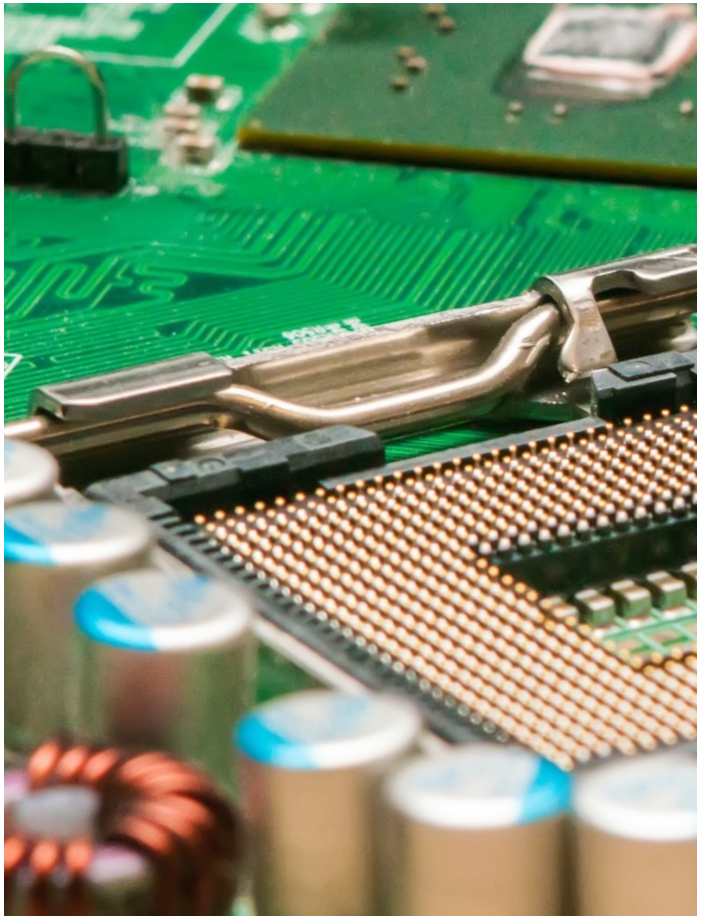
Fundamentals of Electrical & Electronic Engineering

Prof. John G. Breslin, Electrical & Electronic Engineering



Lecture 6

Series-Parallel Circuits

Ollscoil na Gaillimhe
University of Galway

Summary (1 of 40)

Combination Circuits

Most practical circuits have combinations of series and parallel components and a voltage or current source. You can frequently simplify analysis by combining series and parallel components.

Components that are in series share a common path. Components that are in parallel are connected across the same two nodes.

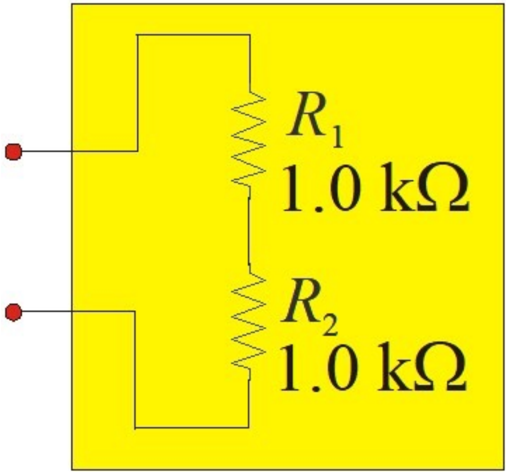
An important analysis method is to form a simpler **equivalent circuit** and use it to solve certain parameters that can be applied to the original circuit.

The next slides illustrate some equivalent resistor combinations.

