

4.2.9

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(a)

$$\begin{aligned} E[X] &= E[I] * E[B] \\ &= q * (E[C] + E[D]) \\ &= 360 \end{aligned}$$

$$\begin{aligned} Var(X) &= E[I] * Var(B) + E[B]^2 * Var(I) \\ &= q * Var(C + D) + (E[C] + E[D])^2 * q(1 - q) \\ &= q * (Var(C) + Var(D) + 2 * Cov(C, D)) + (E[C] + E[D])^2 * q(1 - q) \\ &= 3.5724 \times 10^6 \end{aligned}$$

$$\begin{aligned} E[S] &= E\left[\sum_{i=1}^{500} X_i\right] \\ &= 500 * E[X] \\ &= 1.8 \times 10^5 \end{aligned}$$

$$\begin{aligned} Var(S) &= Var\left(\sum_{i=1}^{500} X_i\right) \\ &= 500 * Var(X), \text{ car indépendants} \\ &= 1.3119504 \times 10^9 \end{aligned}$$

(b)

$$\begin{aligned} E[Y * 1_{\{X > d\}}] &= E[(\sigma Z + \mu * 1_{\{Z > \frac{d-\mu}{\sigma}\}})] \\ &= \sigma E[Z * 1_{\{Z > \frac{d-\mu}{\sigma}\}}] + E[\mu * 1_{\{Z > \frac{d-\mu}{\sigma}\}}] \\ &= \frac{\sigma e^{-\frac{1}{2}(\frac{d-\mu}{\sigma})^2}}{\sqrt{2\pi}} + \mu P(Z > \frac{d-\mu}{\sigma}) \\ &= \frac{\sigma e^{-\frac{1}{2}(\frac{d-\mu}{\sigma})^2}}{\sqrt{2\pi}} + \mu(1 - \Phi(\frac{d-\mu}{\sigma})) \end{aligned}$$

Avec $d = VaR_{\kappa}(Y)$

$$= \frac{\sigma e^{-\frac{1}{2}(\frac{d-\mu}{\sigma})^2}}{\sqrt{2\pi}} + \mu(1 - \kappa)$$

$$\begin{aligned} \Rightarrow TVaR_{\kappa}(Y) &= \frac{E[Y * 1_{\{X > VaR_{\kappa}(Y)\}}]}{1 - \kappa} \\ &= \sigma \frac{e^{-\frac{1}{2}(\frac{VaR_{\kappa}(Y)-\mu}{\sigma})^2}}{\sqrt{2\pi}} \frac{1}{1 - \kappa} + \mu \end{aligned}$$