Quantitative Macroeconomics

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Week 5

Willi Mutschler
willi@mutschler.eu

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Contents

1 Information Criteria For AR(p)

Consider the following information criteria to estimate the order p of an $AR(p)$ model:

$$AIC(n) = \log \tilde{\sigma}^2(n) + \frac{2}{T^{eff}} n$$
$$SIC(n) = \log \tilde{\sigma}^2(n) + \frac{\log T^{eff}}{T^{eff}} n$$
$$HQC(n) = \log \tilde{\sigma}^2(n) + \frac{2 \log \log T^{eff}}{T^{eff}} n$$

where $\tilde{\sigma}^2$ denotes the ML estimate of the variance term based on the OLS residuals $\hat{u}_t(n)$ of the corresponding estimated $AR(p)$ model. n is the number of estimated parameters and $T^{eff} = T - p^{max}$, where p^{max} is the maximum number of lags to consider.

1. Provide intuition between the different criteria. Which one (asymptotically) over- or underestimates the correct order?
2. Write a function `nlag = LagOrderSelectionARp(`y`,`$const$`,`$pmax$`,`$crit$`)` that computes the different order criteria for $p = 1, ..., p^{max}$ using data vector y and possible constant term ($const = 1$) or constant term and linear trend ($const = 2$). $nlag$ should output the recommended lag according to criteria $crit$, which takes a string (AIC, SIC or HQC) as input value.
3. Load the dataset of the simulated AR(4) process given in the CSV file `AR4.csv`. Which model is preferred according to the order selection criteria?

Readings

- Lütkepohl (2004).

2 Portmanteau Test For Residual Autocorrelation

The portmanteau test checks the null hypothesis that there is no remaining residual autocorrelation at lags 1 to h against the alternative that at least one of the autocorrelations is nonzero. In other words, the pair of hypotheses:

$$H_0 : \rho_u(1) = \rho_u(2) = ... = \rho_u(h) = 0$$

versus:

$$H_1 : \rho_u(j) \neq 0 \text{ for at least one } j = 1, ..., h$$

where $\rho_u(j) = Corr(u_t, u_{t-j})$ denotes an autocorrelation coefficient of the residual series.

Consider the Box-Pierce test statistic Q_h

$$Q_h = T \sum_{j=1}^{h} \hat{\rho}_u^2(j)$$

which has an approximate $\chi^2(h - p)$-distribution if the null hypothesis holds and T is the length of the residual series. The null hypothesis of no residual autocorrelation is rejected for large values of the test statistic.

- Load quarterly data for the price index of US Gross National Product given in `gnpdeflator.csv`. This is a chain-type price index with basis year 2005. The data is seasonally adjusted and spans the period from 1954.Q4 to 2007.Q4.
- Compute the inflation series. That is, take the first difference of the log of gnpdeflator.
- Use the Akaike information criteria to determine the lag length $\hat{p}$.
- Estimate two models with OLS: (i) an $AR(\hat{p})$ model and (ii) an $AR(1)$ model.
- Set $h = \hat{p} + 10$ and compute Q_h as well as the corresponding p-value for both models.
- Comment, based on your findings, whether the residuals are white noise.

Readings:

- Lütkepohl (2004).

3 Bootstrap Confidence Interval For AR(1) Coefficient

Consider the AR(1) model with constant

$$y_t = c + \phi y_{t-1} + u_t$$

for $t = 1, \dots, T$ with iid error terms u_t and $E(u_t|y_{t-1}) = 0$. Usually, we construct a $(1-\alpha)\%$-confidence interval for ϕ using the normal (or student's t) approximation:

$$\left[\hat{\phi} - z_{\alpha/2} \cdot SE(\hat{\phi});\ \hat{\phi} + z_{1-\alpha/2} \cdot SE(\hat{\phi})\right]$$

with $\hat{\phi}$ denoting the OLS estimate, $SE(\hat{\phi})$ the estimated standard error of ϕ and $z_{\alpha/2}$ the $\alpha/2$ quantile of the standard normal distribution (or t-distribution). If one does not know the asymptotic distribution of a test statistic (or it has a very complicated form), one often relies on a nonparametric simulation approach. To this end, we are going to do a so-called "*bootstrap*", i.e. we recompute the t-statistics a large number of times on artificial data generated from resampled residuals.

1. What is a "Bootstrap approximation"? Provide insight into the basic idea and possible applications of this statistical technique.

2. Write a program for the following:
   - Simulate $T = 100$ observations with $c = 1$, $\phi = 0.8$ and errors drawn from e.g. the exponential distribution such that $E(u_t) = 0$.
   - Estimate the model with OLS and calculate the t-statistic $\tau = \frac{\hat{\phi}}{SE(\hat{\phi})}$.
   - Store the OLS residuals in a vector $\hat{u} = (\hat{u}_2, \dots, \hat{u}_T)'$.
   - Set $B = 10000$ and initialize the output vector $\tau^* = (\tau_1^*, ..., \tau_B^*)$.
   - For $b = 1, ..., B$:
     - Draw a sample **with replacement** from $\hat{u}$ and save it as $u^* = u_2^*, \dots, u_T^*$.
     - Initialize an artificial time series y_t^* with T observations and set $y_1^* = y_1$.
     - For $t = 2, \dots, T$ generate

       $$y_t^* = \hat{c} + \hat{\phi} y_{t-1}^* + u_t^*$$

     - On this artificial dataset estimate an AR(1) model. Denote the estimated OLS coefficient ϕ^* and corresponding estimated standard deviation $SE(\phi^*)$. Store the following t-statistic in your output vector at position b:

       $$\tau^* = \frac{\phi^* - \hat{\phi}}{SE(\phi^*)}$$

   - Sort the output vector such that $\tau_{(1)}^* \leq ... \leq \tau_{(B)}^*$.
   - The "*bootstrap approximate*" confidence interval for ϕ is then given by

     $$\left[\hat{\phi} - \tau^*_{((1-\alpha/2)B)} \cdot SE(\hat{\phi});\ \hat{\phi} - \tau^*_{((\alpha/2)B)} \cdot SE(\hat{\phi})\right]$$

     Set $\alpha = 0.05$ and compare this with the normal approximation.
   - Redo the exercise for $T = 30$ and $T = 10000$. Comment on your findings.

Readings

- Kilian and Lütkepohl (2017, Ch. 12)

References

Kilian, Lutz and Helmut Lütkepohl (2017). *Structural Vector Autoregressive Analysis*. Themes in Modern Econometrics. Cambridge: Cambridge University Press. ISBN: 978-1-107-19657-5. URL: `https://doi.org/10.1017/9781108164818`.

Lütkepohl, Helmut (2004). "Univariate Time Series Analysis". In: *Applied Time Series Econometrics*. Ed. by Helmut Lütkepohl and Markus Krätzig. First. Cambridge University Press, pp. 8–85. ISBN: 978-0-521-83919-8 978-0-521-54787-1 978-0-511-60688-5. DOI: `10.1017/CBO9780511606885.003`.