Last name	First name	Matriculation number

Midterm 1 Formale Grundlagen der Informatik I (AINF1110, FS18)

March, 22rd 2018
Version A

You have **60** Minutes time for solving the exercise.
In this mandatory exercise you can achieve up to **60** points. You are admitted to the final exam if you can achieve either more than 30 points in a single mandatory exercise or 36 points in both mandatory exercises together.

The following rules apply for this mandatory exercises:

- Answer the questions directly on the exercises sheets.
- Check for completeness at the end of the exercise.
- Usa a pen with blue or black color. No pens with red or green color nor pencils will be considered for correction.
- Always use the notation used in the lecture. However, you can write your answers in **English** or **German**.
- No additional material and supporting devices are allowed. If spotted while cheating, the mandatory exercise will be counted with 0 points.
- Put the student ID ("Legi") in front on your desk.

With this signature I confirm that I have read and understood this rules.

Signature: ______________________

1 (10 P.)	2 (5 P.)	3 (16 P.)	4 (15 P.)	5 (14 P.)	Total

1 True or False? (10 P.)

Check, if the following statements are true or false.

Every correct answer adds a point to your score. Every wrong answer deducts a point from your score. Every answer left blank is not considered for point counting. Allover this task gives a least 0 points.

All fractions of natural numbers with a non-zero denominator are rational. true	☐ true	☐ false
The set $\{2, 2, 3, \{2\}\}$ consists of only 2 elements. false	☐ true	☐ false
$\sim(a \vee b) \equiv \sim a \vee \sim b$. false	☐ true	☐ false
A function from set A to B must map to every value in B. false	☐ true	☐ false
A function from set A to B must map from every value in A. true	☐ true	☐ false
The set $0, 1, 2$ is equal to $\{x \in \mathbf{R} \| -1 < x < 3\}$ false	☐ true	☐ false
$\sim(p \vee q) \equiv \ \sim p \ \wedge \sim q$ is true while $\sim p \ \vee \sim q \equiv \ \sim(p \wedge q)$ is false. false, DeMorgan	☐ true	☐ false
$p \vee \mathbf{t} \equiv \mathbf{t}$, give a variable p and a tautology $\mathbf{t}$. true	☐ true	☐ false
$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$ is not a contradiction. false	☐ true	☐ false
Is the following argument valid? If this is an exam, it is important. This is an exam. $\therefore$This is important. true	☐ true	☐ false

2 Set Theory (5 P.)

Solve the following questions about set theory.

a) (2) How many elements are in the following sets:

(a) $\{1, 1, 2, \{1, 1, 2\}\}$
(b) $\{\{r, g, b\}, \{r\}, \{g\}, b\}$
(c) $\{\{1, 0, 0\}, \{0, 2, 0\}, \{0, 0, 3\}, \{0, 1, 0\}, \{0, 2, 0\}\}$
(d) $\{x \in \mathbb{R} \mid 0 < x < 1\}$

(a) 3
(b) 4
(c) 4
(d) ∞

b) (1) Given two sets $A = -1, 0, 1, 2$ and $B = 0, 1$, write down the ordered pairs of $(x, y) \in A \times B$

c) (1) Given the sets from b), list the pairs that are an element of $\mathbb{R}$, where

$$(x, y) \in \mathbb{R} \text{ means thath } x^2 + y^2 \leq 1$$

.

d) (1) Draw the a graph with the cartesian plane where you indicate the boundary of relation $\mathbb{R}$ form c) and indicate all contained elements in the plane.

3 Logic of Compound Statements (16 P.)

a) (6) Prove or disprove the following equivalence using a truth table for each side of the equation:

$$((\sim p \ \wedge \sim q) \vee (p \wedge q)) \wedge r \equiv (\sim r \vee \sim q) \wedge ((r \vee p) \wedge \sim p)$$

p	q	r	$(\sim p \ \wedge \sim q)$	$(p \wedge q)$	$(\sim p \ \wedge \sim q) \vee (p \wedge q)$	$((\sim p \ \wedge \sim q) \vee (p \wedge q)) \wedge r$
F	F	F	T	F	T	F
F	F	T	T	F	T	T
F	T	F	F	F	F	F
F	T	T	F	F	F	F
T	F	F	F	F	F	F
T	F	T	F	F	F	F
T	T	F	F	T	T	F
T	T	T	F	T	T	T

p	q	r	$(\sim r \vee \sim q)$	$(r \vee p)$	$((r \vee p) \wedge \sim p)$	$(\sim r \vee \sim q) \wedge ((r \vee p) \wedge \sim p)$
F	F	F	T	F	F	F
F	F	T	T	T	T	T
F	T	F	T	F	F	F
F	T	T	F	T	T	F
T	F	F	T	T	F	F
T	F	T	T	T	F	F
T	T	F	T	T	F	F
T	T	T	F	T	F	F

b) (7) Prove or disprove the correctness of the following argument formula:

$$\begin{aligned} & (p \vee q) \wedge r \rightarrow s \wedge \sim p \\ & s \vee \sim p \rightarrow q \wedge \sim r \\ & (p \vee r \vee s) \wedge \sim (p \wedge q) \\ \therefore\ & p \vee (q \wedge \sim r \wedge s) \end{aligned}$$

