College tute Sheet 1

Outstanding

①

	A	B
A	3, 3*	1, 4*
B	4*, 1	2*, 2* ✓

②

	A	B
A	5*, 5*	0, 4
B	4, 0	3*, 3* ✓

③

	A	B
A	1*, -1	-1, 1*
B	-1, 1*	1*, -1

This is a bit ambiguous — see why in class —

a. Regardless of what the other player does, the player would never play ~~&~~ the strictly dominated strategy. ✓ In game 1, any strategy for player 1 that would mean playing action A, is strictly dominated by ~~B~~ the strategy to play action B. Regardless of what player 2 plays, the payoff ~~from play~~ from action B is always larger than from action A. The same is true for player 2 in game 1.

B. A set of strategies is a Nash equilibrium if no player can obtain a higher payoff by choosing a different strategy given everyone else's strategy.

- Game 1:

the Nash equilibrium is when both players do action B. If player 1 changed from B to A their payoff would decrease and the same goes for player 2.

- Game 2:
  - Both players choose A (It would reduce their payoff if they were to deviate).
  - Both players choose B (idem).
  - the mixed ~~equilibrium~~ strategy:

    θ_1 is the probability that player 1 chooses action A.

    then the payoff for player 2 from doing action A is:

$$P(A) = 5\theta_1 + 0(1-\theta_1) = 5\theta_1$$

And the payoff from action B is:

$$P(B) = 4\theta_1 + 3(1-\theta_1) = \theta_1 + 3$$

For player 2 to use a mixed strategy, they must be indifferent ~~to 2~~ between action A and B.

So the payoff must be equal:

$$5\theta_1 = \theta_1 + 3$$

$$\theta_1 = \frac{3}{4}$$

Nice!

~~As~~ As the game is symmetric, θ_2, the chance that player 2 plays action A_1 is also $\frac{3}{4}$.

- Game 3
  - There are no pure ~~...~~ equilibria.
    ~~mixed~~ So in the same way as above.

$$P(A) = -1\theta_1 + 1(1-\theta_1) = 1 - 2\theta_1$$

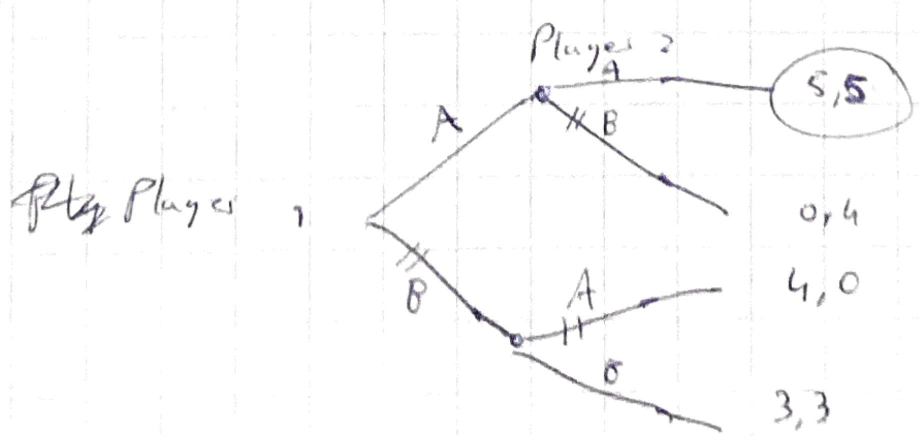
$$P(B) = 1\theta_1 - 1(1-\theta_1) = 2\theta_1 - 1$$

$$1 - 2\theta_1 = 2\theta_1 - 1$$

$$\theta_1 = \frac{1}{2}$$

And as the game is symmetric again $\theta_2 = \theta_1 = \frac{1}{2}$

c.



- If Player 1 would choose A, then Player 2 would also choose A (as it gives Player 2 a higher payoff than B).
- If Player 1 would choose B, the Player 2 would also choose B (idem).

As the father would give Player 1 a higher payoff, they would choose ~~B~~ A.

Good, so the SPE outcome is A, A.

The SPE strategy profile is (A, AB).

if A if B

(5)

a.

	C	N
C		
N		

	C	N
C	$w-1+2b$, $w-1+2b$	$w-1+b$, $w+b^*$
N	$w+b^*$, $w-1+b$	w^*, w^*

Nash equilibrium.

as $0.5 < b < 1$

~~$2b > 1$~~

Not ~~thus~~ Pareto-efficient, as both of them could have gotten a higher payoff if they both contributed.

b. It would stay the same. However, this time the Nash equilibrium would be Pareto-efficient as $2b < 1$, so they both get the highest possible payoff by not contributing.

c.

	C	N
C	$w-1+2b^*, w-1+2b^*$	$w-1+b^*, w+b$
N	$w+b, w-1+b^*$	w, w

Nash equilibrium, ~~which~~ which is also Pareto-efficient as it is the highest possible payoff for both players.

d. $\frac{1}{2} < b < 1$, Because if $b > 1$ then anyone would benefit even if only ~~[illegible]~~ he himself would contribute. And if $b < \frac{1}{2}$, no one should ever contribute as even if everyone were to contribute it still would not be worth it.

