Dealing with interference in random wireless networks

Matthew Aldridge
University of Leeds

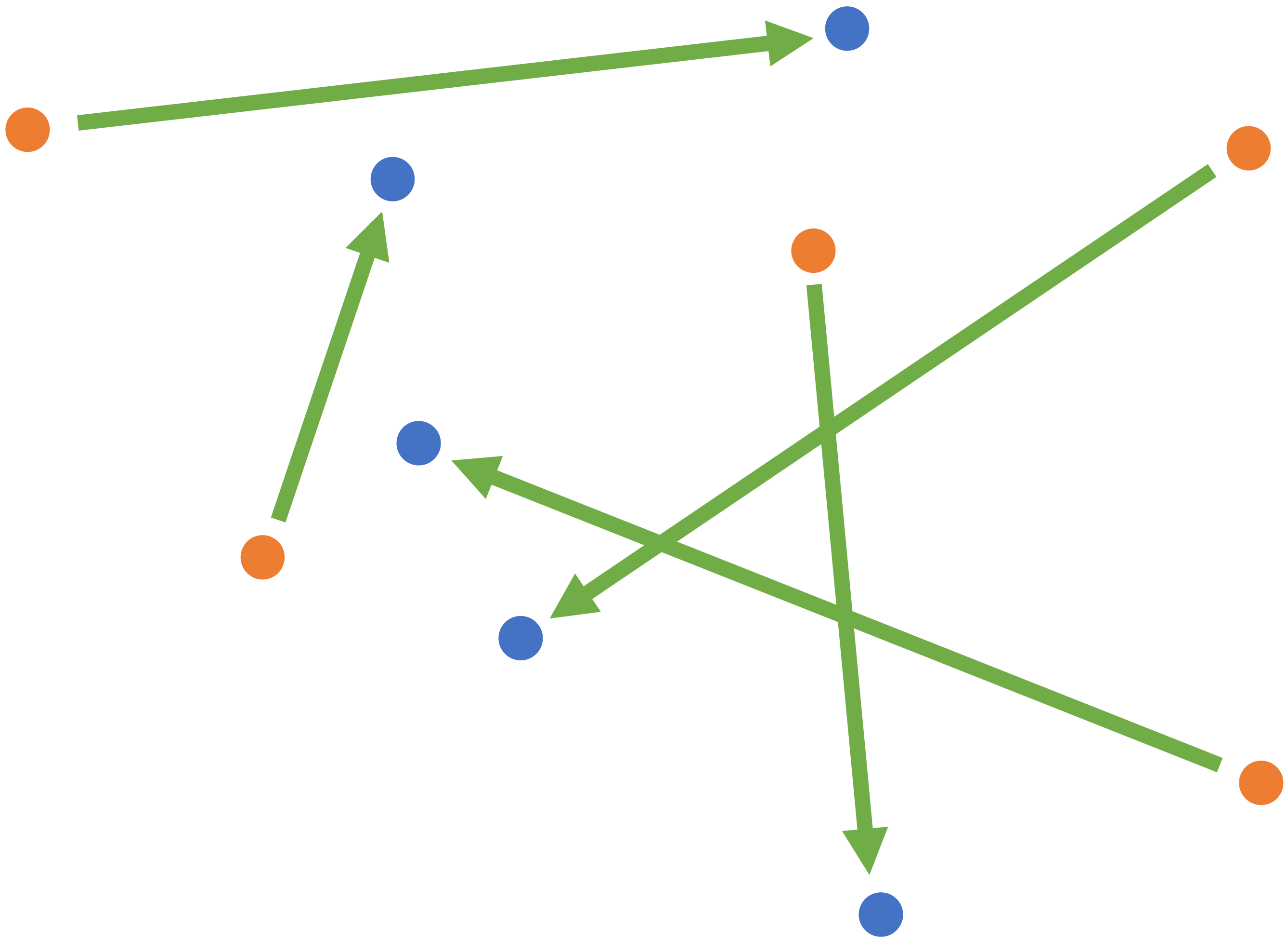
joint work with
Oliver Johnson and Robert Piechocki

supported by
Toshiba Research Europe

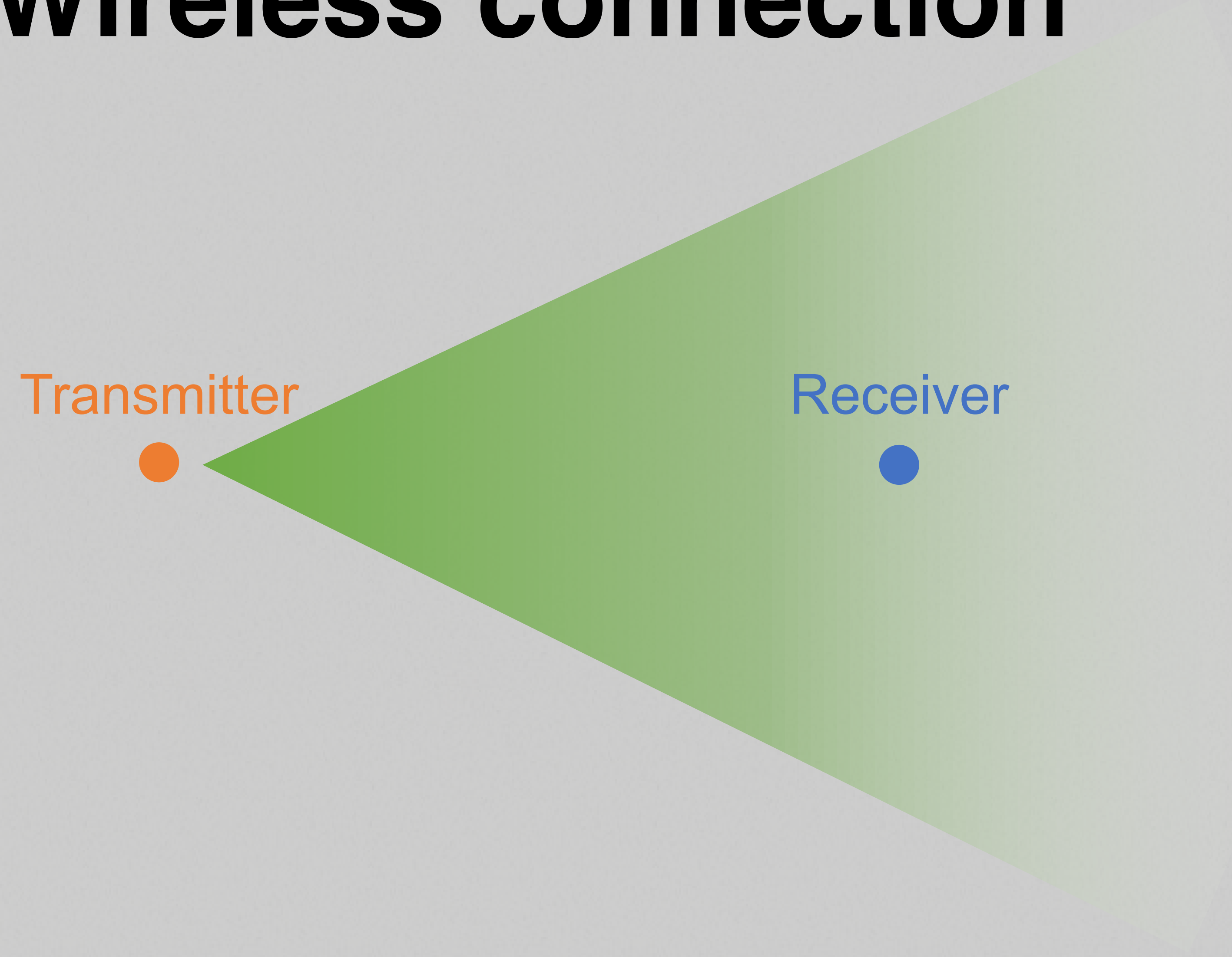
Stochastic Models in Risk Analysis and Queuing
University of Leeds, February 2019

Wired connection

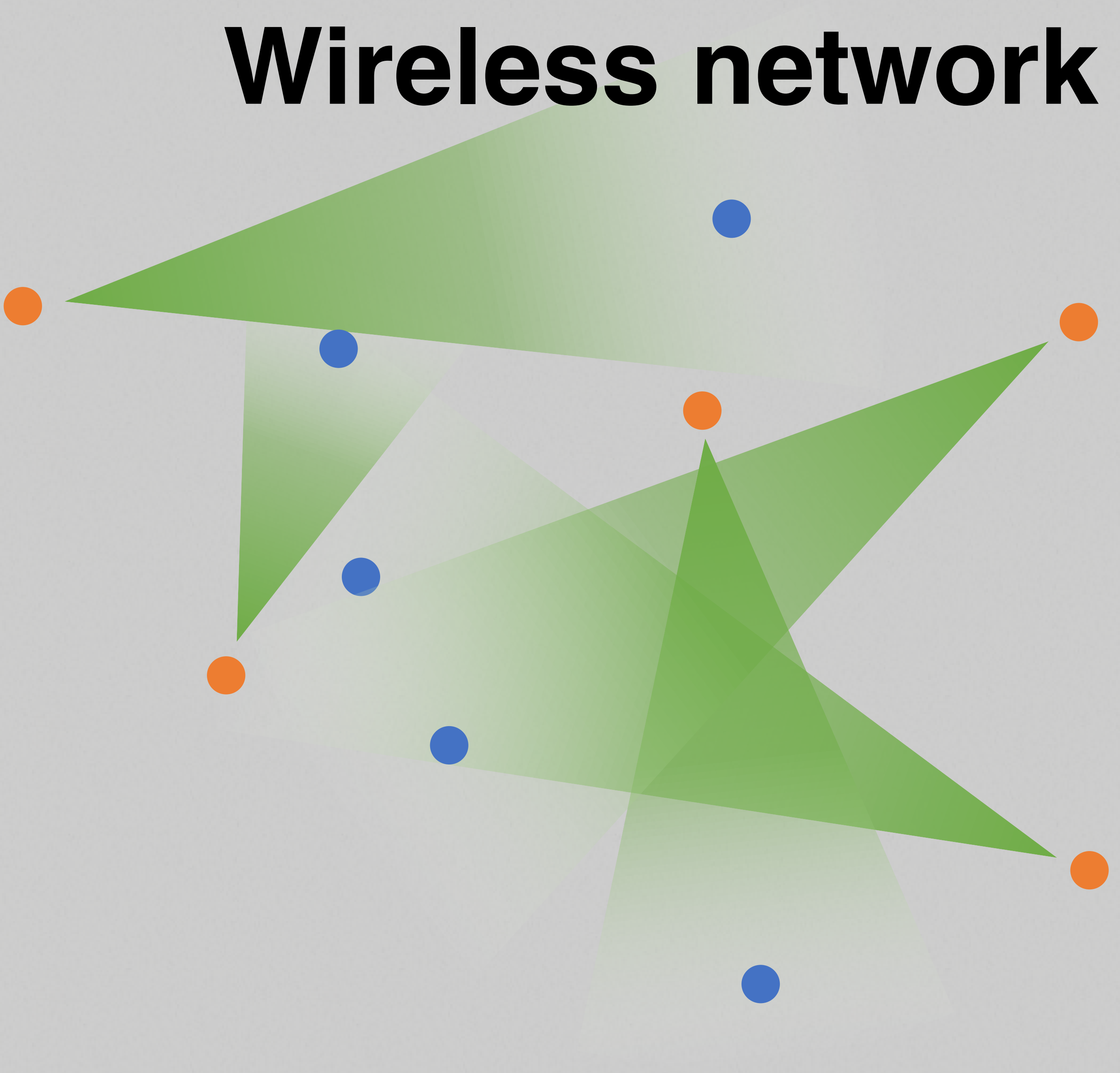
Wired network



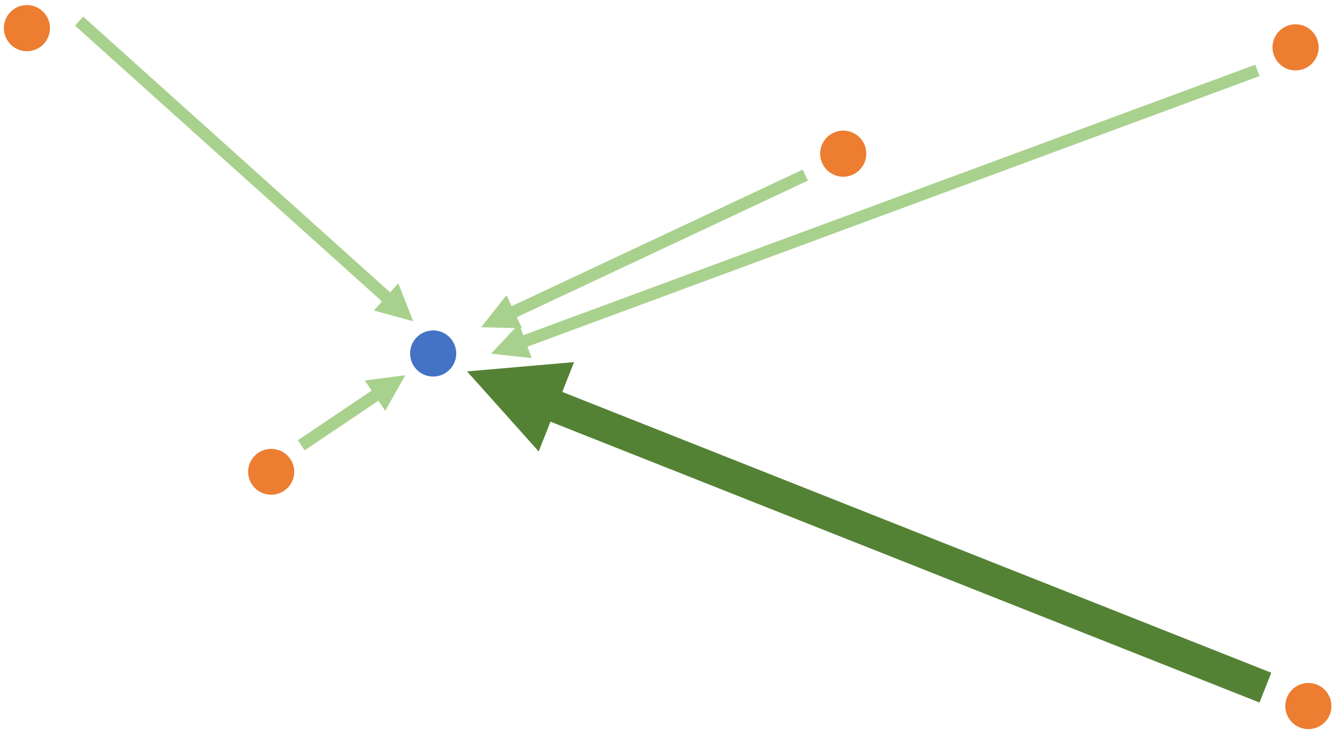
Wireless connection



Wireless network



Wireless network



Two conclusions

Relatively few **“bottleneck” interfering links** tightly bound the total capacity of a network

