4.1.2.1

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(a)

$$S_n \sim BinNegComp(r * n, q, F_B)$$

(b)

$$E[X] = \frac{r * (1-q)}{q} * \frac{\alpha}{\beta}$$
$$E[S_n] = \frac{n * r * (1-q)}{q} * \frac{\alpha}{\beta}$$

(c)

```
r <- 0.5 ; q <- 5/6
a <- 0.5 ; b <- 1/10000
k0 <- 1000

Fs <- function(x,n) dnbinom(0,n*r,q)+
    sum(sapply(seq_len(k0),function(i) dnbinom(i,n*r,q)*pgamma(x,a*i,b)))

Fx <- function(x) Fs(x,1)

Fx(10000)
```

```
## [1] 0.9836086
```

```
Fs(10000,200)
```

```
## [1] 0.0002766746
```

(d)

```
VaR_S <- function(k,n,interval) ifelse(dnbinom(0,n*r,q) >= k,0,uniroot(
    function(x) Fs(x,n)-k,interval)$root )

VaR_X <- function(k,interval) VaR_S(k,1,interval)
```

```
VaR_X(0.5,c(0,10000))
```

```
## [1] 0
```

```
VaR_X(0.99,c(0,100000))
```

```
## [1] 14184.01
```

(e)

```
VaR_S(0.5,200,c(0,100000))
```

```
## [1] 95586.21
```

```
VaR_S(0.99,200,c(0,300000))
```

```
## [1] 211586.3
```

(f)

```
TVaR_S <- function(k,n,interval){
    VaR <- VaR_S(k,n,interval)
    if(VaR == 0) n*r*(1-q)/q*a/b/(1-k)
    else{
        sum(sapply(seq_len(k0),function(i) dnbinom(i,n*r,q)*a*i/b*(1-pgamma(VaR,a*i+1,b))))/(1-k)
    }
}

TVaR_X <- function(k,interval) TVaR_S(k,1,interval)
```

```
TVaR_X(0.5,c(0,10000))
```

```
## [1] 1000
```

```
TVaR_X(0.99,c(0,20000))
```

```
## [1] 23283.01
```

(g)

```
TVaR_S(0.5,200,c(0,10e6))
```

```
## [1] 131492
```

```
TVaR_S(0.99,200,c(0,10e6))
```

```
## [1] 233072
```

(h)

```
200*VaR_X(0.5,c(0,10000)) > VaR_S(0.5,200,c(0,10e4))
```

```
## [1] FALSE
```

(i)

```
200*VaR_X(0.99,c(0,20000)) > VaR_S(0.99,200,c(0,10e6))
```

```
## [1] TRUE
```

(j)

```
200*TVaR_X(0.5,c(0,10000)) > TVaR_S(0.5,200,c(0,10e6))
```

```
## [1] TRUE
```

(k)

```
200*TVaR_X(0.99,c(0,10e6)) > TVaR_S(0.99,200,c(0,10e6))
```

```
## [1] TRUE
```

(l)

On voit en (h) que la VaR n'est pas sous-additive. Pour ce qui est de la TVaR, on peut conclure qu'elle est sous-additive.