Quantitative Macroeconomics

Winter 2022/23

Week 8

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Version: 1.0.0
Latest version available on: GitHub

Contents

1 Identification Problem in Structural Vector Autoregressive Models

Consider a simple 2-variable model:

$$i_t = \beta \pi_t + \gamma_1 i_{t-1} + \gamma_2 \pi_{t-1} + \varepsilon_t^{MP}$$
$$\pi_t = \delta i_t + \gamma_3 i_{t-1} + \gamma_4 \pi_{t-1} + \varepsilon_t^{\pi}$$

where i_t denotes the interest rate set by the central bank and π_t the inflation rate. Assume for the structural shocks: $\varepsilon_t = (\varepsilon_t^{MP}, \varepsilon_t^{\pi})' \sim N(0, \Sigma_\varepsilon)$.

1. Rewrite the model in a compact matrix form $B_0 y_t = B_1 y_{t-1} + \varepsilon_t$. Note that this is a structural VAR(1) model.
2. Since the structural VAR model is not directly observable, derive the reduced-form representation: $y_t = A_1 y_{t-1} + u_t$. What is the relationship between structural shocks ε_t and reduced-form residuals u_t?
3. In your own words, explain the identification problem in SVAR models. Provide intuition behind the popular identification assumptions of short-run, long-run and sign restrictions.

Readings

- Kilian and Lütkepohl (2017, Ch. 7.6)

2 Structural Impulse Response Function

Consider the structural VAR(p) model

$$B_0 y_t = B_1 y_{t-1} + ... + B_p y_{t-p} + \varepsilon_t$$

where the dimension of B_i, $i = 0, ..., p$, is $K \times K$. The $K \times 1$ vector of structural shocks ε_t is assumed to be white noise with covariance matrix $E(\varepsilon_t \varepsilon_t') = I_K$. That is, the elements of ε_t are mutually uncorrelated and also have a clear interpretation in terms on an underlying economic model. The reduced-form VAR(p) model is given by

$$y_t = \underbrace{B_0^{-1} B_1}_{A_1} y_t + ... + \underbrace{B_0^{-1} B_p}_{A_p} y_{t-p} + \underbrace{B_0^{-1} \varepsilon_t}_{u_t}$$

where the reduced-form error covariance matrix is $E(u_t u_t') = \Sigma_u = B_0^{-1} B_0^{-1'}$. Going back and forth between the structural and the reduced-form representation requires knowledge of the structural impact matrix B_0^{-1}. For now, assume that B_0^{-1} is a known matrix. We are interested in the response of each element of y_t to a one-time impulse in ε_t:

$$\frac{\partial y_{t+h}}{\partial \varepsilon_t'} = \Theta_h, \quad h = 0, 1, 2, ..., H$$

where Θ_h is a $K \times K$ matrix with individual elements $\theta_{jk,h} = \frac{\partial y_{j,t+h}}{\partial \varepsilon_{k,t}}$.

1. Usually the objective is to plot the responses of each variable to each structural shock. How many so-called impulse response functions do we need to plot?

2. Consider the VAR(1) representation of the VAR(p) process, i.e.

   $$Y_t = A Y_{t-1} + U_t$$

   where

   $$Y_t = \begin{pmatrix} y_t \\ \vdots \\ y_{t-p+1} \end{pmatrix}, A = \begin{pmatrix} A_1 & A_2 & ... & A_{p-1} & A_p \\ I_k & 0 & ... & 0 & 0 \\ 0 & I_K & ... & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & ... & I_k & 0 \end{pmatrix}, U_t = \begin{pmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

   Show that

   $$y_{t+h} = J A^{h+1} Y_{t-1} + \sum_{j=0}^{h} J A^j J' u_{t+h-j}$$

   where $J = [I_K, 0_{K \times K(p-1)}]$.

3. Derive an expression for Θ_h in terms of J, A and B_0^{-1}.

4. How would you infer from the response of the inflation rate, say Δp_t, the implied response of the log price level p_t?

5. Write a function that plots the IRFs given inputs A, Σ_u, B_0^{-1}, number of lags p and maximum horizon H.

Readings

- Kilian and Lütkepohl (2017, Ch. 4.1)

3 Recursively Identified Models

A popular argument in macroeconomics has been that oil price shocks in particular may act as domestic supply shocks for the U.S. economy. Thus, the question of how oil price shocks affect U.S. real GDP and inflation has a long tradition in macroeconomics. Postulate a VAR(4) model with intercept for the percent changes in the real WTI price of crude oil ($\Delta rpoil_t$), the U.S. GDP deflator inflation rate (Δp_t), and U.S. real GDP growth (Δgdp_t). Consider the dataset given in "USOil.csv". The data are quarterly and the estimation period is 1987q1-2013q2.

1. How can you identify the oil price shock statistically and economically?
2. Estimate the reduced-form vector autoregressive model by least-squares or maximum-likelihood to obtain a consistent estimate of the reduced-form error covariance matrix.
3. Estimate the structural impact multiplier matrix B_0^{-1} based on a lower-triangular Cholesky decomposition of the residual covariance matrix.
4. Estimate the structural impact multiplier matrix B_0^{-1} based on solving the system of nonlinear equations that implicitly defines the elements of B_0^{-1} using a nonlinear equation solver that finds the vector x such that $F(x) = 0$, where $F(x)$ denotes a system of nonlinear equations in x. To this end, vectorize the system of equations $B_0^{-1}B_0^{-1'} - \hat{\Sigma}_u$. The objective is to find the unknown elements of B_0^{-1} such that

$$F_{SR}(B_0^{-1}) = \left[vech\left(B_0^{-1}B_0^{-1'} - \hat{\Sigma}_u\right), \quad b_0^{12}, \quad b_0^{13}, \quad b_0^{23}\right]' = 0$$

   where the vech operator is used to select the lower triangular elements of $B_0^{-1}B_0^{-1'} - \hat{\Sigma}_u$. To this end:

   - Write a function `fSR.m` that computes F_{SR}.
   - Set the termination tolerance `TolX=1e-4`, the termination tolerance on the function value `TolFun` to `1e-9`, the maximum number of function evaluations `MaxFunEvals` to `1e50000`, the maximum number of iterations `MaxIter` to `1000`. Save this set of options:`options = optimset('TolX',TolX, 'TolFun',TolFun,'MaxFunEvals',MaxFunEvals,'MaxIter',MaxIter)`
   - Choose an admissible start value for B_0^{-1} and call the optimization routine `fsolve` to minimize your `fSR` function:
     `[B0inv,fval,exitflag,output]=fsolve('fSR',B0inv0,options,'additional inputs to fSR')`
   - Impose the normalization rule that the diagonal elements of B_0^{-1} are positive.

   Compare this to the Cholesky decomposition. Is there a difference?

5. Plot the impulse response function of an unexpected increase in the real price of crude oil (not its percent change!) using `IRFs.m` and interpret it economically.

Readings

- Kilian and Lütkepohl (2017, Ch. 8-9)

References

Kilian, Lutz and Helmut Lütkepohl (2017). *Structural Vector Autoregressive Analysis*. Themes in Modern Econometrics. Cambridge: Cambridge University Press. ISBN: 978-1-107-19657-5. DOI: 10.1017/9781108164818.