4.2.5

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(a)

$S \sim BinComp(n = 6, q = 0.5, F_B)$

(b)

$$\begin{aligned} E[S] &= E[M] * E[B] \\ &= nq\frac{\alpha}{\beta} \\ &= 36 \end{aligned}$$

$$\begin{aligned} Var(S) &= E[M]Var(B) + E[B]^2 * Var(M) \\ &= nq\frac{\alpha}{\beta^2} + (\frac{\alpha}{\beta})^2 * nq(1-q) \\ &= 576 \end{aligned}$$

(c)

```
Fs <-
    function(x)
    dbinom(0, n, q) + sum(sapply(seq(n), function(i)
    dbinom(i, n, q) * pgamma(x, a * i, b)))

sapply(c(0,10,50,100),Fs)
```

```
## [1] 0.0156250 0.1188027 0.7550784 0.9835211
```

(d)

```
VaR <- function(k) {
    ifelse(dbinom(0, n, q) > k, 0, uniroot(function(x)
    Fs(x) - k, c(0, 100000))$root)
}

k <- c(0.5,0.9,0.99,0.999,0.9999)
sapply(k,VaR)
```

```
## [1]  31.98694  68.45223 108.01183 142.63578 174.81306
```

(e)

```
TVaR <- function(k) {
    v <- VaR(k)
    sum(sapply(seq(n), function(i)
    dbinom(i, n, q) * (a * i) / b * (1 - pgamma(v, a * i + 1, b)))) / (1 -k)
}
sapply(k,TVaR)
```

```
## [1]  54.60673  85.96591 123.17581 156.67898 188.17412
```