(5)

a. $Y = 6K^{\frac{1}{2}}L^{\frac{1}{2}}$

$6(\lambda \cdot K)^{\frac{1}{2}} \cdot (\lambda L)^{\frac{1}{2}} = 6 \cdot \lambda^{\frac{1}{2}} \cdot K^{\frac{1}{2}} \cdot \lambda^{\frac{1}{2}} \cdot L^{\frac{1}{2}}$

$= 6\lambda^{1} K^{\frac{1}{2}} L^{\frac{1}{2}}$

So it has a constant returns to scale

why? What does CRS mean? Define it.

B. $Y = 6K^{\frac{1}{2}}L^{\frac{1}{2}}$

$\cancel{K^{\frac{1}{2}} = \frac{Y}{6} L^{-\frac{1}{2}}}$

$K = \frac{Y^2}{36} L^{-2}$

$K = \left(\frac{Y}{6}\right)^2 L^{-1}$

$L^{\frac{1}{2}} = \frac{Y}{6} K^{-\frac{1}{2}}$

$L = \left(\frac{Y^2}{36}\right) K^{-1}$

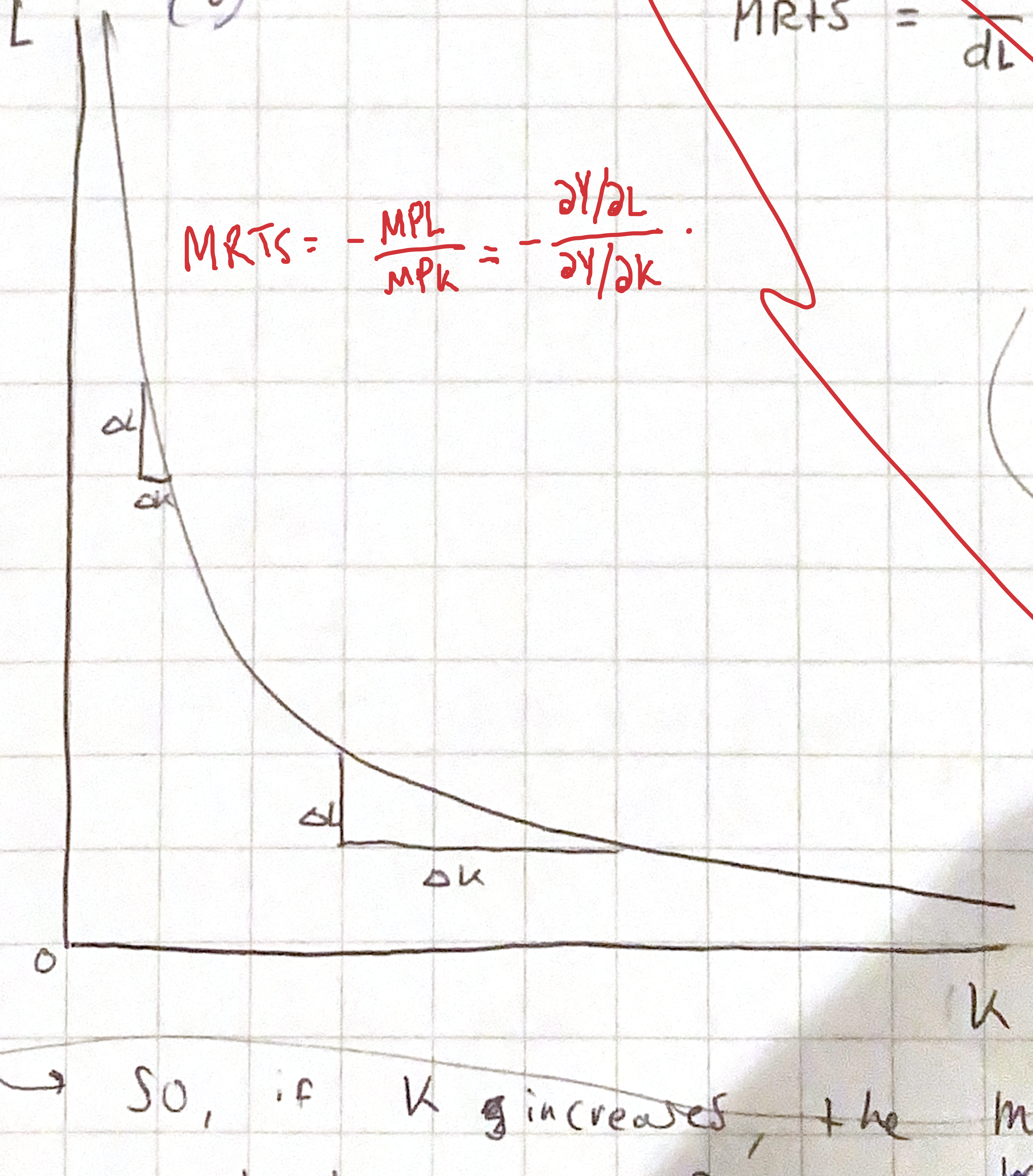
See in class.

