Abstractions for Type-Level Programming

Olivier Blanvillain

Tuesday, 22 March 2022

Example 1: database queries (2016)

Example 1: database queries (2016)

Spark APIs are practically untyped:

```
class DataFrame {
  /** Inner equi-join with another DataFrame on the given column.
   *  The join column will only appear once in the output. */
  def join(right: DataFrame, column: String): DataFrame
}
```

Revisited with type-level programming:

```
class DF[X] {
  def join[Y](df: DF[Y], col: String): DF[col+(X-col)++(Y-col)]
}
```

Example 1: database queries (2016)

Spark APIs are practically untyped:

```
class DataFrame {
  /** Inner equi-join with another DataFrame on the given column.
   *  The join column will only appear once in the output. */
  def join(right: DataFrame, column: String): DataFrame
}
```

Revisited with type-level programming:

```
class DF[X] {
  def join[Y](df: DF[Y], col: String): DF[col+(X-col)++(Y-col)]
}
```

Example 2: regular expressions (2022)

Regular expressions in Scala's standard library:

```
val rational = new Regex("(\\d+)\\.?(\\d+)?")
rational.unapply("3.1415"): Option[Seq[String]]
```

Revisited with type-level programming:

```
rational.unapply("3.1415"): Option[(String, Option[String])]
```

```
class Regex(pattern: String) {
  def unapply(s: String): Option[GroupsOf[pattern]]
}
```

Example 2: regular expressions (2022)

Regular expressions in Scala's standard library:

```
val rational = new Regex("(\\d+)\\.?(\\d+)?")
rational.unapply("3.1415"): Option[Seq[String]]
```

Revisited with type-level programming:

```
rational.unapply("3.1415"): Option[(String, Option[String])]
```

```
class Regex(pattern: String) {
  def unapply(s: String): Option[GroupsOf[pattern]]
}
```

Example 2: regular expressions (2022)

Regular expressions in Scala's standard library:

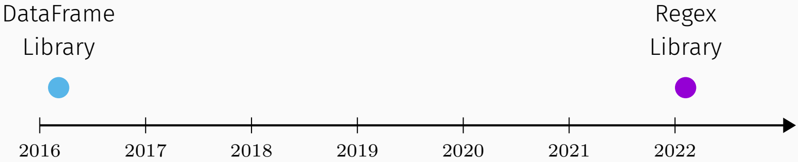
```
val rational = new Regex("(\\d+)\\.?(\\d+)?")
rational.unapply("3.1415"): Option[Seq[String]]
```

Revisited with type-level programming:

```
rational.unapply("3.1415"): Option[(String, Option[String])]
```

```
class Regex(pattern: String) {
  def unapply(s: String): Option[GroupsOf[pattern]]
}
```

What changed between 2016 and 2022?



What changed between 2016 and 2022?

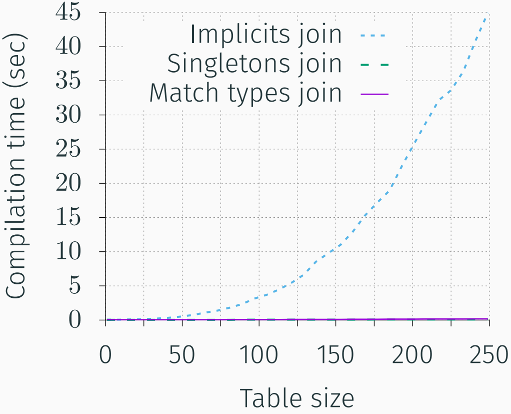
The Spark library uses a hack, implicits:

- convoluted to use
- slow to compile

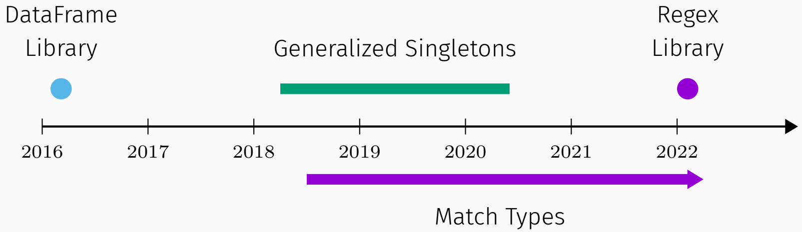
The regex library uses a first-class language construct:

