# **Quantitative Macroeconomics**

# Winter 2022/23

## Week 10

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#### 1 Inference In SVARs Identified By Exclusion Restrictions

Consider an exactly-identified structural VAR model subject to short- and/or long-run restrictions, where the structural impulse response of variable j to structural shock k at horizon h is denoted as  $\theta_{jk,h}$ , which we simply denote as  $\theta$ . We are interested in the distribution of  $\theta$ , in particular deriving  $(1-\gamma)\%$  point-wise confidence intervals given a consistent estimate  $\hat{\theta}$  of  $\theta$ .

1. Consider the asymptotic confidence intervals which are derived using the delta method:

$$\hat{\theta} \pm z_{\gamma/2} \widehat{std}(\hat{\theta})$$

where  $z_{\gamma/2}$  denotes the  $\gamma/2$  percentile of the standard normal distribution and  $\widehat{std}(\hat{\theta})$  a consistent estimate of the standard deviation of  $\theta$ . Name the assumptions and shortcomings of this approach.

- 2. Outline the idea and algorithm of the Standard Residual-Based Recursive-Design Bootstrap approach.
- 3. Name the central idea underlying the Residual-Based Wild Bootstrap.
- 4. Discuss the choice of significance level  $\gamma$ .
- 5. Discuss how to draw initial conditions for a resampling method.
- 6. Given a bootstrap approximation to the distribution of the structural impulse-response function, discuss how to construct bootstrap confidence intervals from this distribution. Particularly, explain
  - a) intervals based on bootstrap standard errors
  - b) Efron's percentile interval
  - c) equal-tailed percentile-t intervals

#### Readings

• Kilian and Lütkepohl (2017, Ch. 12.1-12.5, 12.9)

#### 2 Bootstrapping Standard Deviations of Structural IRFs

Consider an exactly-identified structural VAR model subject to short- and/or long-run restrictions, where the structural impulse response of variable i to shock j at horizon h are simply denoted as  $\theta \equiv \Theta_{ij,h}$ . As an exact closed-form solution for the asymptotic standard errors of  $\theta$  are only available under restrictive assumptions, we will rely on a numerical approximation using a bootstrap approach.

- 1. Reconsider an exercise (of your choice) from the lecture on SVAR models identified with exclusion restrictions and re-estimate the structural impulse response function.
- 2. Compute  $\widehat{std}(\hat{\theta}^*)$  via a bootstrap approximation by following these steps:
  - Write a function BootstrapGDP(VAR) which implements a standard residual-based bootstrap approach using sampling with replacement techniques on the residuals. Furthermore, the initial values should be drawn randomly in blocks. Hint: Use the companion form to do the simulations.
  - Set bootstrap repetitions B equal to 1000 (or higher) and initialize a  $K \times K \times H \times B$  array THETAstar, where the first dimension corresponds to variable i=1,...,K, the second dimension to shock j=1,...,K, the third dimension to the horizon of the IRFs h=0,...,H and the fourth dimension to the bootstrap repetition b=1,...,B.
  - For b = 1, ..., B do the following (you may also try parfor instead of for in order to make use of Matlab's parallel computing toolbox if installed):
    - Compute a bootstrap GDP  $y_t^b$  using the function BootstrapGDP(VAR).
    - Estimate the reduced-form and structural impulse response function on this artificial dataset with the same methodology, settings and identification restrictions as in the estimation of the original dataset.
    - Store the structural IRFs in THETAstar at position (:,:,:,b).
  - Compute the standard deviation of the bootstrap structural IRFs using std(THETAstar,0,4).
- 3. Plot approximate 68% and 95% confidence intervals for the structural impulse response functions according to the delta method:

$$\hat{\theta} \pm z_{\gamma/2} \widehat{std} (\hat{\theta}^*)$$

where  $z_{\gamma/2}$  is the  $\gamma/2$  quantile of the standard normal distribution.

#### Readings

• Kilian and Lütkepohl (2017, Ch. 12.1-12.5, 12.9)

## References

Kilian, Lutz and Helmut Lütkepohl (2017). Structural Vector Autoregressive Analysis. Themes in Modern Econometrics. Cambridge: Cambridge University Press. ISBN: 978-1-107-19657-5. DOI: 10.1017/9781108164818.