Markov Chains in Operations Research

1. Introduction to Markov Chains

Markov Chains are mathematical systems that undergo transitions from one state to another in a chain-like process. They are used in various OR applications such as queuing models, inventory control, reliability systems, and more. Markov processes are memoryless, meaning the next state depends only on the current state and not the past sequence of events.

2. Transition Matrices

A transition matrix shows the probabilities of moving from one state to another. Each row represents a current state, and each column represents a next state. The rows of a transition matrix sum to 1. For example:

	S1	S2
S1	0.7	0.3
S2	0.4	0.6

3. Types of States

- Recurrent: the process is guaranteed to return to the state
- Transient: it is possible that the process will never return
- Absorbing: once entered, cannot be left

4. Steady-State Probabilities

Steady-state probabilities describe the long-run behavior of the system. They are found by solving the system: pi * P = pi, where pi is the steady-state vector and P is the transition matrix, subject to the condition that the sum of all elements in pi equals 1.

5. Applications

- Queueing systems
- Inventory management
- Maintenance and reliability models
- Markov Decision Processes (MDPs)
- Stochastic games