- easier to use
- faster to compile

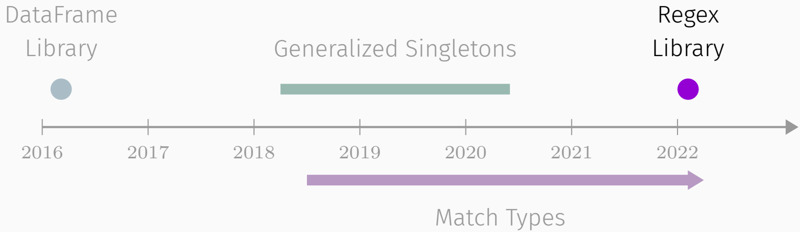
Compilation times for the type-level join operation



Timeline of my PhD

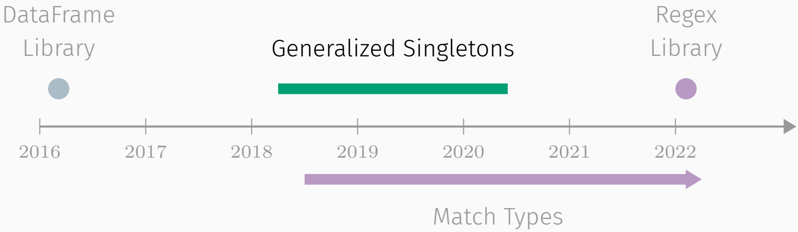


Timeline of my PhD

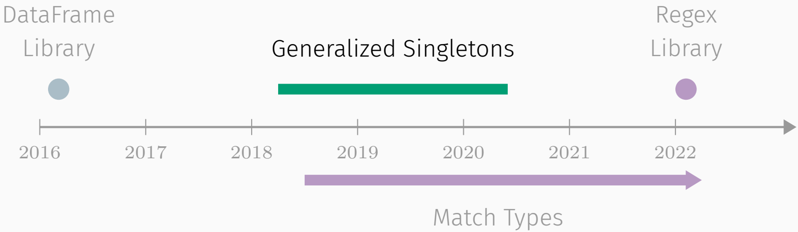


Olivier Blanvillain. "Type-Safe Regular Expressions". In: Proceedings of the ACM SIGPLAN International Symposium on Scala. SCALA'22. New York, NY, USA: ACM, 2022.

Timeline of my PhD



Timeline of my PhD



Georg Stefan Schmid, Olivier Blanvillain, Jad Hamza, and Viktor Kuncak. "Coming to Terms with Your Choices: An Existential Take on Dependent Types". In: CoRR (2020). arXiv: 2011.07653.

Singleton types

Definition

A singleton type is a type that contains exactly one value.

Scala has a few of those:

- `x.type`, the type of the variable `x` (since forever)
- `42`, the type of the integer literal `42` (since 2016)
- `+`, the type of integer addition (since 2020)

Question: How much more of the language can we represent as singleton types?

Singleton types

Definition

A singleton type is a type that contains exactly one value.

Scala has a few of those:

- `x.type`, the type of the variable `x` (since forever)
- `42`, the type of the integer literal `42` (since 2016)
- `+`, the type of integer addition (since 2020)

Question: How much more of the language can we represent as singleton types?

Singleton types

Definition

A singleton type is a type that contains exactly one value.

Scala has a few of those:

- `x.type`, the type of the variable `x` (since forever)
- `42`, the type of the integer literal `42` (since 2016)
- `+`, the type of integer addition (since 2020)

Question: How much more of the language can we represent as singleton types?

Generalized singleton types: a proposal

Our proposal consists of 3 changes:

1. new types for if-then-else, pattern matching, constructors, functions calls
2. “precise mode” of type inference
3. type evaluation, during subtyping

New types

What's the type of `if (x == 0) "zero" else "one"`?

