Monte Carlo Methods for Pricing Options

..., Tim Wolstenholme

March 2025

1 Introduction

2 MCMC samplers

To calibrate the stochastic volatility models for the stock prices, we have used various Monte Carlo Markov Chain (MCMC) sampling techniques. MCMC samplers attempt to find the parameters of some target distribution $\pi(\boldsymbol{\theta})$ by generating candidate values for the parameters ($\boldsymbol{\theta}^*$) from some proposal distribution $q(.|\boldsymbol{\theta})$, and we accept the transition $\boldsymbol{\theta}^{(j+1)} = \boldsymbol{\theta}^*$ with some acceptance probability α, depending on both $\boldsymbol{\theta}^*$ and $\boldsymbol{\theta}$. This process is repeated for a (large) fixed number of iterations (N) to find parameters which fit the model well.

2.1 Random Walk Metropolis Hastings

The simplest MCMC sampler is the Random Walk Metropolis algorithm, which we used for single-asset Black Scholes.
Below are the steps for the Random Walk Metropolis algorithm

1. Initialise the Markov chain $\boldsymbol{\theta}^{(1)} = (\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_{\boldsymbol{n}})$ and set iteration count i=1
2. Generate candidate $\boldsymbol{\theta}^*$ using $q(.|\boldsymbol{\theta}^{(i)})$
3. Calculate the acceptance probability $\alpha = \min\{1, \frac{\pi(\boldsymbol{\theta}^*)q(\boldsymbol{\theta}^{(i)}|\boldsymbol{\theta}^*)}{\pi(\boldsymbol{\theta}^{(i)})q(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{(i)})}\}$
4. Accept the transition $\boldsymbol{\theta}^{(i+1)} = \boldsymbol{\theta}^*$ with probability α, else set $\boldsymbol{\theta}^{(i+1)} = \boldsymbol{\theta}^{(i)}$
5. If i=N stop. Otherwise increment i and repeat from step 2.