Question 1: (10marks)

a)

(x + y + z) ‘= x’y’z’

x	y	z	x’	y’	z’	(x+y+z)	**(x+y+z)’**	**x’y’z’**
0	0	0	1	1	1	0	**1**	**1**
0	0	1	1	1	0	1	**0**	**0**
0	1	0	1	0	1	1	**0**	**0**
0	1	1	1	0	0	1	**0**	**0**
1	0	0	0	1	1	1	**0**	**0**
1	0	1	0	1	0	1	**0**	**0**
1	1	0	0	0	1	1	**0**	**0**
1	1	1	0	0	0	1	**0**	**0**

(xyz)’ = x’ + y’ + z’

x	y	z	x’	y’	z’	xyz	**(xyz)’**	**x’ + y’ + z’**
0	0	0	1	1	1	0	**1**	**1**
0	0	1	1	1	0	0	**1**	**1**
0	1	0	1	0	1	0	**1**	**1**
0	1	1	1	0	0	0	**1**	**1**
1	0	0	0	1	1	0	**1**	**1**
1	0	1	0	1	0	0	**1**	**1**
1	1	0	0	0	1	0	**1**	**1**
1	1	1	0	0	0	1	**0**	**0**

b) The distributive law: x + yz = (x + y)(x + z)

x	y	z	yz	**x + yz**	(x+y)	(x+z)	**(x + y)(x + z)**
0	0	0	0	**0**	0	0	**0**
0	0	1	0	**0**	0	1	**0**
0	1	0	0	**0**	1	0	**0**
0	1	1	1	**1**	1	1	**1**
1	0	0	0	**1**	1	1	**1**
1	0	1	0	**1**	1	1	**1**
1	1	0	0	**1**	1	1	**1**
1	1	1	1	**1**	1	1	**1**

(c) The distributive law: x(y + z) = xy + xz

x	y	z	(y + z)	**x(y + z)**	xy	xz	**xy + xz**
0	0	0	0	**0**	0	0	**0**
0	0	1	1	**0**	0	0	**0**
0	1	0	1	**0**	0	0	**0**
0	1	1	1	**0**	0	0	**0**
1	0	0	0	**0**	0	0	**0**
1	0	1	1	**1**	0	1	**1**
1	1	0	1	**1**	1	0	**1**
1	1	1	1	**1**	1	1	**1**

(d) The associative law: x + (y + z) = (x + y) + z

x	y	z	(y + z)	**x + (y + z)**	(x + y)	**(x + y) + z**
0	0	0	0	**0**	0	**0**
0	0	1	1	**1**	0	**1**
0	1	0	1	**1**	1	**1**
0	1	1	1	**1**	1	**1**
1	0	0	0	**1**	1	**1**
1	0	1	1	**1**	1	**1**
1	1	0	1	**1**	1	**1**
1	1	1	1	**1**	1	**1**

(e) The associative law and x(yz) = (xy)z

x	y	z	yz	**x(yz)**	xy	**(xy)z**
0	0	0	0	**0**	0	**0**
0	0	1	0	**0**	0	**0**
0	1	0	0	**0**	0	**0**
0	1	1	1	**0**	0	**0**
1	0	0	0	**0**	0	**0**
1	0	1	0	**0**	0	**0**
1	1	0	0	**0**	1	**0**
1	1	1	1	**1**	1	**1**

Question 2: (10marks)

Simplify the following Boolean expressions to a minimum number of literals:

(a) ABC + A'B + ABC'

$$ABC + A'B + ABC \overset{Distributive\ law}{=} B(AC + A') + ABC' \overset{Absorption\ law}{=}$$

$$B(C + A') + ABC' \overset{Distributive\ law}{=} BC + BA' + ABC' \overset{Distributive\ law}{=}$$

$$BA' + B(C + AC') \overset{Absorption\ law}{=} BA' + B(A + C) \overset{Distributive\ law}{=}$$

$$BA' + BA + BC \overset{Distributive\ law}{=} B(A + A') + BC \overset{\text{Complement}\ law}{=} B1 + BC \overset{Identity\ Law}{=} B + BC$$

$$\overset{Absorption\ law}{=} B$$

(b) x'yz + xz

$$\boldsymbol{x'yz + xz} \overset{Distributive\ law}{=} z(x'y + x) \overset{Absorption\ law}{=} z(x + y) \overset{Distributive\ law}{=} zx + zy$$

