Henry Saltwater Intrusion Problem

1 Introduction

The Henry saltwater intrusion problem is considered the benchmark analysis for testing density-dependent groundwater flow models. The problem considers a vertical slice through an isotropic, homogenous, confined aquifer. A constant flux of freshwater is applied to the inland boundary, while a seaward boundary is exposed to a stationary body of higher density seawater. Henry (1964) developed a semi-analytical solution for the steady-state distribution of salt concentration within the system. Although the problem has been widely used for benchmarking flow models, a number of authors have proposed modifications to the problem. In particular, Simpson and Clement (2004) suggested that reducing the freshwater flux rate enhances the density-dependent effects; thereby improving the worthiness of the Henry problem as a benchmark analysis. In this example, both the standard and modified Henry saltwater intrusion problems are modeled using a density-dependent CTRAN/W analysis. The results are compared to the semi-analytical solution.

2 Boundary Conditions and Material Properties

Figure 1 presents the model domain and boundary conditions for the problem. The model domain is 2 m in length and 1 m in height. The domain was discretized using predominately quadrilateral elements with a global size of 0.05 m, with refinements to the top-right quadrant where advective transport dominates and the Peclet and Courant criteria is most crucial. The initial pore water pressure conditions in the SEEP/W analyses are defined using a piezometric line drawn at the top of the aquifer, which implies that a hydrostatic condition exists prior to saltwater intrusion. The initial concentration within the model domain is set to zero using the ‘activation concentration’ under the KeyIn materials dialogue box. Boundary conditions for the transient analysis were specified as follows:

1. No flow along the top and bottom boundaries (i.e., q = 0 m/s);
2. Unit flux q = 6.6×10^{-6} and 3.3×10^{-6} m/s for the standard and modified cases, respectively (Simpson and Clement, 2004), applied to the left (inland) freshwater recharge boundary;
3. Hydrostatic pressure distribution (total head = 1 m) along the right side;
4. Constant concentration of 0.0 (i.e. freshwater) on the left boundary; and,
5. Constant concentration of 1.0 mg/L (i.e. saltwater) on the right boundary.

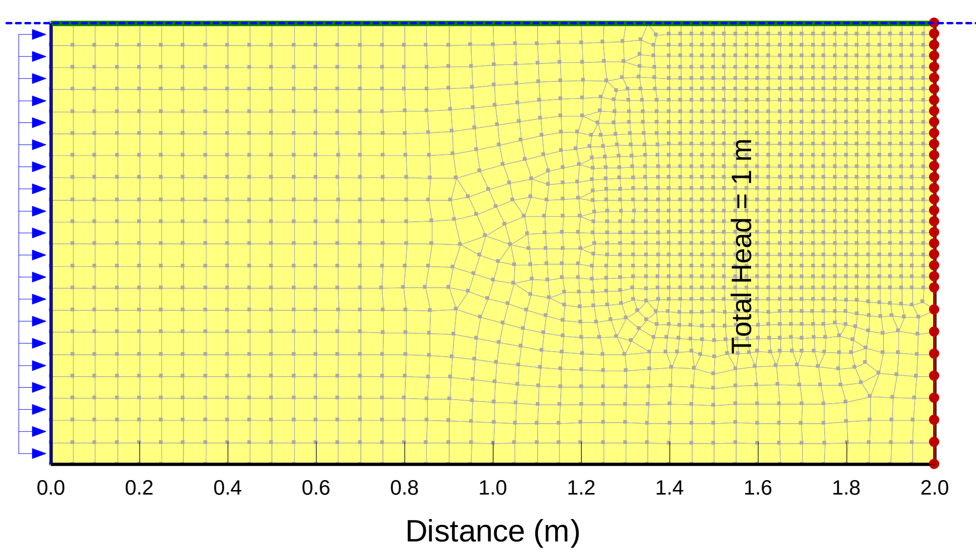

Figure 1 Model domain and boundary conditions.

