

College tote Sheet

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	A	B
A	3, 3*	7, 4*
B	4, 1*	2, 2*

②

	A	B
A	5, 5*	0, 4
B	4, 0	3, 3*

Outstanding

③

	A	B
A	7*, -1	-1, 7*
B	-1, 1*	1, -1

This is a bit ambiguous — see why in class —

- a. Regardless of what the other player does, the player would never play & the Strictly dominated strategy. In game 1, ~~any strategy~~ any strategy for Player 1 that would mean playing action A, is strictly dominated by the strategy to play action B. Regardless of what Player 2 plays, the payoff from action B is always larger than from action A. The same is true for Player 2 in game 1.

B. A set of strategies is a Nash equilibrium if no player can obtain a higher payoff by choosing a different strategy given everyone else's strategy.

- Game 1:

The Nash equilibrium is where both players do action B. If Player 1 changes from B to A their payoff would decrease and the same goes for Player 2.

- Game 2:

- Both players choose A (it would reduce their payoff if they were to deviate).

- Both players choose B (idem).

- The mixed strategy:

θ_1 is the probability that Player 1 chooses action A.

then the payoff for Player

Player 2 is From doing action A: 5

$$P(A) = 5\theta_1 + 0(1 - \theta_1) = 5\theta_1$$

And the payoff from action B is:

$$P(B) = 4\theta_1 + 3(1 - \theta_1) = \theta_1 + 3$$

For Player 2 to use a

mixed strategy, they must be indifferent between action A and B.

So the payoff must be equal:

$$S\theta_1 = \theta_1 + 3$$

$$\theta_1 = \frac{3}{4}$$

Nice!!

Since the game is symmetric,

θ_2 , the chance that Player 2 plays action A_1 is also $\frac{3}{4}$.

- Game 3:

The two no pure equilibria.

So in the same way as above.

$$P(A) = -1\theta_1 + 1(1-\theta_1) = 1 - 2\theta_1$$

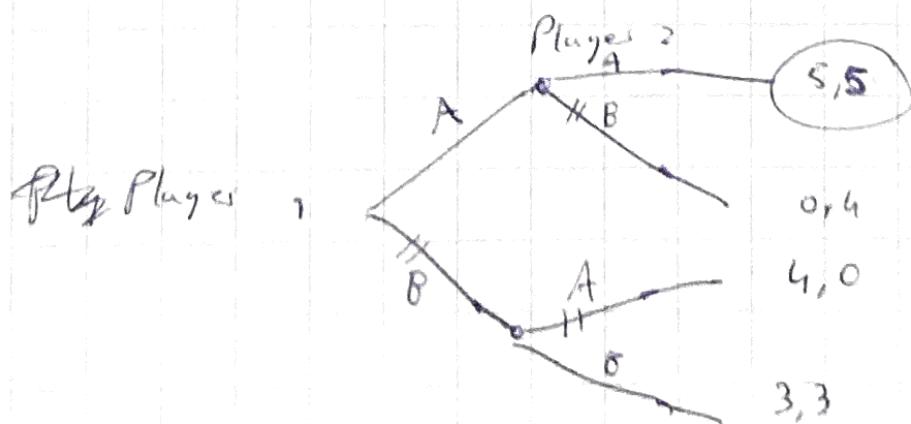
$$P(B) = 1\theta_1 - 1(1-\theta_1) = 2\theta_1 - 1$$

$$1 - 2\theta_1 = 2\theta_1 - 1$$

$$\theta_1 = \frac{1}{2}$$

And at the game is symmetric again $\theta_2 = \theta_1 = \frac{1}{2}$

c.



- IF Player 1 would choose A, then Player 2 would also choose A (as it gives Player 2 a higher payoff than B).

