

(5)

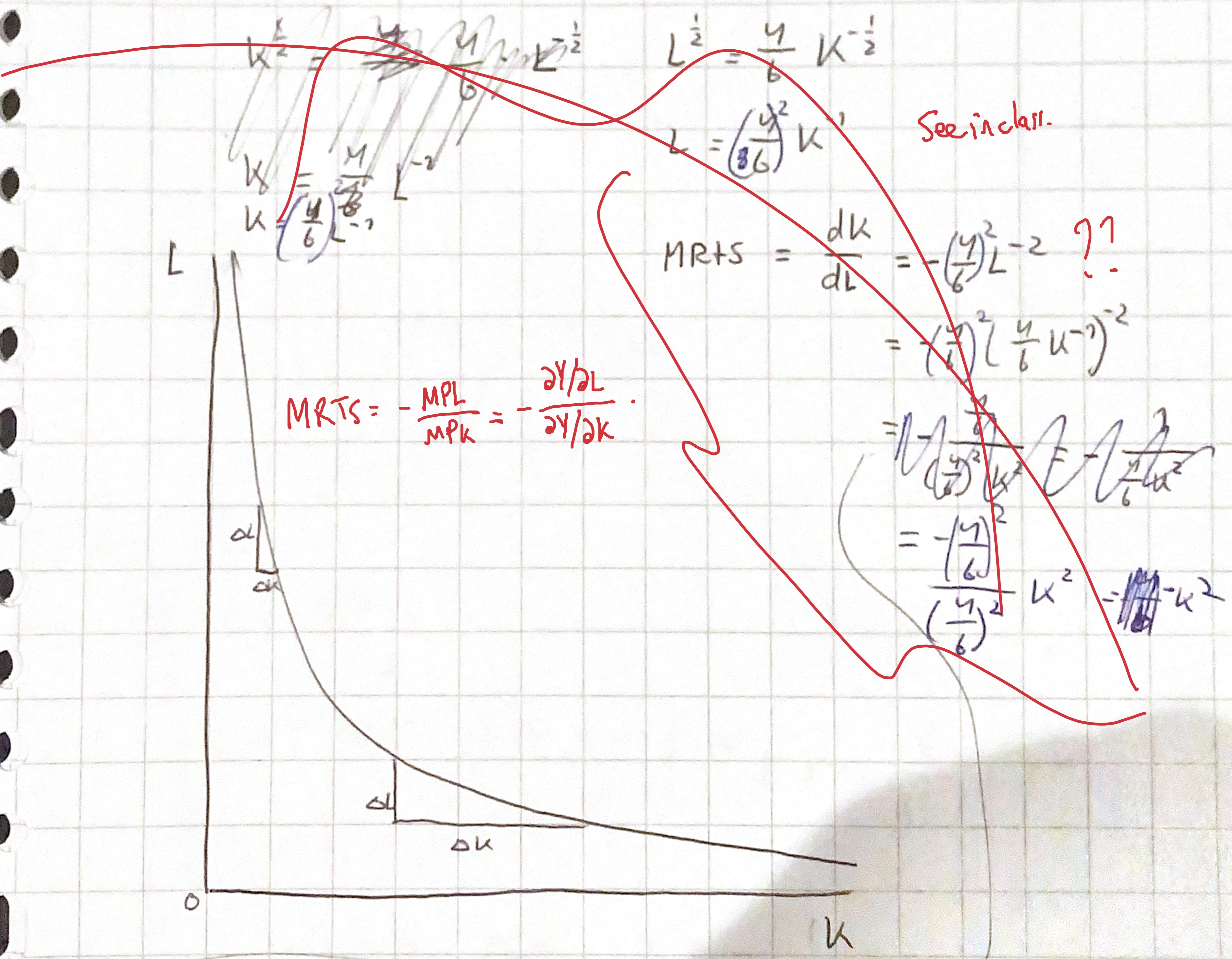
a.  ~~$y = 6K^{\frac{1}{2}}L^{\frac{1}{2}}$~~

$$6(\lambda K)^{\frac{1}{2}} \cdot (\lambda L)^{\frac{1}{2}} = 6 \cdot \lambda^{\frac{1}{2}} \cdot K^{\frac{1}{2}} \cdot \lambda^{\frac{1}{2}} \cdot L^{\frac{1}{2}} \\ = 6\lambda^1 K^{\frac{1}{2}} L^{\frac{1}{2}}$$

[So it has a constant returns to scale  
→ why? What does CRS mean? Define it.]

