## **Quantitative Macroeconomics**

# Winter 2022/23

## Week 2

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### 1 Visualizing Time Series Data

- 1. What sort of variables are we dealing with in macroeconomics?
- 2. List sources and providers to get macroeconomic data.
- 3. Establish a strategy for obtaining macroeconomic data.
- 4. Analyze the quarterly growth rate of gross domestic product (GDP) for Norway. To this end, do the following:
  - Find a source for this data and download it to your computer (preferably as csv or xlsx).
  - Load the data using MATLAB's interactive import tool. Save the dates and data in two separate variables.
  - Create a new figure and plot the quarterly growth rate. Name it "Plot of Q-on-Q growth in Norway".
  - Create a new figure and plot two histograms, one with 10 bins and another one with 30 bins. Call it "Histogram of Q-on-Q growth in Norway". Compare them to a fitted normal distribution.
  - Create a new figure and assess the fit of your data to a normal distribution via a "Q-Q plot". Call it "Normal probability plot".
  - Create a new figure and plot a boxplot of your data. Call it "Boxplot".
  - Give a numerical estimate for the average growth rate and its standard deviation.
- 5. Redo the previous exercise for (i) 1980Q1 to 2012Q4, (ii) 2012Q4 to 2019Q4, and (iii) 2020Q1 to 2022Q2
- 6. Redo the whole exercise for another country of your choice. Do a Pull Request on the course's GitHub repository. to upload the used data file (preferably as csv or xlsx) and your MATLAB script.

#### Readings

- https://de.mathworks.com/help/matlab/import\_export/import-data-interactively.html
- https://de.mathworks.com/help/matlab/import\_export/select-spreadsheet-data-interactively. html
- Bjørnland and Thorsrud (2015, Ch.2)

#### Useful resources

- Economic Data Resources (https://libguides.umn.edu/c.php?g=843682&p=6527336)
- Gould Library The Data Search (https://gouldguides.carleton.edu/c.php?g=147179&p=965273)
- DBNomics Providers (https://db.nomics.world/providers)
- Our World in Data (https://ourworldindata.org)
- FRED (https://fred.stlouisfed.org)

### 2 Definition and Frequencies of Time Series Data

- 1. Briefly define a time series in terms of random variables and stochastic processes.
- 2. Consider data for various time series given in
  - NorwayGDP.xls
  - NorwayInterestRate3m.xls
  - NorwayInterestRate10yrs.xls
  - NorwayOSEBXGR.csv
  - NorwayPopulation.xls
  - NorwayRealHousePrices.xlsx
  - NorwayUnemploymentRate.xls

Open the individual files and make note of the structure and source of the data. Import the data and replicate figure 1.

- 3. What are the data frequencies for each time series? For what kind of economic analysis would you use these frequencies?
- 4. What are the sample sizes? From an economic and/or statistical point of view, is it always better to have a larger sample size?
- 5. Roughly speaking, a time series consists of four components: a trend, a cycle, a season, and noise. To what extent do you find these features in figure 1?
- 6. Consider the plots in figure 1 jointly. What are possible macroeconomic issues that could be analyzed?
- 7. How do you aggregate time series of stock variables (like capital or debt) and of flow variables (like GDP)? That is, if you have monthly data, how do you get a quarterly time series?

#### Readings

• Bjørnland and Thorsrud (2015, Ch.1, Ch.2)

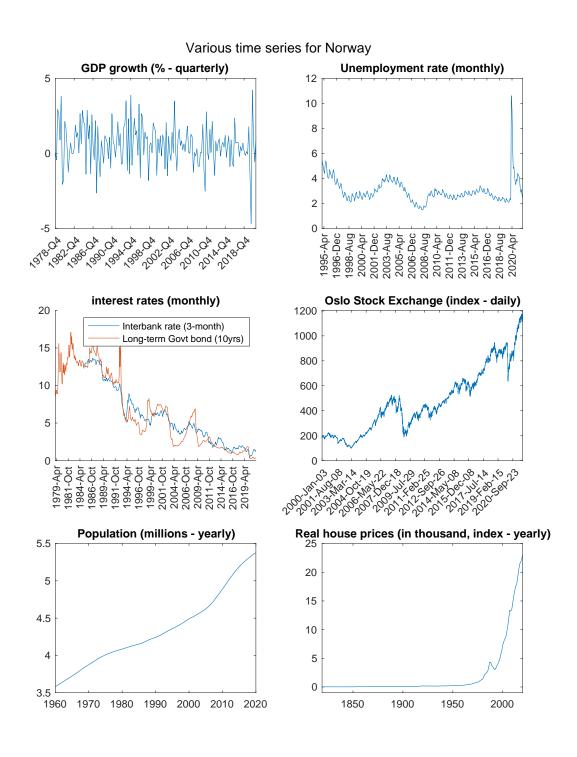


Figure 1: Various Time Series For Norway

### 3 Some Fundamental Concepts Of Univariate Time Series Analysis

- 1. Define the "White Noise Process" labeled shortly as  $\varepsilon_t \sim WN(0, \sigma_{\varepsilon}^2)$ .
- 2. Plot 200 observations of