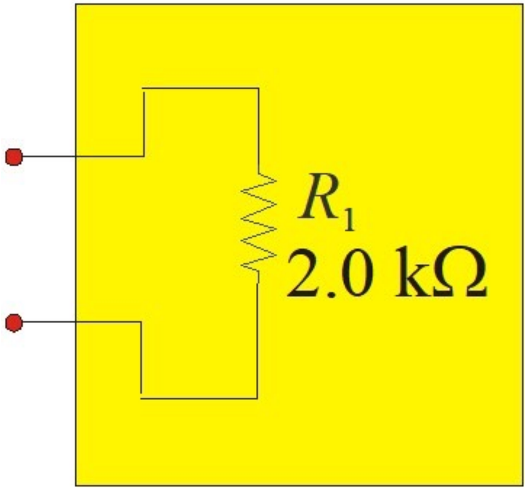
Summary (2 of 40)

Equivalent Combinations

Example:



is equivalent to

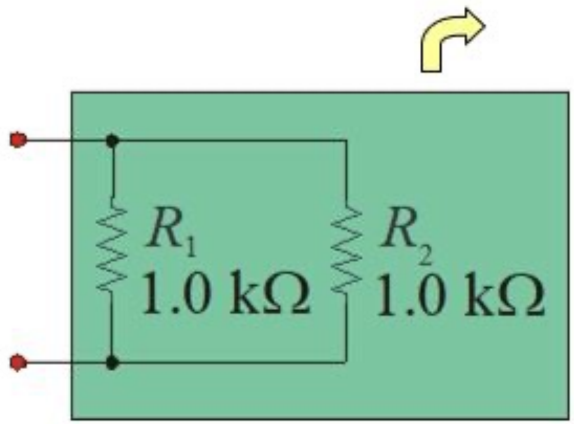


There are no electrical measurements that can distinguish the boxes.

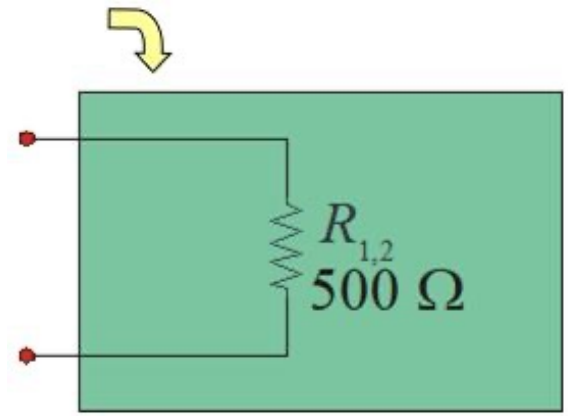
Summary (3 of 40)

Equivalent Combinations

Another example:

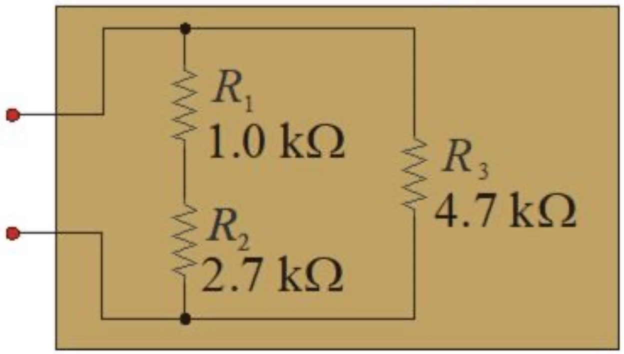


is equivalent to

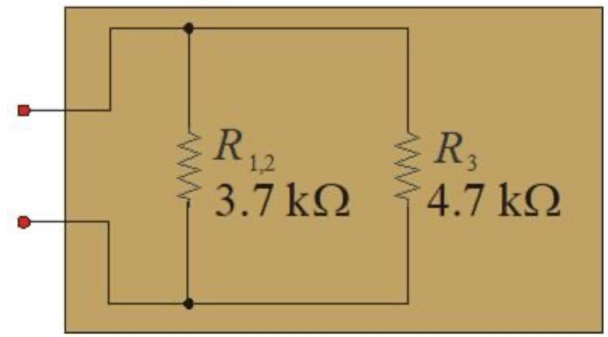


There are no electrical measurements that can distinguish the boxes.

