P1: Predicting Boston Housing Prices

Reported by Yuki Wakatsuki (yuki.oyabu@gmail.com)

1. Statistical Analysis and Data Exploration
   - 1.1. Number of data points (houses): **506**
   - 1.2. Number of features: **13**
   - 1.3. Minimum housing prices: **5**
   - 1.4. Maximum housing prices: **50**
   - 1.5. Mean housing prices: **22.53**
   - 1.6. Median housing prices: **21.2** (Q1: **17.03**, Q3: **25.0**, IQR: **7.98**)
   - 1.7. Standard deviation: **9.19**

2. Evaluating Model Performance
   - 2.1. Which measure of model performance is best to use for predicting Boston housing data and analyzing the errors? Why do you think this measurement most appropriate? Why might the other measurements not be appropriate here?
     - I think mean squared error (MSE) is best to use for predicting Boston housing data and analyzing the errors. As described in section 3.3.4 of scikit-learn's official site, there are total 5 available metrics, such as mean squared error(MSE), mean absolute error(MAE), median absolute error, R^2 and explained variance score, for regression problems. Since R^2 and explained variance score are used to measure goodness of fit, these two metrics are inappropriate to use here. Also, median absolute error is worse than other 2 metrics because it only takes care of robustness to outlier while MSE and MAE take care of all differences between predicted values and target values. Lastly, compared with MAE, MSE puts higher error scores if the differences between predicted and targeted values are getting bigger. Since an accuracy of predicted values to true values is the matter in boston housing prices problem, MSE is more appropriate than MAE.
   - 2.2. Why is it important to split the Boston housing data into training and testing data? What happens if you do not do this?
     - To estimate performance to an independent dataset and check if a model overfits or not, it is important to split the Boston housing data into training and testing data. If we do not split the data, we cannot evaluate model's performance to an unseen data set and cannot avoid making models overfits to a training data set.
   - 2.3. What does grid search do and why might you want to use it?
     - Grid search is a way of systematically working through multiple combinations of parameter tunes, cross-validating as it goes to determine which tune gives the best performance. If we have candidate parameters but don't know which ones should be used, it makes sense to use grid search to estimate best performed parameters.
   - 2.4. Why is cross validation useful and why might we use it with grid search?
     - There are 2 advantages for us to use cross validation. (1) We can maximize data usage because cross validation creates multiple training and test datasets from original dataset. Even if dataset is limited in size, cross validation such as LOOCV allows us to assessing the real potential of algorithm in terms of performance metrics. (2) We can reduce the chance of overfitting because cross validation calculates average error score. If we use traditional train test split approach, a model often overfits when the training dataset is somehow imbalanced. By averaging error score, cross validation can avoid overfitting and average error score itself can well represent average performance to unseen data.
     - If we use cross validation with grid search, there are 2 advantages. (1) We can avoid making unnecessary bugs. If we use grid search and cross validation separately, we need

to remember several rules (i.e. dataset must not be shuffled twice until entire processes finish) and test them to check if they work well. (2) We can automate both parameter search and cross validation with single line of code. This enables us to focus on main logic of the code.

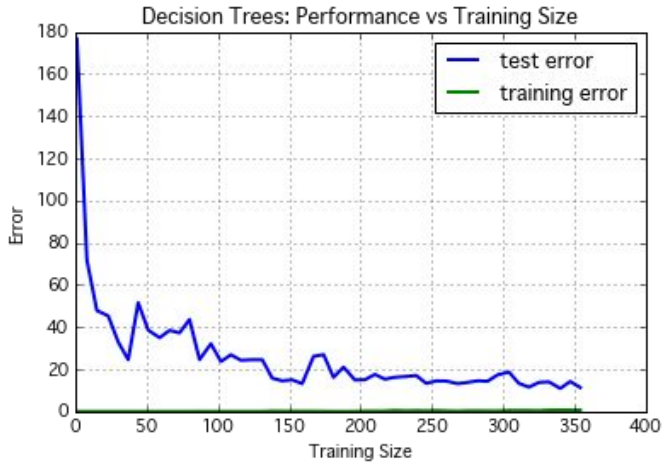
3. Analyzing Model Performance

3.1. Look at all learning curve graphs provided. What is the general trend of training and testing error as training size (n) increases?

- $TrainingError_i \leq TrainingError_j, \ (0 \leq i < j \leq n)$
- $TestError_i \geq TestError_j, \ \ (0 \leq i < j \leq n)$
- $TestError_i - TrainingError_i \geq TestError_j - TrainingError_j, \ \ (0 \leq i < j \leq n)$

