Formale Grundlagen der Informatik

Assignment 3

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3.1
a) i. $a_i = \frac{5-i}{5+i} \to \lim_{i\to\infty}(a_i) = \frac{\underbrace{\frac{5}{i}}_{\to 0} - \underbrace{\frac{i}{i}}_{\to 1}}{\underbrace{\frac{5}{i}}_{\to 0} + 1} = \frac{-1}{1} = -1$ $\quad$ a_i converges to -1

ii. $b_i = \frac{i^3+5}{4-i^2} \to \lim_{i\to\infty}(b_i) = \frac{\frac{i^3}{i^2} + \frac{5}{i^2}}{\frac{4}{i^2} - \frac{i^2}{i^2}} = \frac{\overbrace{i + \frac{5}{i^2}}^{\to \infty}}{\underbrace{\frac{4}{i^2} - 1}_{\to -1}} = \frac{\infty}{-1}$ $\quad$ b_i converges to $-\infty$

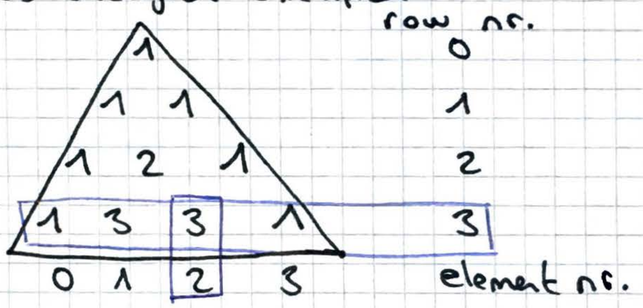
iii. c_i ~~converges to~~ diverges.

3.2
a) The number corresponding to a binomial coefficient can be read from the Pascal's triangle. Example:

given is $\binom{3}{2}$

→ solution:
look in pascal's-Δ at $\boxed{\text{row } 3}$, $\boxed{\text{element } 2}$

$\to \binom{3}{2} = 3$



b)

3.3

a) • It worns by concatinating conditional statements of the form "if this is true, then this also is true"

• It is well suited for Recurrences, for which we know that a certain property holds fore some starting numbers - so we wat to show throw induction, that that property also holds for all other numbers in the sequence.

b) First yom mane a HYPOTHESIS.
Then you perfom the BASIS STEP
followed by the INDUCTIVE STEP.
Then you mane a CONCLUSION to see weather or not the propery holds.

c)
i.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \leftarrow \text{true for } n=1\,?$$

$$\sum_{i=1}^{1} i \stackrel{?}{=} \frac{1\cdot(1+1)}{2}$$

$$= 1 \stackrel{?}{=} \frac{2}{2} = 1, \quad P(1) \text{ holds} \checkmark$$

→ write as $P(k)$

if $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$ holds, then $P(k+1)$ is also true

$$\sum_{i=1}^{k+1} i \stackrel{?}{=} \frac{(k+1)((k+1)+1)}{2}$$

$$\underbrace{\underbrace{1,2,\ldots k}_{\text{we know from before!} = \frac{k(k+1)}{2}}, (k+1)}_{\text{through algebraic transformations}} \stackrel{?}{=} \frac{(k+1)(k+2)}{2}$$

distributive law

$$= \frac{k^2+3k+2}{2} \stackrel{?}{=} \frac{k^2+3k+2}{2}$$

→ both sides are equal, so the condition holds ✓

ii. $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

- show P(n) for n=1

$$\underbrace{\sum_{i=1}^{1} i^2}_{=1} = \frac{1\cdot(1+1)(2\cdot 1+1)}{6} = \frac{2\cdot 3}{6} = \underline{\frac{6}{6} = 1} \checkmark$$

- hypothesis: property is true for k, now show it's true for k+1 (inductive step):

$$\underbrace{\sum_{i=1}^{n+1} i^2}_{} \overset{?}{=} \frac{(n+1)\left((n+1)+1\right)\left(2(n+1)+1\right)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

$$\underbrace{\frac{n(n+1)(2n+1)}{6} + \underbrace{(n+1)^2}_{=n^2+2+2n}}_{\text{Algebra}} \overset{?}{=}$$

$$\frac{(n+1)(n+2)(2n+3)}{6}$$

✓ property holds in inductive step.

Conclusion: Statement (formula) is true.

d) $$\sum_{i=1}^{n} i = \text{sum of all numbers from } 1 \text{ to } n.$$

$$= 1+2+\ldots n$$

★ equals nr. of pairs!

<u>example:</u>

★ → sum of two pairs is contant!

$$\underbrace{1+\underbrace{2+3}_{=5}+4}_{=5} \rightarrow \frac{4}{2}\cdot(\text{sum of pairs}) = 2\cdot\underbrace{5}_{}$$

"pairs"

★ sum of 2 elements (pair) is equal to number of elemets(n) +1

→ from all this logical steps illustrated above, we get the formula "$\frac{\text{nr. of elemets} \cdot (\text{Sum of each pair})}{2 \text{ (so we don't count pairs twice)}}$"

e)

i. $P(2) \Rightarrow 2^2 < (2+1)!$

$$4 < (3)!$$
$$4 < (3 \cdot 2 \cdot 1)$$
$$4 < 6 \leftarrow \underline{\underline{P(2) \text{ is true}}}$$

ii. $P(n) \Rightarrow 2^k < (k+1)!$

iii. $P(k+1) \Rightarrow 2^{k+1} < ((k+1)+1)!$

$$2^k \cdot 2^1 < (k+2)!$$

iv. It must be shown, that the statement of (iii) "$2^k \cdot 2 < (k+2)!$" holds. (So show for $k+1$)