

4.1.2.2

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(a)

$$\begin{aligned}
 F_{X_1}(x) &= P(M=0) + \sum_{i=1}^{\infty} P(M=i) * H(x; i, \beta) \\
 F_{X_1}(0) &= 0.9950125 \\
 F_{X_1}(10) &= 0.995062 \\
 VaR_{0.99}(X_1) &= 0, \text{optimisation} \\
 TVaR_{0.99}(X_1) &= \frac{\sum_{i=1}^{\infty} P(M=i) * \frac{i}{\beta} \bar{H}(VaR_{0.99}(X_1); i+1, \beta)}{1-0.99} \\
 &= 500
 \end{aligned}$$

Code

```

Fx1 <- function(x) dpois(0,1) +
  sum(
    sapply(seq_len(k0),function(i) dpois(i,1) * pgamma(x,i,beta))
  )

VaR <- function(k) ifelse(dpois(0,1) > k,0,uniroot(function(x) Fx1(x)-k,c(0,100))$root)

TVaR <- function(k){
  v <- VaR(k)
  sum(sapply(seq_len(k0),function(i) dpois(i,1)*i/beta*(1-pgamma(v,i+1,beta))))/(1-k)
}

```

(b)

$$\begin{aligned}
 W_n &\sim PoisComp(\lambda^* = n\lambda, F_B^*) B^* \sim Exp(\beta^* = n * \beta) \\
 F_{W_n}(x) &= P(M=0) + \sum_{i=1}^{\infty} P(M=i) * H(x; i, \beta * n) \\
 F_{W_{1000}}(0) &= 0.0067379 \\
 F_{W_{1000}}(10) &= 0.925608 \\
 VaR_{0.99}(W_{1000}) &= 14.4043795, \text{optimisation} \\
 TVaR_{0.99}(W_{1000}) &= \frac{\sum_{i=1}^{\infty} P(M=i) * \frac{i}{\beta * n} \bar{H}(VaR_{0.99}(W_{1000}); i+1, \beta * n)}{1-0.99} \\
 &= 16.35278
 \end{aligned}$$

Code

```
FW <- function(x, n)
  dpois(0, 1 * n) + sum(sapply(seq_len(k0), function(i)
    dpois(i, 1 * n) * pgamma(x, i, beta * n)))

  VaR_W <- function(k, n)
    ifelse(dpois(0, 1 * n) > k , 0 ,
    uniroot(function(x)
      FW(x, n) - k, c(0, 1000))$root)

  TVaR_W <- function(k, n) {
    v <- VaR_W(k, n)
    sum(sapply(seq_len(k0), function(i)
      dpois(i, 1 * n) * i / (beta * n) * (1 - pgamma(v, i + 1, beta * n)))) /
    (1 - k)
  }
```

(c)

$$B_{0.99,1000}^{VaR} = -14.4043795$$

$$B_{0.99,1000}^{TVaR} = 483.64722$$

TVaR sous-additive, VaR non.