4.1.2.3

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(a)

$$\begin{aligned} E[X_1] &= E[M] * E[B] = 60.1805416 \\ VaR_0.99(X_1) &= 0 \\ \rightarrow TVaR_0.99(X_1) &= \frac{E[X_1]}{1-0.99} \\ &= 6018.0541625 \end{aligned}$$

(b)

$$\begin{aligned} S_n &\sim BinNegComp(r^* = r * n, F_B) \\ E[S_n] &= nE[X_1] = \{601.8054162, 6018.0541625, 6018.0541625\} \\ VaR_0.99(S_n) &= \{1.6519959 \times 10^4, 4.061309 \times 10^4, 1.4395913 \times 10^5\} \\ TVaR_0.99(S_n) &= \{2.3088683 \times 10^4, 4.9236978 \times 10^4, 1.5983804 \times 10^5\} \end{aligned}$$

```
FS <-
    function(x, n)
    dnbinom(0, n * r, q) + sum(sapply(seq_len(k0), function(i)
    dnbinom(i, n * r, q) * pgamma(x, a * i, b)))

    VaR <-
    function(k, n)
    ifelse(dnbinom(0, n * r, q) > k, 0 , uniroot(function(x)
    FS(x, n) - k, c(0, 10000000))$root)

    TVaR <- function(k, n) {
    v <- VaR(k, n)
    sum(sapply(seq_len(k0), function(i)
    dnbinom(i, n * r, q) * a * i / b * (1 - pgamma(v, a * i + 1, b)))) / (1 -
    k)
    }
```

(c)

$\sum_{i=1}^{n} VaR_0.99(X_i) = n * VaR_0.99(X) = n * 0 = 0 < VaR_0.99(S_n)$: La VaR n'est pas sous-additive.

(d)

```
sapply(c(10,100,1000),function(n) r*(1-q)/q*a/b/(1-0.99)*n > TVaR(0.99,n))
```

```
## [1] TRUE TRUE TRUE
```

TVaR sous-additive.