Quantitative Macroeconomics

Midterm Exam

Winter 2021/2022

- Answer **all** of the following six exercises in English.
- Hand in your solutions before Friday December, 17 2021 at noon (12:00).
- The solution files should contain your executable (and commented) Matlab functions and script files as well as all additional documentation as `pdf`, not `doc` or `docx`. Your `pdf` files may also include scans or pictures of handwritten notes.
- Please e-mail the solution files to `willi.mutschler@uni-tuebingen.de`
- I will confirm the receipt of your work also by email (typically within the hour). If not, please resend it to me.
- All students must work on their own, please also give your student ID number.
- It is advised to regularly check Ilias and your emails in case of urgent updates.
- If there are any questions, do not hesitate to contact Willi Mutschler.

1 VAR(1) With Time Trend

Consider the VAR(1) model with a constant and time trend

$$y_t = c + d \cdot t + A_1 y_{t-1} + u_t$$

where y_t, c, d and u_t are K-dimensional vectors. Moreover, the Eigenvalues of the $K \times K$ dimensional matrix A_1 are inside the unit circle and u_t is white noise with covariance matrix Σ_u.

1. Compute the unconditional first and second moments, i.e. the unconditional mean, variance, autocovariance and autocorrelation function of y_t.

2. Why is this process not covariance-stationary? How could one proceed to make it covariance-stationary?

2 Maximum Likelihood Estimation Of Laplace AR(1)

Consider the AR(1) model with constant

$$y_t = c + \phi y_{t-1} + u_t$$

Assume that the error terms u_t are i.i.d. Laplace distributed with known density

$$f_{u_t}(u) = \frac{1}{2} \exp\left(-|u|\right)$$

Note that the parametrization of the above density is such that $E(u_t) = 0$ and $Var(u_t) = 2$.

1. Derive the log-likelihood function conditional on the first observation.
2. Write a function that calculates the conditional log-likelihood of c and ϕ.
3. Load the dataset given in the excel file `Laplace.xlsx`. Numerically find the maximum likelihood estimates of c and ϕ by minimizing the negative conditional log-likelihood function.
4. Compare your estimates with the maximum likelihood estimate under the assumption of Gaussianity.
5. Compare your estimates with the ordinary least-squares estimate.

3 Properties Of Lag-Order Selection Criteria

Assume that the true Data-Generating-Process (DGP) follows the following VAR(4) model

$$y_t = \begin{pmatrix} 2.4 & 1.0 \\ 0 & 1.1 \end{pmatrix} y_{t-1} + \begin{pmatrix} -2.15 & -0.9 \\ 0 & -0.41 \end{pmatrix} y_{t-2} + \begin{pmatrix} 0.852 & 0.2 \\ 0 & 0.06 \end{pmatrix} y_{t-3} + \begin{pmatrix} -0.126 & 0 \\ 0 & 0.0003 \end{pmatrix} y_{t-4} + u_t$$

where u_t is a Gaussian white noise with contemporary covariance matrix $\Sigma_u = \begin{pmatrix} 0.9 & 0.2 \\ 0.2 & 0.5 \end{pmatrix}$.

Perform a Monte-Carlo analysis to study both the finite-sample as well as asymptotic properties of the Akaike Information Criterion (AIC) and the Schwarz Information Criterion (SIC):

$$AIC(m) = \log(det(\tilde{\Sigma}_u(m))) + \frac{2}{T}\varphi(m)$$
$$SIC(m) = \log(det(\tilde{\Sigma}_u(m))) + \frac{\log T}{T}\varphi(m)$$

where $\tilde{\Sigma}_u = T^{-1}\sum_{t=1}^{T} \hat{u}_t \hat{u}_t'$ is the residual covariance matrix estimator for a reduced-form VAR model of order m based on OLS residuals $\hat{u}_t$. The function $\varphi(m)$ corresponds to the total number of regressors in the system of VAR equations. The VAR order is chosen such that the respective criterion is minimized over the possible orders $m = 0, ..., p^{max}$. To this end, do the following.

