

## 6.2.1

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(a)

- $F_{X_1}(x) = 0.75 * H(x; 0.25, \frac{1}{4000}) + 0.25 * H(x; 0.25 * 2, \frac{1}{4000})$
- $F_{X_1}(5000) = 0.9361034$
- Optimisation numérique
- $VaR_{0.99}(X_1) = 1.0945267 \times 10^4$
- $TVaR_{0.99}(X_1) = \frac{1}{1-\kappa} \sum_{i=1}^2 P(\Theta = \theta_i) * \frac{\alpha \theta_i}{\beta} \bar{H}(VaR_{\kappa}(X), \alpha * \theta_i + 1, \beta)$
- $TVaR_{0.99}(X_1) = 1.4428388 \times 10^4$

Démonstration TVaR :

$$\begin{aligned} TVaR_{\kappa}(X) &= \frac{E[X * 1_{\{X > VaR_{\kappa}(X)\}}]}{1 - \kappa} \\ &= \frac{E_{\Theta}[E[X * 1_{\{X > VaR_{\kappa}(X)\}} | \Theta]]}{1 - \kappa} \\ &= \frac{E_{\Theta}[\frac{\alpha \theta}{\beta} \bar{H}(VaR_{\kappa}(X), \alpha * \theta + 1, \beta)]}{1 - \kappa} \\ &= \frac{1}{1 - \kappa} \sum_{i=1}^2 P(\Theta = \theta_i) * \frac{\alpha \theta_i}{\beta} \bar{H}(VaR_{\kappa}(X), \alpha * \theta_i + 1, \beta) \end{aligned}$$

Code :

```
theta <- c(1,2)
p_t <- c(0.75,0.25)
a <- 0.25
b <- 1/4000

Fx <- function(x) sum(p_t*pgamma(x,a*theta,b))
VaR <- function(k) uniroot(function(x) Fx(x)-k,c(0,100000))$root
TVaR <-
  function(x)
    sum(p_t * a * theta / b * (1 - pgamma(VaR(x), a * theta + 1, b))) / (1 -
    x)
```

(b)

- $F_{W_n}(x) = 0.75 * H(x; 0.25n, \frac{n}{4000}) + 0.25 * H(x; 0.25 * 2n, \frac{n}{4000})$
- $F_{W_{10}}(5000) = 0.9985591$
- Optimisation numérique
- $Var_{0.99}(W_{10}) = 3884.8365463$
- $TVaR_{0.99}(W_{10}) = \frac{1}{1-\kappa} \sum_{i=1}^2 P(\Theta = \theta_i) * \frac{\alpha\theta_i * 10}{\beta * 10} \bar{H}(VaR_{\kappa}(W), \alpha * \theta_i * 10 + 1, \beta * 10)$
- $TVaR_{0.99}(W_{10}) = 4461.4567885$

Code :

```
Fw <- function(x) sum(p_t*pgamma(x*10,a*theta*10,b))
VaR_W <- function(k) uniroot(function(x) Fw(x) - k, c(0,10000))$root
TVaR_W <- function(x){ sum(p_t*a*theta/b*(1-pgamma(VaR_W(x),a*theta*10+1,b*10)))/(1-x)}
```