B.

$$y = 6K^{\frac{1}{2}}L^{\frac{1}{2}}$$



$$MRCS = -\frac{MPL}{MPK} = -\frac{\partial Y / \partial L}{\partial Y / \partial K}$$

→ So, if  $K$  increases, the marginal rate of technical substitution decreases, increases

this makes sense, if  $K$  <sup>is larger</sup> increased, then more and more Capital is needed ~~to~~ to substitute ~~the~~

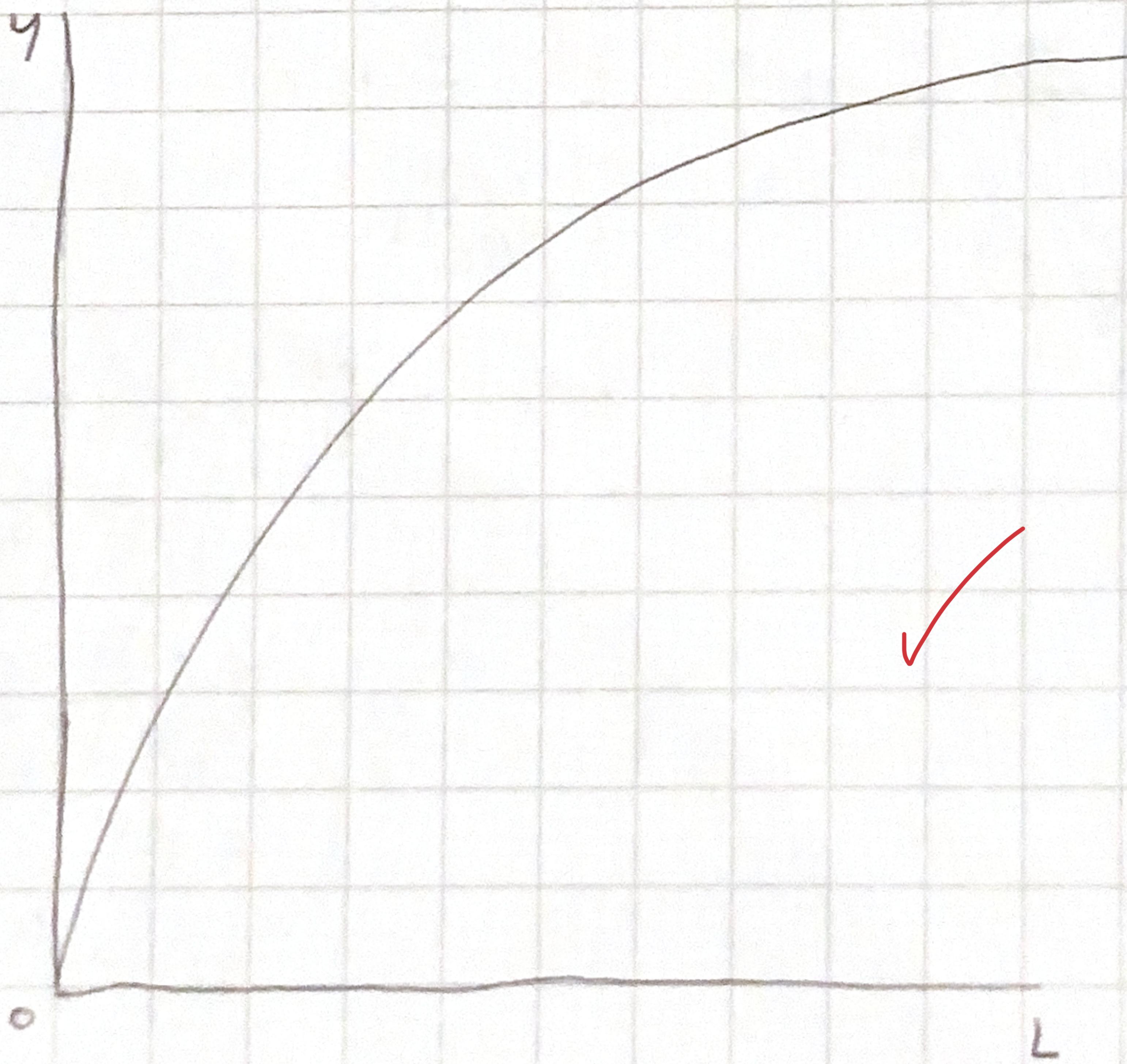
To ~~be~~ a unit of Labour IF you look at the isoquant, if  $K$  is smaller, you need ~~more~~ ~~more~~ ~~you~~ can substitute ~~the~~ 1 unit of Labour with much less Capital than if ~~too~~  $K$  is bigger.