- `String` (Scala 2)
- `"zero" | "one"` (Scala 3)
- `If[x.type == 0, "zero", "one"]` (proposed)
- `{ if (x == 0) "zero" else "one" }` (proposed, syntactic sugar)

Similarly, we add new types for other constructs: `Match[]`, `New[]`, `Call[]`, `TypeTest[]`, and the corresponding syntactic sugar.

New types

What's the type of `if (x == 0) "zero" else "one"`?

- `String` (Scala 2)
- `"zero" | "one"` (Scala 3)
- `If[x.type == 0, "zero", "one"]` (proposed)
- `{ if (x == 0) "zero" else "one" }` (proposed, syntactic sugar)

Similarly, we add new types for other constructs: `Match[]`, `New[]`, `Call[]`, `TypeTest[]`, and the corresponding syntactic sugar.

New types

What's the type of `if (x == 0) "zero" else "one"`?

- `String` (Scala 2)
- `"zero" | "one"` (Scala 3)
- `If[x.type == 0, "zero", "one"]` (proposed)
- `{ if (x == 0) "zero" else "one" }` (proposed, syntactic sugar)

Similarly, we add new types for other constructs: `Match[]`, `New[]`, `Call[]`, `TypeTest[]`, and the corresponding syntactic sugar.

New types

What’s the type of `if (x == 0) "zero" else "one"`?

- `String` (Scala 2)
- `"zero" | "one"` (Scala 3)
- `If[x.type == 0, "zero", "one"]` (proposed)
- `{ if (x == 0) "zero" else "one" }` (proposed, syntactic sugar)

Similarly, we add new types for other constructs: `Match[]`, `New[]`, `Call[]`, `TypeTest[]`, and the corresponding syntactic sugar.

New types

What's the type of `if (x == 0) "zero" else "one"`?

- `String` (Scala 2)
- `"zero" | "one"` (Scala 3)
- `If[x.type == 0, "zero", "one"]` (proposed)
- `{ if (x == 0) "zero" else "one" }` (proposed, syntactic sugar)

Similarly, we add new types for other constructs: `Match[]`, `New[]`, `Call[]`, `TypeTest[]`, and the corresponding syntactic sugar.

New types

What's the type of `if (x == 0) "zero" else "one"`?

- `String` (Scala 2)
- `"zero" | "one"` (Scala 3)
- `If[x.type == 0, "zero", "one"]` (proposed)
- `{ if (x == 0) "zero" else "one" }` (proposed, syntactic sugar)

Similarly, we add new types for other constructs: `Match[]`, `New[]`, `Call[]`, `TypeTest[]`, and the corresponding syntactic sugar.

“Precise mode” of type inference

We need two modes of type inference, for backwards compatibility.

```
def int2str(x: Int) =
  if (x == 0) "zero" else "one"
```

The “precise mode” is best effort: it lifts whatever possible to the type level and approximates the rest.

“Precise mode” of type inference

We need two modes of type inference, for backwards compatibility.

```
def int2str(x: Int): String =
  if (x == 0) "zero" else "one"
```

The “precise mode” is best effort: it lifts whatever possible to the type level and approximates the rest.

“Precise mode” of type inference

We need two modes of type inference, for backwards compatibility.

```
dependent
def int2str(x: Int)
  : { if (x == 0) "zero" else "one" }
  =   if (x == 0) "zero" else "one"
```

The “precise mode” is best effort: it lifts whatever possible to the type level and approximates the rest.

“Precise mode” of type inference

We need two modes of type inference, for backwards compatibility.

```
dependent
def int2str(x: Int)
  : If[x.type == 0, "zero", "one"]
  = if (x == 0) "zero" else "one"
```

The “precise mode” is best effort: it lifts whatever possible to the type level and approximates the rest.

“Precise mode” of type inference

We need two modes of type inference, for backwards compatibility.

```
dependent
def int2str(x: Int)
  : If[x.type == 0, "zero", "one"]
  = if (x == 0) "zero" else "one"
```

The “precise mode” is best effort: it lifts whatever possible to the type level and approximates the rest.