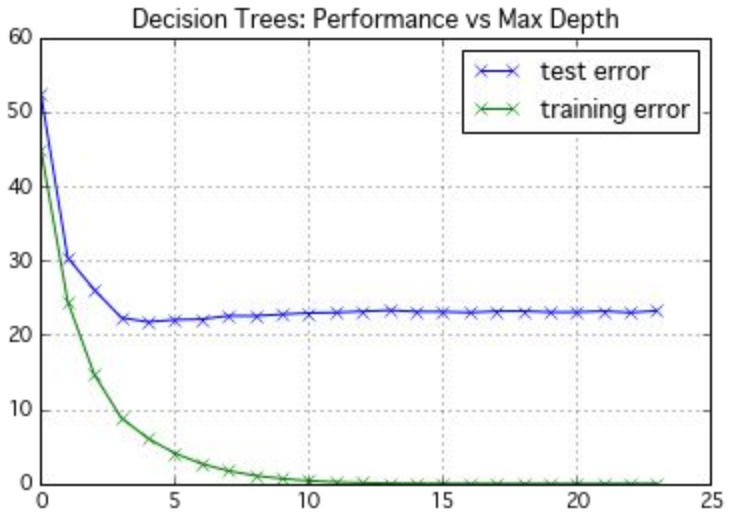
3.2. Look at the learning curves for the decision tree regressor with max depth 1 and 10 (first and last learning curve graphs). When the model is fully trained does it suffer from either high bias/underfitting or high variance/overfitting?

- The regressor with max depth 1 suffers from high bias/underfitting because test errors no longer get smaller since relatively small number of training data and test errors are much higher than one with max depth 10 especially training size becomes over 90. On the other hand, the regressor with max depth 10 suffers from high variance/overfitting because test errors fluctuates and test error is much higher than training error even if training data size is bigger . Please see figures shown below. The first figure shows the learning curves with max depth 1 and the second figure with max depth 10.



3.3. Look at the model complexity graph. How do the training and test error relate to increasing model complexity? Based on this relationship, which model (max depth) best generalizes the dataset and why?

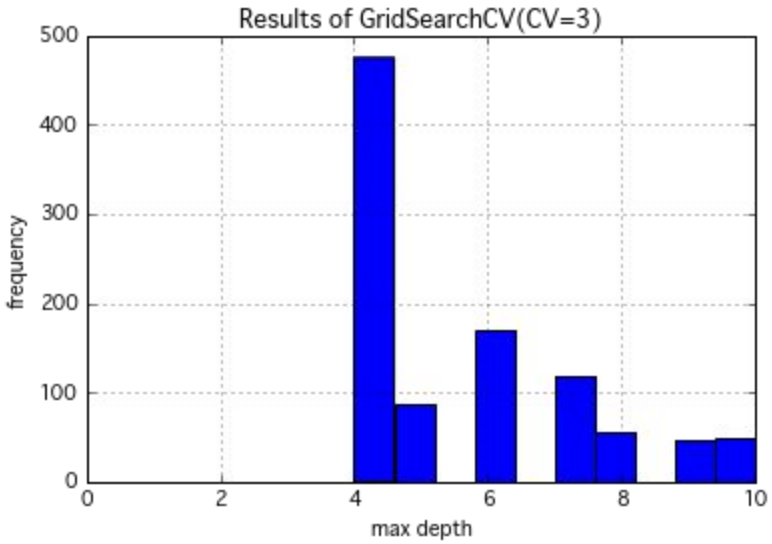
- The figure shown below plots average training and test error from 1000 trials. In the figure, x-axis and y-axis each represent model complexity and error score. There are 3 important points you can see from the figure.
  - Training error continues to decrease as model complexity increases. In the figure training error becomes almost 0 in models with over max depth 10.
  - Test error stops to decrease as model complexity increases. In the figure models with over max depth 4 do not improve its performance against test data.
  - Difference between test and training error increase as model complexity increases. Which means models more overfit as model complexity increases.
- I conclude that models with max depth 4 best generalizes the dataset. In theory, best generalized models can well-perform against test data and keep complexity as lower as possible. By above observations, we found that models with max depth 4 have almost best performance compared with more complex models, and have obviously better performance compared with less complex models.



4. Model Prediction

4.1. Model makes predicted housing price with detailed model parameters (max depth) reported using grid search. Note due to the small randomization of the code it is recommended to run the program several times to identify the most common/reasonable price/model complexity.

- I ran the program 1000 times to identify the most common/reasonable price/model complexity based on distribution of price/model complexity. The figure shown below is histogram about model complexity (max depth) selected by grid searches. It is obvious that max depth 4 is most frequently selected as the best model parameters. As described in 3.3, models with higher max depth more overfit against test data, so it is reasonable to use the models with max depth 4, which is the lowest max depth among candidates, as a common/reasonable model complexity and housing price predicted by the models.



- Model with detailed model parameters reported by grid search:

```
DecisionTreeRegressor(criterion='mse', max_depth=4, max_features=None,
           max_leaf_nodes=None, min_samples_leaf=1, min_samples_split=2,
           min_weight_fraction_leaf=0.0, presort=False, random_state=None,
           splitter='best')
```

- Predicted housing price: **21.68**

4.2. Compare prediction to earlier statistics and make a case if you think it is a valid model.

- Since predicted value is not outlier compared with the earlier statistics, I think it is a valid model. According to the earlier statistics, outliers[1] are fall into below 5.06 calculated by $"Q1 - 1.5 \times IQR"$ or above 36.96 calculated by $"Q3 + 1.5 \times IQR"$. Since the predicted value (21.68) is obviously not within this range, the predicted value is not outlier. Also, the predicted value is between Q1 and Q3, which means the predicted value is more of average housing price than outlier price.

[1] https://en.wikipedia.org/wiki/Interquartile_range, 01/13/2016