→ substitute in order for what to happen?

Make it clear that this is about keeping output constant.

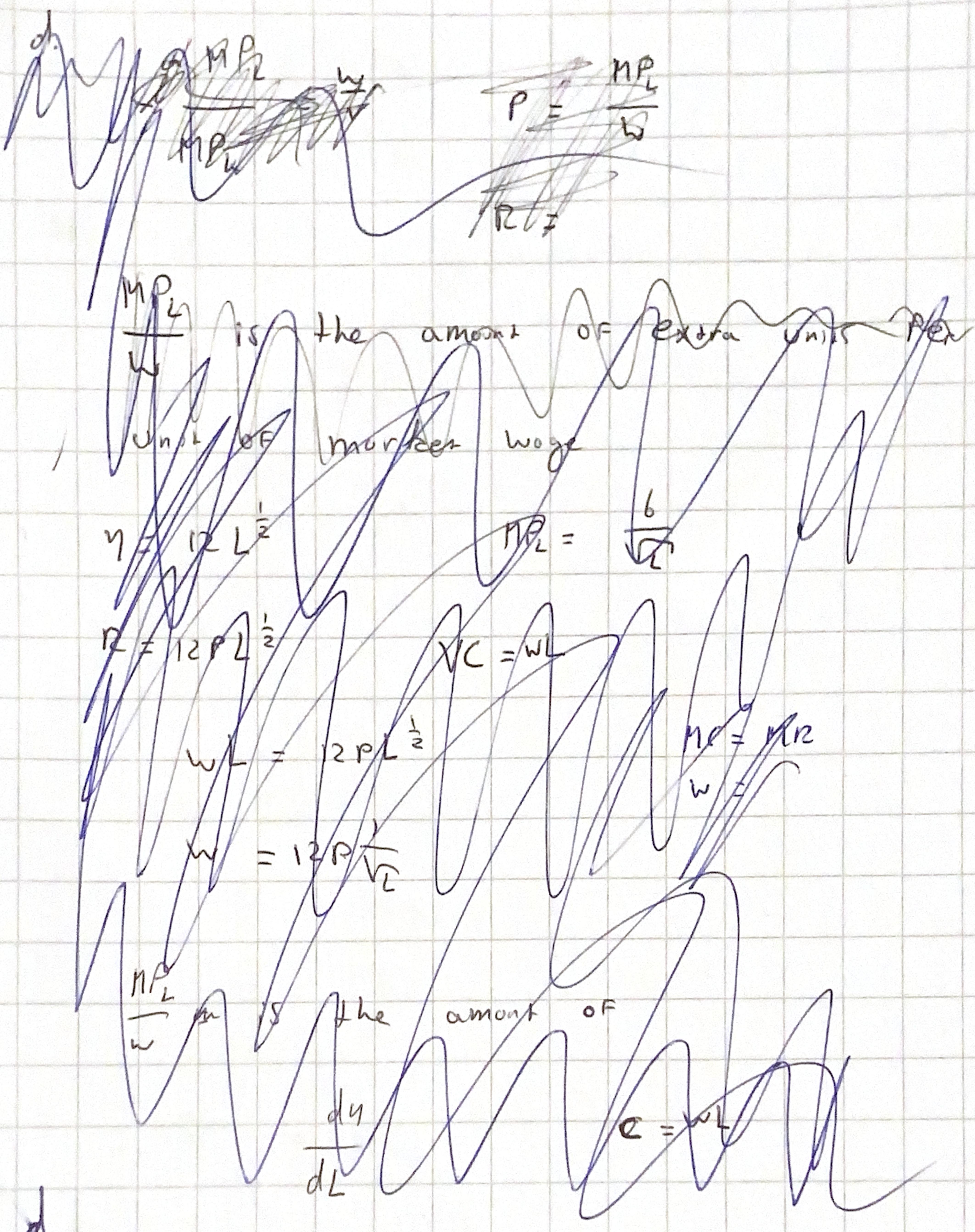
$$C. \bar{K} = a$$

$$y = 6\sqrt{a} L^{\frac{1}{2}} = 12 L^{\frac{1}{2}}$$



$$MP_L = \frac{\partial y}{\partial L} = 6 L^{2\alpha - \frac{1}{2}}$$

$$\frac{dMP_L}{dL} = -3g L^{-\frac{3}{2}} < 0 \text{ so diminishing Product of Labour}$$



d.

$$y = 12 \sqrt{L}$$

$$r = P \cdot \frac{y}{L} - (wL + gr)$$

$$\text{max if } \frac{dr}{dL} = 0 = P \cdot \frac{dy}{dL} - w \rightarrow MP_L$$

$$w = P MP_L$$

$$MP_L = \frac{6}{\sqrt{L}}$$

$$2 = 7 \cdot \frac{6}{\sqrt{L}} \Rightarrow L = 9$$

(4)

$$a. F(\lambda K, \lambda L) = \sqrt{\lambda} V_K + \sqrt{\lambda} V_L$$

$$= \lambda^{\frac{1}{2}} (\sqrt{K} + \sqrt{L})$$

So decreasing returns to scale

Needs explanation.

$$B. MP_L = \frac{\partial F}{\partial L} = \frac{1}{2} L^{-\frac{1}{2}}$$

$$MP_K = \frac{\partial F}{\partial K} = \frac{1}{2} K^{-\frac{1}{2}}$$

$$MRTS = - \frac{MP_L}{MP_K} = - \frac{\frac{1}{2} L^{-\frac{1}{2}}}{\frac{1}{2} K^{-\frac{1}{2}}} = - \frac{\sqrt{L}}{\sqrt{K}}$$

C. ~~Price~~  $\Omega = wR + rwt$

