Graminivorous Beetle Beatdown

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Abstract

In this study we will be analyzing the dynamics of graminivorous beetles competing for resources. The data given is on populations of the Rhizopertha dominica and Oryzaephilus surinamensis, each in their own isolated environment and then in a competitive environment. Mathematical models are then formulated to estimate population parameters. From this, equilibrium populations are determined and stability analysis is done to detemine the nature of each of them.

1 Introduction

The Rhizopertha dominica and Oryzaephilus surinamensis are competing through interspecific competition. Each population will be modeled logistically to estimate the maximum populations and intrinsic growth rates, or the natural growth based on births and deaths. We will then develop a system of equations to analyze the effect of the competition on each species. The completed model is then analyzed to interpret the population dynamics of the beetles.

2 Materials and Methods

The data for the beetle populations, in both isolated and competitive circumstances, is given by:

Isolated			Competitive		
t(days)	Rhizopertha	Oryzaephilus	t(days)	Rhizopertha	Oryzaephilus
0	2	4	0	2	2
14	2	4	14	2	2
28	2	4	28	2	4
35	3	25	35	2	33
42	17	44	42	21	41
49	65	63	49	59	53
63	119	147	63	116	74
77	130	285	77	120	127
91	175	345	91	138	190
105	205	361	105	152	203
119	261	405	119	193	305
133	302	471	133	260	385
147	330	420	147	255	480
161	315	430	161	245	405
175	333	420	175	250	425
189	350	475	189	260	425
203	332	435	203	210	450
231	333	480	231	233	415
245	335	-	245	255	425
259	330	-	259	260	415
273	-	-	287	260	420

When isolated we can model each species logistically. First, by assuming that the growth of each population is proportional to the population, then denoting this by an rX term; Where r is the intrinsic growth rate and X is the population. Also, it is assumed that there exists a maximum population that is a direct result of the specie's environment. This is denoted by the $(1 - \frac{X}{K})$ term, where K is the carrying capacity of the population. Thus the logistic model,

$$\frac{dX}{dt} = rX(1 - \frac{X}{K})$$

. Then, to model their competition we can develop a system of equations with an extra term to account for the negative interactions due to inter-specific competition:

$$\frac{dX}{dt} = r_r X(1 - \frac{X}{K_r}) - a_r XY$$

$$\frac{dX}{dt} = r_o X(1 - \frac{X}{K_o}) - a_o XY$$

Using these equations we will fit the models of each beetle population to the corresponding data for both the isolated and competitive environments.

3 Results

i. First, using non-linear regression, a logistic model is fitted to the data of the isolated Rhizopertha dominica population and the carrying capacity and intrinsic growth rate is estimated.

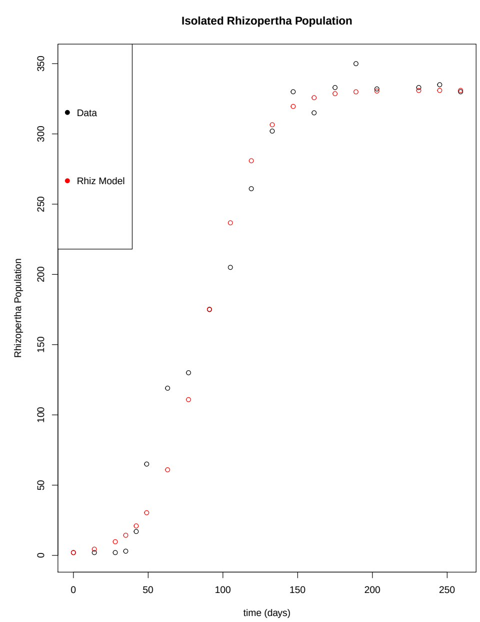


Figure 1: Data and logistic model for the isolated Rhizopertha dominica population.

The intrinsic growth rate evaluated to $r_r = 0.0574$ and the carrying capacity evaluated to $K_r = 331$.

Again using non-linear regression, the same process is carried out for the isolated Oryzaephilus surinamensis population:

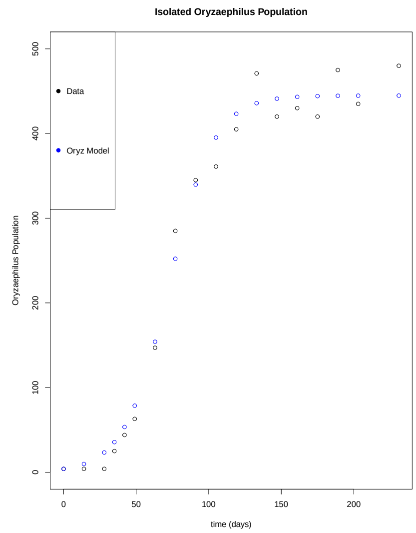


Figure 2: Data and logistic model for the isolated Oryzaephilus surinamensis population.