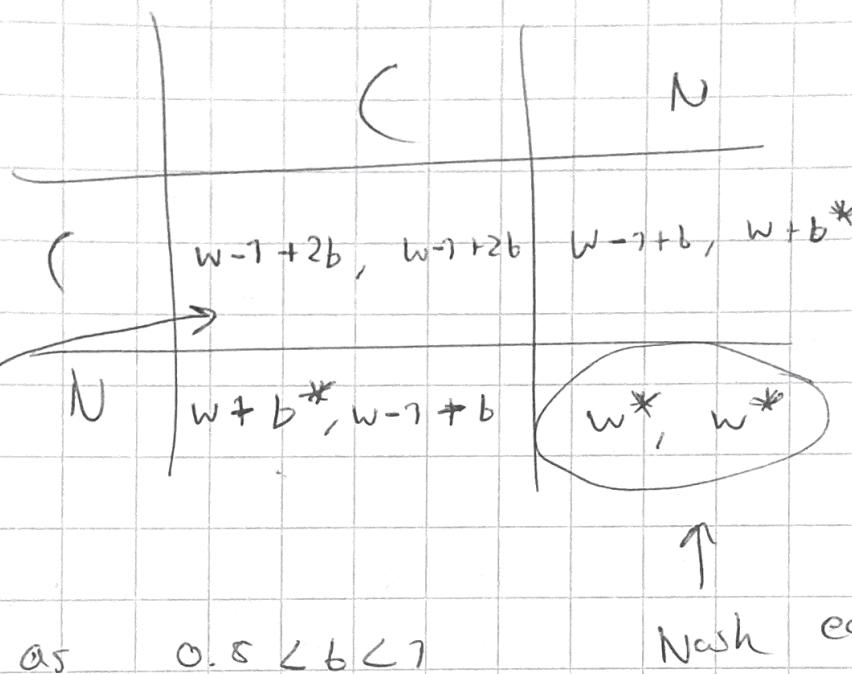
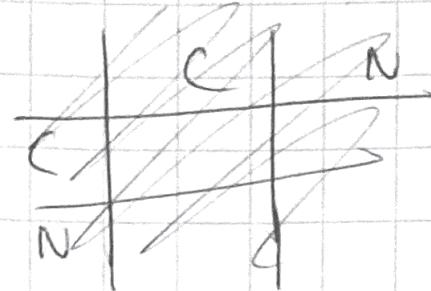
- IF Player 1 would choose B, then Player 2 would also choose B (idem).

As the Father would give Player 1 a higher payoff, they would choose ~~Player~~ A.

Good, so the SPE outcome is A, A. ~~if A if B.~~

The SPE strategy profile is (A, AB) .

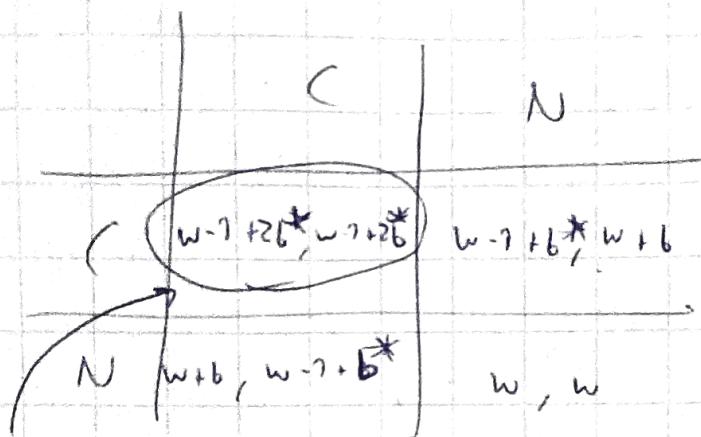
(5)



✓ Not Pareto-efficient, as both of them could have gotten a higher payoff if they both cooperated.)

8. It would stay the same. However, this time the Nash equilibrium would be Pareto-efficient as $2b < 1$, so they both get the highest possible payoff by not competing.

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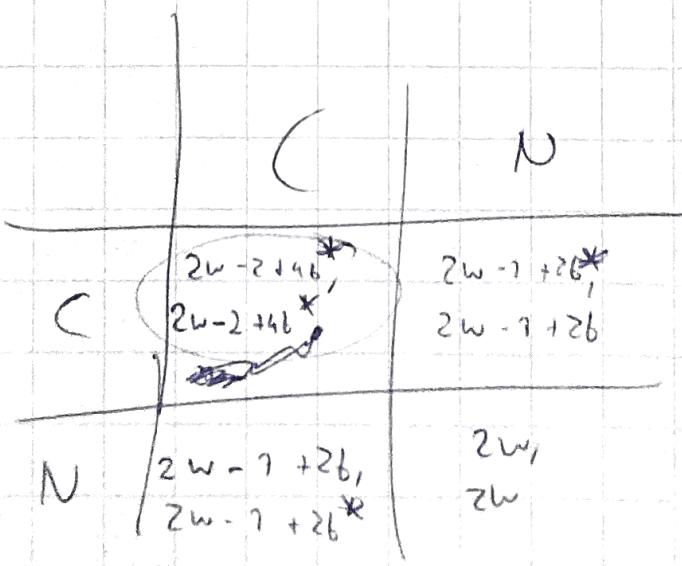


Nash equilibrium, ~~which~~ which is also

Pareto-efficient as it is the highest possible payoff for both players.

d. $\frac{1}{2} < b < 7$, because if $b \geq 7$ everyone would benefit if only ~~contribute~~ ~~he himself~~ would contribute.
And if $b \leq \frac{1}{2}$, no one should ever contribute
as even if everyone were to contribute it still would not be worth it.

Good



(C, C) is the Nash equilibrium now, and by internalizing the externality, it is now also Pareto-efficient.

F. { the externalities should be internalised to get a socially optimal outcome.

I don't know through which specific kind of policy one could do this for the consumer though.

→ see class discussion.

