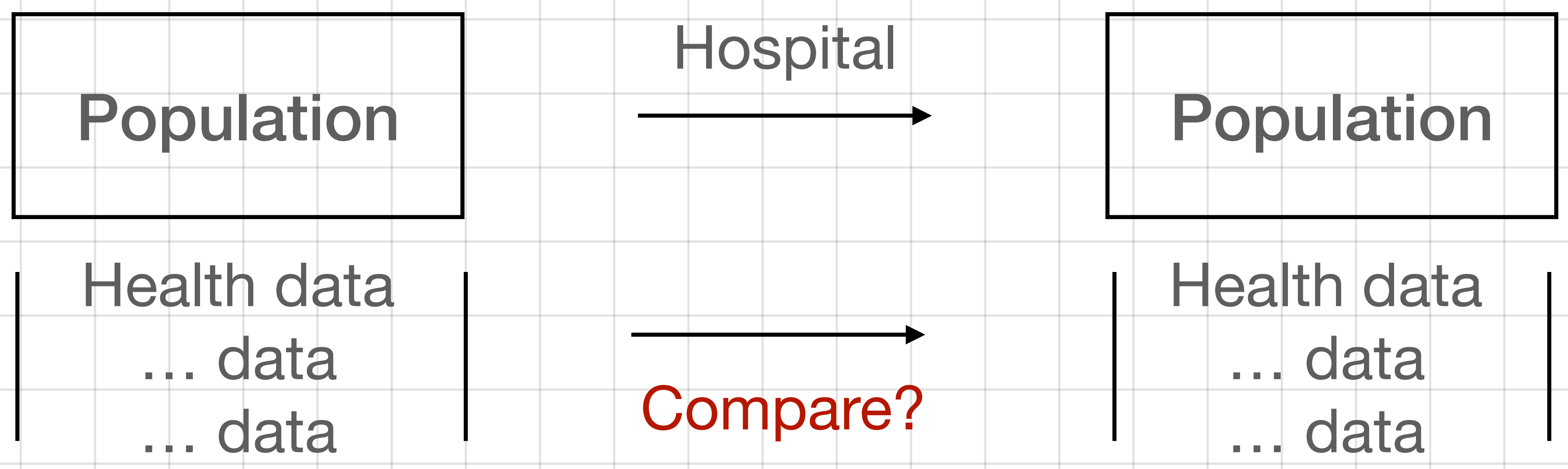
BLP Model

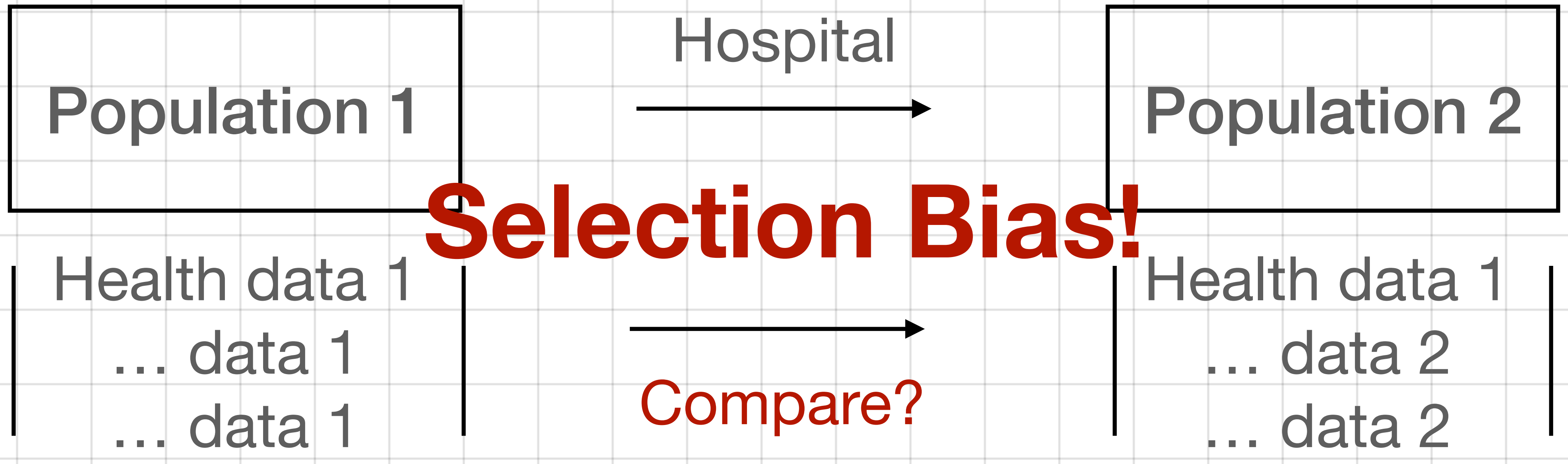
Estimation of Demand & Production function

Berry, Steven, James Levinsohn, and Ariel Pakes. "Automobile Prices in Market Equilibrium." *Econometrica* 63, no. 4 (1995): 841–90. https://doi.org/10.2307/2171802.

