

## 4.2.6

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(a)

$$P(M_i = x) = \begin{cases} \frac{1}{4}, & x = 0 \\ \frac{2}{4}, & x = 1 \\ \frac{1}{4}, & x = 2 \end{cases}, i \in \{1, 2, 3\} \Rightarrow M_i \sim \text{Binom}(n = 2, q = 0.5)$$

(b)

$$S \sim \text{BinComp}(n = 6, q = 0.5, F_B)$$

(c)

$$\begin{aligned} E[S] &= E[M] * E[B] \\ &= nq \frac{\alpha}{\beta} \\ &= 36 \end{aligned}$$

$$\begin{aligned} \text{Var}(S) &= E[M]\text{Var}(B) + E[B]^2 * \text{Var}(M) \\ &= nq \frac{\alpha}{\beta^2} + \left(\frac{\alpha}{\beta}\right)^2 * nq(1 - q) \\ &= 576 \end{aligned}$$

(d)

```
Fs <-
  function(x)
    dbinom(0, n, q) + sum(sapply(seq(n), function(i)
      dbinom(i, n, q) * pgamma(x, a * i, b)))
```

```
sapply(c(0,10,50,100),Fs)
```

```
## [1] 0.0156250 0.1188027 0.7550784 0.9835211
```

(e)

```
VaR <- function(k) {
  ifelse(dbinom(0, n, q) > k, 0, uniroot(function(x)
    Fs(x) - k, c(0, 1000))$root)
}
```

```
k <- c(0.5,0.9,0.99,0.999,0.9999)
sapply(k,VaR)
```

```
## [1] 31.98697 68.45222 108.01181 142.63577 174.81308
```

(f)

```
TVaR <- function(k) {  
  v <- VaR(k)  
  sum(sapply(seq(n), function(i)  
    dbinom(i, n, q) * (a * i) / b * (1 - pgamma(v, a * i + 1, b)))) / (1 - k)  
}  
sapply(k, TVaR)
```

```
## [1] 54.60671 85.96594 123.17594 156.67900 188.17387
```