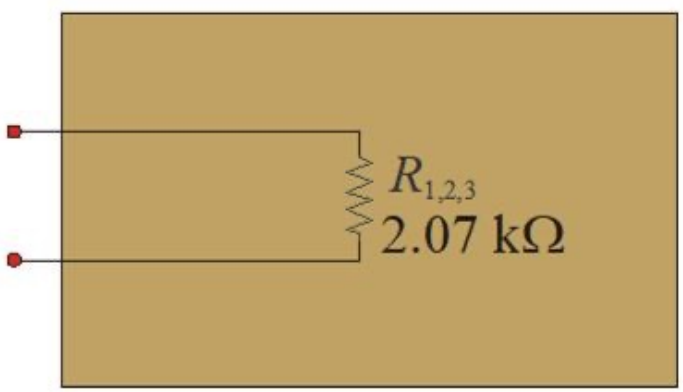
Summary (4 of 40)



is equivalent to



is equivalent to

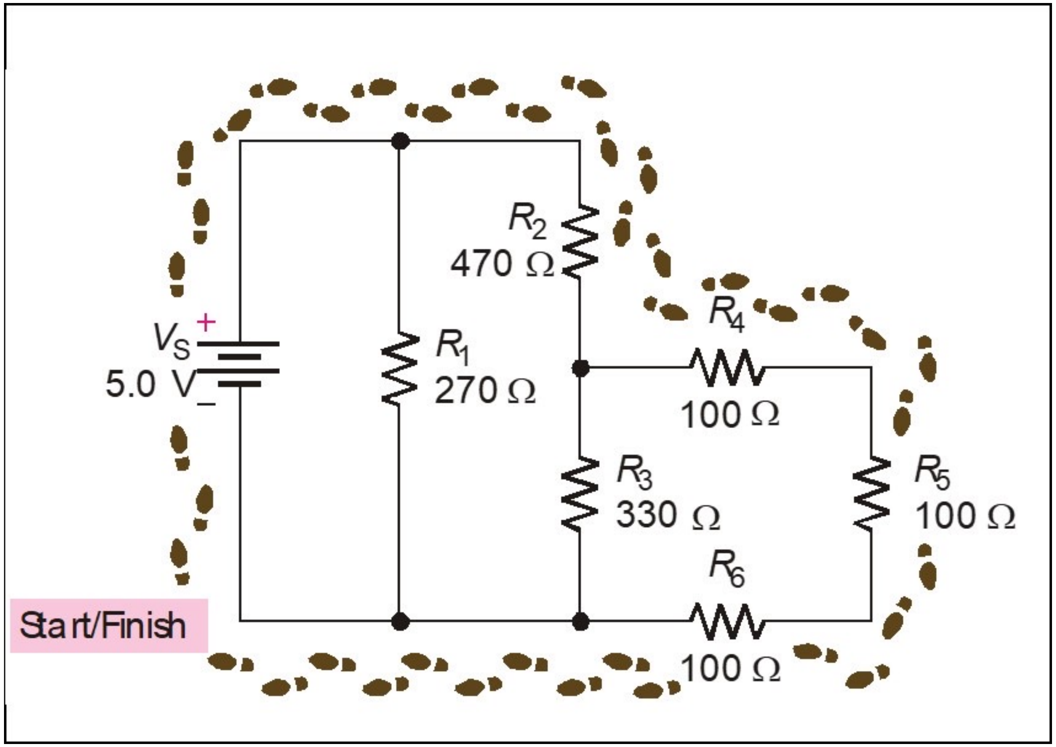


There are no electrical measurements that can distinguish between the three boxes.

Summary (5 of 40)

Kirchhoff's voltage law and **Kirchhoff's current law** can be applied to any circuit, including combination circuits.