(c) (x + y)'(x' + y')

$$(x + y)'(x' + y') \overset{\text{DeMorgan's theorem}}{=} x'y'(x' + y') = x'y'x' + y'x'y' \overset{Idempotent\ Law}{=}$$

$$x'y' + y'x'y' \overset{Idempotent\ Law}{=} x'y' + x'y' \overset{Idempotent\ Law}{=} x'y'$$

(d) xy + x(wz + wz')

$$xy + x(wz + wz') \overset{Distributive\ law}{=} xy + xw(z + z') \overset{\text{Complement}\ law}{=} xy + xw1 \overset{Identity\ Law}{=} xy + xw$$

(e) (BC'+ A'D) (AB'+ CD')

$$(BC' + A'D)(AB' + CD') = BC'(AB' + CD') + A'D(AB' + CD')$$

$$= BC'AB' + BC'CD' + A'D(AB' + CD') \overset{\text{Complement}\ law}{=} 0 + BC'CD' + A'D(AB' + CD') \overset{Identity\ Law}{=}$$

$$BC'CD' + A'D(AB' + CD') \overset{\text{Complement}\ law}{=} 0 + A'D(AB' + CD') \overset{Identity\ Law}{=} A'D(AB' + CD')$$

$$= A'DAB' + A'DCD' \overset{\text{Complement}\ law}{=} 0 + A'DCD' \overset{Identity\ Law}{=} A'DCD' \overset{\text{Complement}\ law}{=} 0$$

(f) (a'+ c') (a + b'+ c')

$$(a' + c')(a + b' + c') = (a + b' + c')a' + (a + b' + c')c'$$

$$= aa' + b'a' + c'a' + (a + b' + c')c' \overset{\text{Complement } law}{=} 0 + b'a' + c'a' + (a + b' + c')c'$$

$$\overset{Identity\ Law}{=} b'a' + c'a' + (a + b' + c')c' = b'a' + c'a' + ac' + b'c' + c'c' \overset{Idempotent\ Law}{=}$$

$$a'b' + a'c' + ac' + b'c' + c' \overset{Absorption\ law}{=} a'b' + a'c' + c' \overset{Absorption\ law}{=} a'b' + c'$$

Question 3: (10 marks) List the truth table of the function: (

(a) F = xy + xy' + y'z

x	y	z	x'	y'	z'	xy	xy'	y'z	**F**
0	0	0	1	1	1	0	0	0	**0**
0	0	1	1	1	0	0	0	1	**1**
0	1	0	1	0	1	0	0	0	**0**
0	1	1	1	0	0	0	0	0	**0**
1	0	0	0	1	1	0	1	0	**1**
1	0	1	0	1	0	0	1	1	**1**
1	1	0	0	0	1	1	0	0	**1**
1	1	1	0	0	0	1	0	0	**1**

(b) F = bc + a'c'

a	b	c	a'	b'	c'	bc	a'c'	**F**
0	0	0	1	1	1	0	1	**1**
0	0	1	1	1	0	0	0	**0**
0	1	0	1	0	1	0	1	**1**
0	1	1	1	0	0	1	0	**1**
1	0	0	0	1	1	0	0	**0**
1	0	1	0	1	0	0	0	**0**
1	1	0	0	0	1	0	0	**0**
1	1	1	0	0	0	1	0	**1**

Question 4: (10 marks) Reduce the following Boolean expressions to the indicated number of literals:

a) A'C' + ABC + AC' to three literals

$$A'C' + ABC + AC' = C'(A' + A) + ABC \overset{\text{Complement } law}{=} C'1 + ABC \overset{Identity\ Law}{=}$$

$$C' + ABC \overset{Absorption\ law}{=} C' + AB$$

(b) (x'y'+ z)'+ z + xy + wz to three literals

$$(x'y' + z)' + z + xy + wz \overset{\text{DeMorgan's theorem}}{=} (x'' + y'')z' + z + xy + wz$$

$$\overset{Involution\ law}{=} (x + y'')z' + z + xy + wz \overset{Involution\ law}{=} (x + y)z' + z + xy + wz$$

$$\overset{Absorption\ law}{=} (x + y)z' + z + xy \overset{Absorption\ law}{=} x + y + z + xy \overset{Absorption\ law}{=} x + y + z$$