“Precise mode” of type inference

We need two modes of type inference, for backwards compatibility.

```
dependent
def int2str(x: Int)
  : If[x.type == 0, "zero", String]
  = if (x == 0) "zero" else readString()
```

The “precise mode” is best effort: it lifts whatever possible to the type level and approximates the rest.

“Precise mode” of type inference

We need two modes of type inference, for backwards compatibility.

```
dependent
def int2str(x: Int)
  : { if (x == 0) "zero" else (_: String) }
  =   if (x == 0) "zero" else readString()
```

The “precise mode” is best effort: it lifts whatever possible to the type level and approximates the rest.

“Precise mode” of type inference: list concatenation example

```
dependent def concat(xs: List, ys: List) <: List =
  xs match
    case x :: xs => x :: concat(xs, ys)
    case Nil => ys

dependent val l1 = "A" :: Nil
dependent val l2 = "B" :: Nil
dependent val l3 = concat(l1, l2)
l3: { "A" :: "B" :: Nil }
```

“Precise mode” of type inference: list concatenation example

```
dependent def concat(xs: List, ys: List) <: List =
  xs match
    case x :: xs => x :: concat(xs, ys)
    case Nil => ys

val l1: List     = "A" :: Nil
dependent val l2 = "B" :: Nil
dependent val l3 = concat(l1, l2)
l3: { concat(l1, l2) }
```

“Precise mode” of type inference: list concatenation example

```
dependent def concat(xs: List, ys: List) <: List =
  xs match
    case x :: xs => x :: concat(xs, ys)
    case Nil => ys

dependent val l1 = "A" :: Nil
val l2: List      = "B" :: Nil
dependent val l3 = concat(l1, l2)
l3: { "A" :: l2 }
```

Type evaluation

During subtyping, we evaluate both sides:

- `A <: B` if eval(`A`) `<:` eval(`B`)

Straightforward for if-then-else:

- eval(`If[true, A, B]`) = `A`
- eval(`If[false, A, B]`) = `B`

More interesting for pattern matching: we "desugar" pattern matching expressions into if-then-else & type tests.

Type evaluation

During subtyping, we evaluate both sides:

- `A <: B` if eval(`A`) `<:` eval(`B`)

Straightforward for if-then-else:

- eval(`If[true, A, B]`) = `A`
- eval(`If[false, A, B]`) = `B`

More interesting for pattern matching: we “desugar” pattern matching expressions into if-then-else & type tests.

Type evaluation

During subtyping, we evaluate both sides:

- `A <: B` if eval(`A`) `<:` eval(`B`)

Straightforward for if-then-else:

- eval(`If[true, A, B]`) = `A`
- eval(`If[false, A, B]`) = `B`

More interesting for pattern matching: we “desugar” pattern matching expressions into if-then-else & type tests.

Type evaluation: type tests

We evaluate type tests using subtyping and type disjointness:

- eval(`x.isInstanceOf[T]`) = `true`   if `x.type` is a subtype of `T`
- eval(`x.isInstanceOf[T]`) = `false`   if `x.type` and `T` are disjoint
- eval(`x.isInstanceOf[T]`) = `x.isInstanceOf[T]`   otherwise

Type evaluation: pattern matching example

```
dependent val foo(x: Any) =
  x match
    case s: String => s
    case i: Int => i+1
```

```
foo(42): { 43 }

foo(readInt()): { (_: Int) + 1 }
```

Type evaluation: pattern matching example

```
dependent val foo(x: Any): {
  x match
    case s: String => s
    case i: Int => i+1
} =
  x match
    case s: String => s
    case i: Int => i+1
```

```
foo(42): { 43 }
```

```
foo(readInt()): { (_: Int) + 1 }
```