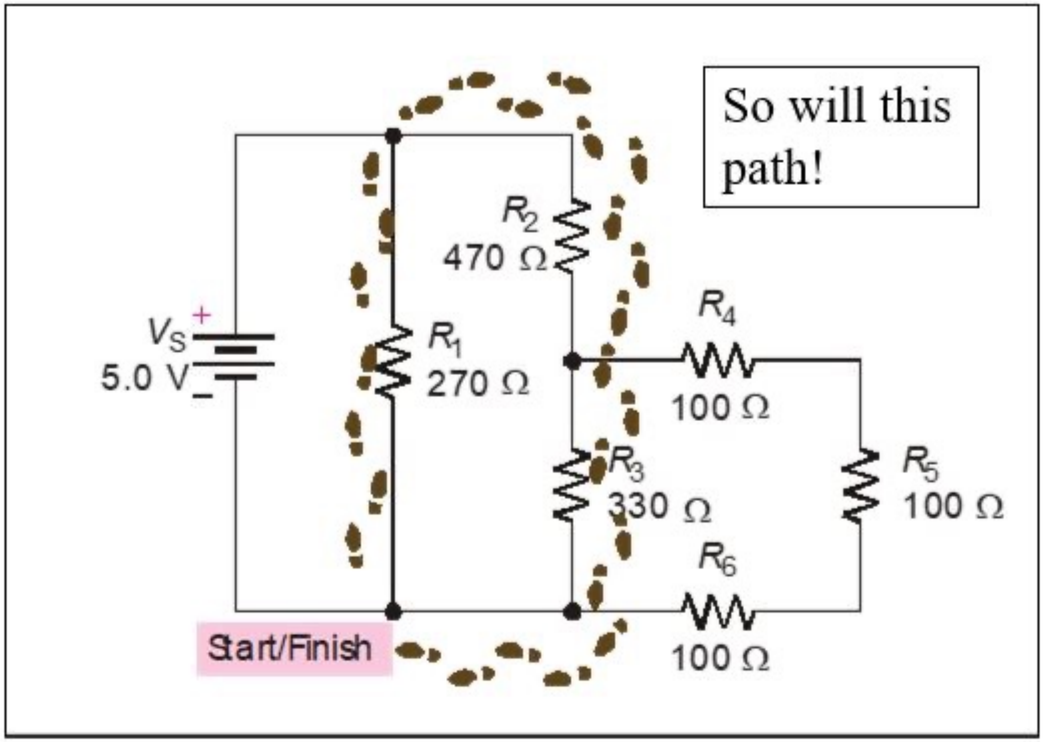
For example, applying KVL, the path shown will have a sum of 0 V.



Summary (6 of 40)

Kirchhoff's voltage law and **Kirchhoff's current law** can be applied to any circuit, including combination circuits.

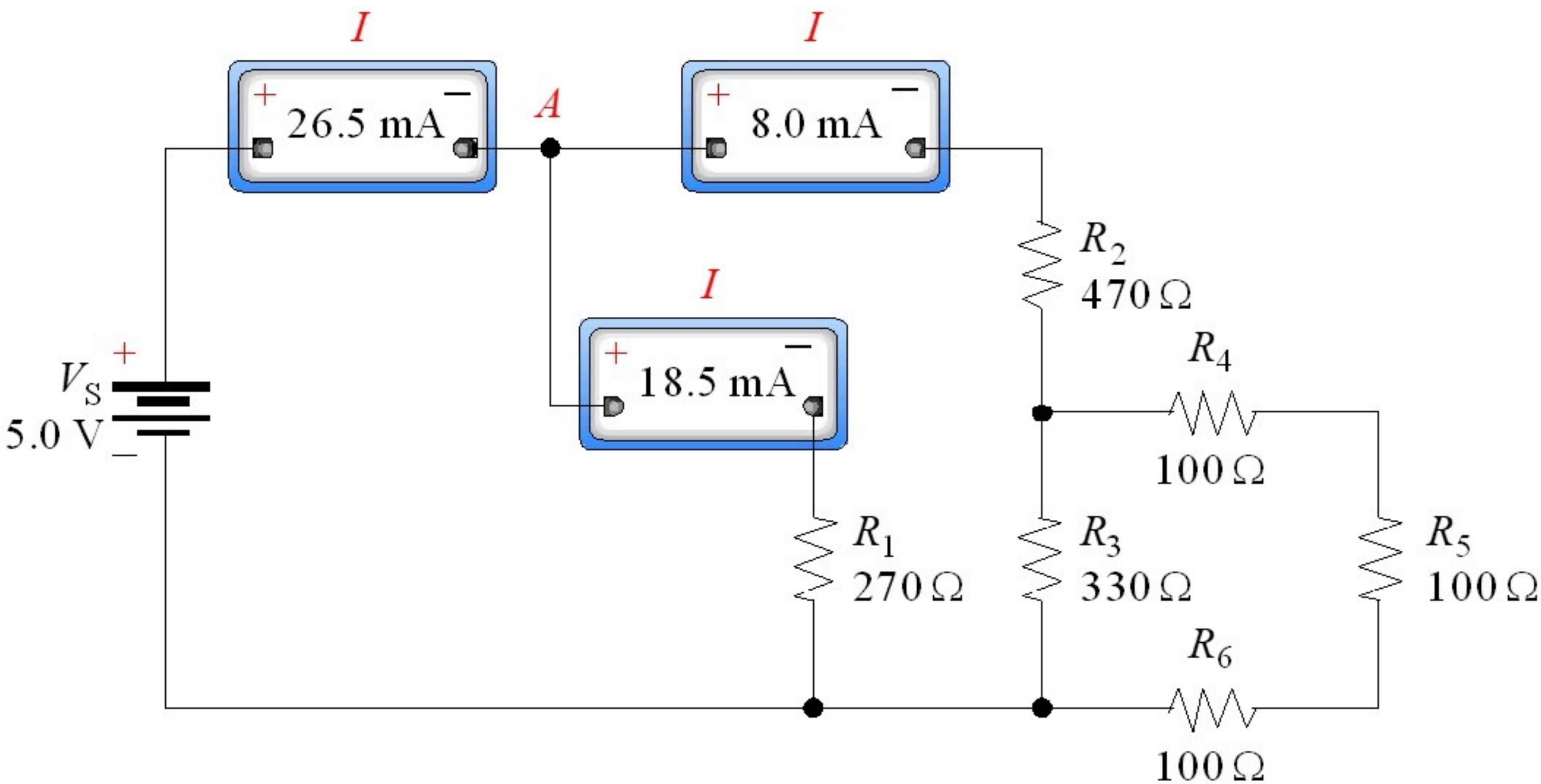
For example, applying KVL, the path shown will have a sum of 0 V.



