M/M/1 Queuing System Theory

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1. Introduction to Queuing Systems

Queueing theory studies the behavior of waiting lines. The M/M/1 model is the most basic single-server queue.
It assumes **exponential (memoryless) interarrival** and **service times**, and a **FIFO queue discipline**.

2. Kendall Notation

The notation A/S/c is used to describe queuing systems:

- A: Arrival process (e.g., M = Markovian = Poisson)
- S: Service time distribution (M = Exponential)
- c: Number of servers

Hence, M/M/1 means:

- **Poisson arrivals**
- **Exponential service times**
- **1 server**

3. Assumptions of M/M/1 Queue

- Interarrival times are **exponentially distributed** with rate **λ**
- Service times are **exponentially distributed** with rate **μ**
- Single server (one service channel)
- FIFO queue discipline
- Infinite queue capacity and calling population

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4. Performance Metrics

Let **ρ = λ / μ** (utilization factor):

- **L** = $\rho / (1 - \rho)$: Average number in the system

- **Lq** = $\rho^2 / (1 - \rho)$: Average number in the queue
- **W** = $1 / (\mu - \lambda)$: Average time in the system
- **Wq** = $\rho / (\mu - \lambda)$: Average waiting time in the queue

The system is **stable** only if **λ < μ**, i.e., $\rho < 1$.

5. Applications of M/M/1

- Bank teller or cashier counters
- Call centers with a single operator
- Machine repair centers with one technician
- Web server handling requests serially