Table 1 presents the aquifer properties used in the analysis. The longitudinal and transverse dispersivity were arbitrarily assigned a value 1×10^{-20} m because the Henry problem does not include the effects of mechanical dispersion. The coefficient of volume change (m_v) was set to 1×10^{-6} 1/kPa despite a value of 0.0 (i.e. incompressible) presented in the original publications. This was done because it is good modeling practice to assign a value for m_v when conducting transient seepage analyses, even if it is a very low number. Finally, the reference concentration and relative density were set to 1.0 and 1.025, respectively. This implies a fluid density of 1025 kg/m^3 (i.e. seawater) at a concentration of 1.0 mg/L. More background information on these inputs can be found in the GeoStudio example file entitled ‘Salt Flow Example’.

Table 1. Aquifer properties used for the analysis.

Parameter	Value	Unit
Coefficient of Molecular Diffusion (D^*)	1.89×10^{-5}	m^2/s
Longitudinal Dispersivity (D_L)	1×10^{-20}	M
Transverse Dispersivity (D_T)	1×10^{-20}	M
Hydraulic Conductivity (K_{sat})	1×10^{-2}	m/s
Coefficient of Volume Change (m_v)	1×10^{-6}	1/kPa
Porosity (n)	0.35	
Reference Concentration	1.0	mg/L
Relative Density	1.025	

The time step sequence consisted of constant time step of 55 and 90 seconds for the standard and modified cases, respectively. Simpson and Clement (2004) report that the system reaches steady-state after approximately 160 minutes, at which time the 25%, 50%, and 75% isochlors become stationary. Accordingly, the standard case was modeled for an elapsed time of 180 minutes. The duration of the modified case was longer at 300 minutes because more time was required to reach steady-state due to the reduced freshwater flux rate.

3 Results and Discussion

3.1 Standard Henry Problem

Figure 2 presents a comparison of the numerical and semi-analytical results for the standard Henry saltwater intrusion problem. The location of the 25%, 50%, and 75% isochlor modeled using CTRAN/W are in agreement with the semi-analytical results. The isochlors intersect the base of the aquifer at distances of 1.186, 1.38, and 1.587 m, respectively, which is in keeping with the values reported by Simpson and Clement (2004).

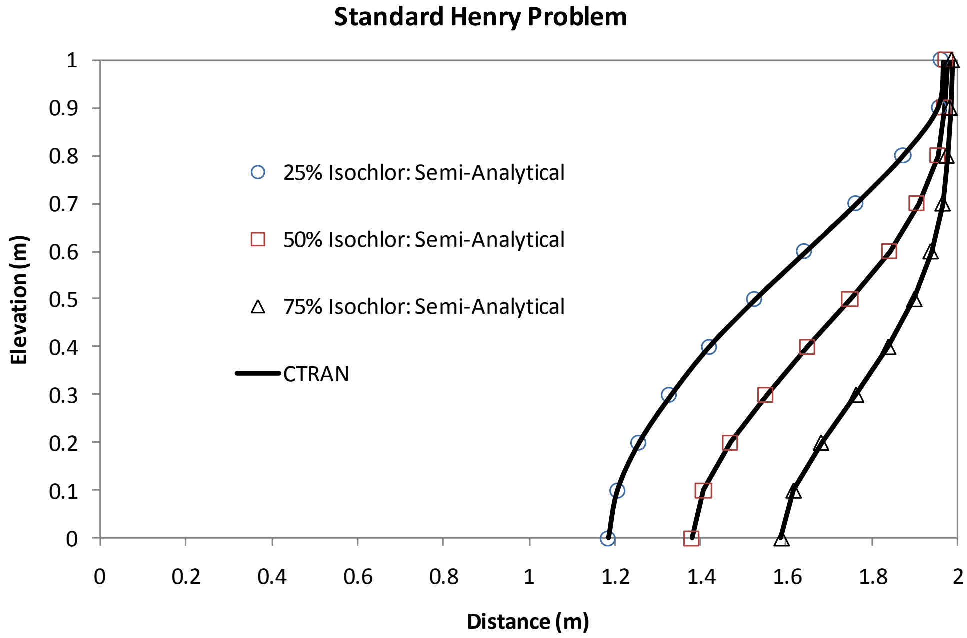

Figure 2 Comparison of numerical and semi-analytical results for the standard Henry problem.

