7.5

Olivier Turcotte

(a)

$$\begin{aligned}
\widetilde{e}_x^{(\frac{1}{h})} &= \frac{h}{2}\Big(\sum_{k=0}^{\frac{w-x}{h}-1} \bar{F}_{Tx}(kh)) + \sum_{k=1}^{\frac{w-x}{h}-1} \bar{F}_{Tx}(kh))\\
&= \frac{h}{2}(1 + \sum_{k=1}^{\frac{w-x}{h}-1} \bar{F}_{Tx}(kh)) + \sum_{k=1}^{\frac{w-x}{h}-1} \bar{F}_{Tx}(kh))\\
&= \frac{h}{2}(1 + 2\sum_{k=1}^{\frac{w-x}{h}-1} \bar{F}_{Tx}(kh))\\
&= \frac{h}{2} + e_x^{(\frac{1}{h})}
\end{aligned}$$

(b)

```
h <- 1/c(1,2,4,12,16,256,1024)
beta <- 5.162 * 1e-5
a <- 9.566 * 1e-4
ga <- log(1.09369)
w <- 130

b <- function(x) beta*exp(ga*x)


kPx <- function(x,t) exp(-b(x)/ga*(exp(ga*t)-1)-a*x)

e_x <- function(x,h) h*sum(sapply(seq((w-x)/h -1 ),function(i) kPx(x,i*h)))
e_x2 <- function(x,h) sapply(h,function(i) i/2+e_x(x,i))

e_x2(40,h)
```

```
## [1] 36.44381 36.43431 36.42959 36.42645 36.42606 36.42496 36.42491
```

(c)

```
e_x2(60,h)
```

```
## [1] 19.47132 19.45672 19.44958 19.44488 19.44430 19.44266 19.44258
```