Planning transmissions so that **interference “aligns”** at each transmitter allows performance close to the bottleneck bound

A1 Single-user channels

A2 Interference networks

B1 Interference alignment

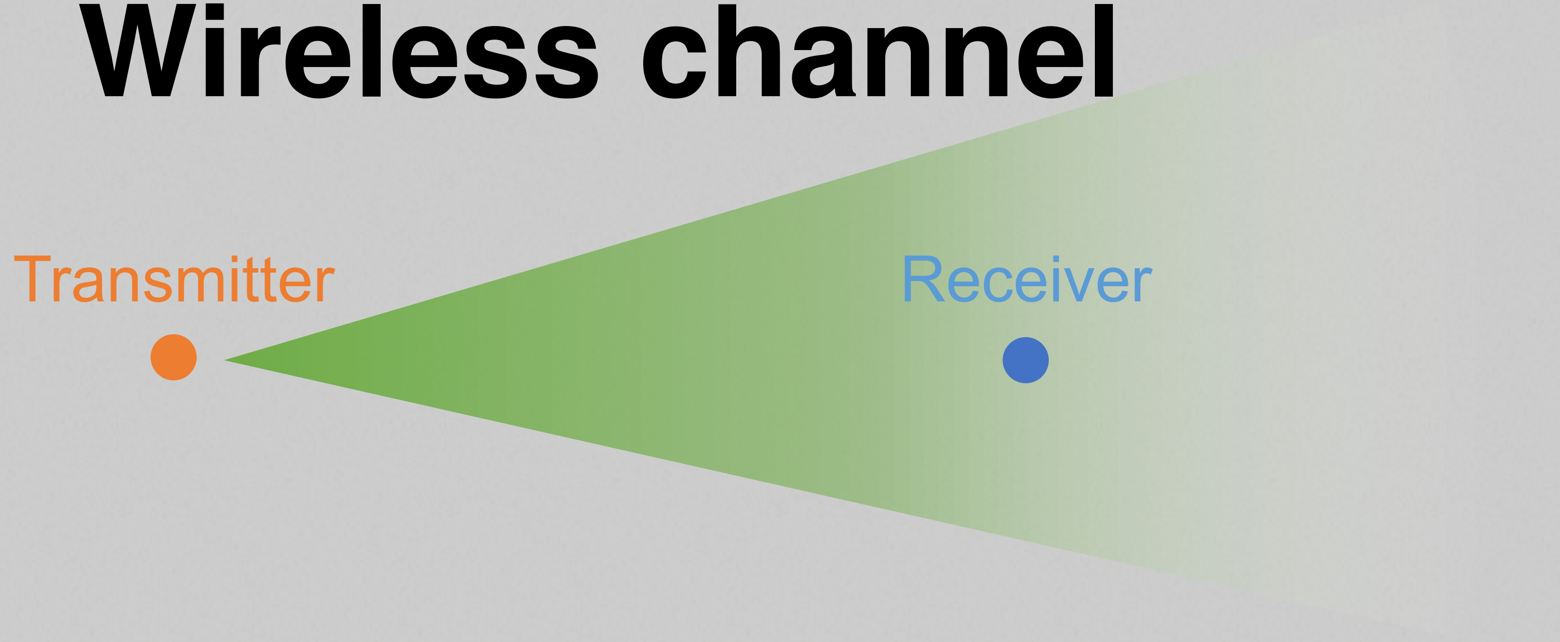
B2 Bottleneck links

C1 Jafar network

C2 Standard dense network

A1
Single-user channels

Wireless channel



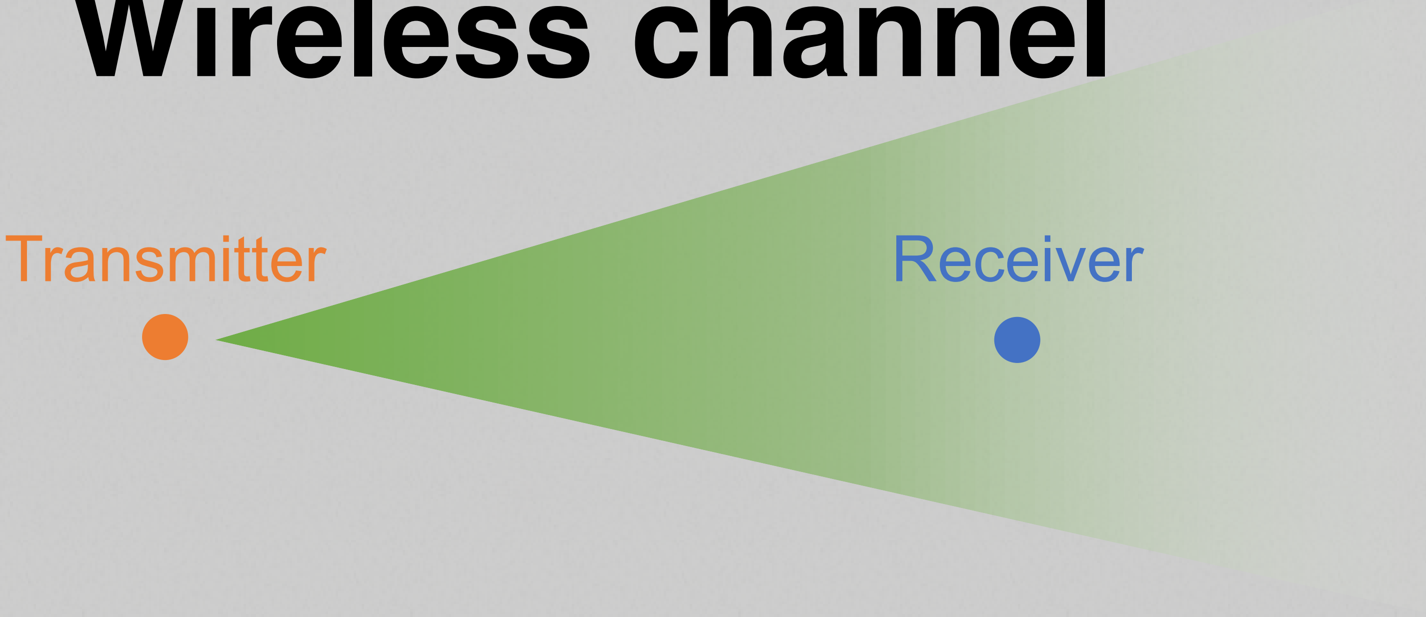
Channel: $Y[t] = H[t]x[t] + Z[t]$

Y, H, x, Z all in $\mathbb{C}$

We assume is $|H[t]|^2$ constant in t

$t = 1, 2, \ldots, T$ indexes channel use (time)

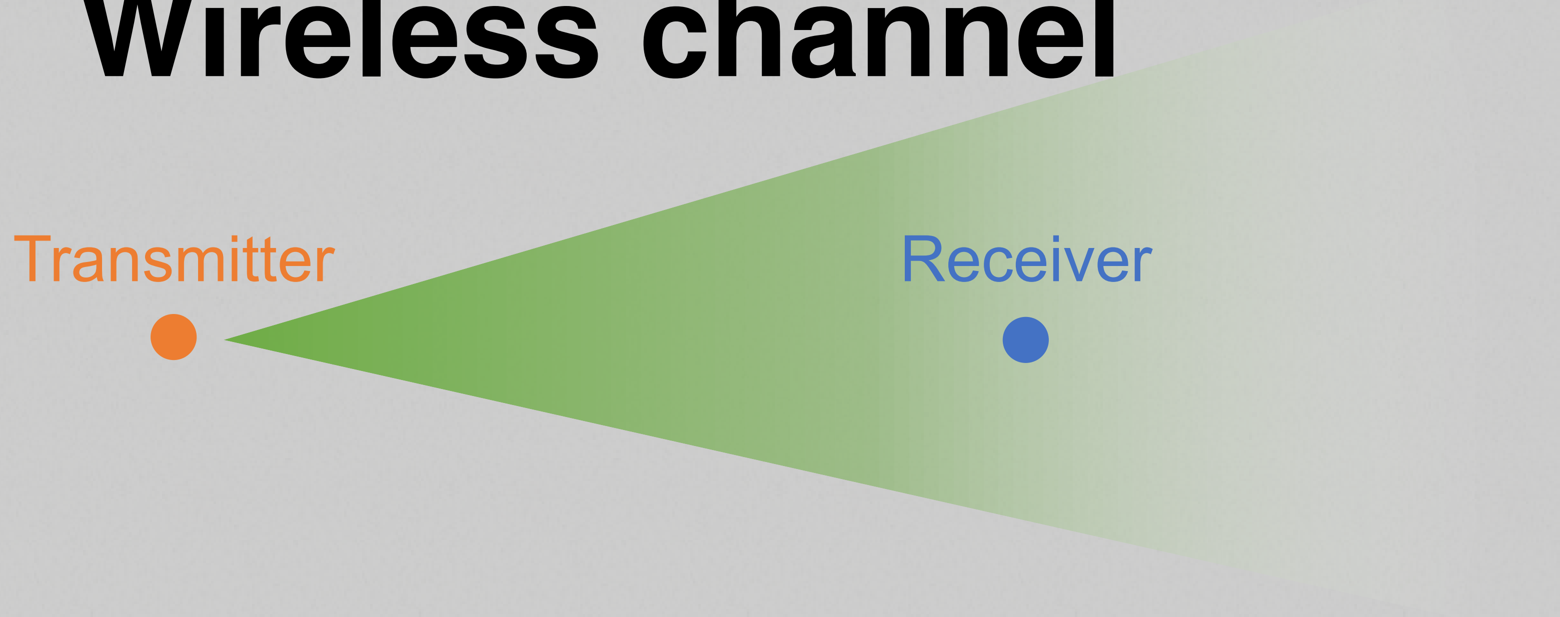
Wireless channel



Channel: $Y[t] = H[t]x[t] + Z[t]$

Background noise: $Z[t] \sim \mathbb{C}\mathrm{N}(0, \sigma^2)$ IID

Wireless channel



Channel: $Y[t] = H[t]x[t] + Z[t]$

Background noise: $Z[t] \sim \mathbb{C}\mathrm{N}(0, \sigma^2)$ IID

Power constraint: $\frac{1}{T}\sum_{t=1}^{T} |x[t]|^2 \leq P$

Wireless channel

$$Y[t] = H[t]x[t] + Z[t]$$

Capacity

the highest rate
at which one can communicate
with arbitrarily low probability of error

Wireless channel

$$Y[t] = H[t]x[t] + Z[t]$$

Capacity

the highest rate
at which one can communicate
with arbitrarily low probability of error

$$C = \log_2\left(1 + \frac{|H|^2 P}{\sigma^2}\right)$$

(Shannon, 1948)

Wireless channel

$$Y[t] = H[t]x[t] + Z[t]$$

transmitted power

$$C = \log_2\left(1 + \frac{|H|^2 P}{\sigma^2}\right)$$

Wireless channel

$$Y[t] = H[t]x[t] + Z[t]$$

received power

$$C = \log_2\left(1 + \frac{|H|^2 P}{\sigma^2}\right)$$

Wireless channel

$$Y[t] = H[t]x[t] + Z[t]$$

received power

$$C = \log_2\left(1 + \frac{|H|^2 P}{\sigma^2}\right)$$

noise power

Wireless channel

$$Y[t] = H[t]x[t] + Z[t]$$

signal-to-noise ratio

$$C = \log_2(1 + \mathrm{SNR})$$

A2
Interference networks

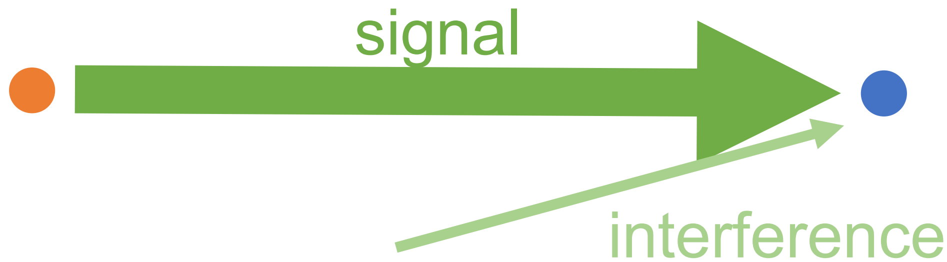
Wireless connection

Wireless network

$$Y_k[t] = \sum_{j=1}^{n} H_{jk}[t]x_j[t] + Z_k[t]$$

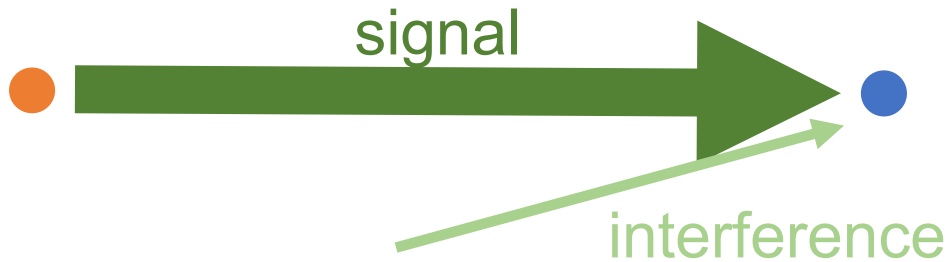
Wireless network

When interference is weak



Wireless network

When interference is weak

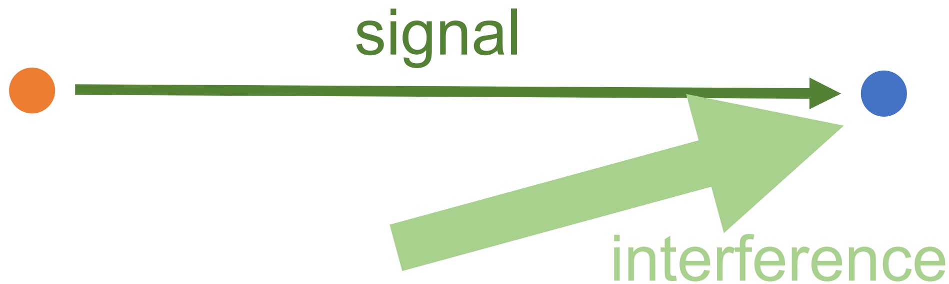


Treat interference as noise

$$R = \log_2\left(1 + \frac{|H_S|^2 P}{|H_I|^2 P + \sigma^2}\right) = \log_2(1 + \mathrm{SINR})$$