(c) A'B(D' + C'D) + B(A + A'CD) to one literal

$$A'B(D' + C'D) + B(A + A'CD) \overset{Absorption\ law}{=} A'B(D' + C') + B(A + A'CD)$$

$$= A'BD' + A'BC' + BA + BA'CD = B(A'D' + A) + A'BC' + BA'CD \overset{Absorption\ law}{=}$$

$$B(D' + A) + BA'(CD + C') \overset{Absorption\ law}{=} B(D' + A) + BA'(D + C')$$

$$= BD' + B(A'(D + C') + A) \overset{Absorption\ law}{=}$$

$$BD' + B(D + C' + A) = BD' + BD + BC' + BA = B(D' + D) + BC' + BA' \overset{\text{Complement } law}{=}$$

$$B1 + BC' + BA \overset{Identity\ Law}{=} B + BC + BA \overset{Absorption\ law}{=} B + BA \overset{Absorption\ law}{=} B$$

(d) (A'+ C) (A'+ C') (A + B + C'D) to four literals

$$(A' + C)(A' + C')(A + B + C'D) = (A' + C')(A + B + C'D)A' + (A' + C')(A + B + C'D)C$$

$$(A + B + C'D)A'A' + (A + B + C'D)A'C' +)A' + C')(A + B + C'D)C \overset{Idempotent\ Law}{=}$$

$$(A + B + C'D)A' + (A + B + C'D)A'C' + (A' + C')(A + B + C'D)C \overset{Absorption\ law}{=}$$

$$(A + B + C'D)A' + (A' + C')(A + B + C'D)C = A'A + A'B + A'C'D + (A' + C')(A + B + C'D)C$$

$$\overset{\text{Complement } law}{=} 0 + A'B + A'C'D + (A' + C')(A + B + C'D)C \overset{Identity\ Law}{=}$$

$$A'B + A'C'D + (A' + C')(A + B + C'D)C = A'B + A'C'D + (A + B + C'D)CA' + (A + B + C'D)C'C$$

$$\overset{\text{Complement } law}{=} A'B + A'C'D + (A + B + C'D)CA' + 0 \overset{Identity\ Law}{=} A'B + A'C'D + (A + B + C'D)CA'$$

$$= A'B + A'C'D + CA'A + +CA'B + CA'C'D \overset{\text{Complement } law}{=} A'B + A'C'D + 0 + CA'B + CA'C'D$$

$$\overset{Identity\ Law}{=} A'B + A'C'D + CA'B + CA'C'D \overset{\text{Complement } law}{=} A'B + A'C'D + CA'B + 0 \overset{Identity\ Law}{=}$$

$$A'B + A'C'D + CA'B \overset{Absorption\ law}{=} A'B + A'C'D$$

(e) ABC'D + A'BD + ABCD to two literals

$$ABC'D + A'BD + ABCD = BD(AC' + A') + ABCD \overset{Absorption\ law}{=} BD(C' + A') + ABCD$$

$$= BDC' + BDA' + ABCD = BDA' + BD(AC + C') \overset{Absorption\ law}{=} BDA' + BD(A + C')$$

$$= BDA' + BDA + BDC' = BD(A + A') + BDC' \overset{\text{Complement law}}{=} BD1 + BDC' \overset{Identity\ Law}{=} BD + BDC'$$

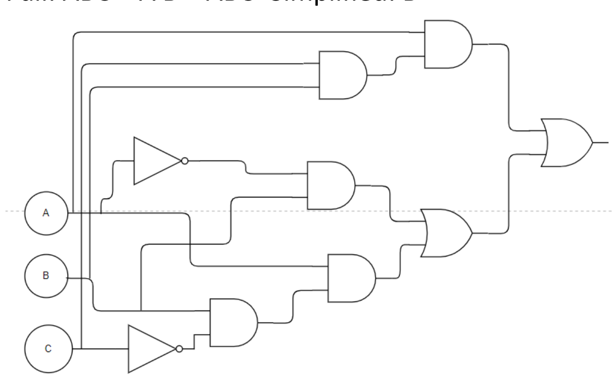
$$\overset{Absorption\ law}{=} BD$$

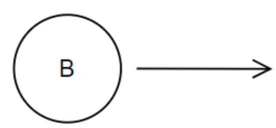
Logic diagrams are drawn using [Visual Paradigm Online]