Requires explanation.  
Where does this come from?

$$-\frac{w}{r} = MRTS = -\frac{\sqrt{K}}{\sqrt{L}}$$

$$\frac{w}{r} = \frac{\sqrt{L}}{\sqrt{K}}$$

$$y = \sqrt{K} + \sqrt{L}$$

$$\sqrt{K} = y - \sqrt{L}$$

$$\frac{w}{r} = \frac{y - \sqrt{L}}{\sqrt{L}} = \frac{y}{\sqrt{L}} - 1$$

$$\frac{y}{\sqrt{L}} = \frac{w}{r} + 1$$

$$\frac{\sqrt{L}}{y} = \frac{1}{\frac{w}{r} + 1}$$

$$\sqrt{L} = \frac{y}{\frac{w}{r} + 1}$$

$$L = \left( \frac{y}{\frac{w}{r} + 1} \right)^2$$

$$\text{and } \sqrt{K} = y - \frac{y}{\frac{w}{r} + 1}$$

$$K = \left( y - \frac{y}{\frac{w}{r} + 1} \right)^2$$

$$d. C = rk + wl$$

$$= r \left( y - \frac{y}{\frac{w}{r} + 1} \right)^2 + w \left( \frac{y}{\frac{w}{r} + 1} \right)^2$$

So, if we move out for it produced  
costs

$$= r \left( y^2 - 2 \cdot \frac{y^2}{\frac{w}{r} + 1} + \frac{y^2}{\left( \frac{w}{r} + 1 \right)^2} \right) + w \cdot \frac{y^2}{\left( \frac{w}{r} + 1 \right)^2}$$

$$= y^2 \left( r - \frac{2r}{\frac{w}{r} + 1} + \frac{r}{\left( \frac{w}{r} + 1 \right)^2} + \frac{w}{\left( \frac{w}{r} + 1 \right)^2} \right)$$

So if  $y$  goes up, costs go up  
exponentially

Yes, why is this the case?