Figure 3 presents the concentration contours for the standard Henry problem after an elapsed time of 180 minutes. The 50% isochlor is marked. As shown above, the seawater intrusion distance is greatest near the bottom right corner, while the isochlors are constricted in the top-right corner. The mechanism controlling the location of the isochlors is related to the density-dependent effect on groundwater flow (discussed below).

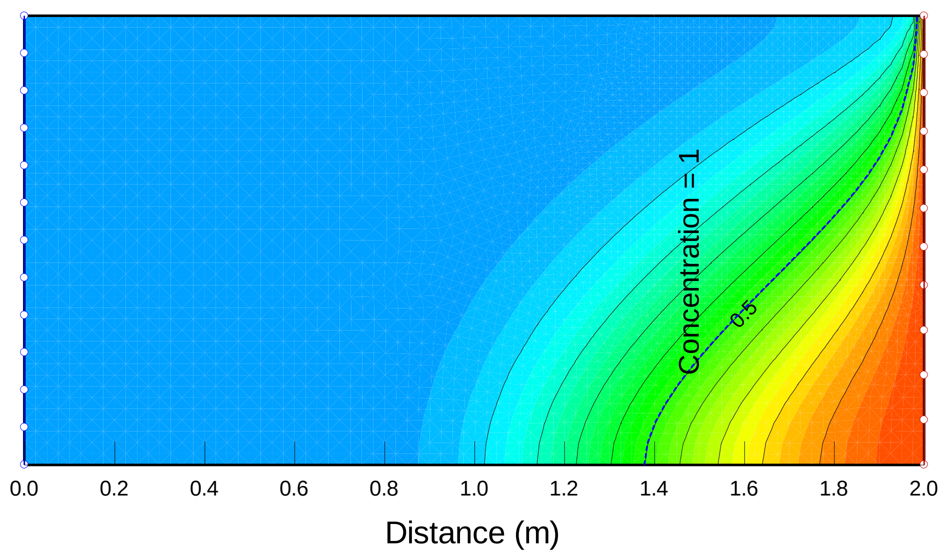

Figure 3 Concentration contours at an elapsed time of 180 minutes for the standard Henry problem.

Figure 4 shows the velocity vectors and contours of equivalent fresh water total head at the end of the analysis. Figure 5 presents a plot of total freshwater hydraulic head verses distance at elevations of 0, 0.5, and 1.0 m at the beginning of the analysis (elapsed time of 275 seconds) and Figure 6 presents a vertical profile of freshwater hydraulic head along the seaward edge. Although the total head on the seawater edge is set to 1 m at the onset of the simulation (i.e. hydrostatic), the freshwater total head increases with depth due to the density of the seawater. At first glance, this would suggest that there is a tendency for upwards flow along the entire right boundary; however, the 'buoyancy' or body force component of the driving force counterbalances the upward hydraulic gradient and there is no upward groundwater flow.