$MRTS = \frac{dK}{dL} = -\left(\frac{Y}{6}\right)^2 L^{-2}$??

$= -\left(\frac{Y}{6}\right)^2 \left(\frac{Y}{6} K^{-1}\right)^{-2}$

$= -\frac{\left(\frac{Y}{6}\right)^2}{\left(\frac{Y}{6}\right)^2 K^{-2}} = -\frac{1}{\frac{Y}{6}K^{-2}}$

$= -\frac{\left(\frac{Y}{6}\right)^2}{\left(\frac{Y}{6}\right)^2} K^2 = -K^2$



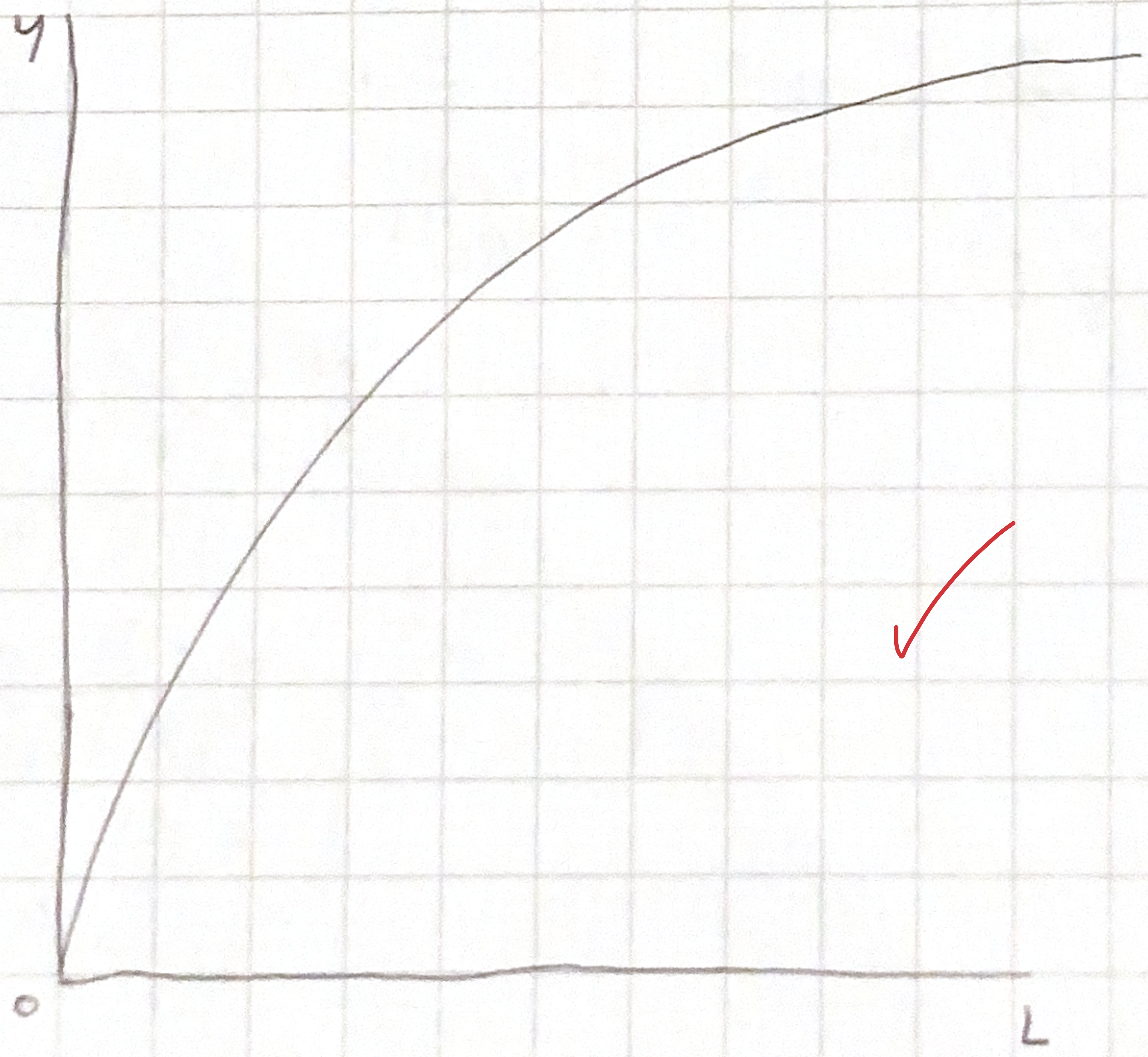
$MRTS = -\frac{MPL}{MPK} = -\frac{\partial Y/\partial L}{\partial Y/\partial K}$