Type evaluation: pattern matching example

```
dependent val foo(x: Any): {
  if (x.isInstanceOf[String]) x.asInstanceOf[String]
  else if (x.isInstanceOf[Int]) x.asInstanceOf[Int] + 1
  else throw new MatchError()
} =
  x match
    case s: String => s
    case i: Int => i+1
```

```
foo(42): { 43 }

foo(readInt()): { (_: Int) + 1 }
```

Type evaluation: pattern matching example

```
dependent val foo(x: Any): {
  if (x.isInstanceOf[String]) x.asInstanceOf[String]
  else if (x.isInstanceOf[Int]) x.asInstanceOf[Int] + 1
  else throw new MatchError()
} =
  x match
    case s: String => s
    case i: Int => i+1

foo(42): { 43 }

foo(readInt()): { (_: Int) + 1 }
```

Type evaluation: pattern matching example

```
dependent val foo(x: Any): {
  if (x.isInstanceOf[String]) x.asInstanceOf[String]
  else if (x.isInstanceOf[Int]) x.asInstanceOf[Int] + 1
  else throw new MatchError()
} =
  x match
    case s: String => s
    case i: Int => i+1
```

```
foo(42): { 43 }
```

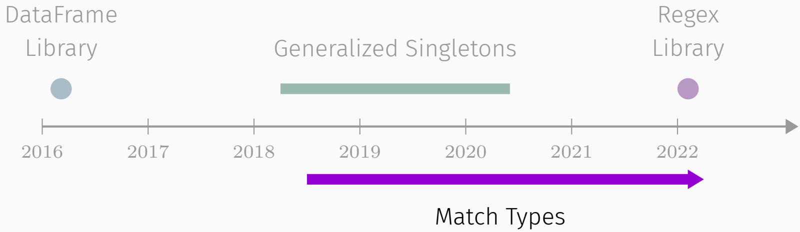
```
foo(readInt()): { (_: Int) + 1 }
```

Generalized singletons: recap

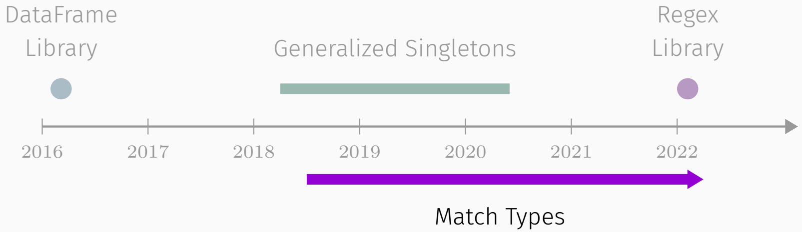
We proposed a generalization of Scala's singleton types:

1. lift a subset of Scala's language constructs to the type level
2. add a "precise mode" of type inference
3. evaluate types during subtyping

Part II: Match Types



Part II: Match Types



Olivier Blanvillain, Jonathan Immanuel Brachthäuser, Maxime Kjaer, and Martin Odersky. “Type-Level Programming with Match Types”. In: Proc. ACM Program. Lang. POPL’22. New York, NY, USA: ACM, 2022.

Pattern matching on types

```
type Elem[X] = X match
  case String => Char
  case List[t] => Elem[t]
  case Any => X
```

Pattern matching on types

```scala
type Elem[X] = X match
  case String => Char
  case List[t] => Elem[t]
  case Any => X
```

Some examples of reduction:

- `Elem[String] =:= Char`
- `Elem[Int] =:= Int`
- `Elem[List[Int]] =:= Int`
- `Elem[Any] =:= Elem[Any]`

Pattern matching on types

```
type Elem[X] = X match
  case String => Char
  case List[t] => Elem[t]
  case Any => X
```

We evaluate type tests using subtyping and type disjointness:

- eval(`X.isInstanceOf[T]`) = `true` if `X` is a subtype of `T`
- eval(`X.isInstanceOf[T]`) = `false` if `X` and `T` are disjoint
- eval(`X.isInstanceOf[T]`) = `X.isInstanceOf[T]` otherwise

Pattern matching on types

```
type Elem[X] = X match
  case String => Char
  case List[t] => Elem[t]
  case Any => X
```