Prof. John G. Breslin, University of Galway

Summary

Kirchhoff's current law can also be applied to the same circuit. The readings for node *A* are shown and illustrate the current law.

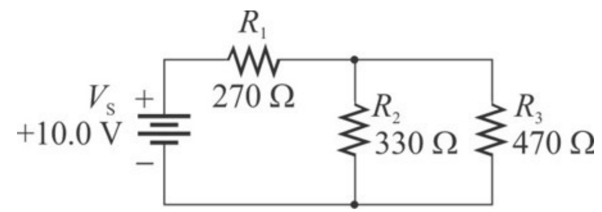


Prof. John G. Breslin, University of Galway

Summary (8 of 40)

Combination Circuits

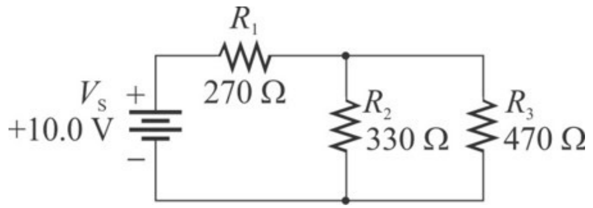
Example:



List a set of steps to find the all voltages, currents, and power in each component for the circuit.

Summary (9 of 40)

Combination Circuits



Example:

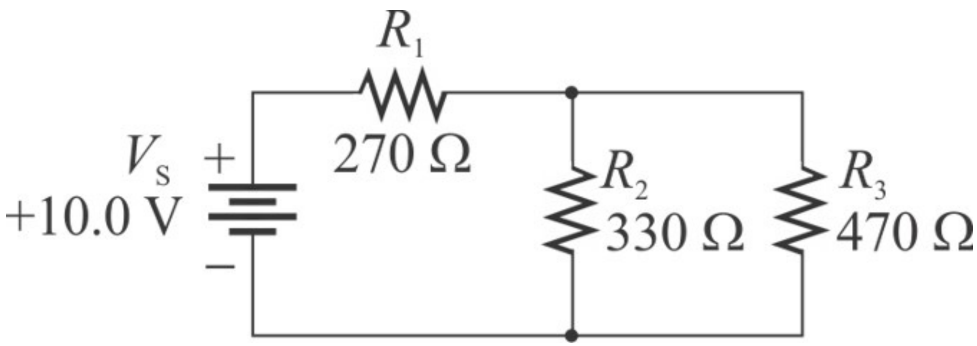
List a set of steps to find the all voltages, currents, and power in each component for the circuit.