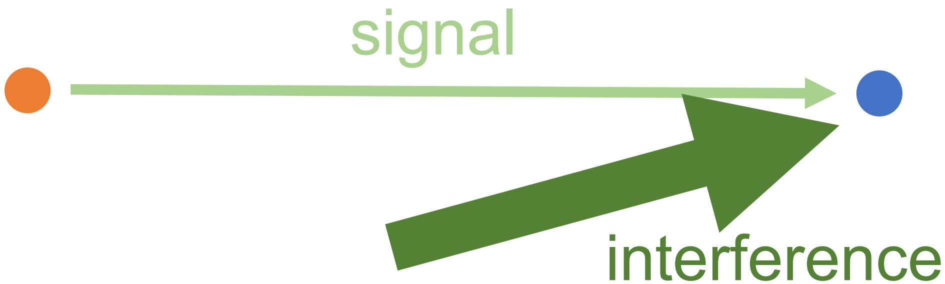
Wireless network

When interference is **strong**



Wireless network

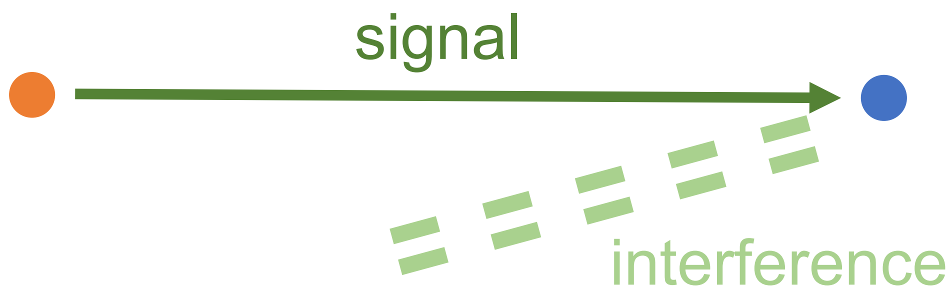
When interference is **strong**



Pretend interference is signal

Wireless network

When interference is **strong**



Pretend interference is signal

Decode and subtract

$$R = \min\left\{\log_2\left(1 + \frac{|H_{\mathrm{I}}|^2 P}{|H_{\mathrm{S}}|^2 P + \sigma^2}\right), \log_2(1 + \mathrm{SNR})\right\}$$

Interference

Weak interference
Treat as noise

Strong interference
Decode and subtract

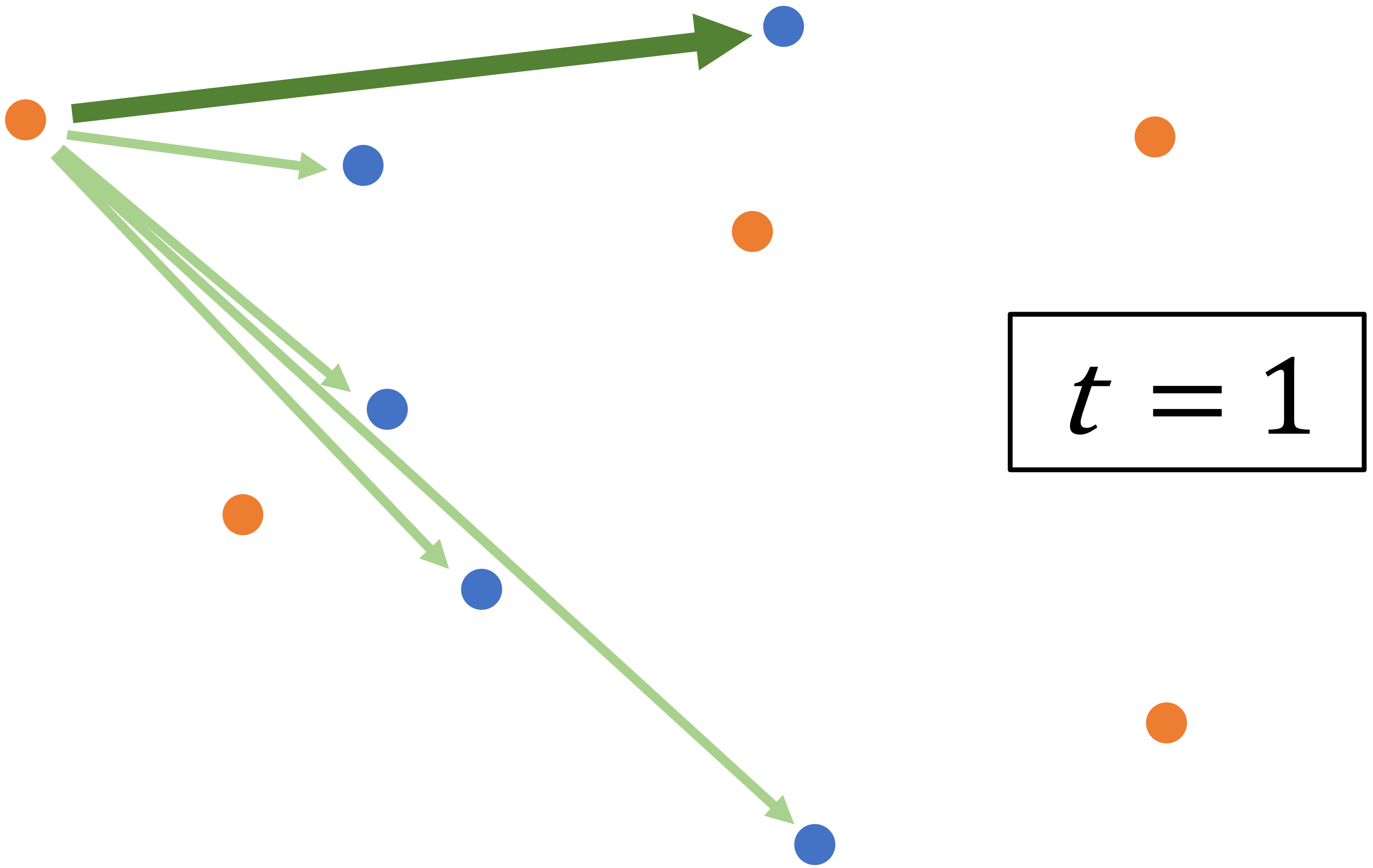
Interference ≈ Signal
???

Interference $\approx$ Signal

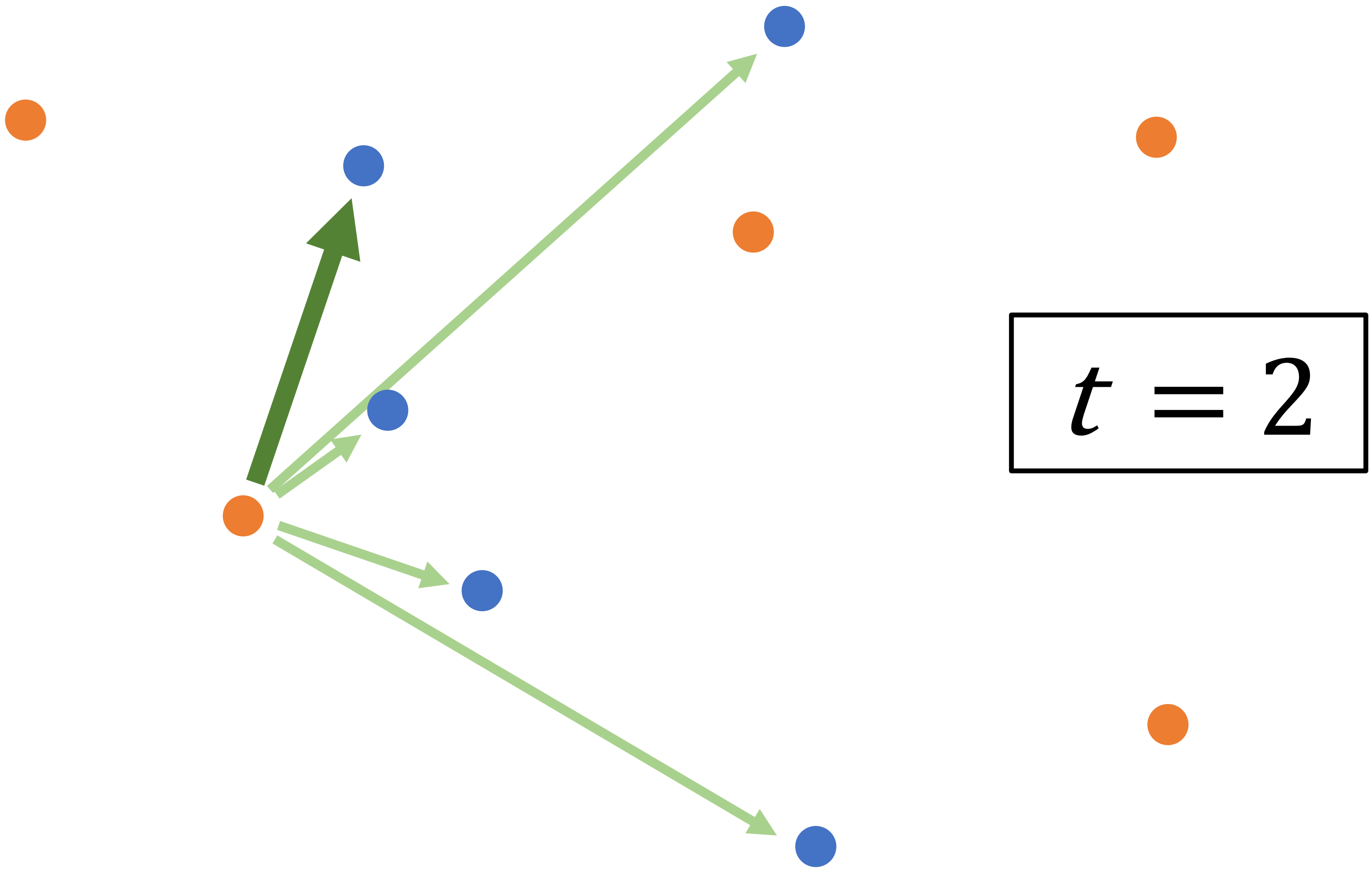
Resource division

(or cake cutting)

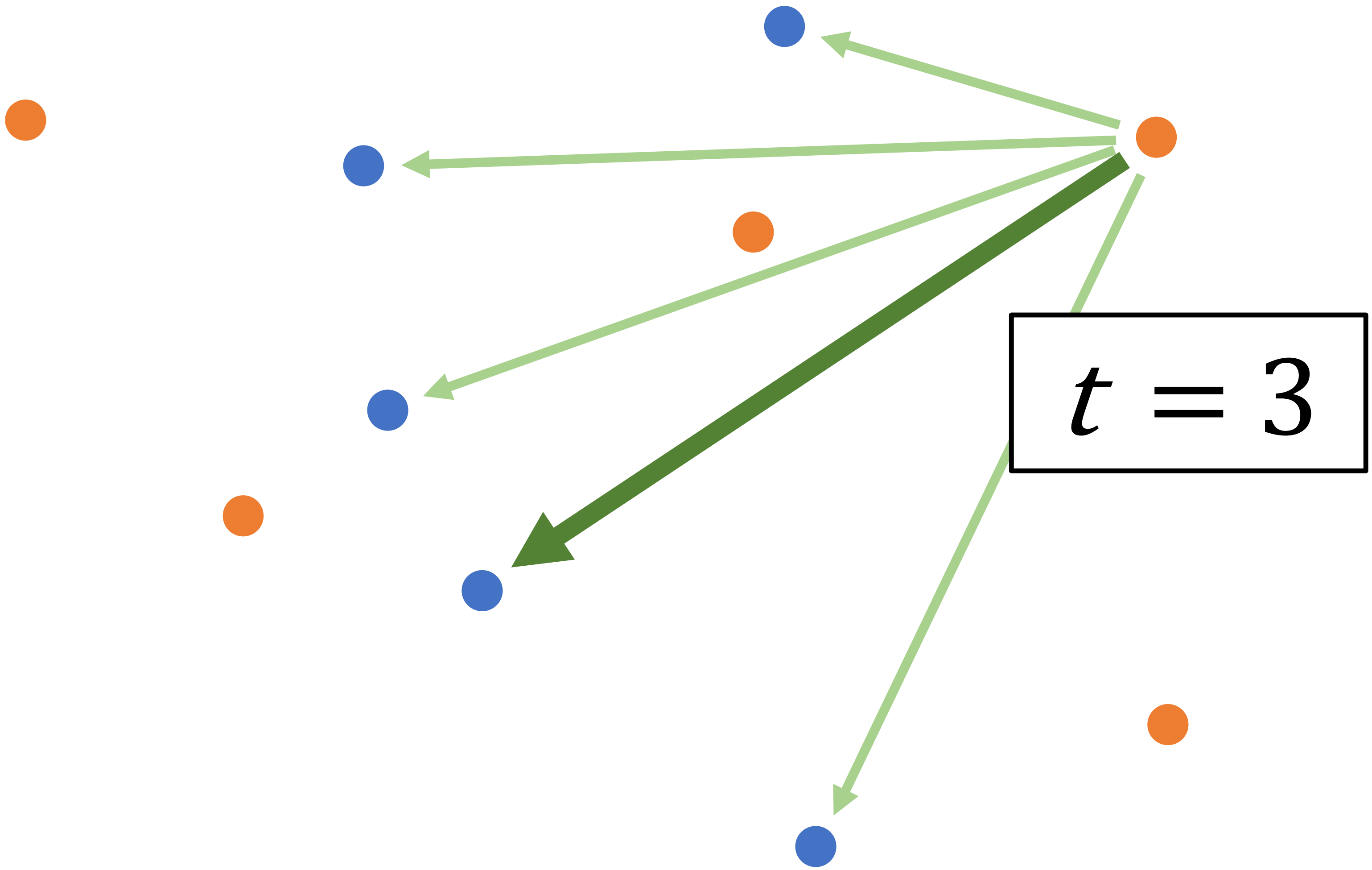
Interference ≈ Signal



Interference ≈ Signal



Interference ≈ Signal



Interference ≈ Signal

Resource division

(or cake cutting)

$$C = \log_2(1 + \mathrm{SNR})$$

Interference ≈ Signal

Resource division
(or cake cutting)

$$R = \frac{1}{n}\log_2(1 + \quad \mathrm{SNR})$$

Only use the channel
an nth of the time

Interference ≈ Signal

Resource division

(or cake cutting)

$$R = \frac{1}{n}\log_2(1 + n\mathrm{SNR})$$

Only use the channel an nth of the time

…but can use n times the power

Interference ≈ Signal

Resource division
(or cake cutting)

$$R_{\mathrm{sum}} = \log_2(1 + n\mathrm{SNR})$$

Sum-rates and **sum-capacity**

Easier to calculate
than the whole "capacity region"