Image on left is full and on the right simplified version

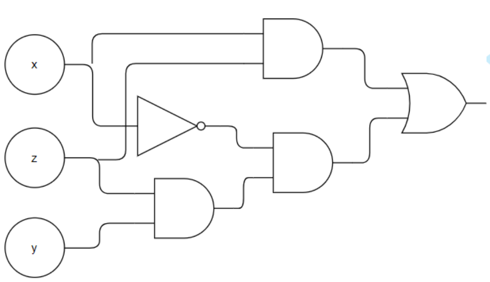
Question 5: (10 marks) Draw logic diagrams of the circuits that implement the original and simplified expressions in Question 2

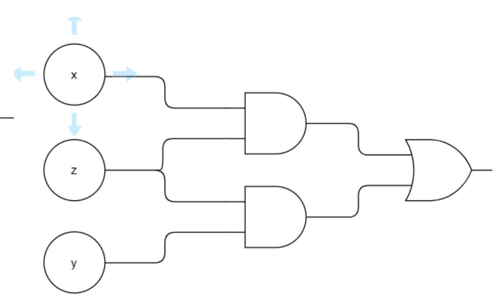
a) Full: ABC + A'B + ABC' Simplified: B



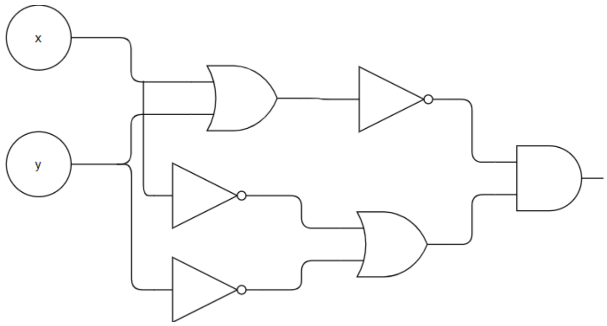


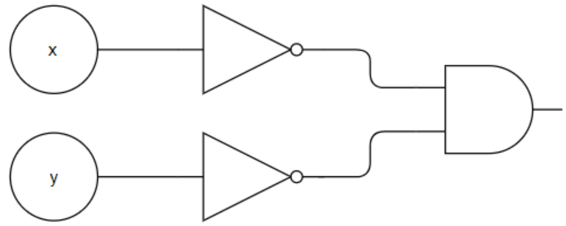
b) Full: x'yz + xz Simplified: zx +zy



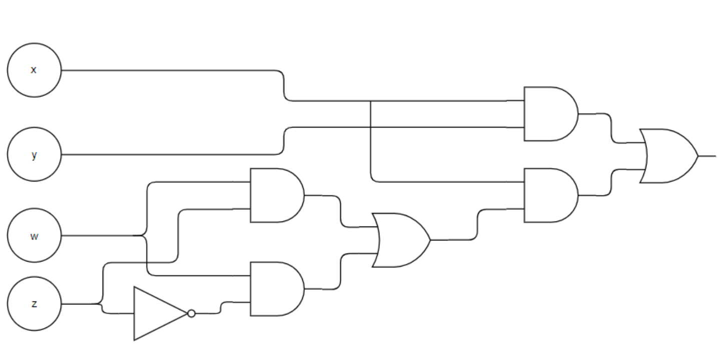


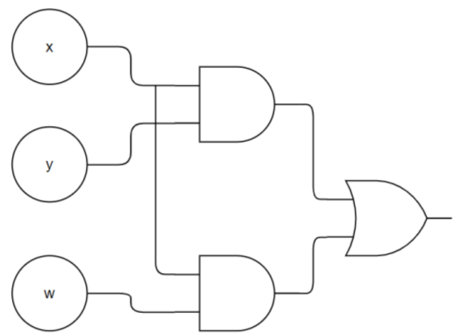
c) Full: (x + y)'(x' + y') Simplified: x'y'