So, if K increases, the marginal rate of technical substitution ~~decreases~~ increases this makes sense, if K is larger, more and more capital is needed to substitute a unit of labour. If you look at the isoquant, if K is smaller, you can substitute 1 unit of labour with much less capital than if K is bigger.

substitute in order for what to happen?
Make it clear that this is about keeping output constant.

c. $\bar{K} = 4$

$$y = 6\sqrt{4}\,L^{\frac{1}{2}} = 12L^{\frac{1}{2}}$$



$$MP_L = \frac{\partial y}{\partial L} = 6L^{-\frac{1}{2}}$$

$$\frac{dMP_L}{dL} = -3L^{-\frac{3}{2}} < 0$$ so diminishing product of labour

~~d.~~

$\cancel{\frac{MP_L}{MP_L} \quad \frac{w}{r}}$ $\quad$ $\cancel{P = \frac{MP_L}{w}}$

~~$\frac{MP_L}{w}$ is the amount of extra units per unit of worker wage~~

$\cancel{y = 12 L^{\frac{1}{2}}}$ $\quad$ $\cancel{MP_L = \frac{6}{\sqrt{L}}}$

$\cancel{\pi = 12 p L^{\frac{1}{2}}}$ $\quad$ $\cancel{VC = wL}$

$\cancel{wL = 12 p L^{\frac{1}{2}}}$ $\quad$ $\cancel{MC = MR}$

$\cancel{w = 12 p \frac{1}{\sqrt{L}}}$ $\quad$ $\cancel{w =}$

~~$\frac{MP_L}{w}$ is the amount of~~

$\cancel{\frac{dy}{dL}}$ $\quad$ $\cancel{C = wL}$

d.

$$y = 12\sqrt{L}$$

$$\pi = p \cdot y - (wL + 4r)$$

max if $\frac{d\pi}{dL} = 0 = p \frac{dy}{dL} - w$ $\quad$ $\frac{dy}{dL} \to MP_L$

$$w = p\, MP_L$$

$MP_L = \frac{6}{\sqrt{L}}$ $\quad$ $2 = 1 \cdot \frac{6}{\sqrt{L}} \Rightarrow L = 9$

(4)

a. $F(\lambda K, \lambda L) = \sqrt{\lambda}\sqrt{K} + \sqrt{\lambda}\sqrt{L}$

$= \lambda^{\frac{1}{2}}(\sqrt{K} + \sqrt{L})$

So decreasing returns to scale

Needs explanation.

B. $MP_L = \frac{\partial F}{\partial L} = \frac{1}{2}L^{-\frac{1}{2}}$

$MP_K = \frac{\partial F}{\partial K} = \frac{1}{2}K^{-\frac{1}{2}}$

$MRTS = -\frac{MP_L}{MP_K} = -\frac{\frac{1}{2}L^{-\frac{1}{2}}}{\frac{1}{2}K^{-\frac{1}{2}}} = -\frac{\sqrt{K}}{\sqrt{L}}$ ✓

C. ~~Q = rK + wL~~

$$\boxed{-\frac{w}{r} = MRTS = -\frac{\sqrt{K}}{\sqrt{L}}}$$

Requires explanation. Where does this come from?