(i) 
$$y_t = \varepsilon_t$$
  
(ii)  $y_t = \frac{1}{5}(\varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_t + \varepsilon_{t+1} + \varepsilon_{t+2})$ 

with  $\varepsilon_t \sim N(0,1)$ . What are the differences?

- 3. Briefly explain the concepts of (i) weak stationarity and (ii) strict stationarity.
- 4. Define the autocovariance and autocorrelation function for a covariance-stationary stochastic process  $\{Y_t\}$ .
- 5. Consider the linear first-order difference equation

$$y_t = \phi y_{t-1} + \varepsilon_t$$

with  $\varepsilon_t \sim N(0,1)$ . Simulate and plot 200 observations of  $(i)|\phi| < 1$ ,  $(ii)\phi = 1$ , and  $(iii)|\phi| > 1$ . What does this imply in terms of stationarity of the process?

6. Briefly explain the Lag-operator and Lag-polynomials. How can we check whether an AR(p) process

$$y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} - \dots - \phi_p y_{t-p} = (1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p) y_t = \varepsilon_t$$

is weakly stationary?

#### Readings

- Bjørnland and Thorsrud (2015, Ch.2)
- Lütkepohl (2004)

## 4 Properties Of AR(1)

Let  $\{\varepsilon_t\}$  be a white noise process with variance  $\sigma_{\varepsilon}^2$ .

1. Consider the first-order autoregressive process AR(1):

$$y_t = \phi y_{t-1} + \varepsilon_t$$

Derive the conditional and unconditional first and second moments.

- 2. Simulate different AR(1) processes and plot the corresponding autocorrelation function (ACF). To this end write a function ACFPlots( $y, p^{max}, \alpha$ ) to plot the ACF of the data vector y with maximum number of lags  $p^{max}$ . Also include an approximate  $(1-\alpha)\%$  confidence interval around zero in your plot. Hints:
  - The empirical autocorrelation function at lag k is defined as  $r_k = c_k/c_0$  where

$$c_k = \frac{1}{T} \sum_{t=k+1}^{T} (y_t - \bar{y})(y_{t-k} - \bar{y})$$

and

$$c_0 = \frac{1}{T} \sum_{t=1}^{T} (y_t - \bar{y})(y_t - \bar{y})$$

You can either use a for-loop to compute the sum or use vectors:  $(y - \bar{y})'(y - \bar{y})$ .

• The sample autocorrelation function is an estimate of the actual autocorrelation only if the process is stationary. If the process is purely random, that is, all members are mutually independent and identically distributed so that  $y_t$  and  $y_{t-k}$  are stochastically independent for  $k \neq 0$ , then the normalized estimated autocorrelations are asymptotically standard normally distributed, i.e.  $\sqrt{T}r_k \to U \sim N(0,1)$  and thus  $r_k \to \tilde{U} \sim N(0,1/T)$ .

#### Readings

- Bjørnland and Thorsrud (2015, Ch.2)
- Lütkepohl (2004)

## 5 Properties AR(1) With Time Trend

Consider the AR(1) model with a constant and time trend

$$y_t = c + d \cdot t + \phi y_{t-1} + u_t$$

where  $u_t \sim WN(0, \sigma_u^2)$ ,  $|\phi| < 1$ ,  $c \in \mathbb{R}$  and  $d \in \mathbb{R}$ .

- 1. Compute the unconditional first and second moments, i.e. the unconditional mean, variance, autocovariance and autocorrelation of  $y_t$ .
- 2. Why is this process not covariance-stationary? How could one proceed to make it covariance-stationary?

#### Readings

• Lütkepohl (2004)

## References

Bjørnland, Hilde Christiane and Leif Anders Thorsrud (2015). Applied Time Series for Macroeconomics. 2. utgave, 1. opplag. Oslo: Gyldendal Akademisk. ISBN: 978-82-05-48089-6.

Lütkepohl, Helmut (2004). "Univariate Time Series Analysis". In: Applied Time Series Econometrics. Ed. by Helmut Lütkepohl and Markus Krätzig. First. Cambridge University Press, pp. 8–85. ISBN: 978-0-521-83919-8 978-0-521-54787-1 978-0-511-60688-5. DOI: 10.1017/CB09780511606885.003.