6

a.  $C = 16 + 4q^2$

$$MC = \frac{dc}{dq} = 8q$$

$$AVC \text{ or } AC = 4q$$

$\min \{AVC\}$  when  $AVC = MC$

$$4q = 8q$$

$$q = 0$$

So if  $q > 0$  ( $AVC'$  is positive)

$$\boxed{P = 8q}$$

$$\boxed{q = \frac{1}{8}P}$$

~~$y = 180 - 2P$~~

$q = 24 \cdot \frac{1}{8}P = 3P$

How can these two things be true?  
Explain your steps.

where does this come from? It requires explanation  
It does not directly follow from the previous line.

$$y = 180 - 2P$$

$$180 - 2P = 3P$$

$$180 = 5P$$

$$P = 36$$

B. In the long run the price is the minimum LR average cost

$$AC = \frac{16}{q} + 4q$$

$$8q = \frac{16}{q} + 4q$$

$$4q = \frac{16}{q}$$

$$4q^2 = 16$$

$$q = 2$$

$\gamma_{\text{tot}}$

$$\gamma_{\text{agg}} = n \cdot \gamma$$

$\Delta \gamma_{\text{tot}}$

$$\rightarrow P = 8\gamma = 8 \cdot 2 = 16$$

$$2n = 180 - 2P$$

$$2n = 180 - 32$$

$$2n = 148 - 16 = 132$$

OK.

Explain:

② College task sheet

a.  $MC = 2y^2 + 1$

II)  $AC = \frac{2}{3}y^2 + 1 + \frac{36}{y}$

$$2y^2 + 1 = \frac{2}{3}y^2 + 1 + \frac{36}{y}$$

$$\frac{4}{3}y^2 - 1 = \frac{36}{y}$$

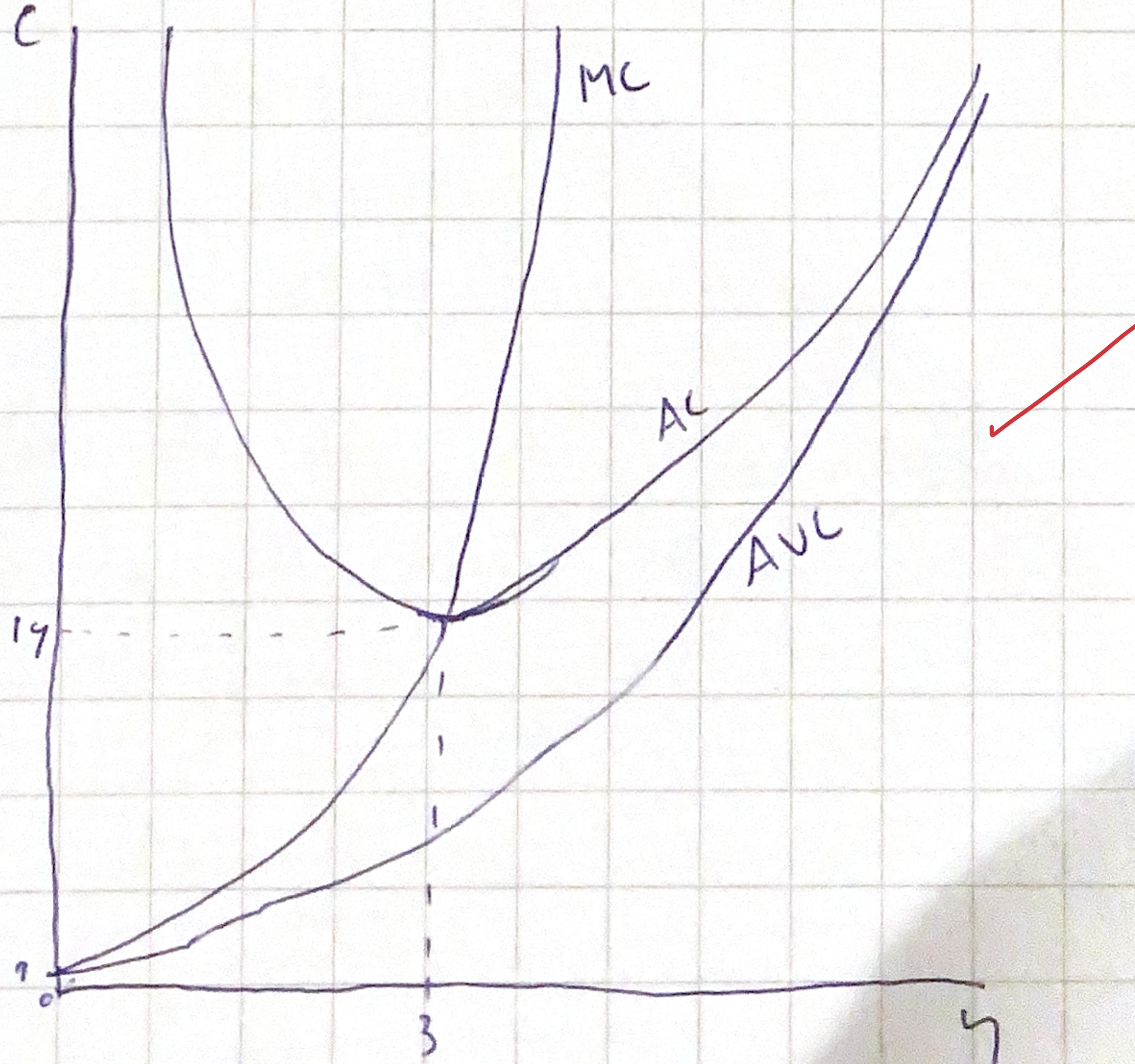
$$\frac{4}{3}y^3 = 36$$

$$y^3 = 27$$

$$y = 3$$

$$MC = 2y^2 + 1 = 2 \cdot 3^2 + 1 = 19 \quad \checkmark$$

III)



$$\text{IV) } AVC = \frac{2}{3}y^2 + 1 \quad MC = 2y^2 + 1$$

$$\frac{2}{3}y^2 + 1 = 2y^2 + 1$$

$$y = 0$$

so for  $y \geq 0$ , the supply is  
the MC:

$$P = MC = 2y^2 + 1$$

$$y = \sqrt{\frac{1}{2}P - \frac{1}{2}}$$

what if  $P < 1$ ?

B.

