Quantitative Macroeconomics

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Week 9

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1 Non-recursively Identified Models By Short-Run Restrictions

Consider a quarterly model of US monetary policy. Let $y_t = (\Delta p_t, \Delta gnp_t, i_t, \Delta m_t)'$, where p_t refers to the log of the GNP deflator, gnp_t to the log of real GNP, i_t to the federal funds rate, averaged by quarter, and m_t to the log of money aggregate M1. The data is given in `Keating1992.csv`.

The structural shock vector $\varepsilon_t = (\varepsilon_t^{AS}, \varepsilon_t^{IS}, \varepsilon_t^{MS}, \varepsilon_t^{MD})$ includes an aggregate supply shock, an IS shock, a money supply shock, and a money demand shock. The structural model can be written as:

$$\begin{pmatrix} u_t^p \\ u_t^{gnp} \\ u_t^i \\ u_t^m \end{pmatrix} = \begin{pmatrix} \varepsilon_t^{AS} \\ -b_{21,0}u_t^p - b_{23,0}u_t^i - b_{24,0}u_t^m + \varepsilon_t^{IS} \\ -b_{34,0}u_t^m + \varepsilon_t^{MS} \\ -b_{41,0}(u_t^{gnp} + u_t^p) - b_{43,0}u_t^i + \varepsilon_t^{MD} \end{pmatrix}$$

Furthermore, it is assumed that $E(\varepsilon_t \varepsilon_t') = \Sigma_\varepsilon$ is NOT the identity matrix; hence, we will use the normalization rule that the diagonal elements of B_0 equal unity.

1. Estimate the reduced-form covariance matrix of a VAR model with four lags and an intercept with ordinary least squares.

2. Provide intuition behind the structural model and derive the B_0 matrix. Note that we are imposing restrictions on both B_0 and Σ_ε, but not on B_0^{-1}.

3. Estimate B_0 and Σ_ε using the nonlinear equation solver `fsolve`. To this end, first set up a function that computes

$$F_{SR}([B_0, diag(\Sigma_\varepsilon)]) = \begin{bmatrix} vech\left(B_0^{-1}\Sigma_\varepsilon B_0^{-1'} - \hat{\Sigma}_u\right) \\ \text{restrictions on } B_0 \end{bmatrix} = 0$$

   where $diag(\Sigma_\varepsilon)$ denotes only the diagonal elements of Σ_ε. Note that these diagonal elements as well as B_0 are stacked into a matrix or vector which is then used as the input for the numerical optimizer. Use feasible options for the optimizer (e.g. as in the previous exercises).

4. Use the estimated structural impact matrix $B_0^{-1}\Sigma_\varepsilon^{1/2}$ to plot the structural impulse response functions using `IRFs.m`. Interpret the effects of an aggregate supply shock on prices, real GNP, the Federal Funds rate and M1.

Readings

- Kilian and Lütkepohl (2017, Ch. 8-9)

2 Long-Run Restrictions

Blanchard and Quah (1989) consider a bivariate model of the U.S. economy, where ur_t denotes the U.S. unemployment rate and gdp_t the log of U.S. real GDP. There is some evidence that ur_t is covariance stationary, whereas gdp_t exhibits a unit root; that is, GDP growth, $\Delta gdp_t = gdp_t - gdp_{t-1}$, is covariance-stationary. Blanchard and Quah (1989) set up a SVAR model for $y_t = (\Delta gdp_t, ur_t)'$ and analyze the effects of two structural shocks, an aggregate supply shock ε_t^{AS} and an aggregate demand shock ε_t^{AD}.