1. Find the total resistance.
2. Use Ohm's law to find the total current. This is also I_1.
3. Use Ohm's law to calculate V_1.
4. Apply KVL to find V_2 and V_3.
5. Use Ohm's law (or the current divider rule) to calculate I_2 and I_3.
6. Apply Watt's law to each resistor and the total circuit to obtain the powers.

Ready to practice?

Summary (10 of 40)

Combination Circuits



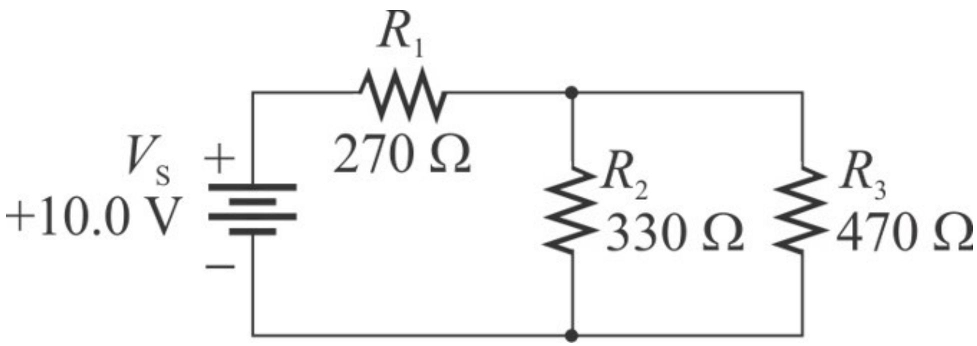
Example:

Following the steps from the last slide, complete the table.

$I_1 =$	$R_1 = 270\ \Omega$	$V_1 =$	$P_1 =$
$I_2 =$	$R_2 = 330\ \Omega$	$V_2 =$	$P_2 =$
$I_3 =$	$R_3 = 470\ \Omega$	$V_3 =$	$P_3 =$
$I_T =$	$R_T =$	$V_S = 10.0\ \text{V}$	$P_T =$

Summary (11 of 40)

Combination Circuits



Example:

Following the steps from the last slide, complete the table.

I_1 = 21.6 mA	R_1 = 270 Ω	V_1 = 5.82 V	P_1 = **126 mW**
I_2 = 12.7 mA	R_2 = 330 Ω	V_2 = 4.18 V	P_2 = **52.9 mW**
I_3 = 8.9 mA	R_3 = 470 Ω	V_3 = 4.18 V	P_3 = **37.2 mW**
I_T = 21.6 mA	R_T = 464 Ω	V_S = 10.0 V	P_T = **216 mW**

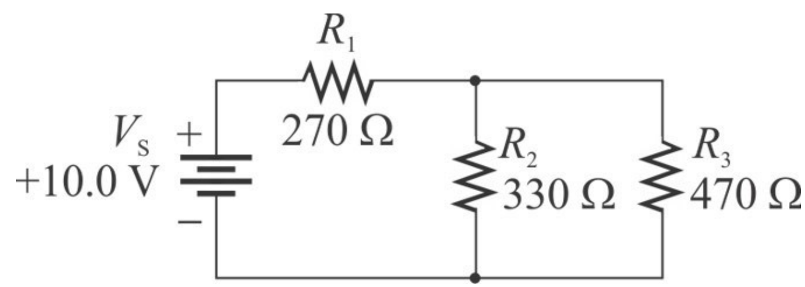
Summary (12 of 40)

Combination Circuits

Kirchhoff's laws can be applied as a check.

KVL: The sum of the voltages around the outside loop is zero.

KCL: I_1 is equal to the sum of the I_2 and I_3.