$$\text{I)} \quad C = Cy + F$$

$$AC = \frac{Cy}{y} + \frac{F}{y} = c + \frac{F}{y}$$

$$\frac{dAC}{dy} = -\frac{F}{y^2} < 0 \text{ so } AC \text{ decreases}$$

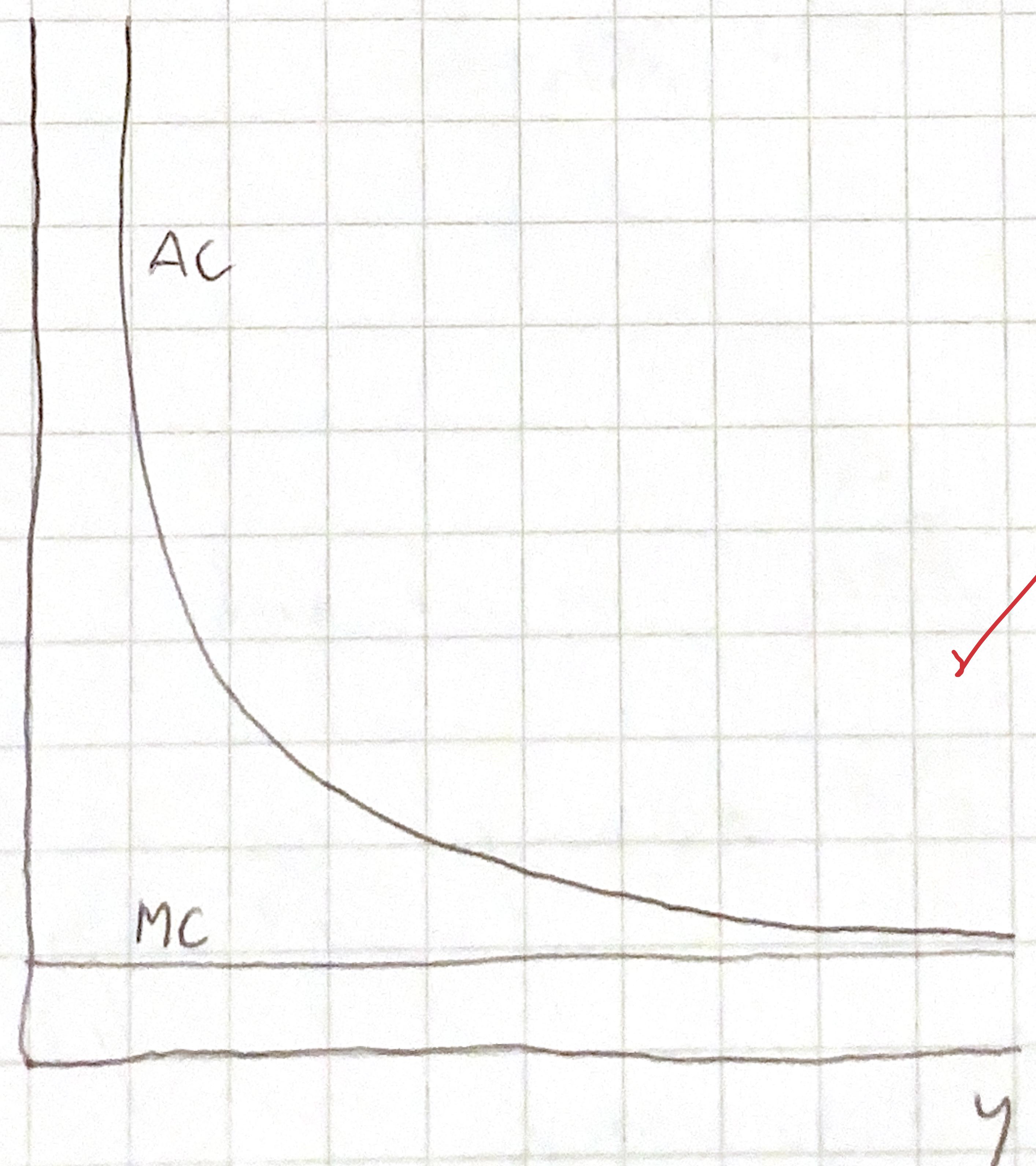
For every  $y (> 0)$

II)

$$MC = c$$

$$AC - MC = c + \frac{F}{y} - c = \frac{F}{y} > 0$$

III)



$$c. \quad MC = \frac{dc}{dy}$$

index 201:

$$\frac{dc}{dy} = \frac{c}{y}$$

$$y = c \cdot \left(\frac{dc}{dy}\right)^{-1}$$

I don't follow the argument  
you've written on this  
so: page.

$$4c = \frac{c}{y} c \cdot \frac{1}{y}$$

$$\text{then } \frac{dMC}{dy} = -\frac{c}{y^2} = 0$$

$$\text{and } \frac{d^2MC}{dy^2} = 2 \frac{c}{y^3} < 0$$

has to be:

$$\frac{dMC}{dy} = \frac{dc}{dy} \cdot \frac{1}{y} + -c \frac{1}{y^2}$$

$$= \frac{dc}{dy} \frac{1}{y} - \frac{c}{y^2} = 0$$

$$0 = \frac{dc}{dy} \cdot \frac{1}{c} - \frac{c}{(c(\frac{dc}{dy})^{-1})^2}$$

Also

$$\frac{c}{c^2 (\frac{dc}{dy})^2} = \frac{\frac{dc}{dy}}{dy} \frac{\frac{dc}{dy}}{c}$$

$$\frac{\left(\frac{dc}{dy}\right)^2}{c} = \frac{\left(\frac{dc}{dy}\right)^2}{c}$$

so it works

Also:  $\frac{d^2MC}{dy^2}$  ~~to do~~ for it to be  
a minimum

$$\frac{d^2MC}{dy^2} = \frac{dc}{dy^2} \cdot \frac{1}{y} + \frac{dc}{dy} \cdot -\frac{1}{y^2} \quad \text{or} \quad \frac{dc}{dy} \cdot \frac{1}{y^2} + c \cdot -2 \frac{1}{y^3}$$

$$= \frac{dc}{dy^2} \cdot \frac{1}{y} - \frac{dc}{dy} \frac{1}{y^2} + \frac{2c}{y^3} = \frac{d^2c}{dy^2} \frac{1}{y} - 2 \frac{dc}{dy} \frac{1}{y^2} + \frac{2c}{y^3}$$