}\left(\sqrt{w(\pi_L)}+5\right)+\frac{2}{5}\sqrt{w(\pi_L)}=9$$

$$\sqrt{w(\pi_L)}=6 \Rightarrow \sqrt{w(\pi_H)}=11$$

$$w(\pi_L)=6^2=36 \qquad w(\pi_H)=11^2=121$$

$$E[w]=\frac{3}{5}\cdot 122+\frac{2}{5}\cdot 36=\cancel{88}\ 87$$

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under $e=0$: Payoff for Principal is:

$$\frac{2}{5}\cdot 198+\frac{3}{5}\cdot 60-64=50$$

under $e=1$:

$$\frac{3}{5}\cdot 198+\frac{2}{5}\cdot 60-87=54$$

~~So a contract with wage~~ so it is worth paying more for the higher profit.

$\rightarrow e=1 \quad w(\pi_H)=121 \quad w(\pi_L)=36$

$$\frac{2}{5}\cdot 168+\frac{3}{5}\cdot 60-64=38 > \frac{3}{5}\cdot 168+\frac{2}{5}\cdot 60-87=36$$

$\rightarrow e=0, \ w=64$

$\pi_H=135$ is even lower so the same

c. it only change with $\pi_{it} = 16c$. Expected wage to induce $e=1$ is higher when effect is unobservable.