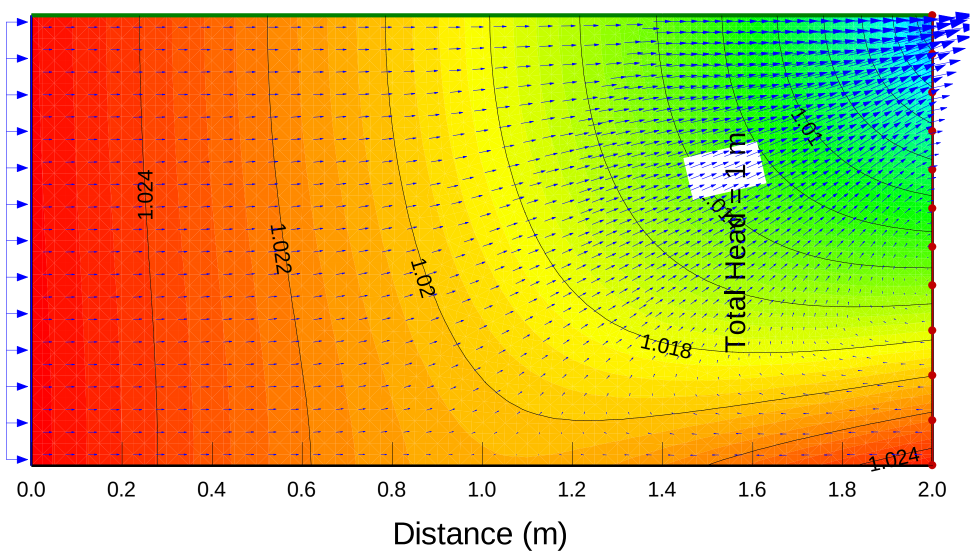

Figure 4 Contours of total freshwater head and vectors of velocity field.

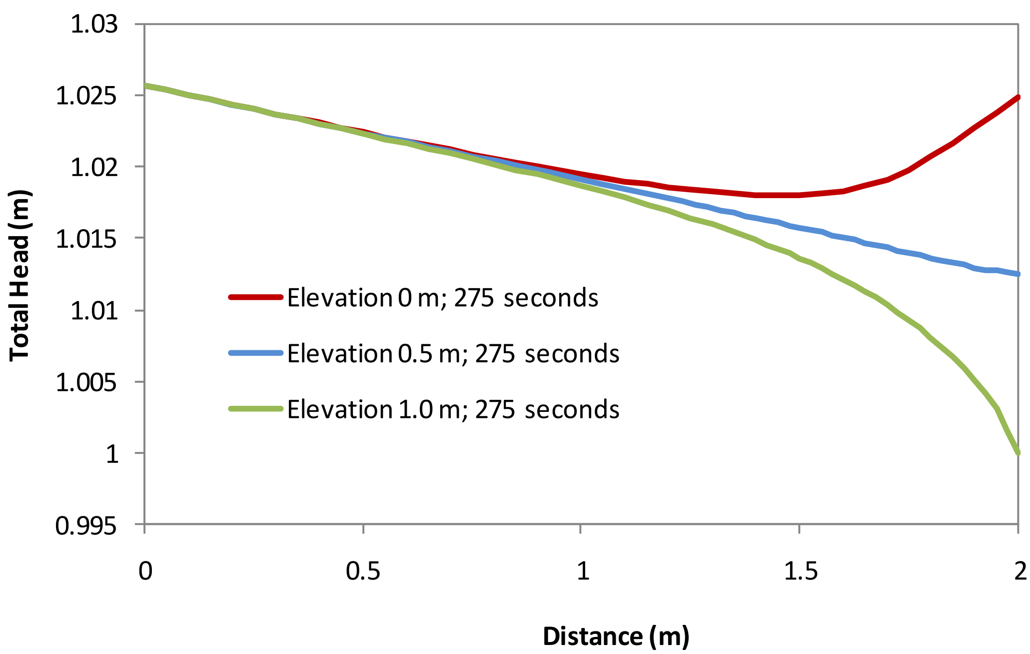

Figure 5 Lateral profiles of total hydraulic head at elevations of 0, 0.5, and 1.0 m (275 seconds).

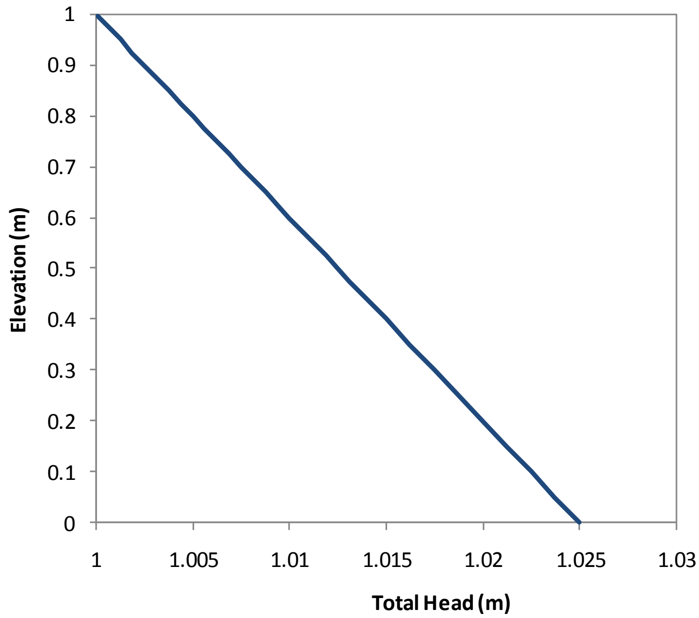

Figure 6 Profile of Total Head (freshwater) along the seaward boundary.