$\frac{w}{r} = \frac{\sqrt{K}}{\sqrt{L}}$

$y = \sqrt{K} + \sqrt{L}$

$\sqrt{K} = y - \sqrt{L}$

$\frac{w}{r} = \frac{y - \sqrt{L}}{\sqrt{L}} = \frac{y}{\sqrt{L}} - 1$

$\frac{y}{\sqrt{L}} = \frac{w}{r} + 1$

$\frac{\sqrt{L}}{y} = \frac{1}{\frac{w}{r} + 1}$

$\sqrt{L} = \frac{y}{\frac{w}{r} + 1}$ and $\sqrt{K} = y - \frac{y}{\frac{w}{r} + 1}$

$L = \left(\frac{y}{\frac{w}{r} + 1}\right)^2$ ✓ $K = \left(y - \frac{y}{\frac{w}{r} + 1}\right)^2$ ✓

d. $C = rK + wL$

$$= r\left(y - \frac{y}{\frac{w}{r}+1}\right)^2 + w\left(\frac{y}{\frac{w}{r}+1}\right)^2$$

~~So, if more output is produced,~~ costs

$$= r\left(y^2 - 2\frac{y^2}{\frac{w}{r}+1} + \frac{y^2}{\left(\frac{w}{r}+1\right)^2}\right) + w \cdot \frac{y^2}{\left(\frac{w}{r}+1\right)^2}$$

$$= y^2\left(r - \frac{2r}{\frac{w}{r}+1} + \frac{r}{\left(\frac{w}{r}+1\right)^2} + \frac{w}{\left(\frac{w}{r}+1\right)^2}\right)$$

So if y goes up, costs go up exponentially ✓

Yes, why is this the case?

6

a.

$C = 16 + 4y^2$

$MC = \frac{dc}{dy} = 8y$

$AVC = 4y$

min AVC when $AVC = MC$

$4y = 8y$

$y = 0$

So if $y > 0$ (AVC' is positive)

$\boxed{P = 8y}$

where does this come from? It require explanation It does not directly follow from the previous line.

$y = \frac{1}{8}P$

~~$y = 2\frac{1}{8}P$~~

$y = 24 \cdot \frac{1}{8}P = 3P$

How can these two things be true? Explain your steps.

$y = 150 - 2P$

$150 - 2P = 3P$

$150 = 5P$

$P = 30$

B. In the long run the Price is the minimum LR average cost

$AC = \frac{16}{y} + 4y$

$8y = \frac{16}{y} + 4y$

$4y = \frac{16}{y}$

$4y^2 = 16$

$y = 2$

$\frac{y}{x}$ ~~y_{tot}~~

Explain: $y_{agg} = n \cdot y$

~~...~~ $\rightarrow P = 8y = 8 \cdot 2 = 16$

$2n = 150 - 2P$

$2n = 150 - 32$

$n = 75 - 16 = 59$

ok.

② College tute sheet

a.