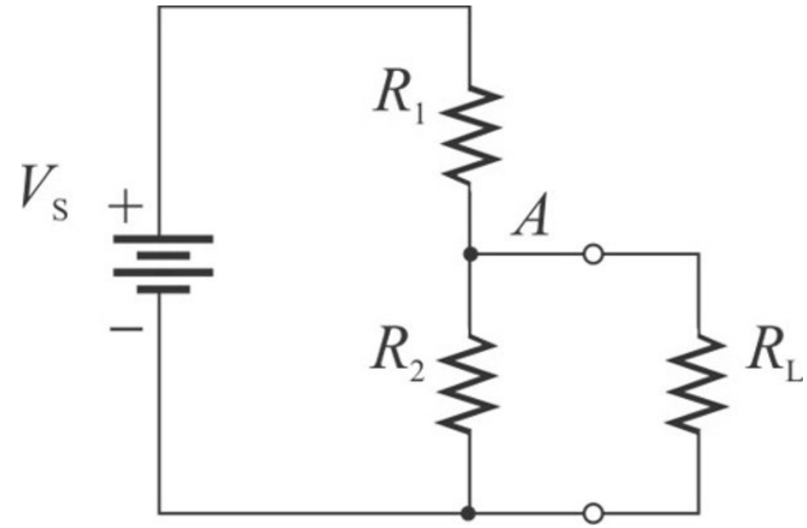
$I_1 = 21.6$ mA	$R_1 = 270\ \Omega$	$V_1 = 5.82$ V	$P_1 = 126$ mW
$I_2 = 12.7$ mA	$R_2 = 330\ \Omega$	$V_2 = 4.18$ V	$P_2 = 52.9$ mW
$I_3 = 8.9$ mA	$R_3 = 470\ \Omega$	$V_3 = 4.18$ V	$P_3 = 37.2$ mW
$I_T = 21.6$ mA	$R_T = 464\ \Omega$	$V_S = 10.0$ V	$P_T = 216$ mW

Summary (13 of 40)

Loaded Voltage Divider

The voltage-divider equation was developed for a series circuit. Recall that the output voltage is given by

$$V_2 = \left(\frac{R_2}{R_T}\right)V_S$$



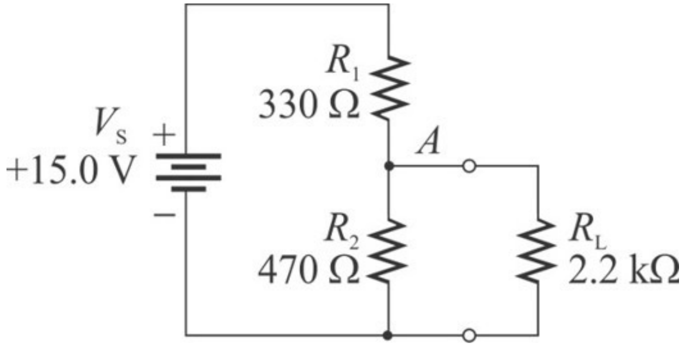
A voltage-divider with a resistive load is a combinational circuit. If R_L is removed, R_1 and R_2 form a normal unloaded voltage divider. The addition of R_L reduces the total resistance from node *A* to ground and the voltage divider is said to be loaded.

Summary (14 of 40)

Loaded Voltage Divider

Example:

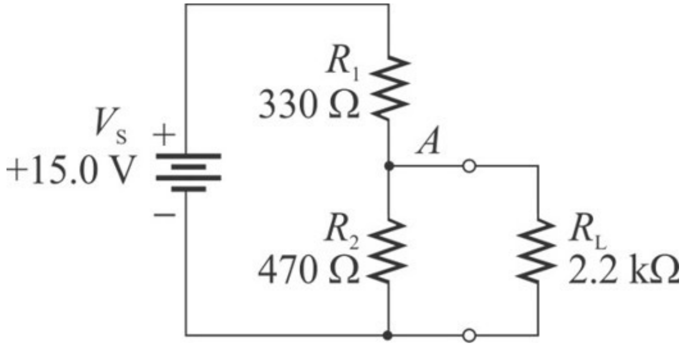
What is the voltage across R_L?