The intrinsic growth rate evaluated to $r_o = 0.0646$ and the carrying capacity evaluated to $K_o = 445$.

ii. Now that the constants for the isolated cases have been found, we can insert these values into our competitive system of equations and estimate the coefficients of competition, a_r and a_o, to futher develop the competitive model. First, the data is plotted.
The model is then developed by fitting the competitive logistic models to the competitive data sets.

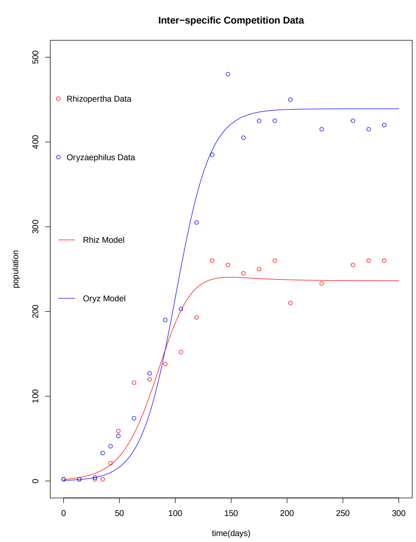


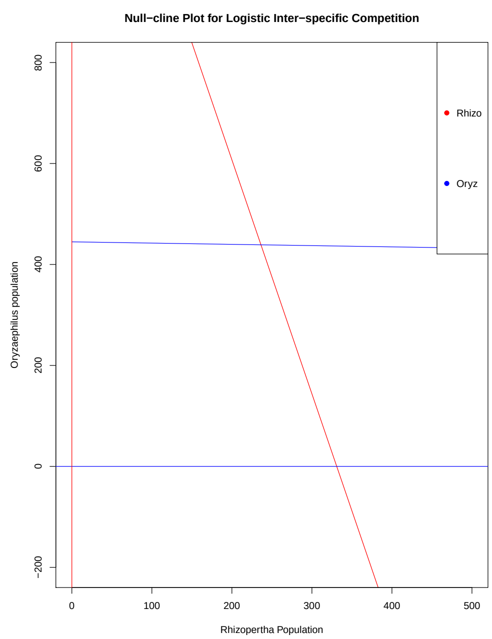
Figure 3: Data along with a competitive logistic model for the inter-specific competition between the Rhizopertha dominica and Oryzaephilus surinamensis populations.

The competitive coefficients were evaluated to be $a_r = 3.74 * 10^{-5}$ and $a_o = 3.62 * 10^{-6}$, implying that the Rhizopertha population experiences a greater effect due to the interspecific competition.

iii. It is notable that both populations increase initially at approximately the same rate, the Rhizopertha seeming slightly faster, and that they part once their individual carrying capacities are reached. From the graph, we can see that in a competitive environment, the Rhizopertha dominica population reaches it's carrying capacity at around $K_r = 225$, a large decrease compared to the isolated case's $K = 331$. Also, the Oryzaephilus surinamensis carrying capacity only slightly decreases from $K = 445$. This also implies that the competitive scenario is more detrimental to the Rhizopertha population.

4 Stability Analysis Using Null-Clines in the Population Plane

i. We can now plot both populations on a plane along with the associated null-clines. The equilibria are represented at the intersections of the different null-clines. By setting the rates of change of both populations to 0, we can locate the equillibria at $(0, 0)$, $(331.008, 0)$, and $(236.3165, 438.9104)$. We can see here that the carrying capacity for each species in a competitive environment is exactly $K_r = 236.3165$ and $K_o = 438.9104$. It is also notable that the Rhizopertha dominica population rests at it's original carrying capacity,$K = 331$, at the equilibrium where the Oryzaephilus surinamensis population is extinct.



ii. We can now evaluate the stability of each equilibrium. First, using linear stability analysis for the trivial equilibrium at $(0, 0)$, the eigenvalues are $\lambda_1 = 0.06456080$ and $\lambda_2 = 0.05736646$. Two positive eigenvalues implies the equilibrium is a source. This can be confirmed from an intuitive sense, that is, if a species is introduced to a new palatable environment, it will reproduce and grow; though this stems from a part of the model that ignores a minimum population requirement. Next, the equilibrium at $(236.3165, 438.9104)$ is analyzed by plotting the trajectories of initial conditions in the population plane.

