Avian Dynamics

Abstract

Three mathematical models were used to model the population of a relatively new bird colony. These models were then compared to the data to determine the model of best fit.

The first model is the logistic growth model, which is useful when modeling asexual organisms. This is also useful when modeling new populations of sexual organisms.

The second and third models modify the logistic growth model to account for how the real world works for sexual animals. Such as predators taking away from the population. Specific to this model, t is assumed that these birds group together to protect themselves from predators, which could imply that there is some minimum population necessary for the population to maintain itself. .

The modifications to the logistic growth model in Model 2 and Model 3 came with improvements. The parameter involved fit the data well in both cases. Though both models grow at slightly faster rates than the data.

Introduction

A relatively new bird colony was surveyed and data on the population size was collected over 10 years; the goal is to find the model of best fit for the given data. Certain characteristics of the population can be estimated using each model; such as the carrying capacity and the intrinsic growth rate. The models will be compared using graphs and by looking at the residual-sum-of-squares between each model and the data. By finding the model of best fit, we can determine which characteristics of the population have the greater effect or attempt to make future predictions for the population.

Outline

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Materials

The data of the population was given by:

n (year)	P_n (population in thousands)
0	5.1
1	5.5
2	5.9
3	6.5
4	7.5
5	8.7
6	10.3
7	12.4
8	14.7
9	16
10	16.2

The models to compare include the logistic growth model, $P_{n+1} = P_n + rP_n(1 - \frac{P_n}{K})$, along with two modification of it. The logistic growth model assumes a linear intrinsic growth rate, r, and a constant carrying capacity K.

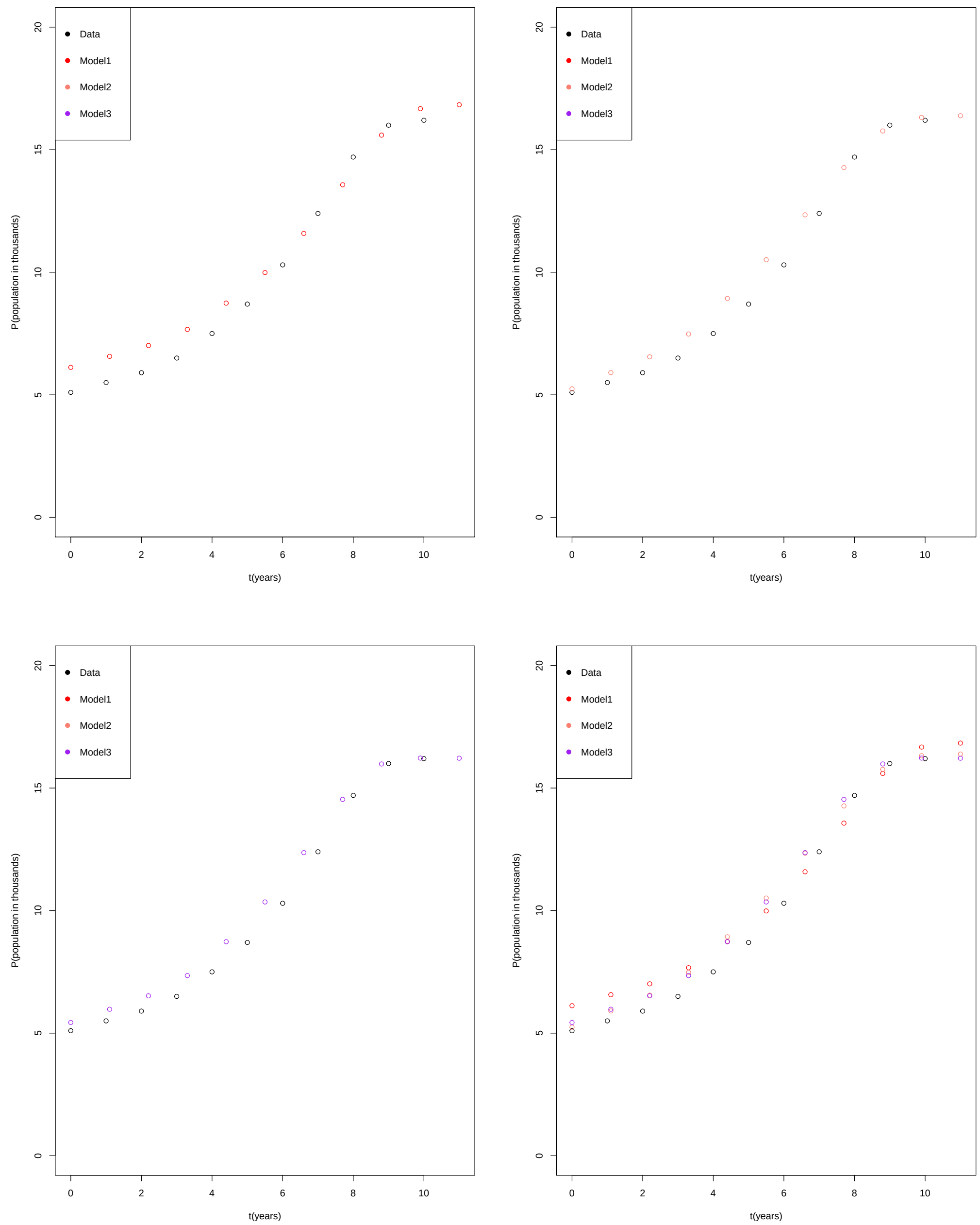
The second model, $P_{n+1} = P_n + rP_n(1 - \frac{P_n}{K}) - m$, assumes the logistic model with the addition of a $(-m)$ term to account for predator's effect on the population. This term is also assumed to be constant.

The last model, $P_{n+1} = P_n + rP_n(1 - \frac{P_n}{K})(\frac{P_n}{L} - 1)$, assumes the logistic model and a term to account for a minimum population requirement, L, where L is a constant.

Using non-linear regression, each model is used to estimate parameters and information about the population.

Results

Each model was then plotted over the data and over each other:


Data
Model1
Model2
Model3
P(population in thousands)
t(years)
0
5
10
15
20
0
2
4
6
8
10

The intrinsic growth rate, carrying capacity, m, and L, were also extracted to a table. The residual sum of squares, the distance between the data points and modeled point square, was also extracted.

Variable	M1	M2	M3
r	0.274	1.31	0.349
K	18.9	21.4	16.2
m	-	4.94	-
L	-	-	4
RSQ	3.671	0.477	0.09141

Conclusion

The first model captures the overall growth of the population, though the initial condition, $P(0)$, and the carrying capacity are skewed as expected. The modifications to the first model in both cases came with improvements. The initial conditions and overall dynamics fit the data well in both cases, though both models grow at a slightly faster rate than the data.

This can be seen dramatically in the intrinsic growth rate and carrying capacity of model 2 where $r = 1.31$ and $k = 21.4$. Despite model 2 graphically fitting the data for our given time-steps, it would not be the best model to use for future predictions.

The third model, although growing at a slightly faster rate than the data, has an accurate carrying capacity, intrinsic growth rate, and initial condition $P(0)$; making it a good model to use for predictions.

By looking at the residual sum of squares, the third model fits the data best. From a biological standpoint, this suggests that having a minimum population has a greater positive effect on the population than the negative effect predatory behavior has during this stage of the population's development. Despite this, both a minimum population, and population loss due to predators are factors that are bound to be present in the population and can be accounted for.