4.2.2

Olivier Turcotte

(a)

$$
\begin{aligned}
F_{X_1}(x) &= P(M=0) + \sum_{i=1}^{\infty} P(M=i) * H(x;i,\beta) \\
F_{X_1}(0) &= 0.9950125 \\
F_{X_1}(10) &= 0.995062 \\
VaR_0.99(X_1) &= 0, \text{optimisation} \\
TVaR_0.99(X_1) &= \frac{\sum_{i=1}^{\infty} P(M=i) * \frac{i}{\beta} \bar{H}(VaR_0.99(X_1);i+1,\beta)}{1-0.99} \\
&= 500
\end{aligned}
$$

Code

```
Fx1 <- function(x) dpois(0,l) +
    sum(
        sapply(seq_len(k0),function(i) dpois(i,l) * pgamma(x,i,beta))
    )

VaR <- function(k) ifelse(dpois(0,l) > k,0,uniroot(function(x) Fx1(x)-k,c(0,100))$root)

TVaR <- function(k){
    v <- VaR(k)
    sum(sapply(seq_len(k0),function(i) dpois(i,l)*i/beta*(1-pgamma(v,i+1,beta))))/(1-k)
}
```

(b)

$$
\begin{aligned}
W_n \sim PoisComp(\lambda^* = n\lambda, F_B^*)\, B^* \sim Exp(\beta^* = n*\beta) \\
F_{W_n}(x) &= P(M=0) + \sum_{i=1}^{\infty} P(M=i) * H(x;i,\beta*n) \\
F_{W_{1000}}(0) &= 0.0067379 \\
F_{W_{1000}}(10) &= 0.925608 \\
VaR_0.99(W_{1000}) &= 14.4043795, \text{optimisation} \\
TVaR_0.99(W_{1000}) &= \frac{\sum_{i=1}^{\infty} P(M=i) * \frac{i}{\beta*n} \bar{H}(VaR_0.99(W_1000);i+1,\beta*n)}{1-0.99} \\
&= 16.35278
\end{aligned}
$$

Code

```
FW <- function(x, n)
    dpois(0, l * n) + sum(sapply(seq_len(k0), function(i)
    dpois(i, l * n) * pgamma(x, i, beta * n)))

    VaR_W <- function(k, n)
    ifelse(dpois(0, l * n) > k , 0 ,
    uniroot(function(x)
    FW(x, n) - k, c(0, 1000))$root)

    TVaR_W <- function(k, n) {
    v <- VaR_W(k, n)
    sum(sapply(seq_len(k0), function(i)
    dpois(i, l * n) * i / (beta * n) * (1 - pgamma(v, i + 1, beta * n)))) /
    (1 - k)

    }
```

(c)

$$B^{VaR}_{0.99,1000} = -14.4043795$$
$$B^{TVaR}_{0.99,1000} = 483.64722$$

TVaR sous-additive, VaR non.