$MC = 2y^2 + 1$

II) $AC = \frac{2}{3}y^2 + 1 + \frac{36}{y}$

$2y^2 + 1 = \frac{2}{3}y^2 + 1 + \frac{36}{y}$

$\frac{4}{3}y^2 = \frac{36}{y}$

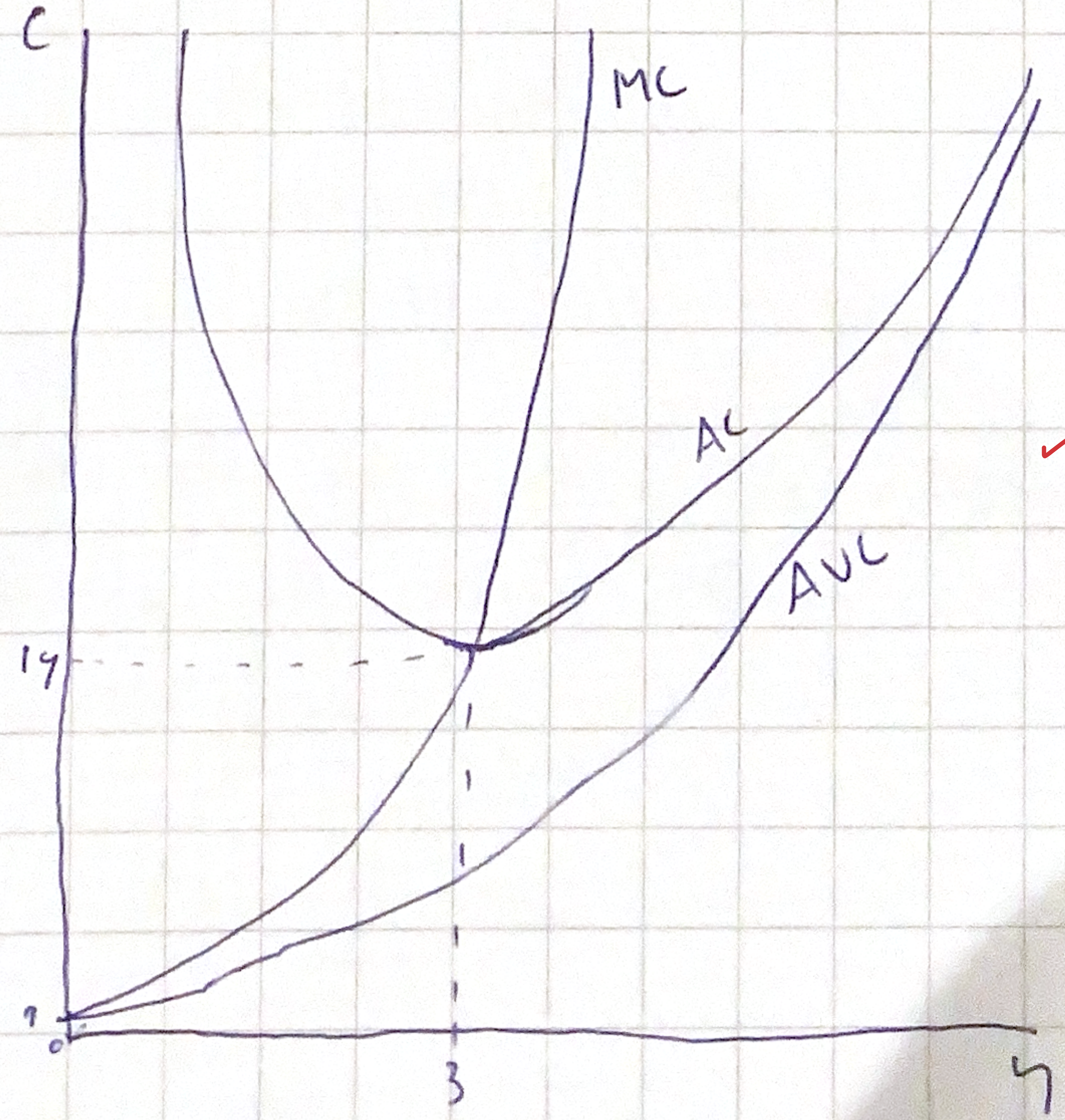
$\frac{4}{3}y^3 = 36$

$y^3 = 27$

$y = 3$

$MC = 2y^2 + 1 = 2 \cdot 9 + 1 = 19$ ✓

III)



✓

IV) $AVC = \frac{2}{3}y^2 + 1$ $\quad MC = 2y^2 + 1$

$$\frac{2}{3}y^2 + 1 = 2y^2 + 1$$

$$y = 0$$

so for $y \geq 0$, the supply is the MC:

$$P = MC = 2y^2 + 1$$

$$y = \sqrt{\frac{1}{2}P - \frac{1}{2}}$$ what if $p < 1$?

8.