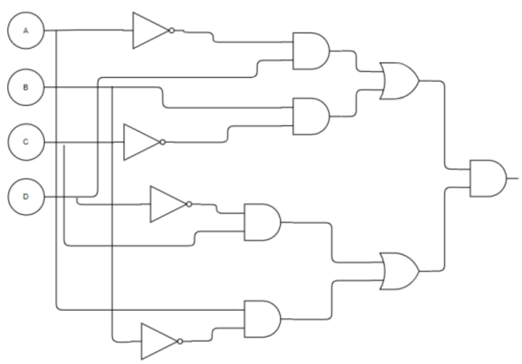


d) Full: xy + x(wz + wz’) Simplified: xy+xw

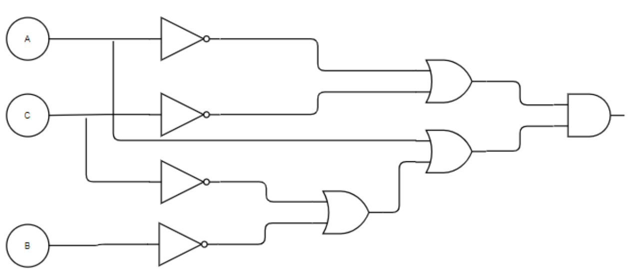


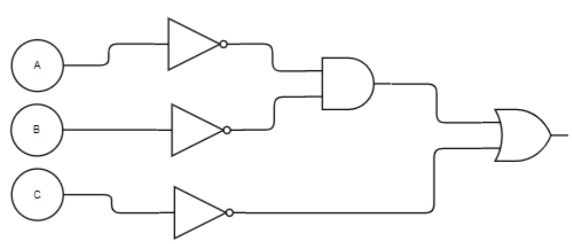


e) Full: (BC’+ A’D) (AB’+ CD’) Simplified: 0



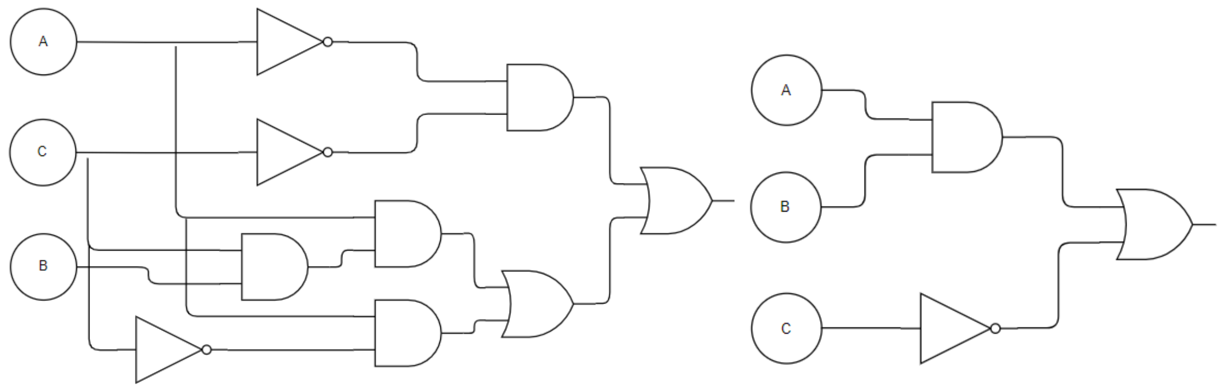
f) Full: (a’+ c’) (a + b’+ c’) Simplified: a' b'+c'



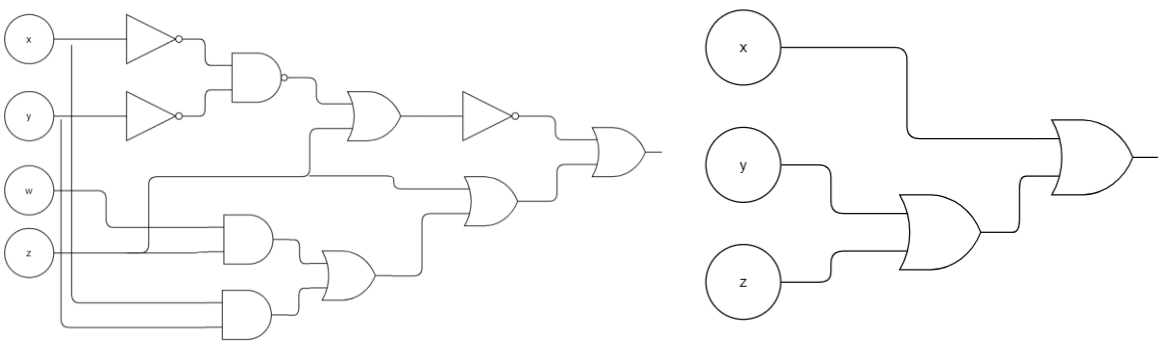


Question 6: (10 marks)Draw logic diagrams of the circuits that implement the original and simplified expressions in Question 4