Interference $\approx$ Signal

Resource division

(or cake cutting)

…by **time** (TDMA)

…by **frequency** (FDMA)

Give each user a separate slice of spectrum

…in **codeword space** (CDMA)

Transmitted signals live in $\mathbb{C}^T$

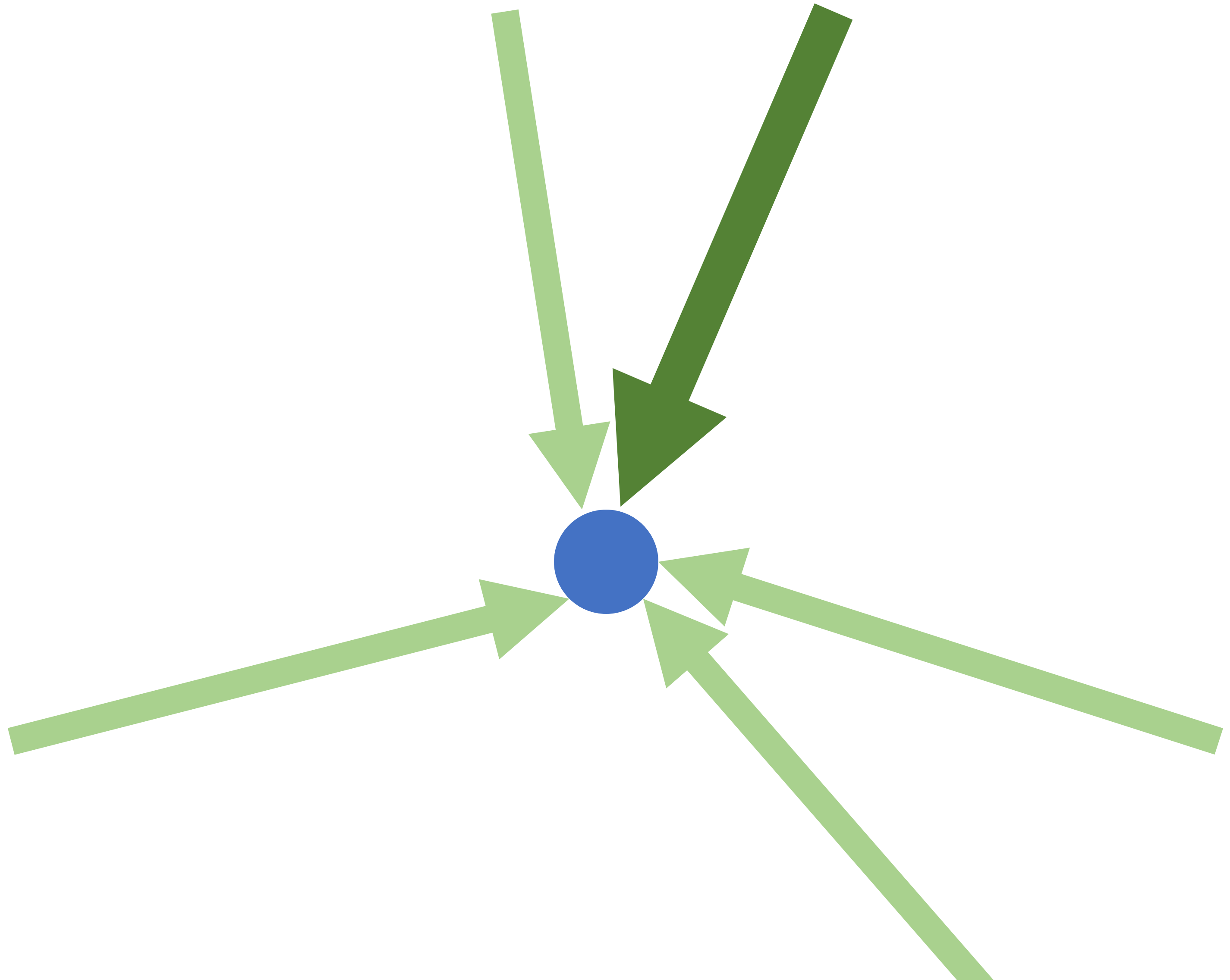
Split this into n orthogonal T/n-dimensional subspaces

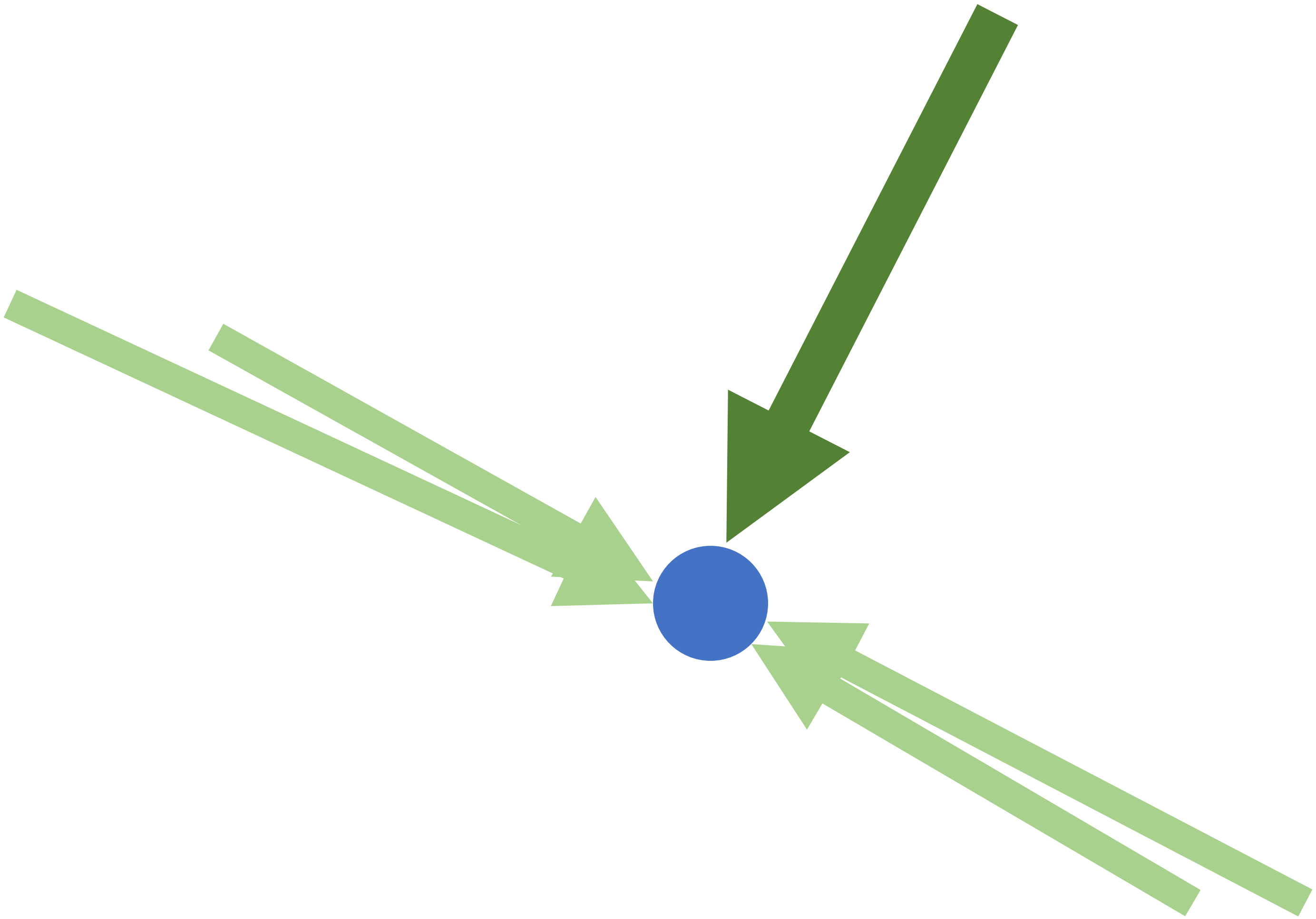
Interference ≈ Signal

Resource division
(or cake cutting)

$$R = \frac{1}{n}\log_2(1 + n\mathrm{SNR})$$

B1
Interference alignment





Interference alignment

$$Y_k = \sum_{j=1}^{n} H_{jk} x_j + Z_k$$

Suppose $H_{kk} = 1$

$$H_{jk} = \mathrm{i}, \quad j \neq k$$

Interference alignment

$$Y_k = \sum_{j=1}^{n} H_{jk} x_k + Z_k$$

Suppose $H_{kk} = 1$

$$H_{jk} = \mathrm{i}, \quad j \neq k$$

And then suppose transmitters send only **real-valued signals** x_k

Interference alignment

Suppose $H_{jj} = 1$

$H_{jk} = \mathrm{i}\,, \quad j \neq k$

And then suppose transmitters send only **real-valued signals** x_k

$$\mathrm{Re}\, Y_k = H_{kk}\, x_k + \mathrm{Re}\, Z_k$$

$$\mathrm{Im}\, Y_k = \sum_{k \neq j} H_{jk}\ x_j + \mathrm{Im}\, Z_k$$

Interference alignment

Interference alignment
(or “everyone gets half a cake”)

$$R = \frac{1}{2}\log(1 + 2\mathrm{SNR}) - O(\log \mathrm{SNR})$$

No matter how many users
it’s as if there’s only two

Interference alignment

Interference alignment
(or “everyone gets half a cake”)

…in **codeword space**
(Cadambe & Jafar, 2008)

…in **time**
(Grokop, Tse & Yates, 2011)

…over the **rational numbers**
(Motahari, Oveis-Gharan, Maddah-Ali & Khandani, 2014)

Interference alignment

Ergodic interference alignment

(Nazer, Gastpar, Jafar & Vishwanath, 2009)

Ensures a rate of

$$R = \frac{1}{2}\log_2(1 + 2\mathrm{SNR})$$

without requiring $\mathrm{SNR} \to \infty$

Interference alignment

High rates

As if there's only two users

"Everyone gets half a cake"

Interference alignment

High rates

As if there's only two users

"Everyone gets half a cake"

