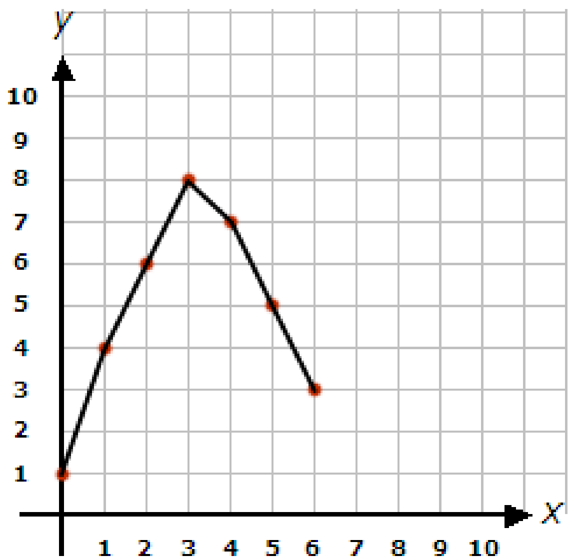
Image Analysis Assignment 2

Olof Harrysson

1.
To figure this out I flip the filter in both the x and y direction.

f1 = A. The filter detects when the image goes from white to black in the direction left to right.
f2 = D. Similarly like f1 the filter detects edges in the y-direction.
f3 = C. Detects the border outside of a black object.
f4 = F. The filter blurs the image which is hard to see for this size.
f5 = E. Detects bottom right edges.
f6 = B. Detects bottom left edges.

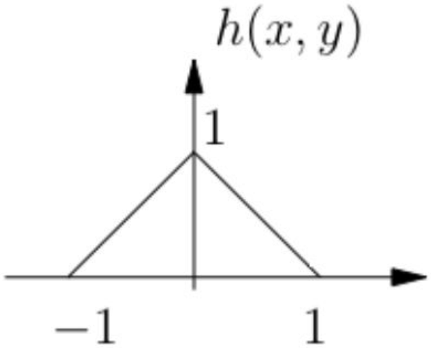
2.
a) Linear interpolation connects sequential points by using straight lines.



b) $g(x) = 0 \; when \; x < -1$

$g(x) = 1 - |x| \; when \; -1 \leq x \leq 1$

$g(x) = 0 \; when \; 1 < x$



c)

```
0.5 interpolates to 2.5
1.5 interpolates to 5.0
2.5 interpolates to 7.0
3.5 interpolates to 7.5
4.5 interpolates to 6.0
5.5 interpolates to 4.0

Deriviate in 0.5 is 3.0
Deriviate in 1.5 is 2.0
Deriviate in 2.5 is 2.0
Deriviate in 3.5 is -1.0
Deriviate in 4.5 is -2.0
Deriviate in 5.5 is -2.0

f * w = [ 3  2  2 -1 -2 -2]
Olofs-MacBook-Pro:2 olof$
```

The derivative vector is the same as f * w.

d) (1) $\frac{\varphi F(k)}{\varphi x} = \sum_i \frac{\varphi g(x-i)}{\varphi x} \times f(k),\ \frac{\varphi g(x-i)}{\varphi x} =$

$0\ when\ x <-1$

$1\ when\ -1 < x < 0$

$-1\ when\ 0 < x < 1$

$0\ when\ 1 < x$

(2) $\sum_i w(x-i) \times f(i)$

If you put equation (1) = (2) you can cross out the f(i) and the sum and what's left is $\frac{\varphi g(j)}{\varphi x} = w(j)$

e) From the previous task we know that $\frac{dg(j)}{dx} = w(j)$.

One such function can be $\frac{dg}{dx} = \frac{-x}{2}$

The primitive of that function determines g(x). $g(x) = \frac{-x^2}{4}$

3.
a) The items in the test data is compared against the training data by looking at the distance between each test and train pair. $err = |test\ item - train\ item|$

The test item is then classified as the class where the error is the smallest.

```
0.4243 is wrongly classified as Class 1
Number of missclassified items are 1
Olofs-MacBook-Pro:2 olof$
```

b) I run each data point through three normal distribution models, one for each class. The data point is classified to the class that corresponds to the highest probability.

```
0.3802 is wrongly classified as Class 1
0.4243 is wrongly classified as Class 1
Number of missclassified items are 2
Olofs-MacBook-Pro:2 olof$
```

4.
a)
$P(X|Y = 1) = 0.1^1 * 0.9^3 = 0.0729$
$P(X|Y = 2) = 0.1^3 * 0.9^1 \approx 9 * 10^{-4}$
$P(X|Y = 3) = 0.1^2 * 0.9^2 \approx 8.1 * 10^{-3}$

Multiply these with the prior probability of the images.