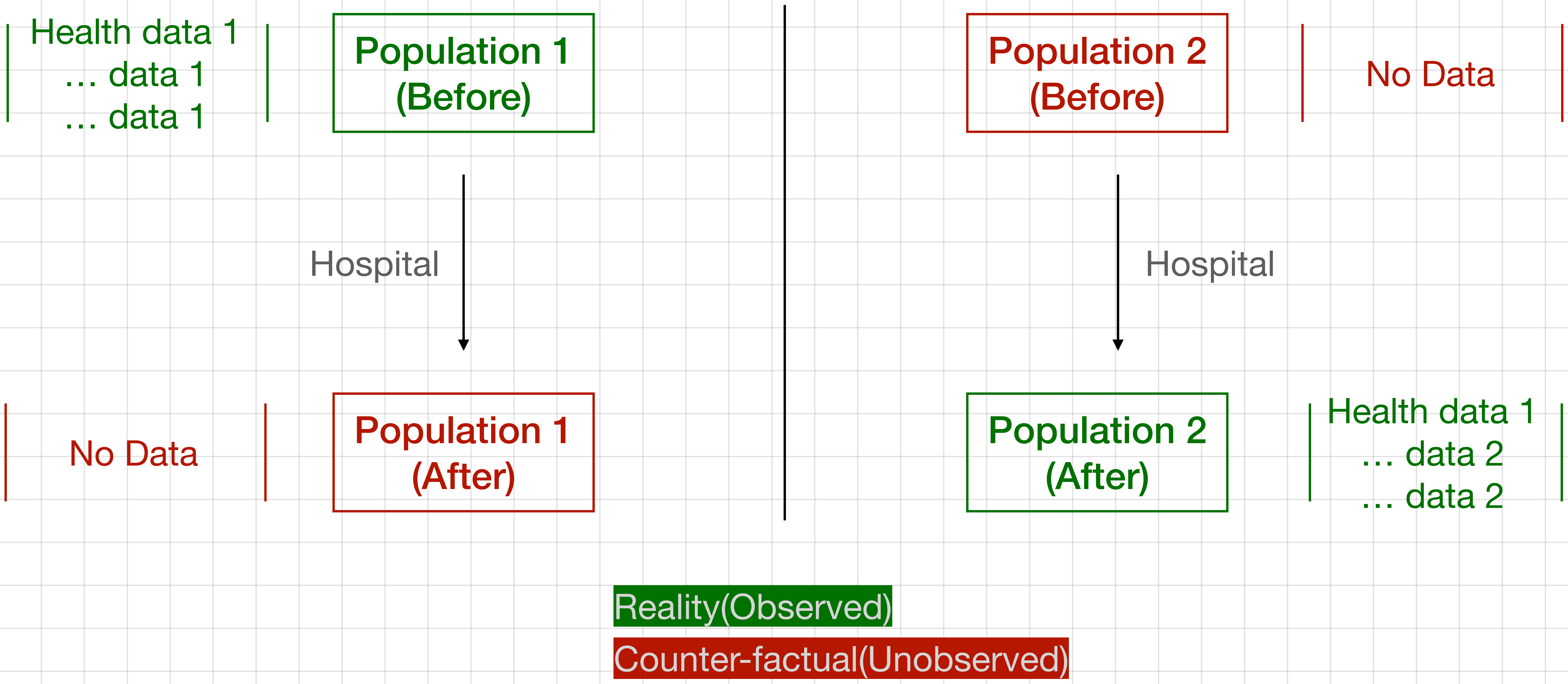
Counter-factual Analysis



Counter-factual Analysis



Counter-factual Analysis



DID

Find

comparable groups

one with treatment, one without

Usually sufficient for a good research

What would be...

If I impose a tax on this merchandise?

If I produce a new product going to sell in the market?