⑥

a. The utility $\stackrel{\text{of Player 1}}{v_1}$ depends on how much of the stake it gets, but gets ~~reduced~~ reduced between any of the players, ~~from 2 to the~~ when $\alpha = 0$, it is the most ~~then~~ economies as ~~they~~ player 1 only cares about their own share.

$$\bar{x} = \frac{1}{2} \text{ so}$$

$$B. v_1 = x_1 - \frac{\alpha}{2} \left((x_1 - \frac{1}{2})^2 + (x_2 - \frac{1}{2})^2 \right)$$

$$= x_1 - \frac{\alpha}{2} \left(x_1^2 - x_1 + \frac{1}{4} + x_2^2 - x_2 + \frac{1}{4} \right)$$

$$= x_1 - \frac{\alpha}{4} (2x_1^2 - 2x_1 + 2x_2^2 - 2x_2 + 1)$$

Since $x_2^2 = (1-x_1)$

Therefore

$$(x_2 - x_2)^2 = (x_1 - (1-x_1))^2 = (2x_1 - 1)^2$$

$$= 4x_1^2 - 4x_1 + 1$$

$$x_1 - \frac{\alpha}{4} (2x_1^2 - 2x_1 + 2(1-x_1)^2 - 2(1-x_1) + 1)$$

$$x_1 - \frac{\alpha}{4} (2x_1^2 - 2x_1 + 2(1-2x_1+x_1^2) - 2 + 2x_1 + 1)$$

$$x_1 - \frac{\alpha}{4} (2x_1^2 + 2 - 4x_1 + 2x_1^2 - 2 + 1)$$

$$x_1 - \frac{\alpha}{4} (4x_1^2 - 4x_1 + 1) = x_1 - \frac{\alpha}{4} (x_1 - x_2)^2$$

c. Plug in works to maximize utility.

$$\max_{x_1} u_1 = x_1 - \frac{\alpha}{4} (x_1 - (1-x_1))^2$$

$$u_1 = x_1 - \frac{\alpha}{4} (2x_1 - 1)^2$$

$$\text{to maximize } \frac{du_1}{dx_1} = 1 - \frac{\alpha}{4} \cdot 2 \cdot 2(2x_1 - 1) = 0$$

$$1 - 2\alpha x_1 + \alpha = 0$$

$$2\alpha x_1 = 1 + \alpha$$

$$x_1 = \frac{1+\alpha}{2\alpha}$$

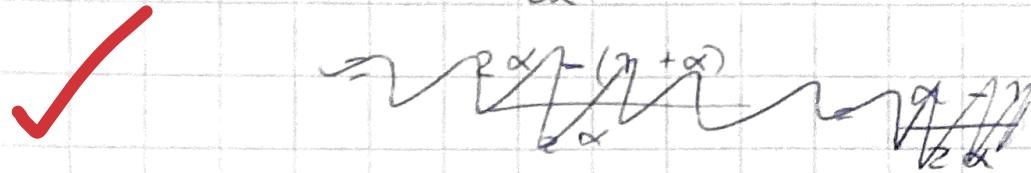
$$\text{So } x_1 = \frac{1+\alpha}{2\alpha} \text{ if } \frac{1+\alpha}{2\alpha} \leq 1, \text{ otherwise } x_1 = 0$$

Nice!

and $x_2 = 1 - \frac{1+\alpha}{2\alpha}$ if $\frac{1+\alpha}{2\alpha} \leq 1, \text{ otherwise } x_2 = 0$

d. the offer to player 2 is

$$x_2 = \frac{1}{2} - \frac{1+\alpha}{2\alpha} = \frac{1}{2} - \frac{1}{2\alpha}$$



$$\frac{dx_2}{d\alpha} = \frac{1}{2} \alpha^{-2} = \frac{1}{2\alpha^2}$$

$\frac{dx_2}{d\alpha} > 0$

as $\alpha \geq 0$, $\frac{1}{2\alpha^2} \geq 0$ for every α . So, x_2 increases as α increases.

e. $x_2 = \frac{1}{2} - \frac{1}{2\alpha}$

as $\lim_{\alpha \rightarrow \infty} \frac{1}{2\alpha} = 0$ so if

α becomes very large, the share that x_2 gets comes closer and closer to $\frac{1}{2}$. So, it explains

Yes // Why Player 1 would give up to $\frac{1}{2}$, but not more.

f.

$$x_2 = \frac{1}{2} - \frac{1}{2\alpha} = \frac{1}{n}$$

∴

$$\frac{1}{2\alpha} = \frac{1}{n}$$

$$2\alpha = n$$

$\alpha = 2$

II)
 $\frac{1}{2} - \frac{1}{2\alpha} = 0$

$$\frac{1}{2\alpha} = \frac{1}{2}$$

$\alpha = 1$

$\alpha \in [0, 1]$