$w_1 = 0.0729 * 0.25 \approx 1.8 * 10^{-2}$
$w_2 = 9 * 10^{-4} * 0.25 = 2.25 * 10^{-4}$
$w_3 = 8.1 * 10^{-3} * 0.5 = 4.05 * 10^{-3}$

Here we can already determine the most likely class, by choosing the max of wi. The likelihood of the different images are the following.

$P(1) = 1.8 * 10^{-2} / (1.8 * 10^{-2} + 2.25 * 10^{-4} + 4.05 * 10^{-3}) \approx 0.808$
$P(2) = 2.25 * 10^{-4} / (1.8 * 10^{-2} + 2.25 * 10^{-4} + 4.05 * 10^{-3}) \approx 0.01$
$P(3) = 4.05 * 10^{-3} /(1.8 * 10^{-2} + 2.25 * 10^{-4} + 4.05 * 10^{-3}) \approx 0.182$

b)
$P(X|Y = 1)$ = $P(X|Y = 2)$ = $P(X|Y = 3)$ = $0.5^4 = 0.0625$

$w_1 = 0.0625 * 0.25 \approx 0.0156$
$w_2 = 0.0625 * 0.25 \approx 0.0156$
$w_3 = 0.0625 * 0.5 \approx 0.0313$

$P(1) = P(2) = 0.0156 / (0.0156 * 2 + 0.0313) \approx 0.25$
$P(3) = 0.0313 / (0.0156 * 2 + 0.0313) \approx 0.5$

These values correspond to the prior probabilities which is logical. If the error rate for every pixel is 50% it doesn't add any information and the probabilities are entirely made up of the priors.

5.
The probability of the image looking the way it does given that the line was in column i is the probability of the correct pixels multiplied with the incorrect ones. For example, if the line would be in column 1 the top row of pixels would all be correct. The second row would have two incorrect pixels as the two most leftward pixels are swapped.

$P(X|Y = 1) = 0.8^{10} * 0.2^6 \approx 6.872 * 10^{-6}$
$P(X|Y = 2) = 0.8^{12} * 0.2^4 \approx 1.099 * 10^{-4}$
$P(X|Y = 3) = 0.8^{10} * 0.2^6 \approx 6.872 * 10^{-6}$
$P(X|Y = 4) = 0.8^8 * 0.2^8 \approx 4.295 * 10^{-7}$

Multiply these with the prior probability of the line being in the column.

$w_1 = 6.872 * 10^{-6} * 0.3 = 2.062 * 10^{-6}$
$w_2 = 1.099 * 10^{-4} * 0.2 = 2.198 * 10^{-6}$
$w_3 = 6.872 * 10^{-6} * 0.2 = 1.3744 * 10^{-6}$
$w_4 = 4.295 * 10^{-7} * 0.3 = 1.289 * 10^{-6}$

It turns out that the column two has the highest number and therefor probability. To calculate the probability of column 2, we take w2 and divide with the sum of all w.

The probability of the line being in column 2 is 86%

6.

Similarly to the last exercise we want to figure out what the probability of the image looking the way it does given the original image.

$$P(X|Y=B) = 0.2^{5} * 0.3^{0} * 0.7^{5} * 0.8^{5} \approx 1.762 * 10^{-5}$$
$$P(X|Y=0) = 0.2^{5} * 0.3^{2} * 0.7^{5} * 0.8^{3} \approx 2.478 * 10^{-6}$$
$$P(X|Y=8) = 0.2^{3} * 0.3^{1} * 0.7^{7} * 0.8^{4} \approx 8.096 * 10^{-5}$$