At the onset of the analysis, there is also an inward lateral hydraulic gradient (freshwater) in the bottom right corner of the domain and an outward hydraulic gradient in the top-right quadrant. The inward gradient is caused by the sharp contrast between the freshwater total head at the seaward edge and immediately inside the model domain. For example, the freshwater total head is 1.025 m at the bottom right corner on the seaward side, but immediately inside the model domain it is 1 m (at the start of the analysis). The sharp gradient quickly gets smoothed because the concentrations are increasing further into the domain (Figure 5). The lateral inward gradient diminishes towards the middle of the slice, and then reverses direction in the top half. Above an elevation of 0.5, the density effects become less pronounced and water flow is outward. A relatively stagnant zone exists along the right boundary between elevations of about 0.25 m and 0.5 m because the lateral inward gradient caused by the density effects is counterbalanced by flow from the inland side.

There are four additional noteworthy observations. First, the hydraulic gradient increases – as evidenced by the tighter spacing of head contours – in the top-right quadrant of the model domain due to convergence of flow originating from the inland edge and the bottom right quadrant of the model domain. The groundwater flux rate therefore increases (as indicated by the velocity vectors) and advection dominates transport in this region. Secondly, the convergence of flow creates a convective cell. Third, the density effects near the bottom-right edge of the column accelerate salt intrusion. This can be further illustrated by noting that the concentration contours would be perfectly vertical if this problem was modeled without density effects, as shown in Figure 7. In this case, the intrusion distance of the 50% isochlor would be less than 0.1 m because advective transport (outflow) is counteracting inward diffusion. Again, it is also apparent that the increased flux in the top right quadrant attenuates salt ingress, as the isochlors are more constricted in this region for the Henry problem (Figure 3). Finally, the flow vectors within the bottom right quadrant (Figure 4) are not perpendicular to the freshwater total head contours. This occurs because of the buoyancy or 'body' force component of the dynamic driving force (see 'Salt Flow Example'). In fact, the vectors are nearly horizontal in the bottom-right corner because the upward fresh water total head gradient is nearly balanced by the buoyancy force.

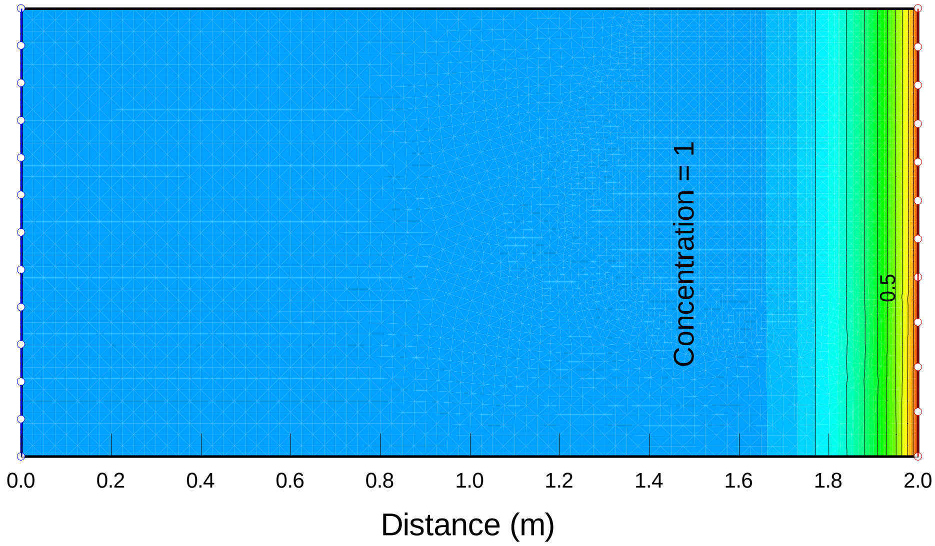

Figure 7 Concentration profiles for analysis without density-dependent effects.