If two company merge?

If the interest rate grows?

In a products differentiated market?

Product 1
Product 2
Product 3
Product 4
Product List
Product 5
Product 6
Product 7
...
Or Buy **Nothing**

If the income increases,
which would the consumer buy?
Give each product a **linear model**?

$$purchase_j = a_j - b_j p_j + \epsilon_j$$

Unacceptable.

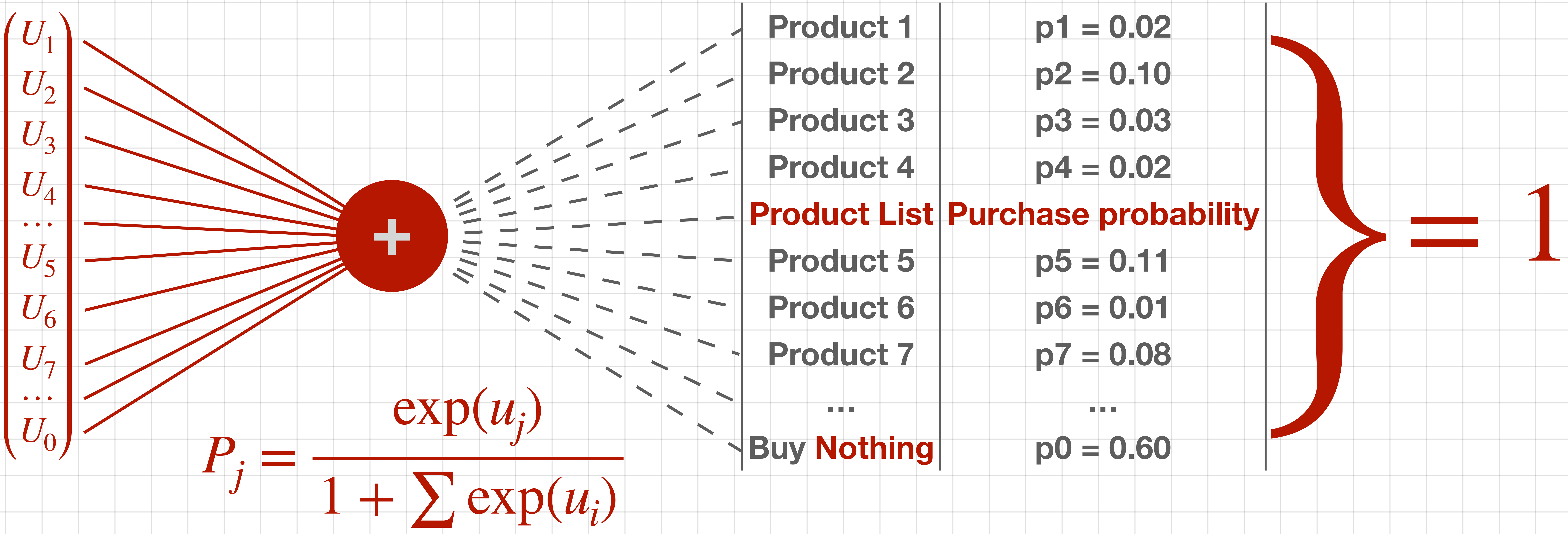
Hard to depict the **cross elasticity**:
when a new product comes in,
all other parameters will alter

$$\sum \left| \begin{array}{c} \text{Product 1} \\ \text{Product 2} \\ \text{Product 3} \\ \text{Product 4} \\ \textbf{Product List} \\ \text{Product 5} \\ \text{Product 6} \\ \text{Product 7} \\ \cdots \\ \text{Or Buy Nothing} \end{array} \right| = 1$$

Constraint 1:
For each consumer,
the **probabilities** of choosing each product **sum to one**

What then can be used to describe **the tendency** of choosing particular product?

Latent Utility Model / Random Coefficient Model / Multinomial Login Model



Latent Utility Model /
Random Coefficient Model /
Multinomial Login Model

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ \cdots \\ U_5 \\ U_6 \\ U_7 \\ \cdots \\ U_0 \end{pmatrix}$$

What is the real market looks like?