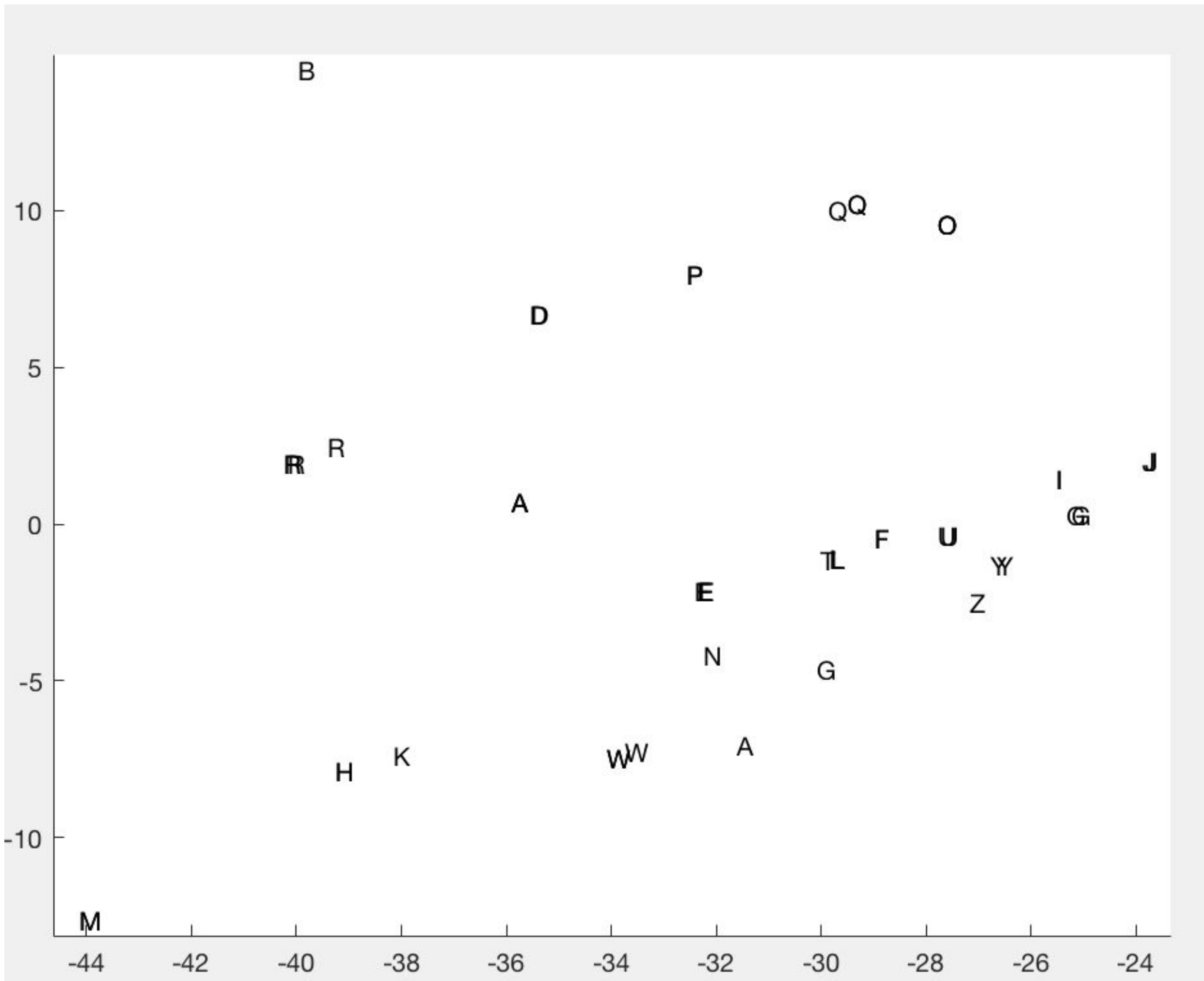
$$w_{b} = 1.762 * 10^{-5} * 0.3 = 5.286 * 10^{-6}$$
$$w_{0} = 2.478 * 10^{-6} * 0.4 = 9.912 * 10^{-7}$$
$$w_{8} = 8.096 * 10^{-5} * 0.3 = 2.429 * 10^{-5}$$

$$P(B) = 5.286 * 10^{-6} / (5.286 * 10^{-6} + 9.912 * 10^{-7} + 2.429 * 10^{-5}) \approx 0.173$$
$$P(0) = 9.912 * 10^{-7} / (5.286 * 10^{-6} + 9.912 * 10^{-7} + 2.429 * 10^{-5}) \approx 0.032$$
$$P(8) = 2.429 * 10^{-5} / (5.286 * 10^{-6} + 9.912 * 10^{-7} + 2.429 * 10^{-5}) \approx 0.794$$

