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Formale Grundlagen der Informatik I - Assignment 3

Hand out: 28.03.2019 - Due to: 18.04.2019

Please upload your solutions to the Olat system.

3.1 Sequences and Sums

a) Do the following sequences converge or diverge for $\lim\limits_{i \to \infty}$? If they converge, please give the converging value. In any case, give a short reasoning (instead of a full proof).

   i. $a_i = \frac{5-i}{5+i}$, $i \in \mathbb{N}$.

   ii. $b_i = \frac{i^3+5}{4-i^2}$, $i \in \mathbb{N}_{\geq 3}$.

   iii. $c_i = \sin(i) + \cos(i)$, $i \in \mathbb{N}$.

3.2 Binomial Coefficients

a) Without proof: What is the relation between Pascal's triangle and the binomial coefficients?

b) With respect to the relation you found before, please give a recursive formula to calculate $\binom{n}{k}$ with $n, k \in \mathbb{N}$ that appears reasonable. Please give some reasoning.

3.3 Mathematical Induction and Proofs

a) Please describe, how induction works as a proof. For what kind of problems is it well suited and for what kind of problems is it badly applicable and why?

b) Name the four steps that you have to perform in every induction (including the ones in the exam!)

c) Prove the following statements using induction:

i. $\sum\limits_{i=1}^{n} i = \frac{n(n+1)}{2}, \forall n \in \mathbb{N}^+$.

ii. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \forall n \in \mathbb{N}^+.$

d) Now prove $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \forall n \in \mathbb{N}$ without induction.

e) Given $P(n) = (2^n < (n+1)!)\forall n \in \mathbb{N}^+.$ (P takes a positive integer and returns a boolean.)

   i. Write $P(2)$, is $P(2)$ true?

   ii. Write $P(k)$.

   iii. Write $P(k+1)$.

   iv. In a proof by mathematical induction that this inequality holds for all integers $n \geq 2$, what must be shown in the inductive step?