Match types can be explained in terms of generalized singletons:

```
type Elem[X] = { (_: X) match
  case _: String => (_: Char)
  case _: List[t] => (_: Elem[t])
  case _: Any => (_: X)
}
```

Pattern matching on types

```
type Elem[X] = X match
  case String => Char
  case List[t] => Elem[t]
  case Any => X
```

The type/term correspondence dictates match type semantics:

```
def elem[X](x: X): Elem[X] = x match
  case x: String => x.charAt(0)
  case x: List[t] => elem(x.head)
  case x: Any => x
```

Pattern matching on types

```scala
type Elem[X] = X match
  case String => Char
  case List[t] => Elem[t]
  case Any => X
```

The type/term correspondence dictates match type semantics:

```scala
def elem[X](x: X): Elem[X] = x match
  case x: String => x.charAt(0)
  case x: List[t] => elem(x.head)
  case x: Any => x

elem[Any]("foo")
```

Formalization

System FM is an extension of System $F_{<:}$ with

- classes (defined externally)
- pattern matching
- match types

Design goals:

- explain the essence of match types
- give us confidence in our design
- be as simple as possible

Formalization

System FM is an extension of System $F_{<:}$ with

- classes (defined externally)
- pattern matching
- match types

Design goals:

- explain the essence of match types
- give us confidence in our design
- be as simple as possible

System FM is parametric

System FM is parametrized by a set of externally defined classes:

FM(C, Ψ, Ξ)

- C $\coloneqq$ set of classes
- Ψ $\coloneqq$ class inheritance relation
- Ξ $\coloneqq$ class disjointness relation

For example:

`class A; class B extends A`	$C = \{A, B\}$	$\Psi = \{(B, A)\}$	$\Xi = \varnothing$
`class A; class D`	$C = \{A, D\}$	$\Psi = \varnothing$	$\Xi = \{(A, D)\}$
`class A; trait T`	$C = \{A, T\}$	$\Psi = \varnothing$	$\Xi = \varnothing$

System FM is parametric

System FM is parametrized by a set of externally defined classes:

$\mathrm{FM}(C, \Psi, \Xi)$

- $C \coloneqq$ set of classes
- $\Psi \coloneqq$ class inheritance relation
- $\Xi \coloneqq$ class disjointness relation

For example:

`class A; class B extends A`	$C = \{A, B\}$	$\Psi = \{(B, A)\}$	$\Xi = \varnothing$
`class A; class D`	$C = \{A, D\}$	$\Psi = \varnothing$	$\Xi = \{(A, D)\}$
`class A; trait T`	$C = \{A, T\}$	$\Psi = \varnothing$	$\Xi = \varnothing$

System FM's syntax

$t ::=$		$T ::=$	
x	*variable*	X	*type variable*
$\lambda x{:}T.\ t$	*abstraction*	$T \to T$	*type of functions*
$\lambda X{<:}T.\ t$	*type abstraction*	$\forall X{<:}T.\ T$	*universal type*
$t\ t$	*application*	Top	*maximum type*
$t\ T$	*type application*	C	*class*
$\mathit{new}\ C$	*constructor call*	$\{\mathit{new}\ C\}$	*constructor singleton*
$t\ \mathit{match}\{\overline{x{:}C \Rightarrow t}\}\mathit{or}\ t$	*match expr.*	$T\ \mathit{match}\{\overline{T \Rightarrow T}\}\mathit{or}\ T$	*match type*
$v ::=$		$\Gamma ::=$	
$\lambda x{:}T.\ t$	*abstraction*	$\emptyset$	*empty context*
$\lambda X{<:}T.\ t$	*type abstraction*	$\Gamma, x{:}T$	*term binding*
$\mathit{new}\ C$	*constructor call*	$\Gamma, X{<:}T$	*type binding*