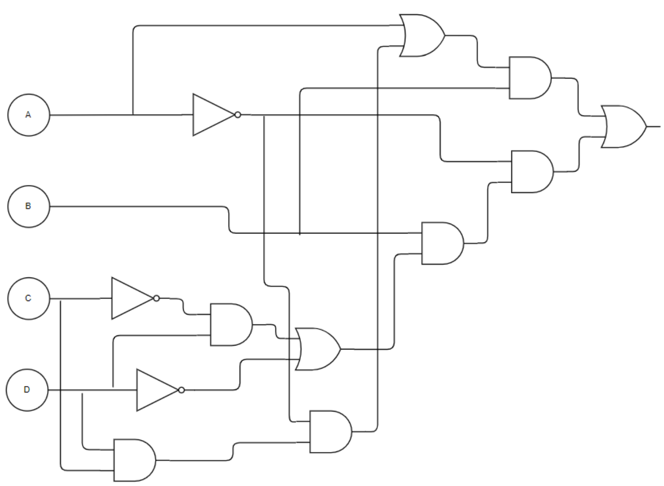
(a) Full A'C' + ABC + AC' Simplified C'+AB



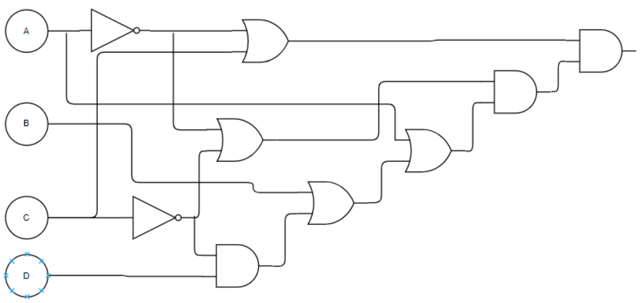
(b) Full (x'y'+ z)'+ z + xy + wzto Simplified x + y +z

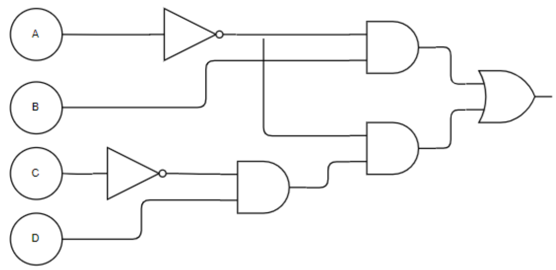


(c) Full A'B(D'+ C'D) + B(A + A'CD) Simplified B

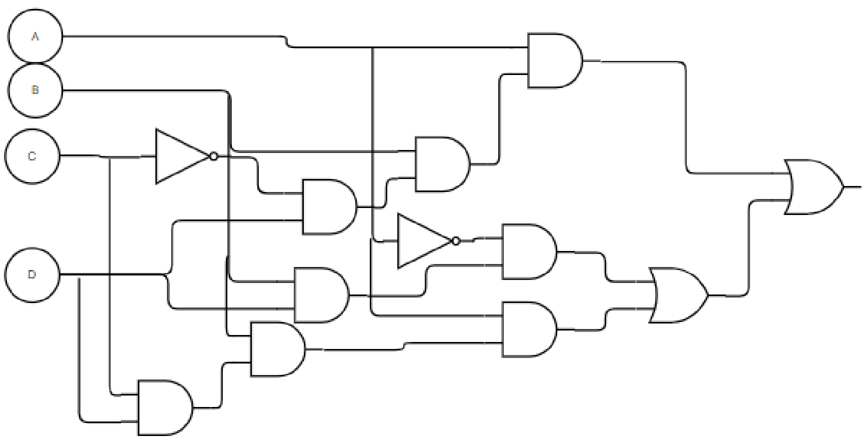


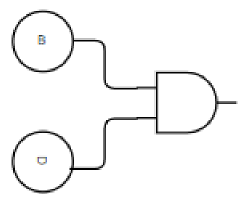
(d) Full (A'+ C) (A'+ C') (A + B + C'D) Simplified A' B+A'C'D





(e) Full ABC'D + A'BD + ABCD Simplified BD





Question 7: (10 marks) Simplify the following Boolean functions, using three-variable maps:

(a) F (x, y, z) = Σ (0, 1, 5, 7)

yz / x	00	01	11	10
0	1	1	0	0
1	0	1	1	0

$F(x, y, z) = x'y' + xz$

(b) F (x, y, z) = Σ (1, 2, 3, 6, 7)

yz / x	00	01	11	10
0	0	1	1	1
1	0	0	1	1

$F(x, y, z) = x'z + y$

(c) F(x, y, z)= Σ (2, 3, 4, 5)

yz / x	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$$F(x, y, z) = x'y + xy'$$

(d) F(x, y, z)= Σ (1, 2, 3, 5, 6, 7)

yz / x	00	01	11	10
0	0	1	1	1
1	0	1	1	1

$$F(x, y, z) = y + z$$

(e) F(x, y, z)= Σ (0, 2, 4, 6)

yz / x	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$$F(x, y, z) = z'$$

(f) F(x, y, z)= Σ (3, 4, 5, 6, 7)

yz / x	00	01	11	10
0	0	0	1	0
1	1	1	1	1

$$F(x, y, z) = x + yz$$

Question 8: (10 marks) Simplify the following Boolean expressions, using four-variable maps:

m		A	B	C	D	
0	0000	A’	B’	C’	D’	
1	0001	A’	B’	C’	D	
2	0010	A’	B’	C	D’	
3	0011	A’	B’	C	D	
4	0100	A’	B	C’	D’	
5	0101	A’	B	C’	D	
6	0110	A’	B	C	D’	
7	0111	A’	B	C	D	
8	1000	A	B’	C’	D’	
9	1001	A	B’	C’	D	
10	1010	A	B’	C	D’	
11	1011	A	B’	C	D	
12	1100	A	B	C’	D’	
13	1101	A	B	C’	D	
14	1110	A	B	C	D’	
15	1111	A	B	C	D	

CD / AB	00	01	11	10
00	M0	M1	M3	M2
01	M4	M5	M7	M6
11	M8	M9	M11	M10
10	M12	M13	M15	M14

(a) A'B'C'D'+ AC'D'+ B'CD'+ A'BCD + BC'D

CD \ AB	00	01	11	10
00	1			1
01		1	1	
11	1	1		
10	1			1

$$B'D' + A'BD + ABC'$$

(b) xz + w xy + w(xy + xy) = xz +wxy

wz \ xy	00	01	11	10
00				
01		1	1	
11		1	1	1
10				

(c) ABCD + ABD + ABC + ABCD + ABC

CD \ AB	00	01	11	10
00				
01				
11		1	1	1
10				

$$ABD + ABC$$

(d) ABCD + BCD + ACD + ABCD + ACD=BCD+ACD

CD \ AB	00	01	11	10
00				
01			1	
11			1	
10			1	

Question 9: (10 marks) Find the minterms of the following Boolean expressions by first plotting each function in a map:

(a) xy + yz + xy'z

x \ yz	00	01	11	10
0			1	
1		1	1	1

$$F(x, y, z) = \sum(3,5,6,7)$$

(b) C'D + ABC'+ ABD'+ A'B'D

AB \ CD	00	01	11	10
00		1	1	
01		1		
11	1	1		1
10		1		

$$F(A, B, C, D) = \sum(1, 3, 5, 9, 12, 13, 14)$$

(c) wyz+ w'x'+ wxz'

wx \ yz	00	01	11	10
00	1	1	1	1
01				
11	1		1	1
10			1	

$$F(w, x, y, z) = \sum(0, 1, 2, 3, 11, 12, 14, 15)$$

(c) A'B + A'CD + B'CD + BC'D'

AB \ CD	00	01	11	10
00			1	
01	1	1	1	1
11	1			
10			1	

$$F(A, B, C, D) = \sum(3, 4, 5, 6, 7, 11, 12)$$

Question 10

Convert the following Boolean function from a sum-of-products form to a simplified product-of-sums form. F(w, x, y, z) = Σ (0, 1, 2, 5, 8, 10, 13)

$$F(w,x,y,z) = \sum(0,1,2,5,8,10,13) = \prod(3,4,6,7,9,11,12,14,15)$$

yz / wx	00	01	11	10
00			0	
01	0		0	0
11	0		0	0
10		0	0	

$$F' = yz + wx'z + xz'$$

$$F = (yz + wx'z + xz')' = (yz)' * (wx'z)' * (xz')' =$$

$$= (y' + z')(w' + x + z')(x'z)$$