U_1	Product 1	p1 = 0.02		s1 = 2%
U_2	Product 2	p2 = 0.10		s2 = 10%
U_3	Product 3	p3 = 0.03		s3 = 3%
U_4	Product 4	p4 = 0.02		s4 = 2%
$\dots$	**Product List**	**Purchase probability**	Population	**Market Share**
U_5	Product 5	p5 = 0.11	ns = np	s5 = 11%
U_6	Product 6	p6 = 0.01		s6 = 1%
U_7	Product 7	p7 = 0.08		s7 = 8%
$\dots$	...	...		...
U_0	Buy **Nothing**	p0 = 0.60		s0 = 60%

See Product
(Each have a utility)

Making aggregated
decisions

Result Market Share
(Observable)

Product Info.(obs) <— Utility Info. <—Market Info.(obs.)

Problem?

Latent Utility Model /
Random Coefficient Model /
Multinomial Login Model

People have different tastes!

Interaction between
Consumer's characteristics
and product characteristics

e.g. the rich and the poor prefer diff. premium
e.g. large family prefer large cars, youth may not

What then should be the utility function looks like?

Random Coefficient Model

$$u_{ij} = x_j\beta + \nu_i\sigma + \epsilon_{ij}$$

Random Coefficient Model with Interactions

$$u_{ij} = x_j\beta + \sigma\nu_i x_j + \epsilon_{ij}$$

Random Coefficient Model with Interactions and unobservables

$$u_{ij} = x_j\beta + \xi_j + \sigma\nu_i x_j + \epsilon_{ij}$$

ν_i: vector of consumer characteristics
x_j vector of product characteristics
ξ_j: product j's unobserved characteristics
ϵ_{ij}: random error term

Individual i
Product j

Problem 1 [No interaction]:
Given product, consumer's chars.
have no influence on its choice

Problem 2 [Omitted variable]:
Many product chars. are not observable to scholars,
but obs. to consumer and manufacturer,

Problem 3 [Aggregated data]:
We only have aggregated data.
We do not have information about individual purchase and their chars.

We do not have **individual** characteristics ν_i

but we have their **statistics** P_0

$$\nu_i \sim P_0 \simeq P_{ns}$$

sample ns from millions of demographical data
from *Statistical Abstract of the U.S.*

Then we can **integrate out** the random term

(β, σ): parameter to estimate

$$u_{ij}(x_j, \xi_j, \nu_i; \beta, \sigma) = x_j\beta + \xi_j + \sigma\nu_i x_j + \epsilon_{ij}$$

Integrate out the error term ϵ_{ij}

$$f_j(x_j, \xi_j, \nu_i, \beta, \sigma) = \frac{\exp(x_j\beta + \xi_j + \sigma\nu_i x_j)}{1 + \sum \exp(x_j\beta + \xi_j + \sigma\nu_i x_j)}$$

Integrate out the individual chars. ν_i

$$s_j^{model}(x_j, \xi_j; \beta, \sigma) = \int f_j(x_j, \xi_j, \nu_i) P_0(d\nu_i)$$

We have **data of x_j**, **distribution $P_{ns}(d\nu_i)$**, the **real market share s_j^{data}**

assume (for now) we have information about the un.obs. ξ_j

then we can **search for the parameter (β, σ)** such that $s^{model} = s^{data}$

Problem 1: Real market share?

Problem 2: Unobserved chars.?

Problem 3: Searching space?

Problem 4: Demographical distribution?

$$s_j^{model}(x_j, \xi_j; \beta, \sigma) = \int f_j(x_j, \xi_j, \nu_i) P_0(d\nu_i) = \int \frac{\exp(x_j\beta + \xi_j + \sigma\nu_i x_j)}{1 + \sum \exp(x_j\beta + \xi_j + \sigma\nu_i x_j)} P_0 d\nu_i$$

(β, σ): parameter to estimate

How much do we know about ξ_j?

1. Not a random variable. Just unobserved
2. For a given product, $x_j\beta + \xi_j$ is determined

That is to say

$(x_j\beta + \xi_j)$ can be regarded as a constant term $\delta_j(\cdot)$

and use numerical method to solve

$$T(s, \theta, P)[\delta_j] = \delta_j + \ln(s_j^{data}) - \ln[s_j^{model}(p, x, \delta, P; \theta)]$$

Contraction Mapping

$T(s, \theta, P) : R^J \rightarrow R^J$ is a contraction mapping with modulus less than one

1. Begin by initial guess of δ_j
2. Calculate the model implied market share
3. Calculate the new δ_j'
4. Substitute back
5. …
6. Iterating until $\delta_j' - \delta_j < e$

1. After we obtain δ_j and s_j^{model}

2. How to recover ξ_j?

3. How to solve β?

And more importantly

4. How do we know whether it is a correct σ?

We need to find a target function!