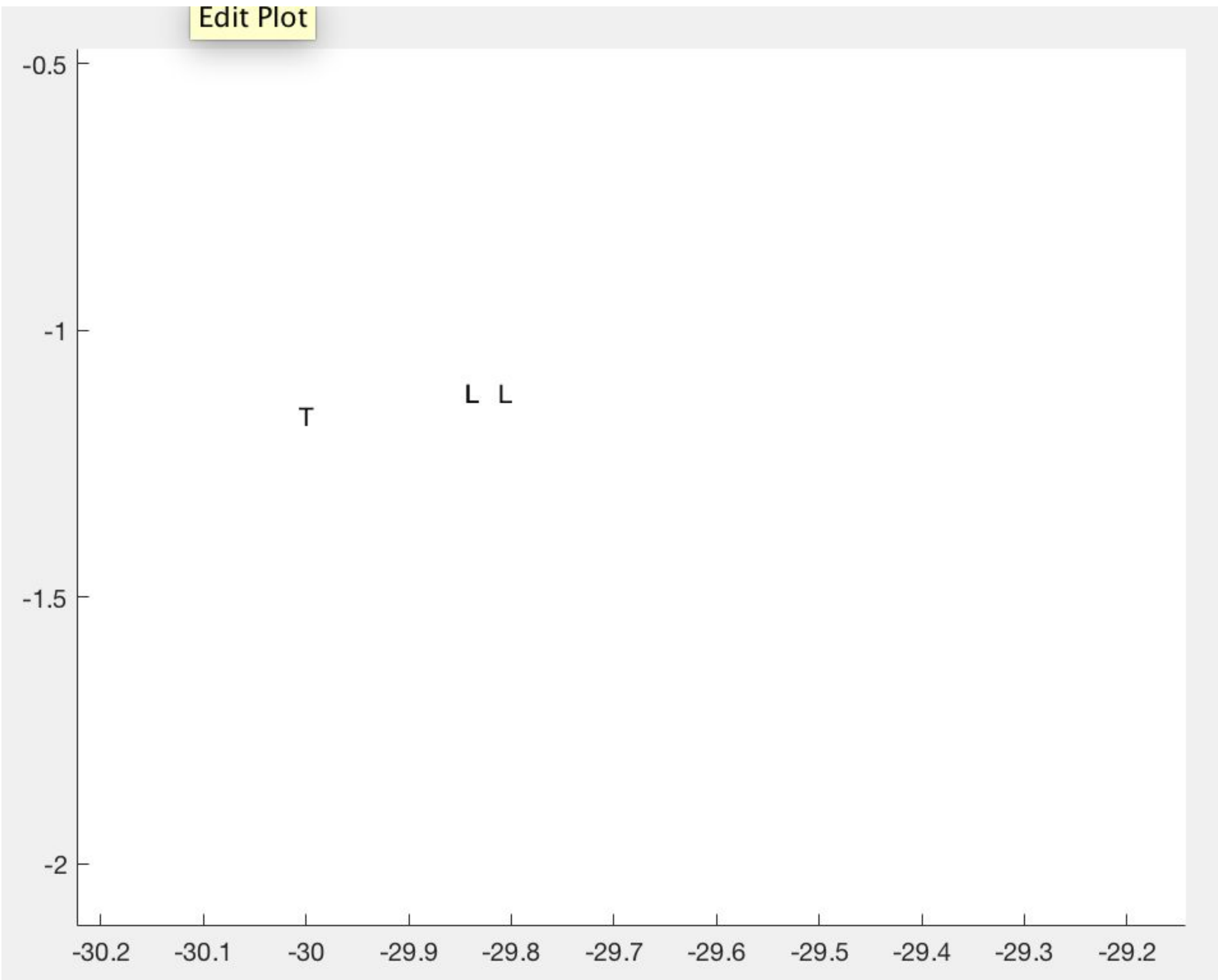
7.

```
function features = segment2features(input_img)
% features = segment2features(I)
features = zeros(6,1);

% Number of white pixels in the image
% I imagine this could separate for example I and B as B has more
% white pixels.
brightness = sum(sum(input_img));
features(1) = brightness / 10;

% Number of white pixels in the whitest column
% Could detect if the letter countains a long vertical line.
% Compare C vs P
brightest_col = max(sum(input_img));
features(2) = brightest_col;

% Number of white pixels in the whitest row
% Similar to the previous feature but horizontional.
% Compare T vs J
brightest_row = max(sum(input_img, 2));
features(3) = brightest_row;

% Number of segments in the inverted image
% Some letters such as A or B encloses part of the image within
% themselves that isn't a part of letter pixels. By inverting the
% image and running the bwlabel, we can detetc how many such
% segments there are. J has 0, A has 1, B has two enclosed segments.
compl_img = imcomplement(input_img);
```

```
[labels, number_seg] = bwlabel(compl_img, 8);
features(4) = number_seg * 10;


% Number of "feet"
% Taking the first row that countains a white pixel, starting from
% the bottom, and check how many segments that is.
% Compare A vs B

% Feet width
% Similar to the previous feature I check the bottom row that contains
% a white pixel. I then take the sum of that row.
% Compare L vs C
nbr_feet = 0;
feet_length = 0;
for index_seg = size(input_img,1): -1 : 1
    if sum(input_img(index_seg,:)) ~= 0
        [segment_label, nbr_segments] = bwlabel(input_img(index_seg,:), 4);
        nbr_feet = nbr_segments;
        feet_length = sum(input_img(index_seg,:));
        break;
    end
end
features(5) = nbr_feet * 10;
features(6) = feet_length;
```