System FM's disjointness relation

$$\frac{(C_1, C_2) \in \Xi}{\Gamma \vdash \mathrm{disj}(C_1, C_2)} \quad (\text{D-Xi})$$

$$\frac{(C_1, C_2) \notin \Psi}{\Gamma \vdash \mathrm{disj}(\{\mathit{new}\ C_1\}, C_2)} \quad (\text{D-Psi})$$

$$\frac{\Gamma \vdash S <: U \qquad \Gamma \vdash \mathrm{disj}(U, T)}{\Gamma \vdash \mathrm{disj}(S, T)} \quad (\text{D-Sub})$$

$$\Gamma \vdash \mathrm{disj}(T_1 \to T_2, C) \quad (\text{D-Arrow})$$

$$\Gamma \vdash \mathrm{disj}(\forall X <: T_1.\ T_2, C) \quad (\text{D-All})$$

System FM's evaluation rules

We evaluate term-level matches by querying the class inheritance relation:

$$\frac{(\mathrm{C}, \mathrm{C}_n) \in \Psi \qquad \forall m < n.\ (\mathrm{C}, \mathrm{C}_m) \notin \Psi}{\mathit{new}\,\mathrm{C}\ \ \mathit{match}\{\mathrm{x}_i : \mathrm{C}_i \Rightarrow \mathrm{t}_i\}\,\mathit{or}\ \ \mathrm{t}_d \longrightarrow [\mathrm{x}_n \mapsto \mathit{new}\,\mathrm{C}]\mathrm{t}_n}$$

System FM's evaluation rules

We evaluate term-level matches by querying the class inheritance relation:

$$\frac{(\mathrm{C}, \mathrm{C}_n) \in \Psi \quad \forall m < n.\ (\mathrm{C}, \mathrm{C}_m) \notin \Psi}{\mathit{new}\,\mathrm{C}\ \ \mathit{match}\{\mathrm{x}_i : \mathrm{C}_i \Rightarrow \mathrm{t}_i\}\mathit{or}\ \ \mathrm{t}_d \longrightarrow [\mathrm{x}_n \mapsto \mathit{new}\,\mathrm{C}]\mathrm{t}_n}$$

We evaluate type-level matches using subtyping and disjointness:

$$\frac{\Gamma \vdash \mathrm{T}_s <: \mathrm{S}_n \quad \forall m < n.\ \Gamma \vdash \mathrm{disj}(\mathrm{T}_s, \mathrm{S}_m)}{\Gamma \vdash \mathrm{T}_s\ \ \mathit{match}\{\mathrm{S}_i \Rightarrow \mathrm{T}_i\}\mathit{or}\ \ \mathrm{T}_d =:= \mathrm{T}_n}$$

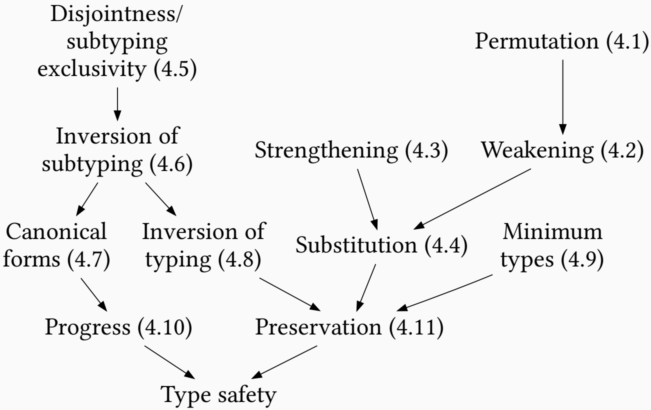
System FM's type safety

We show type safety through the progress and preservation.

Our proof comes in two versions:

1. Pen and paper (~30 pages)
2. Coq mechanization (~6000 LOC)

Structure of the type safety proof



Formalization vs implementation

Our implementation goes much further than System FM:

- recursion / non-termination
- type parameter bindings (`case List[t]`)
- empty types
- variance

Empty types

Definition

An empty type is a type that contains no values.

Our system does not like empty types.

`Nothing`, in particular, is both subtype and disjoint from every type.

```
type M[X] = X match
  case Int => String
  case String => Int

class C:
  type X
  def f(bad: M[X & String]): Int = bad

class D extends C:
  type X = Int
```

Empty types

Definition

An empty type is a type that contains no values.

