

4.1

= Free

Formulae

Sentence

I)  $\forall x (P^1_{1x} \rightarrow Q^1_y)$  ✓ ✗ ✓

II)  $\exists x \neg (\exists y P^1_y \wedge \neg \neg \neg \neg R^2_{xa})$  ✓ ✓ ✓

III)  $P^0$  ✓ ✓ ✓

IV)  $\forall x \exists y \exists z (R^3_{25} x y z)$  ✓ ✓ ✓

No, because of the brackets

V)  $\forall x \exists x Q^2_{xx}$  According to TLM it is

Interesting, let's discuss in class. According to Halbach this is straightforwardly a sentence

Sentence & a Formulae.  
According to the rule in  
the slides it is not.

VI)  $\neg (\neg (\exists x P^1_x \wedge \exists y Q^1_y))$  ✗ ✗ ✓

VII)  $\forall x (\exists y (P^2_{xy} \wedge P^2_x) \vee Q^3_{\underline{x} y \underline{x}})$  ✓ ✗ ✓

4.2

I)  $(\forall x \forall y (P^2_{xy} \rightarrow (P^2_{y \underline{x}} \wedge R^1_x)))$

 $\forall x \forall y (P_{xy} \rightarrow (P_{yx} \wedge R_x))$ 

II)  $(\forall x R^3_{\underline{x} \underline{x} z} \wedge \exists y R^3_{\underline{x} \underline{z} y})$  ✓

III)  $(\forall z_2 R^2_{\underline{x} \underline{z} \underline{z}})$

No brackets, well done on the free variables

IV)  $(\forall x \forall y ((P^2_{xy} \vee R^2_{yx}) \vee R^2_{\underline{z} \underline{y} \underline{z}}))$

 $\forall x \forall y ((P_{xy} \vee R_{yx}) \vee R_{zy})$

4.3

I)  $a: \text{London}$

$P^1: \dots, \text{is big}$

$R^1: \dots, \text{is ugly}$

$P^1_a \wedge R^1_a$



II)

$c: \text{Culham}$

$P^1: \dots, \text{is a large village}$

$P^1_c$



III)

$P^1: \dots, \text{is a city}$

$Q^1: \dots, \text{is a city hall}$

$R^2: \dots, \text{has} \dots_2$

$\forall x (P^1_x \rightarrow \exists y (Q^1_y \wedge R^2_{xy}))$



IV)

$P^1: \dots, \text{is a material object}$

$Q^1: \dots, \text{is divisible}$

$\forall x (P^1_x \rightarrow Q^1_x)$



V)  $a: \text{Tom}$

$P^1: \dots, \text{is a car}$

$Q^2: \dots, \text{owns} \dots_2$

$\exists x (P^1_x \wedge Q^2_{ax})$



VI)  $a: \text{tom}$

$P^1: \dots, \text{ is a car}$

$Q^2: \dots, \text{ owns } \dots$

$R^2: \dots, \cancel{\text{will sell}} \dots$

$$\exists x (P_x^1 \wedge Q_{ax}^2 \wedge R_{ax}^2)$$

Yes, although you could have had  $R: \dots \text{ will sell } \dots$   
and then put in a negative

VII)

$P^1: \dots, \text{ is a man}$

$Q^2: \dots, \text{ is a country}$

$R^2: \dots, \text{ has visited } \dots$

$$\exists x (P_x^1 \wedge \forall y (Q_y^2 \rightarrow R_{xy}^2))$$

(u.4)

I) tom acts freely.

II)  $\forall a$  Either tom ~~is~~ acts freely, or ~~is not~~  
it is not the case that tom is  
a person.

III) Every person acts freely.

IV) Someone is a person if and only if  
they act freely.

V) It is not the case that there exists  
someone who acts freely.

Yes, careful with the 'someone'. That implies person, which isn't specified yet. In both iv and v it would  
be better to say 'something is a person iff ...' or 'nothing acts freely'

4.5

I) It is not the case that there exists a set. Yes. Alt.: 'there are no sets'

II) It is not the case that every set has at least one element.

III) There exists a set with no elements.

IV) There does ~~not~~ exist a set that contains everything as an element.

4.2

I)  $P^? : \dots, \text{ is a book author}$

$Q^? : \dots, \text{ is famous}$

Yes, alternatively you can include

a book predicate:

$\neg \forall x (\exists y (Py \wedge Qxy) \rightarrow Rx)$  where

P: ... is a book

Q: ... is the author of ...