Need more assumptions!

Nash Equilibrium

Price p_j, chars. x_j, un.obs chars. ξ_j came out of market equilibrium

Market data is the Nash equilibrium that both maximize manufacturer's profit and consumer's utility
Use this information to find moment conditions.

Cost function

w_j: observed cost characteristics
ω_j unobserved cost characteristics
mc_j: marginal cost

Assumption: product characteristics (x_j, ξ_j) are not free

$$\ln(mc_j) = w_j\gamma + \omega_j$$

x_j is a prt of w_j

ξ_j is a part of ω_j

Profit function

M: Market size
s_j: market share of
$\mathcal{F}_f$: a company's product set

$$\Pi_f = \sum_{j \in \mathcal{F}_f} (p_j - mc_j) M s_j(x, \xi; \beta, \sigma)$$

(β, σ): parameter to estimate

Profit function

M: Market size
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$\mathscr{F}_f$: a company's product set

$$\Pi_f = \sum_{j \in \mathscr{F}_f} (p_j - mc_j) M s_j(x, \xi; \beta, \sigma)$$

Company's best response

$$s_j(p, x, \xi; \beta, \sigma) + \sum_{r \in \mathscr{F}_f} (p_r - mc_r) \frac{\partial s_r(p, x, \xi; \beta, \sigma)}{\partial p_j} = 0$$

(β, σ): parameter to estimate

Instrument Variable

We assume that some characteristics are not correlated with others

$$\begin{cases} z = [x_j, w_j] \\ E[\xi_j \mid z] = 0 \\ E[\omega_j \mid z] = 0 \end{cases}$$

Generalized Method of Moments

A set of conditions that cannot satisfy in reality
Weighting them and set them as close to zero as possible

ξ_j: unobserved product chars.
ω_j: unobserved cost chars.
z, x, w: observable chars.

Generalized Method of Moments

Assumption

Finite variance $E[(w,\omega)'(w,\omega)] = \Omega(z)$

Moment conditions

[All the conditions put together]:

$$g(\theta) = \frac{1}{J}\sum_{r}^{J} f(x_r, w_r, \omega_r, \xi_r; \theta) = 0$$

Usually the number of functions are not equal to number of variables

We set this to be as close to zero as possible

ξ_j: unobserved product chars.
ω_j: unobserved cost chars.
z, x, w: observable chars.

Generalized Method of Moments

Any transformation of Instruments can be used as instrument.

z is instrument => $H(z)$ is also instrument

Standardize: $T(z)'T(z) = \Omega(z)^{-1}$

Then the Moment conditions:

$$G^J(\theta) = E\left[H_j(z)T(z_j)\begin{pmatrix}\xi_j(\theta)\\ \omega_j(\theta)\end{pmatrix}\right]$$

This should be $G^J(\theta^*) = 0$ when it is the correct parameter.

Target function

Generalized Method of Moments

$$\min_{\theta} \|G^J(\theta; s, P_0)\|$$

For all the parameters we want to estimate, we separate them into two group $\theta = (\theta_1, \theta_2)$

[Outer Loop] Enumerate possible θ_2,

1. [Inner Loop] Using market share data to solve linear utility
    1. Starting from initial δ
    2. calculating market share implied by the model s^{model}
    3. Compare with real market share s^{data}, calculate new δ'
    4. Stop when $\delta \sim \delta'$, in the same time $s^{model} \sim s^{data}$
2. Solve unobservable $\xi_j = \delta_j - \beta x_j$, while obtaining θ_1

    by using instrumental variable $E[Z\xi_j] = 0$
3. Calculating GMM target function $||G^J(\theta_1, \theta_2)||$

Try another θ_2 till $||G^J(\theta_1, \theta_2)|| \to 0$

In reality

full function form

$$u_{ij} = \underbrace{\alpha \ln(y_i - p_j)}_{\text{Income is log-normal}} + x_j\beta + \xi_j + x_j'\sigma_{ij}\nu_i + \epsilon_{ij}$$