- Set the number of Monte Carlo repetitions $R = 100$ and $p^{max} = 8$.
- Initialize output matrices aic and sic each of dimension $R \times 5$.
- For $r = 1, ..., R$ do the following:
  - Simulate 10100 observations for the DGP given above and discard the first 100 observations as burn-in phase. Save the remaining 10000 observations in a matrix Y.
  - Compute the lag criteria for 5 different sample sizes $T = \{80, 160, 240, 500, 10000\}$, i.e. use the last T observations of your simulated data matrix Y for computations.
  - Save the chosen lag order in the corresponding output object at position $[r, j]$ where $j = 1, ..., 5$ indicates the corresponding sample size.
- Look at the frequency tables of your output objects for the different subsamples. Hint: `tabulate(aic(:,1))` displays a frequency table for the AIC criterion with sample size equal to 80.

Given your results, do you agree with the following (general) findings?

1. AIC is not consistent for the true lag order, whereas SIC is consistent.
2. AIC never (asymptotically) selects a lag order that is lower than the true lag order.
3. In finite samples, we usually have $\hat{p}_{SIC} \leq \hat{p}_{AIC}$.
4. In finite samples, AIC has a tendency to overestimate the lag order, SIC has a tendency to underestimate the lag order; hence, one should rely on AIC in finite samples.

4 Bayesian Estimation Of VAR(2) and Zero-Lower-Bound

Re-consider the exercise on Bayesian Estimation of a VAR(2) model for the US economy which includes (in this ordering) the federal funds rate, government bond yield, unemployment and inflation. The sample period consists of 2007m1 to 2010m12. Data is given in the excel sheet "USZLB" in the file `data.xlsx`.

As the sample period includes the financial crisis, re-estimate the model with Bayesian methods and a Minnesota prior, but now use a small prior variance to reflect the view that monetary policy is at the zero-lower-bound and hence unlikely to respond to changes in the other variables. Compare both the mean as well as the 16th and 84th percentiles of the posterior distribution of the regression coefficient matrix A and the reduced-form covariance matrix Σ_u with the estimates you get if you do not explicitly adjust the prior for the lower-zero-bound.

5 How Well Does the IS-LM Model Fit Postwar US Data?

Consider a quarterly model for $y_t = (\Delta gnp_t, \Delta i_t, i_t - \Delta p_t, \Delta m_t - \Delta p_t)'$, where gnp_t denotes the log of GNP, i_t the nominal yield on three-month Treasury Bills, Δm_t the growth in M1 and Δp_t the inflation rate in the CPI. There are four shocks in the system: an aggregate supply (AS), a money supply (MS), a money demand (MD) and an aggregate demand (IS) shock. Ignoring the lagged dependent variables for **expository** purposes ($B_1 = ... = B_p = 0$), the unrestricted structural VAR model can be simply written as $B_0 y_t = \varepsilon_t$. That is:

$$\Delta gnp_t = -b_{12}\Delta i_t - b_{13}(i_t - \Delta p_t) - b_{14}(\Delta m_t - \Delta p_t) + \varepsilon_t^{AS} \tag{1}$$

$$\Delta i_t = -b_{21}\Delta gnp_t - b_{23}(i_t - \Delta p_t) - b_{24}(\Delta m_t - \Delta p_t) + \varepsilon_t^{MS} \tag{2}$$

$$i_t - \Delta p_t = -b_{31}\Delta gnp_t - b_{32}\Delta i_t - b_{34}(\Delta m_t - \Delta p_t) + \varepsilon_t^{MD} \tag{3}$$

$$\Delta m_t - \Delta p_t = -b_{41}\Delta gnp_t - b_{42}\Delta i_t - b_{43}(i_t - \Delta p_t) + \varepsilon_t^{IS} \tag{4}$$

where b_{ij} denotes the ijth element of B_0. Consider the following identification restrictions:

- Money supply shocks do not have contemporaneous effects on output growth, i.e.
$$\frac{\partial \Delta gnp_t}{\partial \varepsilon_t^{MS}} = 0$$
- Money demand shocks do not have contemporaneous effects on output growth, i.e.
$$\frac{\partial \Delta gnp_t}{\partial \varepsilon_t^{MD}} = 0$$
- Monetary authority does not react contemporaneously to changes in the price level.
Hint: compute
$$\frac{\partial \Delta i_t}{\partial \Delta p_t} = 0$$
from equation (2).
- Money supply shocks, money demand shocks and aggregate demand shocks do not have long-run effects on the log of real GNP:
$$\frac{\partial gnp_t}{\partial \varepsilon_t^{MS}} = 0, \qquad \frac{\partial gnp_t}{\partial \varepsilon_t^{MD}} = 0, \qquad \frac{\partial gnp_t}{\partial \varepsilon_t^{IS}} = 0$$
- The structural shocks are uncorrelated with covariance matrix $E(\varepsilon_t \varepsilon_t') = \Sigma_\varepsilon$. The variances are **not** normalized.

Solve the following exercises:

1. Derive the implied exclusion restrictions on the matrices B_0, B_0^{-1} and $\Theta(1)$.
2. Consider the data given in the excel file `gali1992.xlsx`. Estimate a VAR(4) model with constant term.
3. Estimate the structural impact matrix using a nonlinear equation solver, i.e. the objective is to find the unknown elements of B_0^{-1} and the diagonal elements of Σ_ε such that
$$\begin{bmatrix} vech(B_0^{-1}\Sigma_\varepsilon B_0^{-1'} - \hat{\Sigma}_u) \\ \text{short-run restrictions on } B_0 \text{ and } B_0^{-1} \\ \text{long-run restrictions on } \Theta(1) \end{bmatrix}$$
is minimized. Normalize the shocks such that the diagonal elements of B_0^{-1} are positive.
4. Use the implied estimates of B_0^{-1} and Σ_ε to plot the structural impulse responses functions for (i) real GNP, (ii) the yield on Treasury Bills, (iii) the real interest rate and (iv) real money growth.

6 Posterior distribution of sign-identified structural IRFs

Consider data for $y_t = (\Delta gnp_t, \Delta p_t, i_t)'$, where gnp_t denotes the log of U.S. real GNP, p_t the consumer price index in logs, and i_t the federal funds rate. The sample period consists of 1970Q1 to 2011Q1. Data is given in the Excel sheet "MonPolData" in the file `data.xlsx`.

1. Estimate the parameters of a VAR(4) model with constant by using Bayesian methods, i.e. a Gibbs sampling method. To this end:

   - Assume a Minnesota prior for the VAR coefficients, where the prior mean should reflect the view that the VAR follows a random walk. Set the hyperparameters for the prior covariance matrix V_0 such that the tightness parameter on lags of own and of other variables are both equal to 0.5, and the tightness parameter on the constant term is equal to 100.
   - Assume an inverse Wishart prior for the covariance matrix with degrees of freedoms equal to the number of variables and the identity matrix as prior scale matrix.
   - Initialize the first draw of the covariance matrix with OLS values.
   - Draw in total 30100 times from the conditional posteriors, where you discard draws of the coefficient matrix that are unstable.
   - Save the last $n_G = 100$, draws (A^r, Σ_u^r) $(r = 1, ..., n_G)$ for inference on the structural model.

2. Estimate the posterior of the structural impulse response function by considering the following sign restrictions pattern on the impact matrix

$$B_0^{-1} = \begin{bmatrix} + & + & - \\ + & - & - \\ + & * & + \end{bmatrix}$$

   where $*$ denotes unrestricted values.

   - For each draw of the posterior (A^r, Σ_u^r) $(r = 1, ..., n_G)$ find $n_Q = 200$ rotation matrices that provide impact matrices B_0^{-1} and implied impulse response functions that are in accordance with the sign restrictions pattern given above. Note that in the end you should have $n_Q \cdot n_G = 20000$ accepted draws from the posterior of structural impulse response functions.
   - Display the vector of point-wise posterior medians and the vectors of point-wise 68% error bands of the structural impulse responses. Interpret your results for one structural shock (of your choice) on (i) the level of real gnp, (ii) the consumer price index and (iii) on the federal funds rate.
   - Name two shortcomings of using the median response function as a measure of central tendency in sign-identified SVARs.