Good

e.

	C	N
C	$2w-2+4b^*,$ $2w-2+4b^*$	$2w-1+2b^*,$ $2w-1+2b$
N	$2w-1+2b,$ $2w-1+2b^*$	$2w,$ $2w$

(C, C) is the Nash equilibrium now, and by internalising the externality, it is now also Pareto-efficient.

F. The externalities should be internalised to get a socially optimal outcome. I don't know through which specific kind of policy one could do this for the consumer though.

see class discussion.

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a. The utility of Player 1 is dependent on how much of the stake it gets, but gets ~~[illegible]~~ reduced by ~~[illegible]~~ the difference in portion size between any of the ~~[illegible]~~ players' portion and the average portion size. ~~[illegible]~~ When $\alpha = 0$, it is the home economics as ~~[illegible]~~ then Player 1 only cares about their own share.

$\bar{x} = \frac{1}{2}$ so

B. $$u_1 = x_1 - \frac{\alpha}{2}\left(\left(x_1 - \frac{1}{2}\right)^2 + \left(x_2 - \frac{1}{2}\right)^2\right)$$

$$= x_1 - \frac{\alpha}{2}\left(x_1^2 - x_1 + \frac{1}{4} + x_2^2 - x_2 + \frac{1}{4}\right)$$

$$= x_1 - \frac{\alpha}{4}\left(2x_1^2 - 2x_1 + 2x_2^2 - 2x_2 + 1\right)$$

Since $x_2 = (1-x_1)$ $\quad (x_1 - x_2)^2 = x_1^2 - 2x_1x_2 + x_2^2$

therefore

$$(x_1 - x_2)^2 = (x_1 - (1-x_1))^2 = (2x_1 - 1)^2$$

$$= 4x_1^2 - 4x_1 + 1$$

$$x_1 - \frac{\alpha}{4}\left(2x_1^2 - 2x_1 + 2(1-x_1)^2 - 2(1-x_1) + 1\right)$$

$$x_1 - \frac{\alpha}{4}\left(2x_1^2 \cancel{-2x_1} + 2(1 - 2x_1 + x_1^2) - 2 + \cancel{2x_1} + 1\right)$$

$$x_1 - \frac{\alpha}{4}\left(2x_1^2 + \cancel{2} - 4x_1 + 2x_1^2 \cancel{-2} + 1\right)$$

$$x_1 - \frac{\alpha}{4}\left(4x_1^2 - 4x_1 + 1\right) = x_1 - \frac{\alpha}{4}(x_1 - x_2)^2$$

c. Player 1 wants to maximize utility:

$$\max_{x_1} \quad u_1 = x_1 - \frac{\alpha}{4}(x_1 - (1-x_1))^2$$

$$u_1 = x_1 - \frac{\alpha}{4}(2x_1 - 1)^2$$

to maximize $\quad \frac{du_1}{dx_1} = 1 - \frac{\alpha}{4} \cdot 2 \cdot 2(2x_1 - 1) = 0$

$$1 - 2\alpha x_1 + \alpha = 0$$

$$2\alpha x_1 = 1 + \alpha$$

$$x_1 = \frac{1+\alpha}{2\alpha}$$

Nice!

So $x_1 = \frac{1+\alpha}{2\alpha}$ if $\frac{1+\alpha}{2\alpha} \leq 1$, otherwise

$x_1 = 1$

and $x_2 = 1 - \frac{1+\alpha}{2\alpha}$ if $\frac{1+\alpha}{2\alpha} \leq 1$, otherwise $x_2 = 0$

d. the offer to Player 2 is

$$x_2 = 1 - \frac{1+\alpha}{2\alpha} = \frac{1}{2} - \frac{1}{2\alpha}$$

$$\frac{dx_2^*}{d\alpha} = \frac{1}{2}\alpha^{-2} = \frac{1}{2\alpha^2}$$

as $\alpha \geq 0$, $\frac{1}{2\alpha^2} \geq 0$ for every α. So, x_2 increases as α increases.

e. $x_2 = \frac{1}{2} - \frac{1}{2\alpha}$

as $\lim_{\alpha \to \infty} \frac{1}{2\alpha} = 0$ so if α Becomes very large, the share that x_2 gets comes closer and closer to $\frac{1}{2}$. So it explains why Player 1 would give up to $\frac{1}{2}$, But not more.

Yes

f. I) $x_2 = \frac{1}{2} - \frac{1}{2\alpha} = \frac{1}{4}$

$$\frac{1}{2\alpha} = \frac{1}{4}$$

$$2\alpha = 4$$

$$\alpha = 2$$

II) $\frac{1}{2} - \frac{1}{2\alpha} = 0$

$$\frac{1}{2\alpha} = \frac{1}{2}$$

$$\alpha = 1$$

$\alpha \in [0, 1]$