but…

Impractical

Requires coordination

Requires knowledge of channel coefficients

Requires very long blocklengths

etc

B2

Bottleneck links

First, some housekeeping

$$Y = Hx + Z$$

$$C = \log\left(1 + \frac{|H|^2 P}{\sigma^2}\right)$$

First, some housekeeping

$$Y = Hx + Z$$

$$C = \log\left(1 + \frac{|H|^2 P}{\sigma^2}\right)$$

Pick units so $\sigma^2 = 1$

Absorb P into $|H|$, to allow $P = 1$

Write $|H| = \sqrt{\mathrm{SNR}}$

First, some housekeeping

$$Y = \sqrt{\mathrm{SNR}}\mathrm{e}^{\mathrm{i}\Theta} x + Z$$

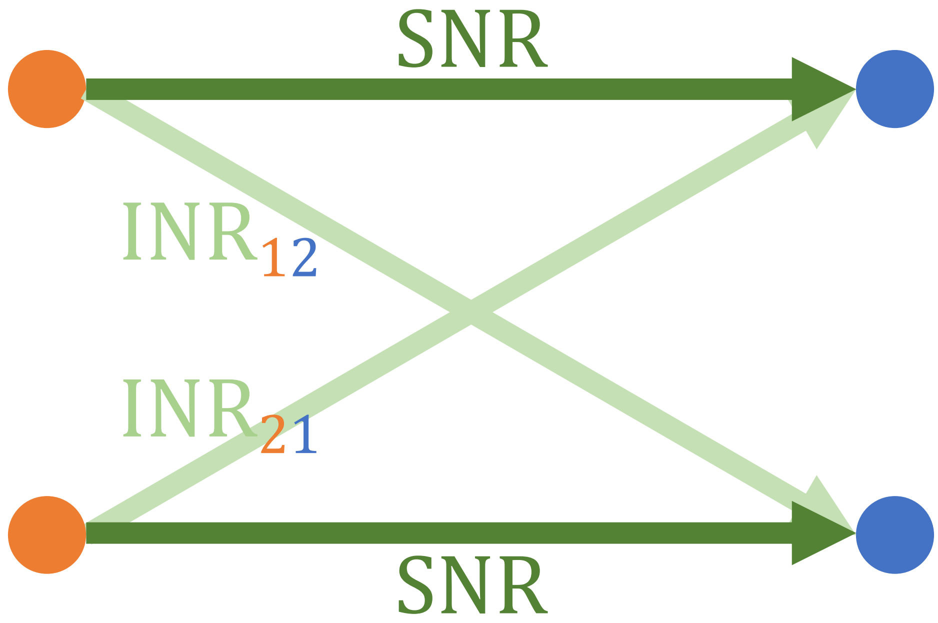
$$C = \log(1 + \mathrm{SNR})$$

From now on, we assume $\Theta \sim \mathrm{U}[0,2\pi)$
and is IID over time

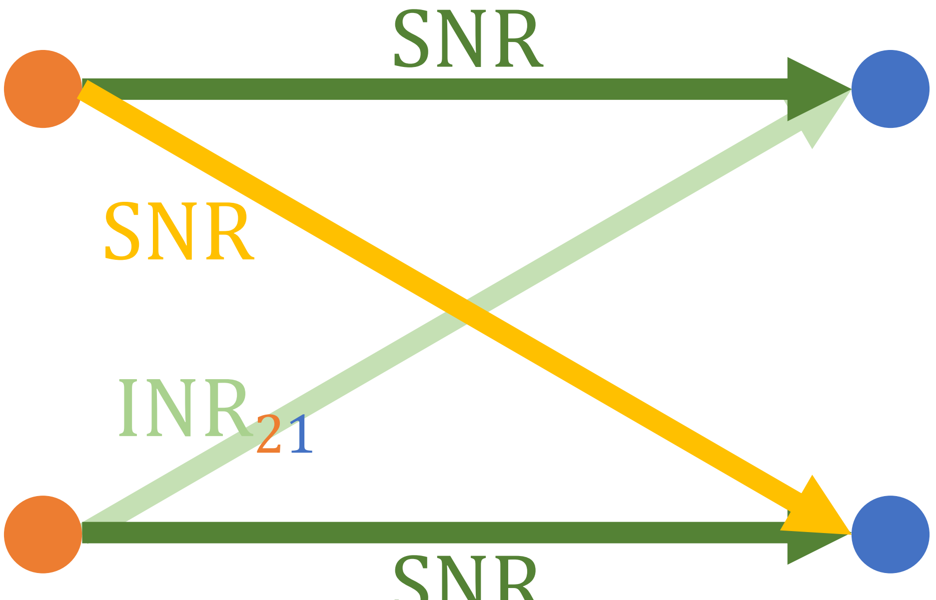
First, some housekeeping

$$Y_k = \sqrt{\mathrm{SNR}_k}\ \mathrm{e}^{\mathrm{i}\Theta_{kk}}\ x_k + \sum_{k \neq j} \sqrt{\mathrm{INR}_{jk}}\ \mathrm{e}^{\mathrm{i}\Theta_{jk}}\ x_j + Z_k$$

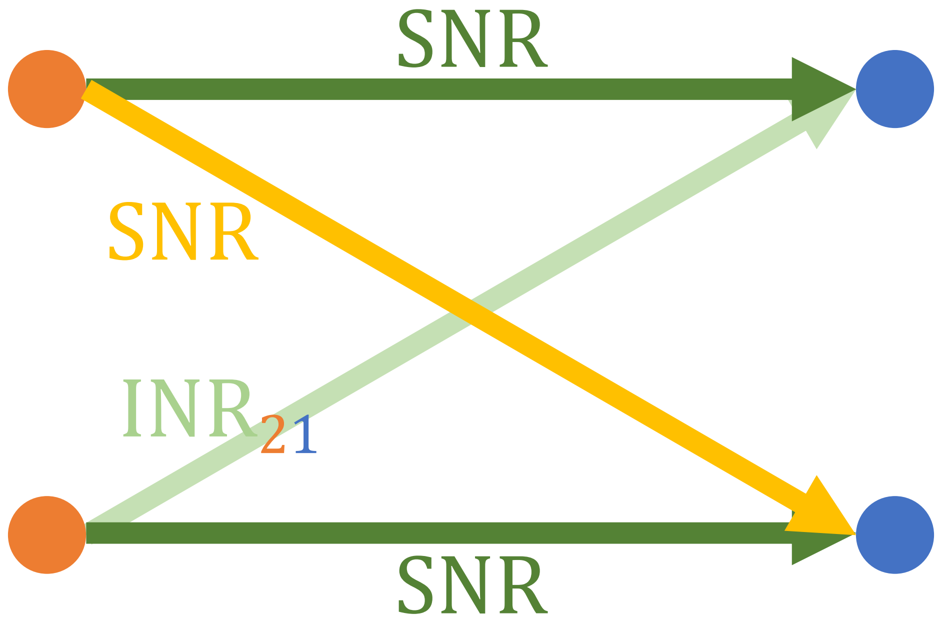
A two-user network



A bottleneck link



A bottleneck link

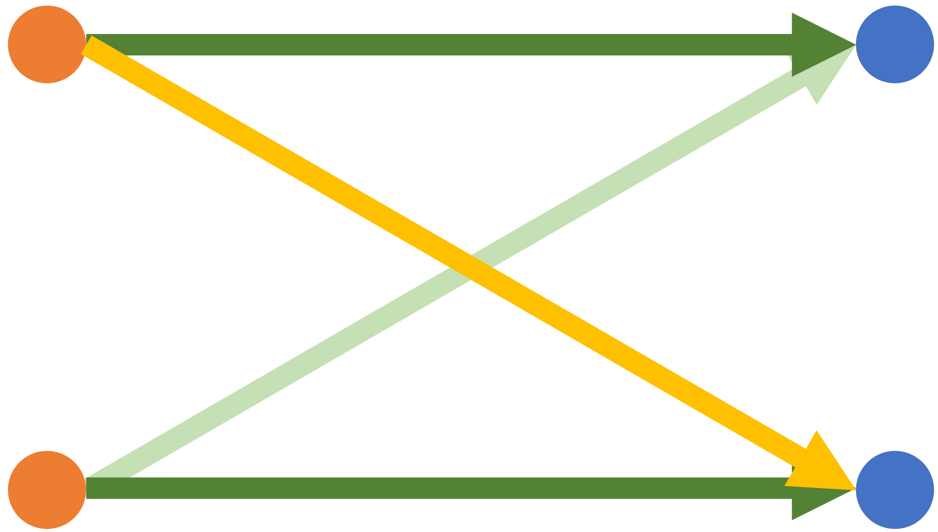


$$C_{\text{sum}} = \log(1 + 2\text{SNR})$$

regardless of the value of INR_{21}

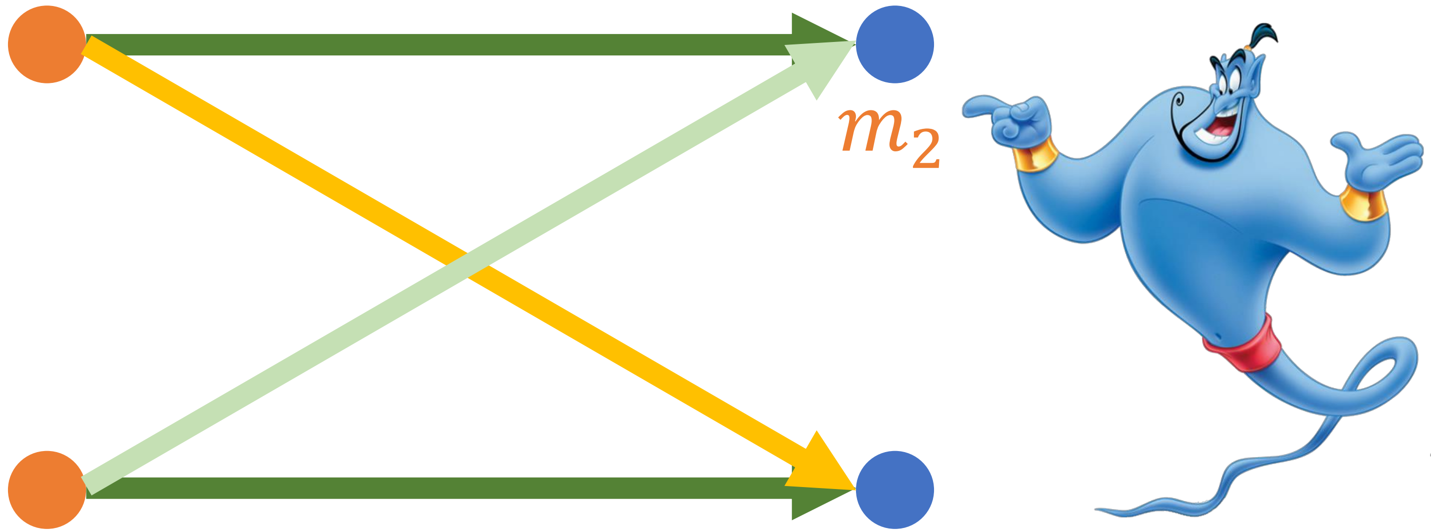
(Jafar, 2011)