Our system does not like empty types.

`Nothing`, in particular, is both subtype and disjoint from every type.

```
type M[X] = X match
  case Int => String
  case String => Int

class C:
  type X
  def f(bad: M[X & String]): Int = bad

class D extends C:
  type X = Int
```

Empty types

Definition

An empty type is a type that contains no values.

Our system does not like empty types.

`Nothing`, in particular, is both subtype and disjoint from every type.

```
type M[X] = X match
  case Int => String
  case String => Int

class C:
  type X
  def f(bad: M[X & String]): Int = bad

class D extends C:
  type X = Int
```

Variance

Definitions

`F` is covariant in `T`, written `F[+T]`, if `T1 <: T2` implies `F[T1] <: F[T2]`.
`G` is contravariant in `T`, `G[-T]`, if `T1 <: T2` implies `G[T1] >: G[T2]`.

We cannot prove disjointness when variance is involved, `F[T1]` and `F[T2]` always overlap:

$\forall$ T1, T2. `F[Nothing] <: F[T1]` and `F[Nothing] <: F[T2]`
$\forall$ T1, T2. `G[Any] <: G[T1]` and `G[Any] <: G[T2]`

We make an exception for covariant type that are used as class fields:

```
case class Some[+A](value: A)
```

`Some[String]` and `Some[Int]` are disjoint, since there is no runtime value of type `Some[Nothing]`.

Variance

Definitions

`F` is covariant in `T`, written `F[+T]`, if `T1 <: T2` implies `F[T1] <: F[T2]`.
`G` is contravariant in `T`, `G[-T]`, if `T1 <: T2` implies `G[T1] >: G[T2]`.

We cannot prove disjointness when variance is involved, `F[T1]` and `F[T2]` always overlap:

∀ T1, T2. `F[Nothing] <: F[T1]` and `F[Nothing] <: F[T2]`
∀ T1, T2. `G[Any] <: G[T1]` and `G[Any] <: G[T2]`

We make an exception for covariant type that are used as class fields:

```
case class Some[+A](value: A)
```

`Some[String]` and `Some[Int]` are disjoint, since there is no runtime value of type `Some[Nothing]`.

Variance

Definitions

`F` is covariant in `T`, written `F[+T]`, if `T1 <: T2` implies `F[T1] <: F[T2]`.
`G` is contravariant in `T`, `G[-T]`, if `T1 <: T2` implies `G[T1] >: G[T2]`.

We cannot prove disjointness when variance is involved, `F[T1]` and `F[T2]` always overlap:

$\forall$ T1, T2. `F[Nothing] <: F[T1]` and `F[Nothing] <: F[T2]`
$\forall$ T1, T2. `G[Any] <: G[T1]` and `G[Any] <: G[T2]`

We make an exception for covariant type that are used as class fields:

```
case class Some[+A](value: A)
```

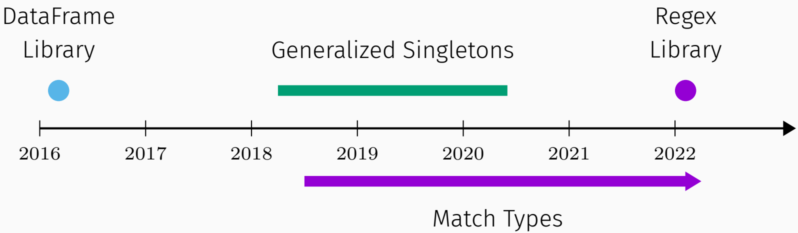
`Some[String]` and `Some[Int]` are disjoint, since there is no runtime value of type `Some[Nothing]`.

Match types: recap

We presented match types, a new language construct for type-level programming:

- implemented in the Scala 3 compiler
- formalized in an extension of System $F_{<:}$
- already in active use

Thank you!


DataFrame Library
Generalized Singletons
Regex Library
2016
2017
2018
2019
2020
2021
2022
Match Types