The conclusion can only be wrong if $p \vee (q \wedge \sim r \wedge s)$ is false, so either p is false or $(q \wedge \sim r \wedge s)$ is false therefore we only need to check the following seven configurations: The second premise $s \vee \sim p$ is for all seven configurations true (since p is always false), while $q \wedge \sim r$ is only true if q is true and r is false. The only configuration, in which q is true and r is false, is the one with s being false. This configuration invalidates the third premise, therefore there is no configuration in which the premises are all true but the conclusion is not. The argument formula is proven.

p	q	r	s
F	F	F	F
F	F	F	T
F	F	T	F
F	F	T	T
F	T	F	F
F	T	T	F
F	T	T	T

c) (3) Show if the following logical equivalence is a tautology:

$$p \to (\sim q \to r) \;\equiv\; \sim p \lor (q \lor r)$$

$(q \to r) \equiv (\sim q \lor r)$, respectively $(\sim q \to r) \equiv (q \lor r)$ can be used to simplify the equivalence, as the following truth table also shows.

p	q	r	$(\sim q \to r)$	$p \to (\sim q \to r)$	$(q \lor r)$	$\sim p \lor (q \lor r)$
F	F	F	F	T	F	T
F	F	T	T	T	T	T
F	T	F	T	T	T	T
F	T	T	T	T	T	T
T	F	F	F	F	F	F
T	F	T	T	T	T	T
T	T	F	T	T	T	T
T	T	T	T	T	T	T

4 Number theory (15 P.)

Prove of disprove the following statements

a) (5) $10 - 3 * \sqrt[3]{9}$ is irrational.

If it is rational there must exist integers a and b such that:

$$10 - 3 \cdot \sqrt[3]{9} = \frac{a}{b} \tag{1}$$

$$-3 \cdot \sqrt[3]{9} = \frac{a}{b} - 10 \tag{2}$$

$$-3 \cdot \sqrt[3]{9} = \frac{a - 10b}{b} \tag{3}$$

$$\sqrt[3]{9} = \frac{a - 10b}{-3b} \tag{4}$$

$$\tag{5}$$

while the righthand side is based on integers and rational, the lefthand side is an irrational, therefore the original statement was also irrational. .

b) (3) The product of any two irrational numbers is irrational.

Counter example: $x := \sqrt{2}, y := -\sqrt{2}$. x and y are irrational numbers, but $x * y = -2$, which is rational.

c) (7) There exists a rational x such that $100 = \left(x^2 + 2\right)^2$.

vgl. 4.7. h19 in book (update solutions), or more like simple proof by example of an existential statement

5 Circuits (14 P.)

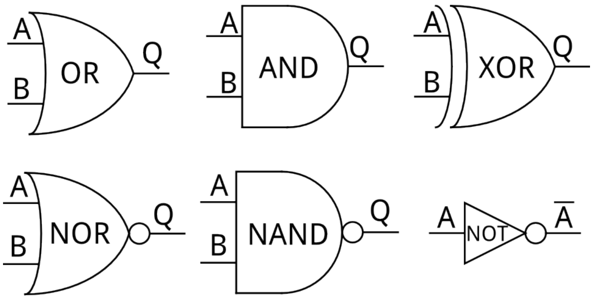
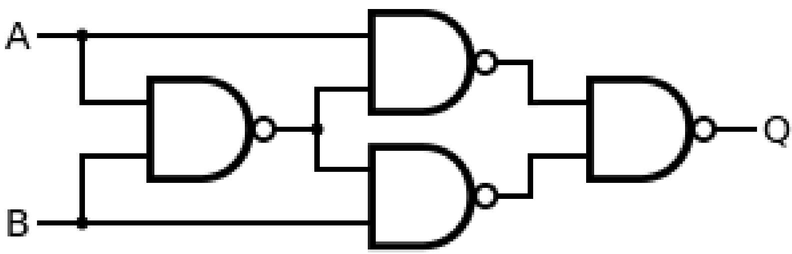


Figure 1: A overview of possible logic circuits

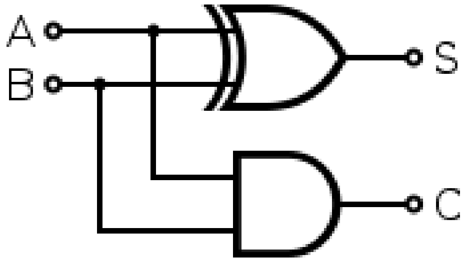
a) (3) You are given many NOT gates and one OR gate. Use these gates to build an AND gate. Draw the gate and show that it is correct.

The circuit corresponds to the formula (obtained by De Morgan's law) $\sim(\sim A \vee \sim B)$, which is equivalent to $A \wedge B$.

b) (3) You are given many NAND gates. Build a XOR gate (exclusive or).



c) (3) Build a half adder circuit using XOR and AND logic. The circuit has 2 inputs and 2 output, whih sum up the inputs. Therefore, if one onput is true, the output should be the bits 01, it both inputs are true it should be 10 and if non ar true it should be 00.



d) (5) Now build the same half adder again, but this time only using NAND logic.