A bottleneck link



Suppose the rates R_1 and R_1 are achievable

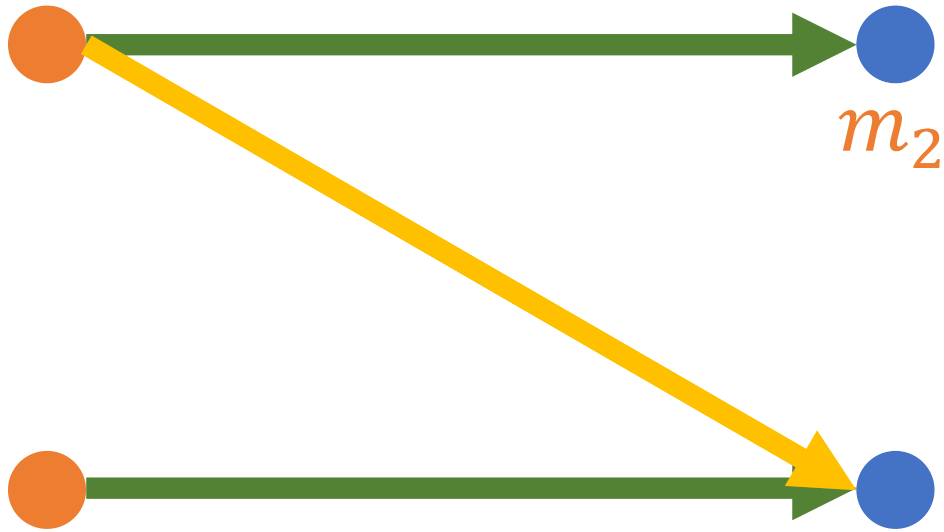
A bottleneck link



Suppose the rates R_1 and R_2 are achievable

Genie provides Rx 1 with message m_2

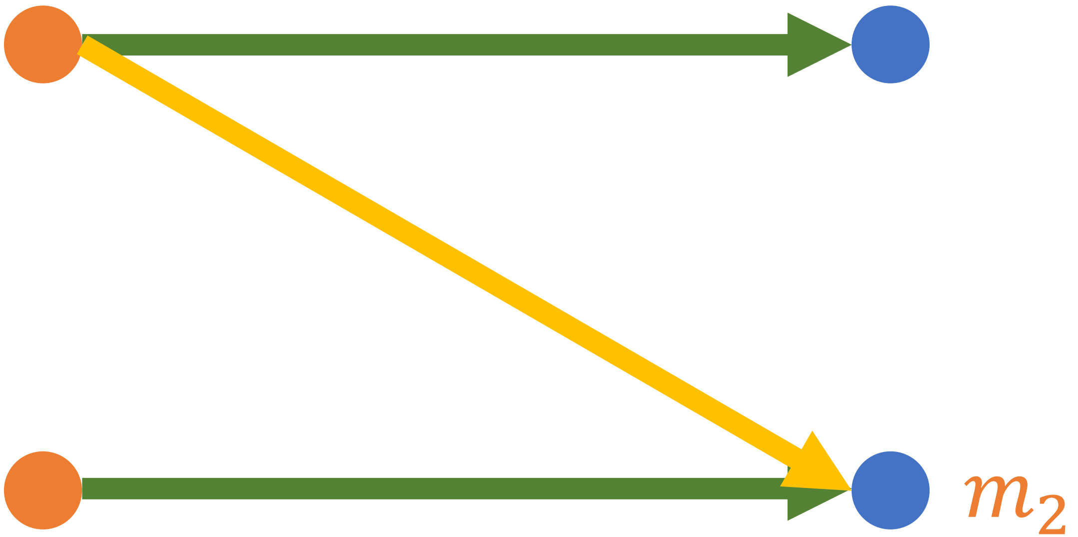
A bottleneck link



Suppose the rates R_1 and R_2 are achievable

Genie provides Rx 1 with message m_2
Rx 1 can decode and subtract

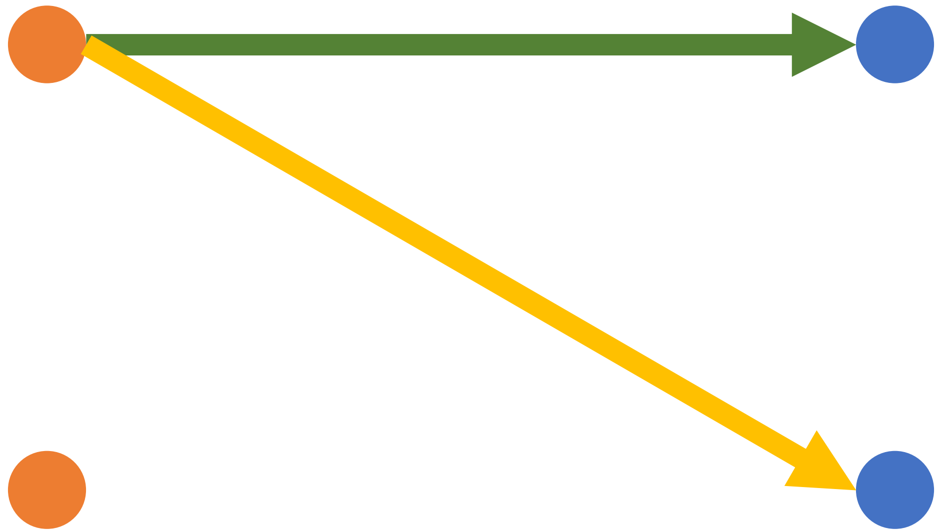
A bottleneck link



Suppose the rates R_1 and R_2 are achievable

Rx 2 can decode and subtract message m_2

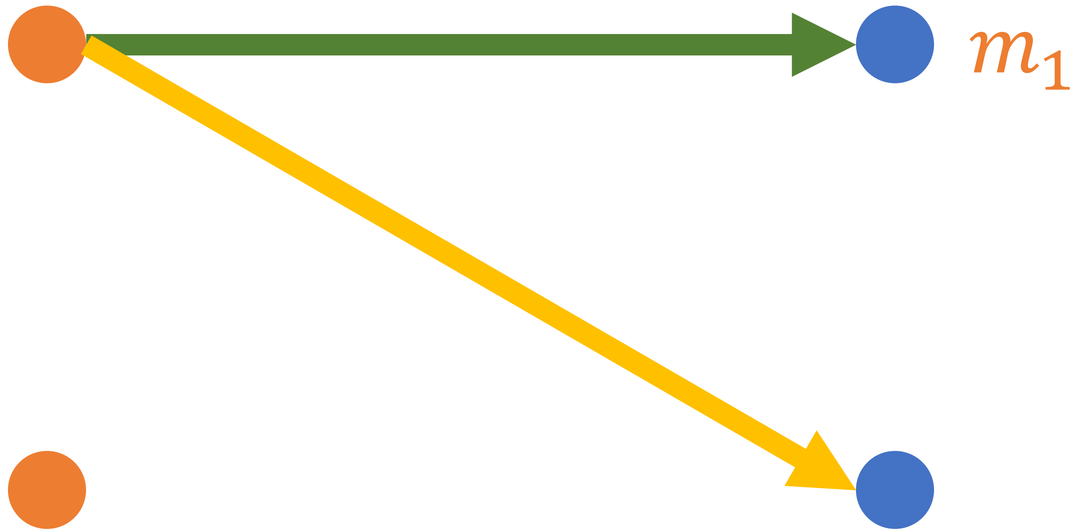
A bottleneck link



Suppose the rates R_1 and R_2 are achievable

Rx 2 can decode and subtract message m_2

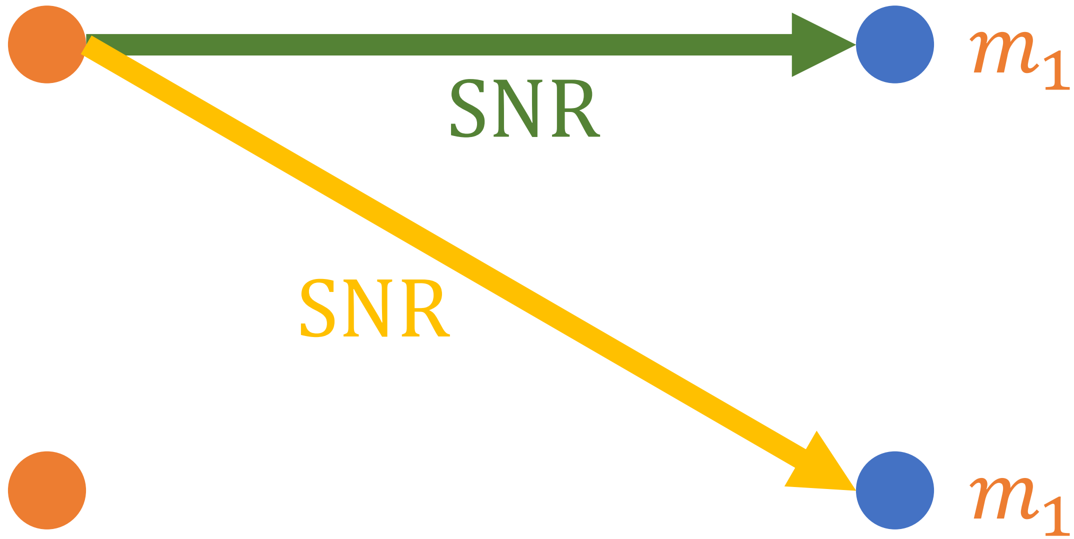
A bottleneck link



Suppose the rates R_1 and R_2 are achievable

Rx 1 can decode and subtract message m_1

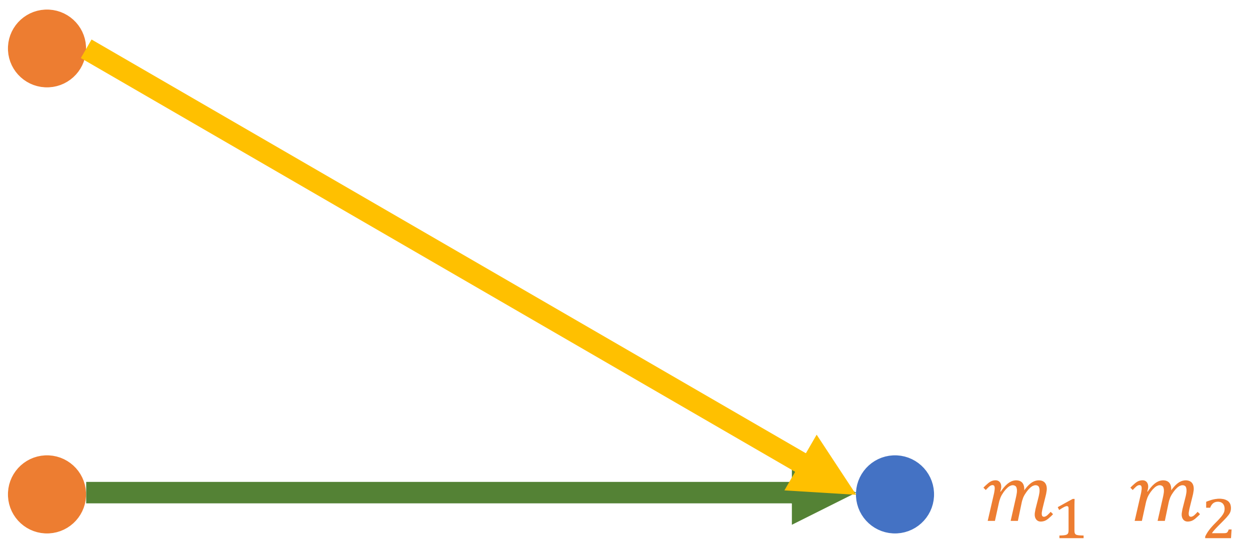
A bottleneck link



Suppose the rates R_1 and R_2 are achievable

Rx 1 can decode and subtract message m_1
So Rx 2 can also

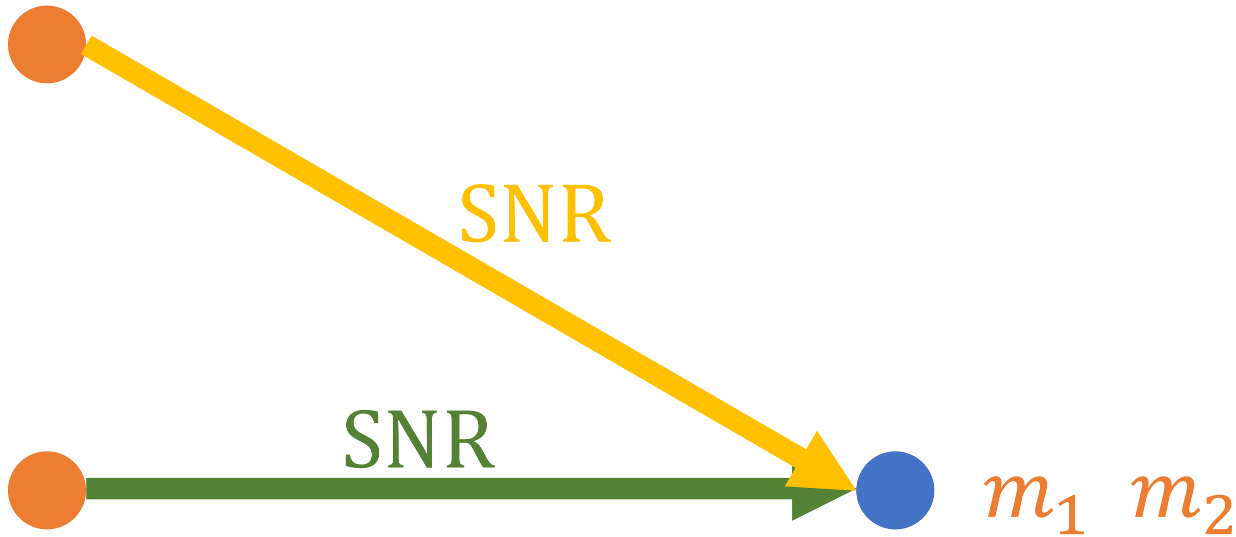
A bottleneck link



Suppose the rates R_1 and R_2 are achievable

So Rx 2 has decoded **both messages**
This is a **multiple access channel**

A bottleneck link

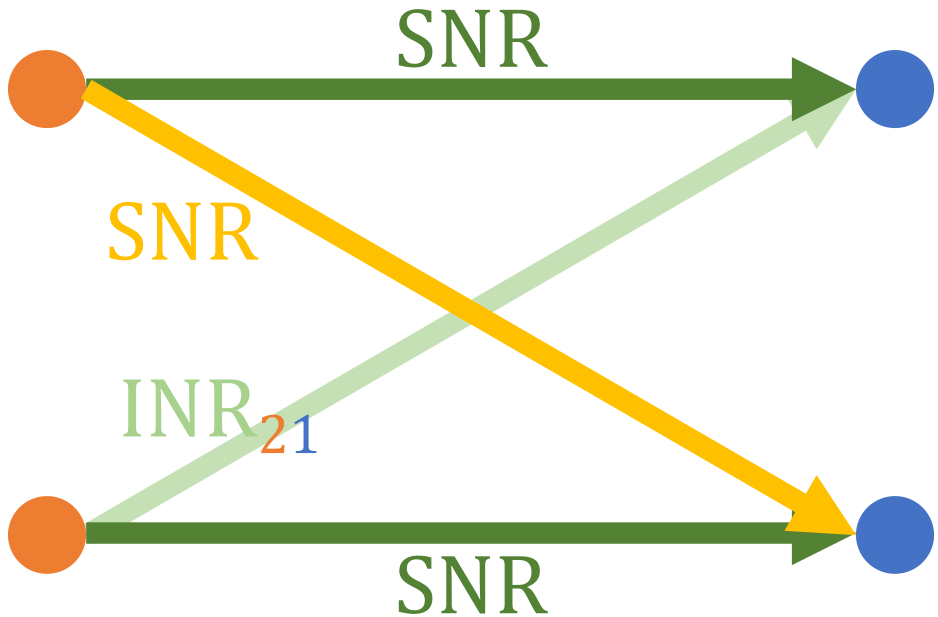


Suppose the rates R_1 and R_2 are achievable

So Rx 2 has decoded **both messages**

So $\mathrm{R}_1 + \mathrm{R}_2 \leq \log(1 + \mathrm{SNR} + \mathrm{SNR})$

A bottleneck link



$$C_{\mathrm{sum}} = \log(1 + 2\mathrm{SNR})$$

regardless of the value of INR_{21}

(Jafar, 2011)

C1

Jafar

networks

Jafar network

(Jafar, 2011)

In a **wireless network**, we might expect:

SNRs roughly the same

INRs vary, but can be similar to SNR

Jafar network

(Jafar, 2011)

In a **wireless network**, we might expect:

SNRs roughly the same

INRs vary, but can be similar to SNR

In a **Jafar network:**

SNRs identical and deterministic

INRs IID from a distribution supported at SNR

Jafar network

(Jafar, 2011)

$$Y_k = \sqrt{\mathrm{SNR}_k}\; \mathrm{e}^{\mathrm{i}\Theta_{kk}}\, x_k + \sum_{k \neq j} \sqrt{\mathrm{INR}_{jk}}\; \mathrm{e}^{\mathrm{i}\Theta_{jk}}\, x_j + Z_k$$

In a **Jafar network:**

SNRs identical and deterministic

INRs IID from a distribution supported at SNR

Jafar network

SNRs identical and deterministic

INRs IID from a distribution supported at SNR

Theorem (Jafar, 2011)

The sum-capacity of the

n-user Jafar network satisfies

$$\frac{C_{\text{sum}}}{n} \xrightarrow{\mathbb{P}} \frac{1}{2}\log(1 + 2\text{SNR})$$

as $n \to \infty$

Jafar network

Theorem (Jafar, 2011)
The sum-capacity of the
n-user Jafar network satisfies

$$\frac{C_{\mathrm{sum}}}{n} \xrightarrow{\mathbb{P}} \frac{1}{2}\log(1+2\mathrm{SNR})$$

Lower bound:
Achievable using ergodic interference alignment.