3.2 Modified Henry Problem

Figure 8 presents the concentration contours for the modified Henry saltwater intrusion problem after an elapsed time of 300 minutes. Again, the density-dependent CTRAN/W results are in agreement with the closed-form solution. The reduction in freshewater recharge means that the velocity and advective transport of solute is reduced, particularly near the outflow region in the top-right quadrant. Accordingly, the isochlors are less constricted toward the outflow region of the aquifer. As noted by Simpson and Clement (2004), it is therefore easier to identify the relative accuracy of the numerical solution in this region. Finally, the density effects are more pronounced near the bottom of the aquifer. The 50% isochlor intersects the base of the aquifer at a distance of 1.073 m, verses 1.38 m for the standard Henry problem.

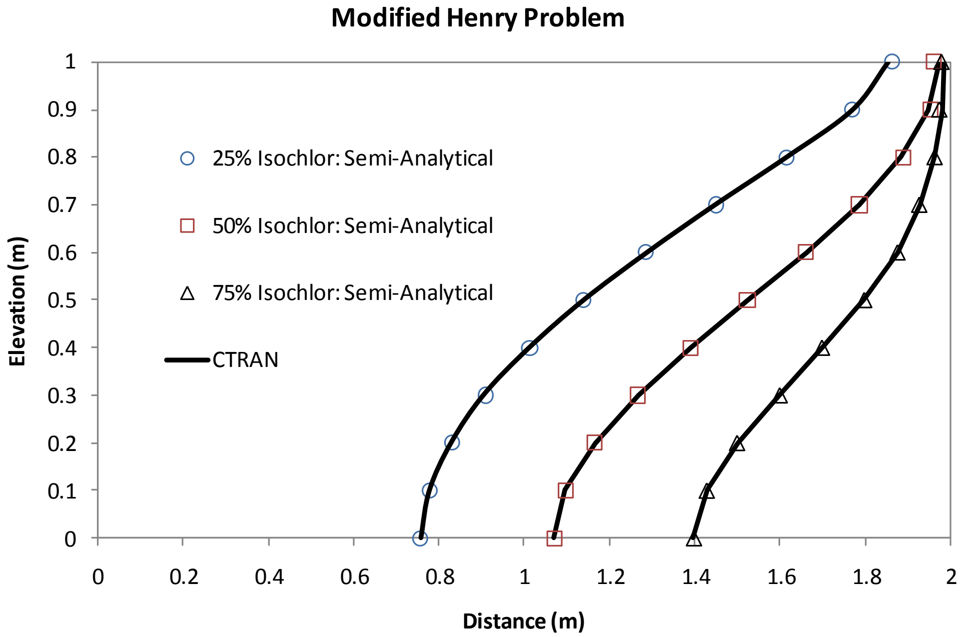

Figure 8 Comparison of numerical and semi-analytical results for the modified Henry problem.

4 Concluding Remarks

The results from a density-dependent CTRAN/W analysis are compared to the closed-form solution for the Henry saltwater intrusion problem (1964). The Henry saltwater problem is considered as the benchmark analysis for testing finite element code. The problem considers seawater intrusion in a confined coastal aquifer with freshwater recharge on the inland side. The problem can also be modified to improve the worthiness of the test by reducing the flux rate along the inland side, thereby enhancing the density-dependent effects. The results from CTRAN/W compare well to the semi-analytical solution for both the standard and modified cases, which demonstrates the reliability of GeoStudio for modeling density-dependent transport problems.

5 References

Henry, H. R. 1964. Effects of dispersion on salt encroachment in coastal aquifers. Sea Water in Coastal Aquifers, U.S. Geol. Surv. Supply Pap., 1613-C, C71-C84.

Simpson, M.J. and Clement, T.B. 2004. Improving the worthiness of the Henry problem as a benchmark for density-dependent groundwater flow models. Water Resources Research, 40 (W01504).