4

Olivier Turcotte

(a)

$$\tilde{e}_x^{\left(\frac{1}{h}\right)} = \frac{h}{2}\left(\sum_{k=0}^{\frac{w-x}{h}-1} \bar{F}_{Tx}(kh)) + \sum_{k=1}^{\frac{w-x}{h}-1} \bar{F}_{Tx}(kh) \right)$$

$$= \frac{h}{2}(1 + \sum_{k=1}^{\frac{w-x}{h}-1} \bar{F}_{Tx}(kh)) + \sum_{k=1}^{\frac{w-x}{h}-1} \bar{F}_{Tx}(kh))$$

$$= \frac{h}{2}(1 + 2 \sum_{k=1}^{\frac{w-x}{h}-1} \bar{F}_{Tx}(kh))$$

$$= \frac{h}{2} + e_x^{\left(\frac{1}{h}\right)}$$

(b)

```r
h <- 1/c(1,2,4,12,16,256,1024)
beta <- 0.00006148
ga <- log(1.09159)
b <- function(x) beta*exp(ga*x)

w <- 130
kPx <- function(x,t) exp(-b(x)/ga*(exp(ga*t)-1))

e_x <- function(x,h) h*sum(sapply(seq((w-x)/h -1 ),function(i) kPx(x,i*h)))
e_x2 <- function(x,h) sapply(h,function(i) i/2+e_x(x,i))

e_x2(40,h)
```

```
## [1] 37.41102 37.41089 37.41086 37.41085 37.41085 37.41085 37.41085
```

(c)

```r
e_x2(60,h)
```

```
## [1] 20.33472 20.33398 20.33380 20.33374 20.33374 20.33373 20.33373
```