B
10
QQ
O
P
D
5
R
RR
J
A
I
0
GG
F
U
TL
YY
E
Z
N
-5
G
A
WW
K
H
-10
M
-44
-42
-40
-38
-36
-34
-32
-30
-28
-26
-24




```
Studying the character A
There are 4 examples in the database.
The feature vectors for these are:

ans =

    9.3000    9.4000    9.4000    9.4000
   10.0000   10.0000   10.0000   10.0000
   14.0000   14.0000   14.0000   14.0000
   10.0000   20.0000   20.0000   20.0000
   20.0000   20.0000   20.0000   20.0000
   14.0000   14.0000   14.0000   14.0000
```

```
Studying the character E
There are 4 examples in the database.
The feature vectors for these are:

ans =

   10.6000   10.6000   10.8000   10.7000
   17.0000   17.0000   17.0000   17.0000
   16.0000   16.0000   16.0000   16.0000
   10.0000   10.0000   10.0000   10.0000
   10.0000   10.0000   10.0000   10.0000
   16.0000   16.0000   16.0000   16.0000
```

```
Studying the character H
There are 2 examples in the database.
The feature vectors for these are:

ans =

   13.4000   13.4000
   17.0000   17.0000
   18.0000   18.0000
   10.0000   10.0000
   20.0000   20.0000
   18.0000   18.0000
```

```
Studying the character L
There are 3 examples in the database.
The feature vectors for these are:

ans =

    7.7000    7.7000    7.6000
   17.0000   17.0000   17.0000
   14.0000   14.0000   14.0000
   10.0000   10.0000   10.0000
   10.0000   10.0000   10.0000
   14.0000   14.0000   14.0000
```

```
Studying the character O
There are 3 examples in the database.
The feature vectors for these are:

ans =

   10.4000   10.4000   10.4000
   13.0000   13.0000   13.0000
    7.0000    7.0000    7.0000
   20.0000   20.0000   20.0000
   10.0000   10.0000   10.0000
    7.0000    7.0000    7.0000
```

```
Studying the character Q
There are 3 examples in the database.
The feature vectors for these are:

ans =

   13.0000   13.1000   13.0000
   16.0000   16.0000   16.0000
    9.0000    9.0000    9.0000
   20.0000   20.0000   20.0000
   10.0000   10.0000   10.0000
    3.0000    4.0000    3.0000
```

```
Studying the character W
There are 3 examples in the database.
The feature vectors for these are:

ans =

   15.4000   15.4000   15.4000
   11.0000   11.0000   11.0000
   22.0000   22.0000   22.0000
   10.0000   10.0000   10.0000
   20.0000   20.0000   20.0000
    4.0000    3.0000    4.0000
```

```
Studying the character T
There are 1 examples in the database.
The feature vectors for these are:

ans =

    8.7000
   17.0000
   16.0000
   10.0000
   10.0000
   11.0000
```

The chosen features seem to do a good job of separating many of the characters. Some of the letters such as T & L aren't separated nicely. This surprises me as I thought the feet length feature would take care of that. Comparing those feature vectors I can see that the feet length for T=11 and L=14. The feature for H=18 which tells me that this isn't working as intended.

I’m guessing that looking at the bottom row which contain a white pixel isn’t robust enough. If the letter is tilted slightly you can get big differences between two “L”s. It might be better to look at the bottom 4 rows and take an average.

I also tried convolving the image but the conv2 function gave me errors that I was unable to fix