R: ... is famous

$$\neg \forall x (P^? x \rightarrow Q^? x)$$

II)

$P^? : \dots, \text{ is a book}$

$Q^? : \dots, \text{ is famous}$

$$\exists x (P^? x \wedge Q^? x)$$

III)

$P^? : \dots, \text{ is a book}$

$Q^? : \dots, \text{ is famous}$

$R^? : \dots, \text{ is well written}$

$$\forall x (P^? x \rightarrow (Q^? x \leftrightarrow R^? x))$$

iv)

a: Tom

$\neg P^1$

b: Tom is a book author.

c: Tom is famous.

$P^2$

d: Tom does not believe that every book author is famous.

Half Pbk  $\Rightarrow$

$\exists x (\forall y \forall z Q^1_x)$

a: Tom

$P^1$ : believes that not every book author is famous.

$\neg P^1 a$



a: Tom

b: the Believe that not every book author is famous

$P^2$

c: Believes ...

$P^2_{ab}$

You could make the case that

b refers to just a single thing, multiple people can have the same belief.)

7.3

I)

b: Ben

$P^2$ :  $\dots \rightarrow$  deupsies  $\dots_2$

$Q^2$ :  $\dots$  is a logician

$\forall x (Q^2_x \rightarrow P^2_{bx})$

$\exists x (Q^2_x \wedge P^2_{bx})$

II)

a: Harry

b: Ron

$P^2$ :  $\dots$  is a parent of  $\dots_2$

$Q^2$ :  $\dots$  stands  $\dots_2$

$Q^2_{ab} \wedge \forall x (P^2_{xa} \rightarrow Q^2_{ax})$

or or:

$Q^2_{ab} \wedge \forall x (P^2_{xb} \rightarrow Q^2_{ax})$

III)

$P^2$ :  $\dots$  is a Student

$Q^2$ :  $\dots$  is a Tutor

$R^2$ :  $\dots$  is better than  $\dots_2$

$\forall x (P^2_x \rightarrow \forall y (Q^2_y \rightarrow R^2_{xy}))$

or:

$\forall x (P^2_x \rightarrow \exists y (Q^2_y \wedge R^2_{xy}))$

or:

$\exists x (P^2_x \wedge \forall y (Q^2_y \rightarrow R^2_{xy}))$

or:

$\exists x (P^2_x \wedge \exists y (Q^2_y \wedge R^2_{xy}))$

Well done! Present this in class?

IV)

$P^1$ : ... is rich

Only rich Germans  
Buy

$Q^1$ : ... is German

$R^1$ : ... is a house

$P_1^1$ : ... is in Munich

~~$P^2$~~ : ... buys ...

One house vs. many houses

$$\forall x ((R^1 x \wedge P_1^1 x) \rightarrow \exists y (P^1 y \wedge Q^1 y \wedge P^2 y x))$$

Only rich Germans buy houses in Munich

→ If a house is in Munich, then there exists a rich German who buys it.

However, it might also mean  
possible that the houses get bought  
by nobody. So:

If a house is in Munich, then  
there does not exist a rich  
German someone who is not rich  
and German who buys the house:

$$\forall x ((R^1 x \wedge P_1^1 x) \rightarrow \neg \exists y ((P^1 y \wedge Q^1 y \wedge P^2 y x)))$$

You might have slightly overcomplicated this. Consider: 'Everybody buying a house in Munich is rich and German.'  $\forall x (Q x \rightarrow P x \wedge R x)$  and 'Every German buying a house in Munich is rich.'  $\forall x (P x \rightarrow (Q x \rightarrow R x))$ , where P: ... is a German, Q: ... buys a house in Munich, and R: ... is rich.

IV)

$\alpha$ : Jane

$P^1$ : ... is a fast car

$Q^2$ : ... likes ...

$\forall x (P^1_x \rightarrow Q^2_{\alpha x})$

or:  $\exists x (P^1_x \wedge Q^2_{\alpha x})$

IV)

$P^1$ : ... is a mistake

$Q^2$ : ... is a thing that  $\epsilon$  is capable  
of making mistakes

interesting differentiation from Halbach's 'is a person'

$R^2$ : ... made ...

$\exists x (P^1_x \wedge \forall y (Q^2_y \rightarrow R^2_{yx}))$

or:  $\forall x (\omega^1_x \rightarrow \exists y (P^1_y \wedge R^2_{xy}))$