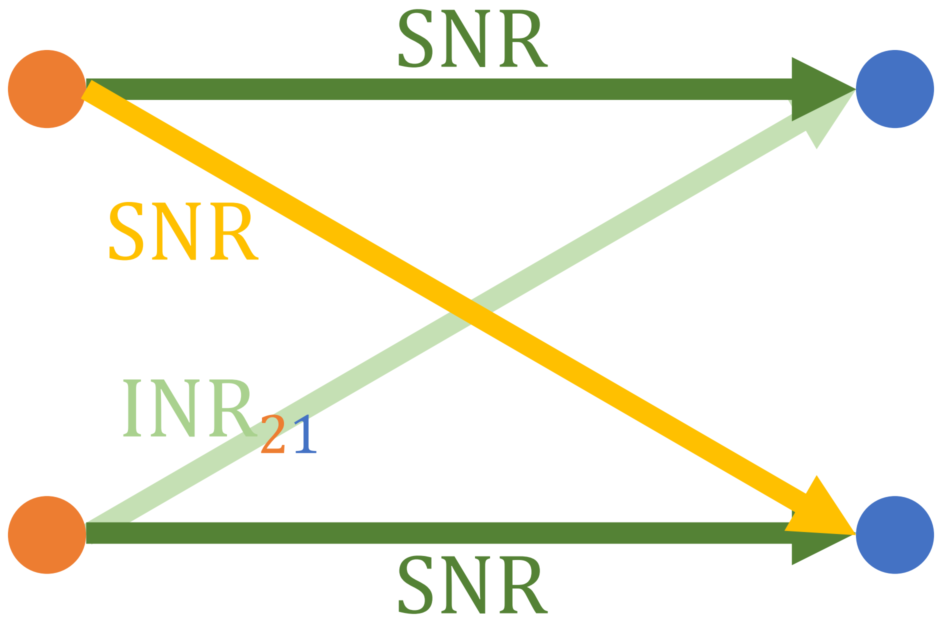
Jafar network

Theorem (Jafar, 2011)
The sum-capacity of the
n-user Jafar network satisfies

$$\frac{C_{\text{sum}}}{n} \xrightarrow{\mathbb{P}} \frac{1}{2}\log(1 + 2\mathrm{SNR})$$

Lower bound: ergodic interference alignment
Upper bound: bottleneck links

A bottleneck link

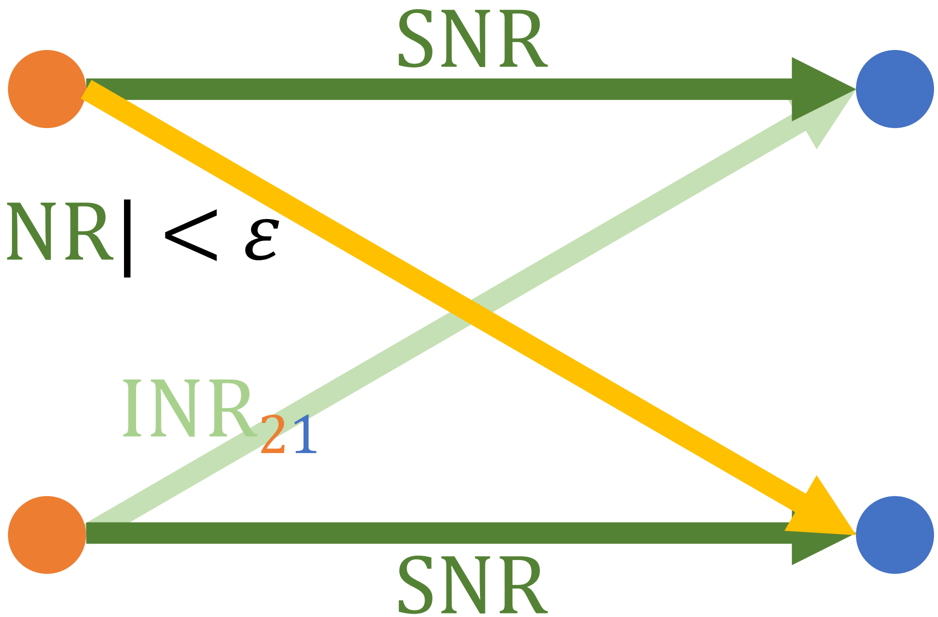


$$C_{\mathrm{sum}} = \log(1 + 2\mathrm{SNR})$$

regardless of the value of INR_{21}

(Jafar, 2011)

A ε-bottleneck link



$|\mathrm{INR}_{21} - \mathrm{SNR}| < \varepsilon$

$$R_1 + R_2 \leq \log(1 + 2\mathrm{SNR}) + \varepsilon$$

regardless of the value of INR_{21}

Jafar network

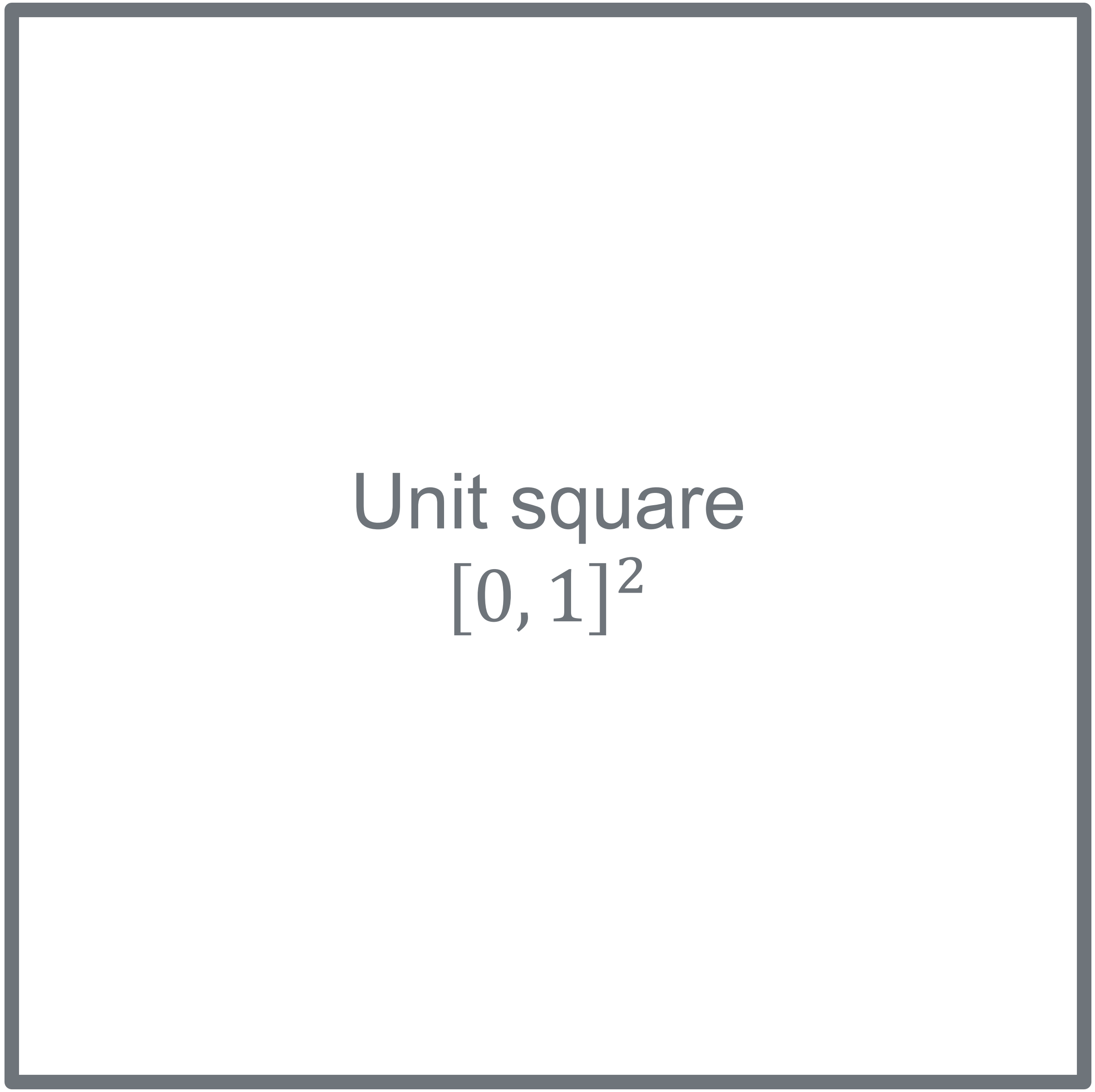
Theorem (Jafar, 2011)
The sum-capacity of the
n-user Jafar network satisfies

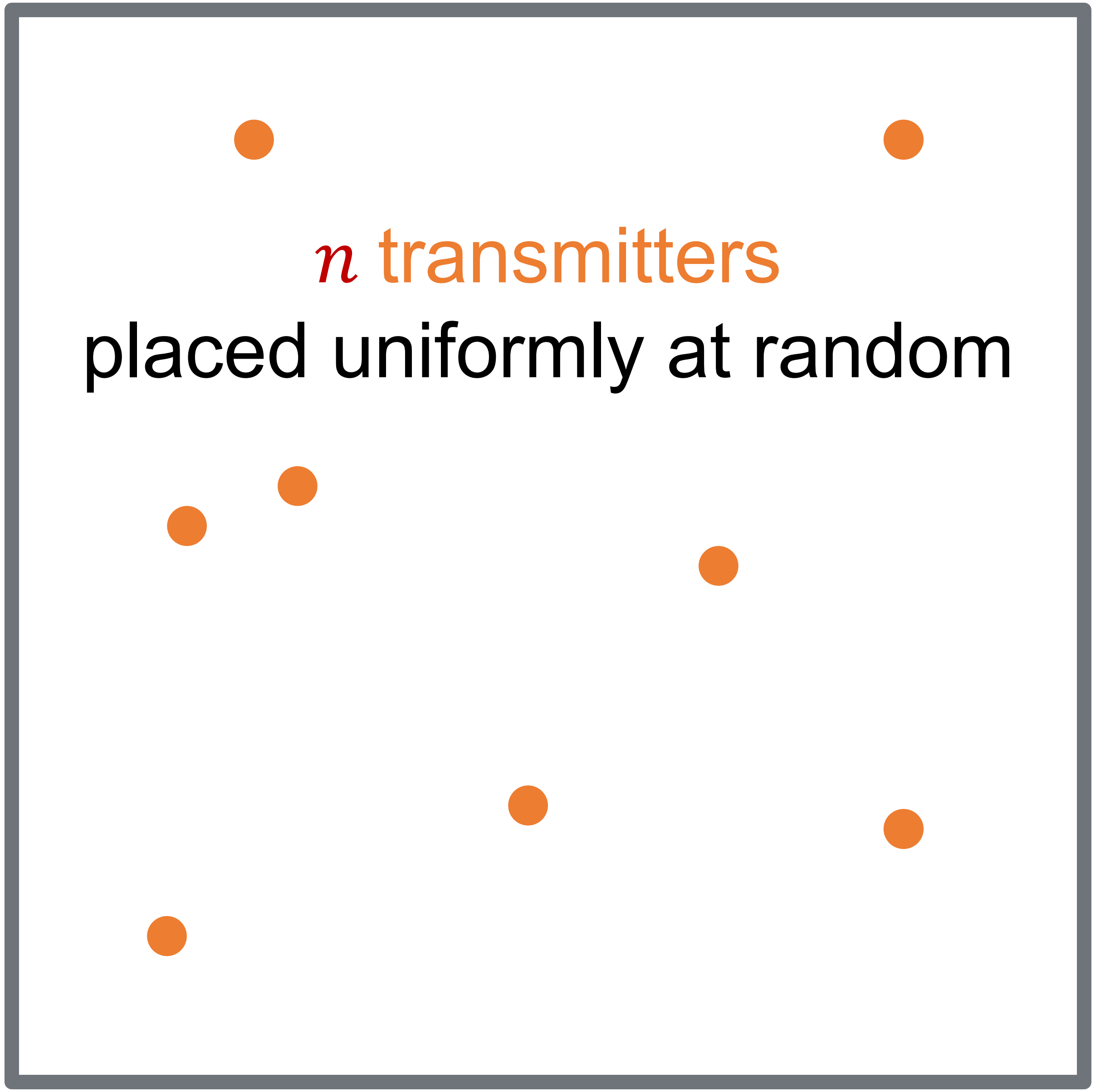
$$\frac{C_{\mathrm{sum}}}{n} \xrightarrow{\mathbb{P}} \frac{1}{2}\log(1 + 2\mathrm{SNR})$$

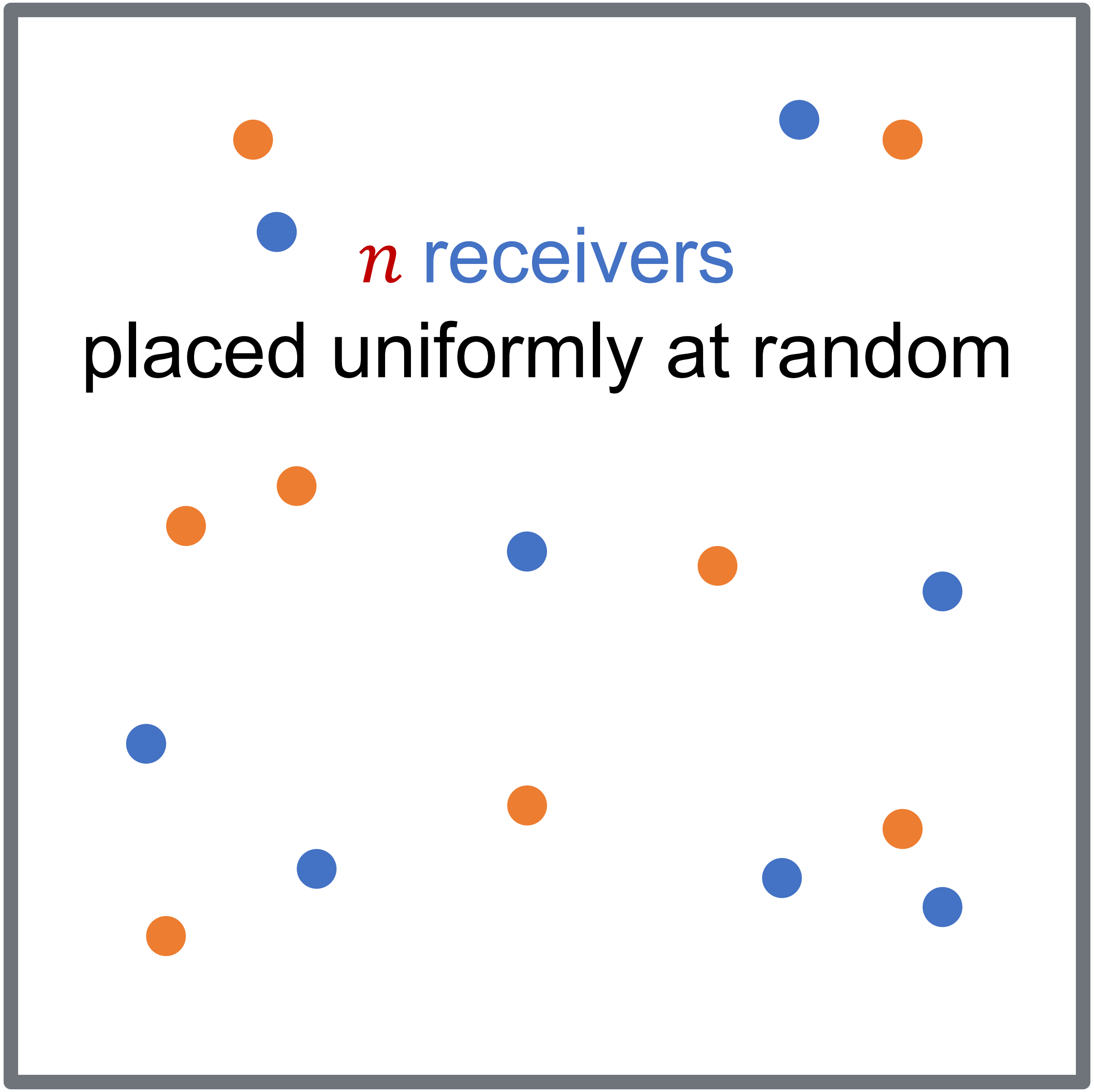
Upper bound:
Probabilistic (Jafar, 2011)
Combinatorial (Johnson, Aldridge, Piechocki, 2011)

