4.2.9

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(a)

$$\begin{aligned}
E[X] &= E[I] * E[B] \\
&= q * (E[C] + E[D]) \\
&= 360
\end{aligned}$$

$$\begin{aligned}
Var(X) &= E[I] * Var(B) + E[B]^2 * Var(I) \\
&= q * Var(C + D) + (E[C] + E[D])^2 * q(1 - q) \\
&= q * (Var(C) + Var(D) + 2 * Cov(C, D)) + (E[C] + E[D])^2 * q(1 - q) \\
&= 3.5724 \times 10^6 \\
E[S] &= E[\sum_{i=1}^{500} X_i] \\
&= 500 * E[X] \\
&= 1.8 \times 10^5 \\
Var(S) &= Var(\sum_{i=1}^{500} X_i) \\
&= 500 * Var(X), \text{ car indépendants} \\
&= 1.7862 \times 10^9
\end{aligned}$$

(b)

$$\begin{aligned}
E[Y * 1_{\{X>d\}}] &= E[(\sigma Z + \mu * 1_{\{Z>\frac{d-\mu}{\sigma}\}}) \\
&= \sigma E[Z * 1_{\{Z>\frac{d-\mu}{\sigma}\}}] + E[\mu * 1_{\{Z>\frac{d-\mu}{\sigma}\}}] \\
&= \frac{\sigma e^{\frac{-1}{2}(\frac{d-\mu}{\sigma})^2}}{\sqrt{2\pi}} + \mu P(Z > \frac{d-\mu}{\sigma}) \\
&= \frac{\sigma e^{\frac{-1}{2}(\frac{d-\mu}{\sigma})^2}}{\sqrt{2\pi}} + \mu(1 - \Phi(\frac{d-\mu}{\sigma})) \\
\text{Avec } d = VaR_\kappa(Y) & \\
&= \frac{\sigma e^{\frac{-1}{2}(\frac{d-\mu}{\sigma})^2}}{\sqrt{2\pi}} + \mu(1 - \kappa)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow TVaR_\kappa(Y) &= \frac{E[Y * 1_{\{X>VaR_\kappa(Y)\}}]}{1-\kappa} \\
&= \sigma \frac{e^{\frac{-1}{2}(\frac{VaR_\kappa(Y)-\mu}{\sigma})^2}}{\sqrt{2\pi}} \frac{1}{1-\kappa} + \mu
\end{aligned}$$