Results

TABLE VI

A SAMPLE FROM 1990 OF ESTIMATED OWN- AND CROSS-PRICE SEMI-ELASTICITIES: BASED ON TABLE IV (CRTS) ESTIMATES

	Mazda 323	Nissan Sentra	Ford Escort	Chevy Cavalier	Honda Accord	Ford Taurus	Buick Century	Nissan Maxima	Acura Legend	Lincoln Town Car	Cadillac Seville	Lexus LS400	BMW 735i
323	−125.933	1.518	8.954	9.680	2.185	0.852	0.485	0.056	0.009	0.012	0.002	0.002	0.000
Sentra	0.705	−115.319	8.024	8.435	2.473	0.909	0.516	0.093	0.015	0.019	0.003	0.003	0.000
Escort	0.713	1.375	−106.497	7.570	2.298	0.708	0.445	0.082	0.015	0.015	0.003	0.003	0.000
Cavalier	0.754	1.414	7.406	−110.972	2.291	1.083	0.646	0.087	0.015	0.023	0.004	0.003	0.000
Accord	0.120	0.293	1.590	1.621	−51.637	1.532	0.463	0.310	0.095	0.169	0.034	0.030	0.005
Taurus	0.063	0.144	0.653	1.020	2.041	−43.634	0.335	0.245	0.091	0.291	0.045	0.024	0.006
Century	0.099	0.228	1.146	1.700	1.722	0.937	−66.635	0.773	0.152	0.278	0.039	0.029	0.005
Maxima	0.013	0.046	0.236	0.256	1.293	0.768	0.866	−35.378	0.271	0.579	0.116	0.115	0.020
Legend	0.004	0.014	0.083	0.084	0.736	0.532	0.318	0.506	−21.820	0.775	0.183	0.210	0.043
TownCar	0.002	0.006	0.029	0.046	0.475	0.614	0.210	0.389	0.280	−20.175	0.226	0.168	0.048
Seville	0.001	0.005	0.026	0.035	0.425	0.420	0.131	0.351	0.296	1.011	−16.313	0.263	0.068
LS400	0.001	0.003	0.018	0.019	0.302	0.185	0.079	0.280	0.274	0.606	0.212	−11.199	0.086
735*i*	0.000	0.002	0.009	0.012	0.203	0.176	0.050	0.190	0.223	0.685	0.215	0.336	−9.376

Note: Cell entries i, j, where i indexes row and j column, give the percentage change in market share of i with a \$1000 change in the price of j.

Cross-price elasticities

Substitute for outside good

TABLE VII

SUBSTITUTION TO THE OUTSIDE GOOD

Model	Given a price increase, the percentage who substitute to the outside good (as a percentage of all who substitute away.) Logit	BLP
Mazda 323	90.870	27.123
Nissan Sentra	90.843	26.133
Ford Escort	90.592	27.996
Chevy Cavalier	90.585	26.389
Honda Accord	90.458	21.839
Ford Taurus	90.566	25.214
Buick Century	90.777	25.402
Nissan Maxima	90.790	21.738
Acura Legend	90.838	20.786
Lincoln Town Car	90.739	20.309
Cadillac Seville	90.860	16.734
Lexus LS400	90.851	10.090
BMW 735*i*	90.883	10.101

Company Markups for each model

TABLE VIII

A SAMPLE FROM 1990 OF ESTIMATED PRICE-MARGINAL COST MARKUPS AND VARIABLE PROFITS: BASED ON TABLE 6 (CRTS) ESTIMATES

	Price	Markup Over MC $(p - MC)$	Variable Profits (in $'000's) $q * (p - MC)$
Mazda 323	$5,049	$ 801	$18,407
Nissan Sentra	$5,661	$ 880	$43,554
Ford Escort	$5,663	$1,077	$311,068
Chevy Cavalier	$5,797	$1,302	$384,263
Honda Accord	$9,292	$1,992	$830,842
Ford Taurus	$9,671	$2,577	$807,212
Buick Century	$10,138	$2,420	$271,446
Nissan Maxima	$13,695	$2,881	$288,291
Acura Legend	$18,944	$4,671	$250,695
Lincoln Town Car	$21,412	$5,596	$832,082
Cadillac Seville	$24,353	$7,500	$249,195
Lexus LS400	$27,544	$9,030	$371,123
BMW 735*i*	$37,490	$10,975	$114,802

Comments

- Data requirement is extremely low.
    - Market Level & Industry level data. No need for individuals
- Also we could conduct counter-factual analysis.
    - Assuming that the parameters behind do not vary in a short time
    - Welfare from new goods, and merger evaluation
    - Geographic differentiation, accurate marketing.
- Most interesting part is the Hotz-Miller Inversion, use the market share data to solve the market share s^{model} and random utility δ_j simultaneously