① a $U_1 = W_0 + kA \log(1 + e_1 + e_2) - e_1$

outstanding.

B. $\max_{e_1} \; U_1 = W_0 + kA \log(1 + e_1 + \bar{e}_2) - e_1$

$$\frac{\partial U_1}{\partial \bar{e}_1} = \frac{kA}{1 + e_1 + \bar{e}_2} - 1 = 0$$

(set to 0 to optimize)

$$kA = 1 + e_1 + \bar{e}_2$$

$$e_1 = kA - 1 - \bar{e}_2$$

C the workers are identical so $e_1 = \bar{e}_2$:

$$e_1 = kA - 1 - e_1$$

$$e_1 = \frac{kA - 1}{2}$$

You're told to look for a symmetric equlibrium.

Symmetric implies $e_1 = e_2$. So you can take this as given.

Equilibrium implies $e_i = \bar{e}_i$.

Together with $e_i = kA - 1 - \bar{e}_j$, you get your solution.

Do I need to proof this? Is there a proof? (It's the only way to do it I think because if you do it the other way around and get $e_2 = ka - 1 - e_1$ it is the same equation. So 2 unknowns with 1 equation.)

d. If we merge the utility of the workers:

$$U_{merge} = 2w_0 + 2kA\log(1+e_1+e_2) - (e_1+e_2)$$

$\max_{e_1,e_2} U_{merge}$

$$\frac{\partial U_{merge}}{\partial e_1} = \frac{2kA}{1+e_1+e_2} - 1 = 0 \quad \text{(set to 0 to maximize)}$$

$$2kA = 1+e_1+e_2$$

$$e_1+e_2 = 2kA - 1$$

~~With the~~ With the Nash equilibrium, $e_1+e_2 = kA-1$

$$U_{Nash,merged} = 2W_0 + 2kA\log(1+kA-1) - kA + 1$$

$$U_{merged} = 2W_0 + 2kA\log(2kA) - 2kA + 1$$

$$U_{merged} - U_{Nash,merged} = (2kA\log(2kA) - kA) - 2kA\log(kA)$$

$$= 2kA\log(2) + 2kA\log(kA) - kA - 2kA\log(kA)$$

$$= 2kA\log(2) - kA$$

$$= kA(\log(4) - 1)$$

$$= kA\log(4/e) > 0$$

Because $kA > 1$ and $\log(4/e) \approx 0{,}3$

So $U_{merged} > U_{merged,Nash}$ so the two workers can obtain a higher utility. Good

② $\pi = P_F \dfrac{F(b)}{b} - P_F \cdot \dfrac{1}{4}$

B. that means that people will send boats until profits are 0, so until

$$P_F \frac{F(\hat{b})}{\hat{b}} - P_F \frac{1}{4} = 0$$

$$P_F \left(\frac{F(\hat{b})}{\hat{b}} - \frac{1}{4} \right) = 0$$

$$\cancel{P_F = 0} \quad \vee \quad \frac{F(\hat{b})}{\hat{b}} = \frac{1}{4} \Rightarrow \hat{b} = 4\, F(\hat{b})$$

So until either the price of fish is zero or until the number of boats is 4 times the amount of fish caught. However, that also means that eventually, everyone will make 0 profit.

c. $\max\limits_{b^*} \quad \pi = P_F\, F(b^*) - \frac{P_F}{4} b^*$

$$0 = \frac{\partial \pi}{\partial b^*} = P_F\, F'(b^*) - \frac{P_F}{4}$$

(set to 0 to maximize)

$$P_F = 0 \quad \vee \quad F'(b^*) = \frac{1}{4}$$

So it will continue sending out boats until the amount of extra fish caught is $\frac{1}{4}$ of a ton.

d. Profits will be maximized so it is ~~the~~ Pareto-efficient. Compared to the decentralised case, everyone will keep sending out ~~Bo~~ boats until profits are 0, so even if they would share the profit and go to this Pareto-efficient case, it would be a Pareto improvement, everyone would be better off.

e. $b^{\wedge} = 4F(b) = 4 \cdot 8 \cdot b^{\frac{1}{4}}$

$b^{\frac{3}{4}} = 32$

$b^{\wedge} \approx 101,6$

$F'(b^*) = \frac{1}{4}$

$2b^{-\frac{3}{4}} = \frac{1}{4}$

$b^{*-\frac{3}{4}} = \frac{1}{8}$

$b^* = 16$

F. the tax will make sure that profits are 0 when $b = 16$, so:

$$\pi = P_F \frac{8b^{\frac{1}{4}}}{b} - \frac{P_F}{4} - t = 0$$

$$P_F \cdot 8b^{-\frac{3}{4}} - \frac{P_F}{4} = t$$

$$t = P_F \cdot 8 \cdot 16^{-\frac{3}{4}} - \frac{P_F}{4}$$

$$t = \frac{3}{4} P_F$$

③ a. $m_S = 0$ and as she will maximize her utility and U_S is an increasing function,

$D = 100$ so $U_S = 10 \cdot \sqrt{100} + 0 = 100$

B. By merging utility functions:

$$\max_D U_{merge} = 10 D^{\frac{1}{2}} + m_S + 10(100 - D)^{\frac{1}{2}} + m_R$$

$$\frac{\partial U_{merge}}{\partial D} = 5D^{-\frac{1}{2}} - 5(100 - D)^{-\frac{1}{2}} = 0$$ (set to 0 to maximize)

$$D^{-\frac{1}{2}} = (100 - D)^{-\frac{1}{2}}$$

$$D = 100 - D$$

$$D = 50$$

c. If P is the amount she pays, then in order for the ~~student~~ fellow to be indifferent to paying or to not paying:

$$10(100-50)^{\frac{1}{2}} + 100 - P = 10(100-100)^{\frac{1}{2}} + 100$$

$$10\sqrt{50} + 100 - P = 100$$

$$P = 10\sqrt{50} = 50\sqrt{2} \approx 70,7$$

So that is the maximum.

If she pays 50, then:

$$U_S = 10 \cdot \sqrt{50} + 50 \approx 120,7 > 100$$

So the student would be better off by taking the money.