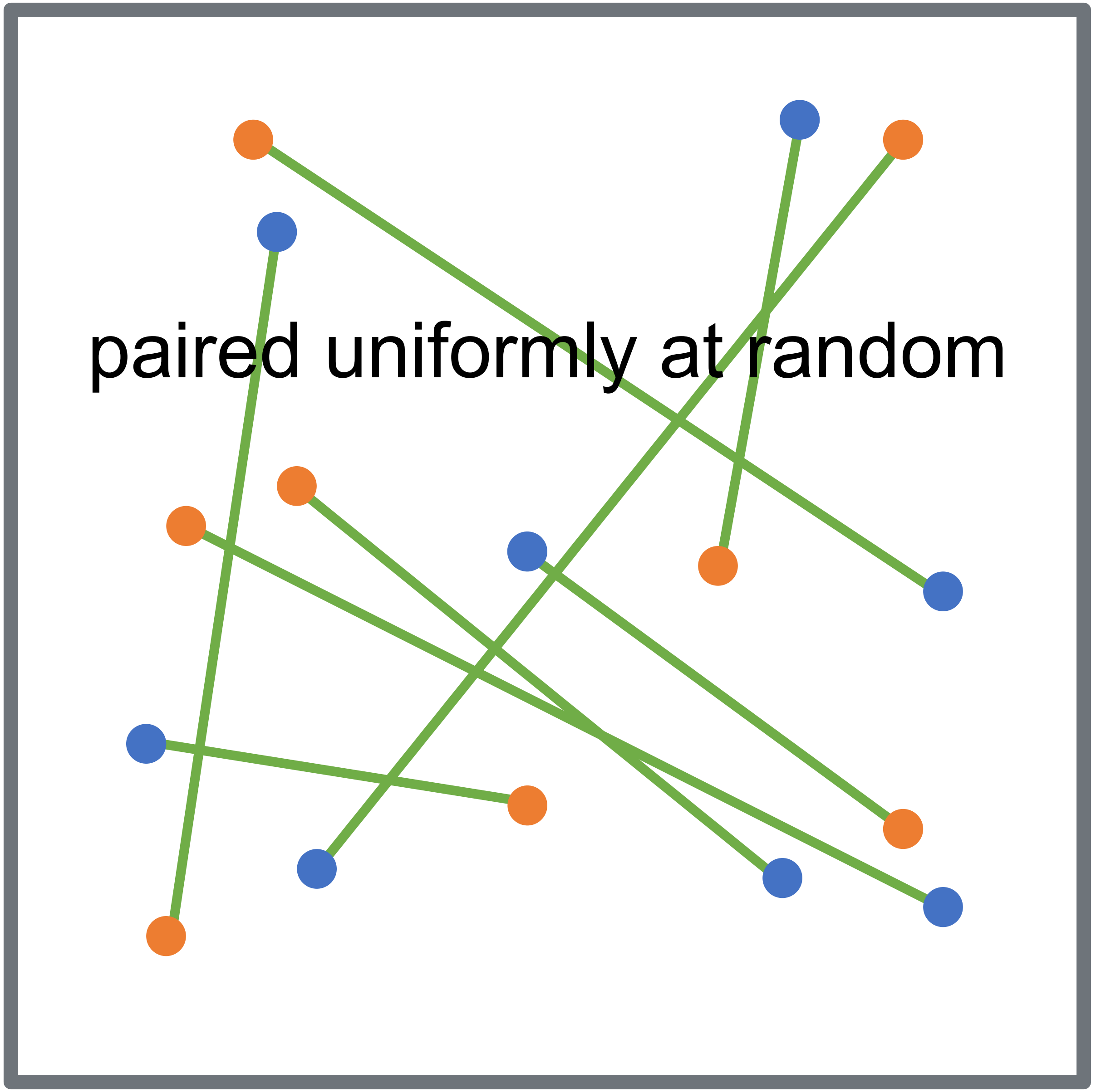
C2

The standard dense network


Unit square
$[0,1]^2$

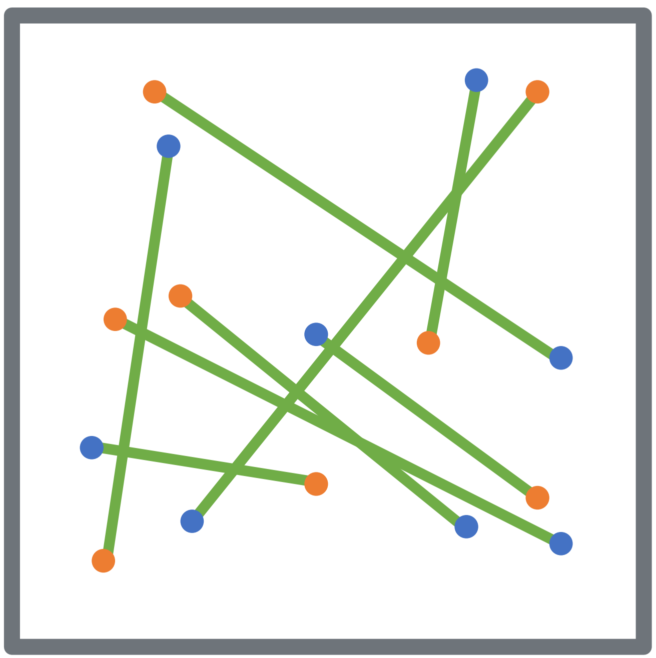

n transmitters
placed uniformly at random


n receivers
placed uniformly at random


paired uniformly at random

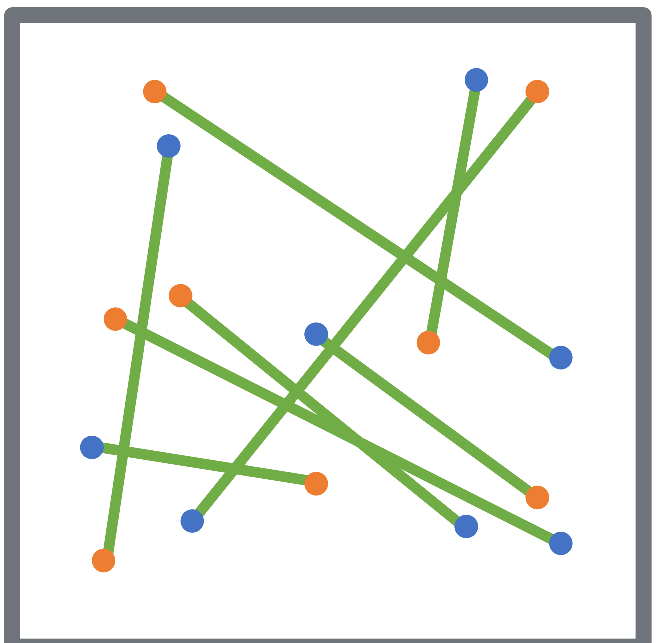
Standard dense network

$$Y_k = \sqrt{\mathrm{SNR}}_k\ \mathrm{e}^{\mathrm{i}\Theta_{kk}}\ x_k + \sum_{k \neq j} \sqrt{\mathrm{INR}}_{jk}\ \mathrm{e}^{\mathrm{i}\Theta_{jk}}\ x_j + Z_k$$



Standard dense network

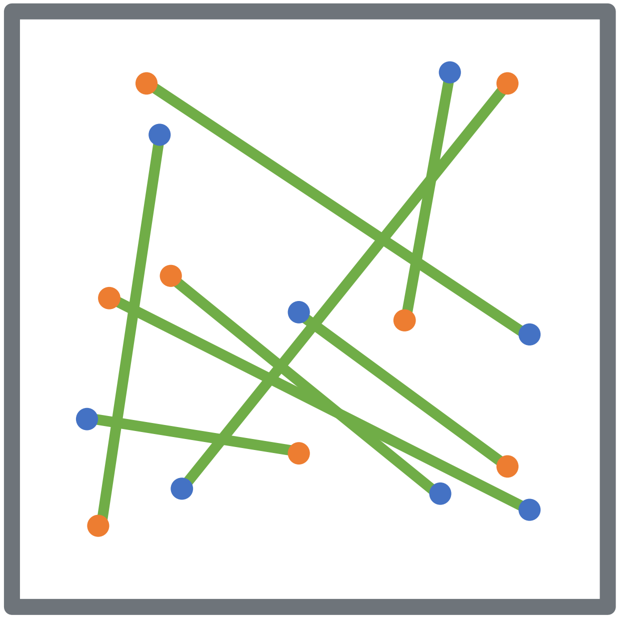
$$Y_k = \sqrt{\mathrm{SNR}_k}\ \mathrm{e}^{\mathrm{i}\Theta_{kk}}\ x_k + \sum_{k \neq j} \sqrt{\mathrm{INR}_{jk}}\ \mathrm{e}^{\mathrm{i}\Theta_{jk}}\ x_j + Z_k$$



$$\mathrm{SNR}_k = C\ d(\mathrm{Tx}\ k, \mathrm{Rx}\ k)^{-\alpha}$$

$$\mathrm{INR}_{jk} = C\ d(\mathrm{Tx}\ j\ , \mathrm{Rx}\ k)^{-\alpha}$$

Standard dense network



$$\mathrm{SNR}_k = C\, d(\mathrm{Tx}\ k, \mathrm{Rx}\ k)^{-\alpha}$$
$$\mathrm{INR}_{jk} = C\, d(\mathrm{Tx}\ j, \mathrm{Rx}\ k)^{-\alpha}$$

So SNRs and INRs are identically distributed
(but not independent)

Write $E = \frac{1}{2}\mathbb{E}\log(1 + 2\mathrm{SNR}) = \frac{1}{2}\mathbb{E}\log(1 + 2\mathrm{INR})$

Standard dense network

Theorem (Johnson, Aldridge & Piechocki, 2011)
The sum-capacity of the
n-user standard dense network satisfies

$$\frac{C_{\text{sum}}}{n} \xrightarrow{\mathbb{P}} E = \frac{1}{2}\,\mathbb{E}\log(1 + 2\,\text{SNR})$$

as $n \to \infty$

Standard dense network

Theorem (Johnson, Aldridge & Piechocki, 2011)
The sum-capacity of the
n-user standard dense network satisfies

$$\frac{C_{\text{sum}}}{n} \xrightarrow{\mathbb{P}} E = \frac{1}{2}\,\mathbb{E}\log(1 + 2\,\mathrm{SNR})$$

Lower bound: ergodic interference alignment
Upper bound: bottleneck links

Standard dense network

$$\frac{C_{\mathrm{sum}}}{n} \xrightarrow{\mathbb{P}} E = \frac{1}{2}\mathbb{E}\log(1 + 2\mathrm{SNR})$$

Lower bound: ergodic interference alignment

$$\sum_{j=1}^{n} \frac{1}{2}\log\left(1 + 2\mathrm{SNR}_j\right) \xrightarrow{\mathbb{P}} E$$

by the weak law of large numbers

Bottleneck links

We say that (j, k) is an **ε-bottleneck link** if

$$\frac{1}{2}\log\left(1 + 2\mathrm{SNR}_j\right) \le E + \frac{\varepsilon}{2}$$

$$\frac{1}{2}\log\left(1 + 2\mathrm{INR}_{jk}\right) \le E + \frac{\varepsilon}{2}$$

$$\frac{1}{2}\log\left(1 + 2\mathrm{INR}_{kj}\right) \le \frac{1}{2}\log(1 + 2\mathrm{SNR}_k)$$

Bottleneck links

Lemma (Johnson, Aldridge & Piechocki, 2011)
If (j, k) is an **ε-bottleneck link**,
then any achievable rates satisfy

$$R_j + R_k \leq 2E + \varepsilon$$

Proof
As before, genie + multiple access channel.

Upper bound

Proof 1 (Johnson, Aldridge & Piechocki, 2011)
Discretize space
look for zones with bottleneck links

Proof 2 (Aldridge, Johnson & Piechocki, 2010)
Probabilistic existence proof
Along the lines of Jafar's proof earlier

Standard dense network

Theorem (Johnson, Aldridge & Piechocki, 2011)
The sum-capacity of the
n-user standard dense network satisfies

$$\frac{C_{\mathrm{sum}}}{n} \xrightarrow{\mathbb{P}} E = \frac{1}{2}\,\mathbb{E}\log(1 + 2\mathrm{SNR})$$

Lower bound: ergodic interference alignment
Upper bound: bottleneck links

Two conclusions

Relatively few **"bottleneck" interfering links** tightly bound the total capacity of a network

Planning transmissions so that **interference "aligns"** at each transmitter allows performance close to the bottleneck bound

Open Questions
Engineering

How can we make interference alignment more plausible?

Imperfect channel knowledge
Shorter time delays
Less precise arithmetic
Less precise timing

How do more realistic assumptions affect the sum-capacity?

Open Questions

Mathematical

Can we prove similar results for more physically realistic networks?

Preferential attachment

Nodes with movement

What about “finite n” results?

Can we use combinatorics to give a proof with an exponential rate of convergence?