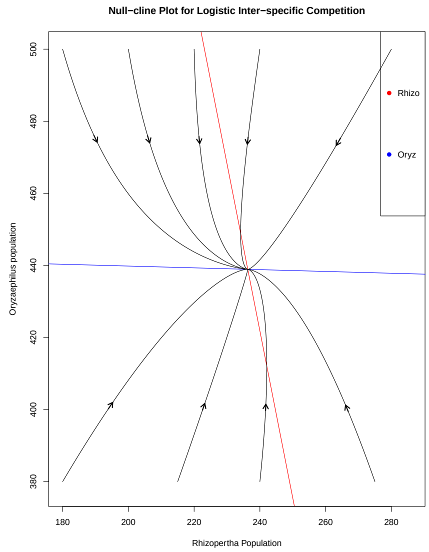


Figure 4: A plot of null-clines with initial conditions and their trajectories; focused at (236.3165,438.9104).

iii. Based on the image the equilibrium seems to be a sink. Using linear stability analysis, the eigenvalues are $\lambda_1 = -0.06430658$ and $\lambda_2 = -0.04035539$. Thus, the equilibrium is a sink.
Also, consider the trajectory starting at approximately $(245, 500)$. From this, we can conclude that the Rhizopertha dominica close to carrying capacity competes well enough with high populations of Oryzaephilus surinamensis to force it down to it's carrying capacity with minimal effect on it's own population. The equilibrium at $(331.008, 0)$ is now examined using trajectories of initial conditions.

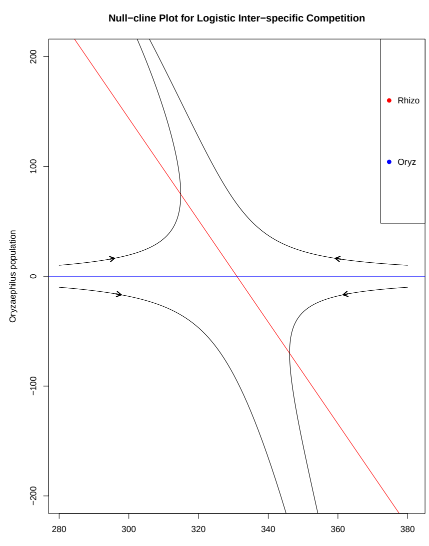


Figure 5: A plot of null-clines with initial conditions and their trajectories; focused at (331,0).

Although negative values for population do not exist, the local sensitivity of the equilibrium insists that it is a saddle point. Again, we can confirm this using linear stability analysis. The eigenvalues for this equillibrium are $\lambda_1 = 0.063362$ and $\lambda_2 = -0.057366$, confirming that is it a saddle point. It seems to be stable in the direction of a constant Oryzaephilus surinamensis population, and unstable in the direction of a constant Rhizopertha dominica population. This could correspond to the Rhizopertha dominica population reproducing much quicker than the Oryzaephilus surinamensis population. This also illustrates how at a small population of Oryzaephilus surinamensis, the competing Rhizopertha dominica population will reach it's carrying capacity first, and the Oryzaephilus surinamensis population will then begin to increase. This increasing population competes with the Rhizopertha beetles and decreases their numbers. This is also notable in figure 4, at the initial trajectory of approximately $(240, 380)$.

5 Conclusion

The interspecific competition between the Rhizopertha dominica and Oryzaephilus surinamensis beetles surely effects the Rhizopertha dominica population more based on the change of carryin

g capacities from the isolated cases to the competitive cases.(K_r reduces by 106, K_o by 6) Also, even at low populations, the Oryzaephilus surinamensis competes well enough with a population of Rhizopertha that is well above carrying capacity, that it decreases the population of Rhizopertha as it reaches it's own carrying capacity. The ability of the Oryzaephilus surinamensis to compete seems to have a limit. When the Rhizopertha dominica population is at or around it's competitive carrying capacity with a population of Oryzaephilus surinamenses that is much above it's competitive carrying capacity, it maintains it's population while the Oryzaephilus surinamenses decreases. This limit could be due to the Rhizopertha dominica's ability to reproduce much quicker. Also, since the Oryzaephilus surinamenses's isolated and competitive carrying capacities are so close, we can conclude that this decrease is due to environmental features other than the inter-specific competition with the Rhizopertha dominica.