1. Why are short-run restrictions sometimes (or even often) *problematic*? What about long-run restrictions?
2. Assume for simplicity a VAR(1) model for y_t. Derive the effect of the structural shocks on the behavior of ur_{t+h}, Δgdp_{t+h} and gdp_{t+h} for $h = 0, 1, 2, ...$ What happens in the long-run, i.e. for $h \to \infty$?
3. Discuss the implications on the structural impulse responses of requiring gdp_t to return to its initial level in the long-run in response to an aggregate demand shock.
4. Given knowledge of the reduced-form VAR model parameters, show how to recover the short-run impact matrix B_0^{-1} from the long-run structural impulse response matrix $\Theta(1) = (I - A_1 - ... - A_p)^{-1} B_0^{-1} = A(1)^{-1} B_0^{-1}$, where $A(1)$ denotes the lag polynomial evaluated at $L = 1$.
5. Consider the data given in `BlanchardQuah1989.csv`. Estimate a SVAR(8) model with a constant term. The structural shocks are identified by imposing that ε_t^{AD} has no long-run effect on the level of real GDP. Estimate the impact matrix B_0^{-1} using
   a) the Cholesky decomposition on $\hat{A}(1)^{-1} \hat{\Sigma}_u \hat{A}(1)^{-1'} = \Theta(1)\Theta(1)'$
   b) a nonlinear equation solver that minimizes
   $$F(B_0^{-1}) = \begin{bmatrix} vech(B_0^{-1} B_0^{-1'} - \hat{\Sigma}_u) \\ \text{restrictions on } \Theta(1) \end{bmatrix}$$
   where $\Theta(1) = (I - A_1 - ... - A_p)^{-1} B_0^{-1} = A(1)^{-1} B_0^{-1}$. Assume that $E(\varepsilon_t \varepsilon_t') = I_2$ and the diagonal elements of B_0^{-1} are positive.
6. Plot the structural impulse response functions using `IRFs.m` for the level of GDP and the unemployment rate. Interpret your results in economic terms.

Readings

- Kilian and Lütkepohl (2017, Ch. 10.1, 10.3, 11.1, 11.2)

3 Combining Short-Run And Long-Run Restrictions

Consider a stylized VAR(4) model of U.S. monetary policy with only three quarterly variables. Let $y_t = (\Delta gnp_t, i_t, \Delta p_t)'$ be stationary variables, where gnp_t denotes the log of U.S. real GNP, p_t the corresponding GNP deflator in logs, and i_t the federal funds rate, averaged by quarter. The estimation period is restricted to 1954q4-2007q4 in order to exclude the period of unconventional monetary policy measures. Defining $\varepsilon_t = (\varepsilon_t^{policy}, \varepsilon_t^{AD}, \varepsilon_t^{AS})'$, the identifying restrictions can be summarized as

$$B_0^{-1} = \begin{bmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \text{ and } \Theta(1) = A(1)^{-1}B_0^{-1} = \begin{bmatrix} 0 & 0 & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

The long-run restrictions are imposed on the cumulated impulse responses.

1. Provide intuition given the above identifying restrictions.
2. Consider the data given in `RWZ2010.csv`. Estimate a VAR(4) model with a constant term.
3. Estimate the structural impact matrix using a nonlinear equation solver, i.e. the objective is to find the unknown elements of B_0^{-1} such that

$$\begin{bmatrix} vech(B_0^{-1}B_0^{-1'} - \hat{\Sigma}_u) \\ \text{short-run restrictions on } B_0^{-1} \\ \text{long-run restrictions on } \Theta(1) \end{bmatrix}$$

   is minimized where the normalization $\Sigma_\varepsilon = I_3$ is imposed. Furthermore, use the following insight to normalize the signs of the columns of B_0^{-1}:

   - a monetary policy shock (first column) raises the interest rate (second row) (monetary tightening)
   - a positive aggregate demand shock (second column) does not lower real GNP (first row) and inflation (third row)
   - a positive aggregate supply shock (3rd column) does not lower real GNP (first row) and does not raise inflation (third row)

4. Use the implied estimate of the structural impact matrix to plot the structural impulse response functions for the level of real GNP, the Federal Funds rate and the Deflator Inflation with response to a tightening in monetary policy. Interpret your results economically.

Readings

- Kilian and Lütkepohl (2017, Ch. 10.4, 10.5, 11.3)

References

Blanchard, Olivier J. and Danny Quah (Sept. 1989). “The Dynamic Effects of Aggregate Demand and Supply Disturbances”. In: *American Economic Review* 79.4, pp. 655–673.

Kilian, Lutz and Helmut Lütkepohl (2017). *Structural Vector Autoregressive Analysis*. Themes in Modern Econometrics. Cambridge: Cambridge University Press. ISBN: 978-1-107-19657-5. DOI: 10.1017/9781108164818.