I) $C = cy + F$

$AC = \frac{cy}{y} + \frac{F}{y} = c + \frac{F}{y}$

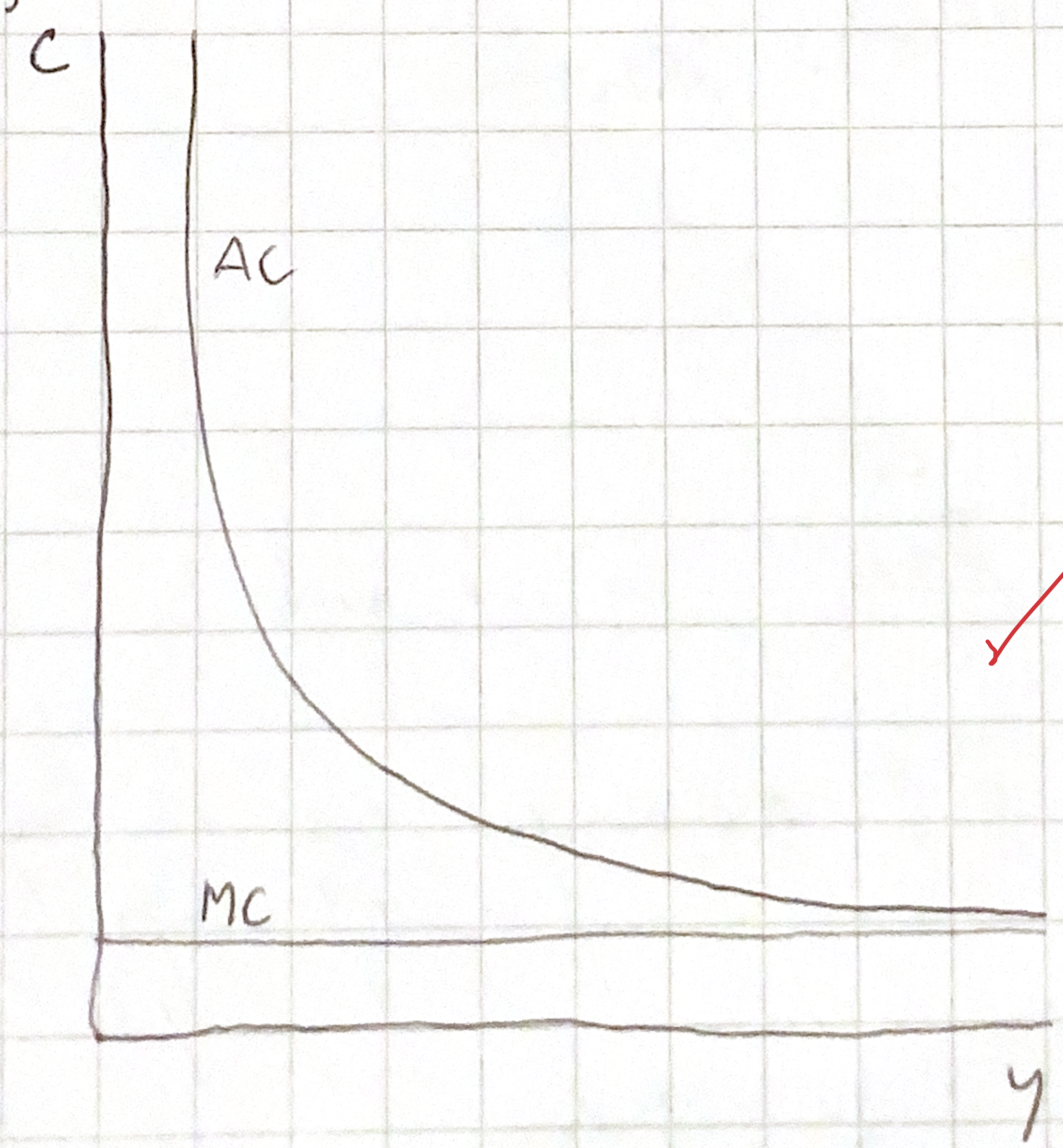
$\frac{dAC}{dy} = -\frac{F}{y^2} < 0$ so AC decreases

for every $y \ (>0)$

II) $MC = c$

$AC - MC = c + \frac{F}{y} - c = \frac{F}{y} > 0$

III)



c. $MC = \frac{dc}{dy}$

$AC = c \cdot \frac{1}{y}$

has to be: $\frac{dAC}{dy} = -\frac{c}{y^2} = 0$

and $\frac{d^2AC}{dy^2} = 2\frac{c}{y^3} < 0$

intersect:

$\frac{dc}{dy} = \frac{c}{y}$

$y = c \cdot \left(\frac{dc}{dy}\right)^{-1}$

I don't follow the argument you've written on this page.

so:

$0 = \frac{dc}{dy} \cdot \frac{1}{c\left(\frac{dc}{dy}\right)^{-1}} - \frac{c}{\left(c\left(\frac{dc}{dy}\right)^{-1}\right)^2}$

$\frac{c}{c^2\left(\frac{dc}{dy}\right)^{-2}} = \frac{dc}{dy} \frac{\frac{dc}{dy}}{c}$

$\frac{\left(\frac{dc}{dy}\right)^2}{c} = \frac{\left(\frac{dc}{dy}\right)^2}{c}$ so it works

has to be:

$\frac{dAC}{dy} = \frac{dc}{dy} \cdot \frac{1}{y} + -c\frac{1}{y^2}$

$= \frac{dc}{dy}\frac{1}{y} - \frac{c}{y^2} = 0$

that

Also: $\frac{d^2AC}{dy^2} > 0$ for it to be a minimum

so.

$\frac{d^2AC}{dy^2} = \frac{d^2c}{dy^2} \cdot \frac{1}{y} + \frac{dc}{dy} \cdot -\frac{1}{y^2} - \left(\frac{dc}{dy} \cdot \frac{1}{y^2} + c \cdot -2\frac{1}{y^3}\right)$

$= \frac{d^2c}{dy^2}\frac{1}{y} - \frac{dc}{dy}\frac{1}{y^2} - \frac{dc}{dy}\frac{1}{y^2} + \frac{2c}{y^3} = \frac{d^2c}{dy^2}\frac{1}{y} - 2\frac{dc}{dy}\frac{1}{y^2} + \frac{2c}{y^3}$ pa