Loaded Voltage Divider

Example:

What is the voltage across R_L?



Solution:

Form an equivalent series circuit by combining R_2 and R_L; then apply the voltage-divider formula to the equivalent circuit:

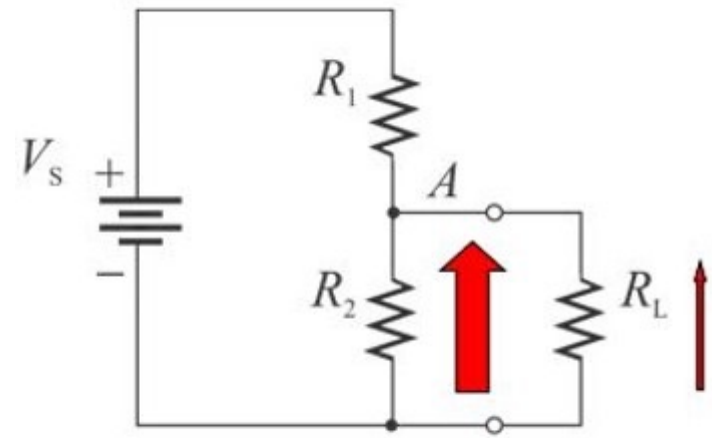
$$R_{2,L} = R_2 \parallel R_L = 470\,\Omega \parallel 2.2\,\text{k}\Omega = 387\,\Omega$$

$$V_L = V_{2,L} = \left(\frac{R_{2,L}}{R_1 + R_{2,L}}\right) V_S = \left(\frac{387\,\Omega}{330\,\Omega + 387\,\Omega}\right) 15\text{ V} = 8.10\text{ V}$$

Summary (16 of 40)

Stiff Voltage Divider

A **stiff voltage-divider** is one in which the loaded voltage is nearly the same as the no-load voltage. To accomplish this, the load current must be small compared to the current in the divider resistors (called the *bleeder* current).

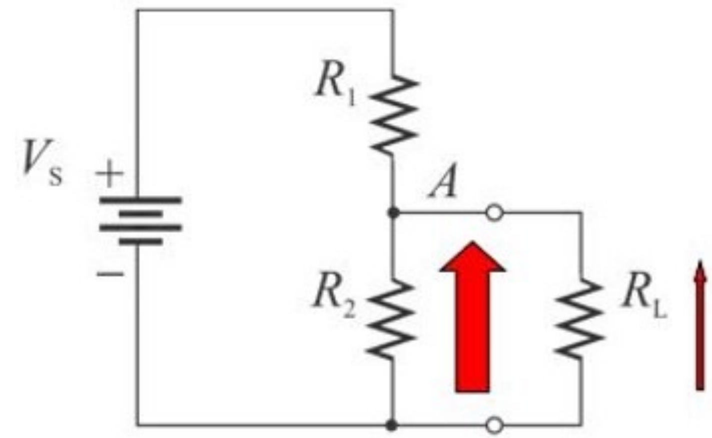


Example: If $R_1 = R_2 = 1.0\,\text{k}\Omega$, what value of R_L will make the divider a stiff voltage divider? What fraction of the unloaded voltage is the loaded voltage?

Summary (17 of 40)

Stiff Voltage Divider

A **stiff voltage-divider** is one in which the loaded voltage is nearly the same as the no-load voltage. To accomplish this, the load current must be small compared to the current in the divider resistors (called the *bleeder* current).



Example: If $R_1 = R_2 = 1.0\ \text{k}\Omega$, what value of R_L will make the divider a stiff voltage divider? What fraction of the unloaded voltage is the loaded voltage?

Solution: $R_L > 10\text{x}R_2; R_L$ should be $10\ \text{k}\Omega$ or greater. For a $10\ \text{k}\Omega$ load:

$$V_L = \left(\frac{R_2 \,||\, R_L}{R_1 + R_2 \,||\, R_L}\right)V_S = \left(\frac{0.91\,\text{k}\Omega}{1.0\ \text{k}\Omega + 0.91\,\text{k}\Omega}\right)V_S = 0.476\,V_S$$

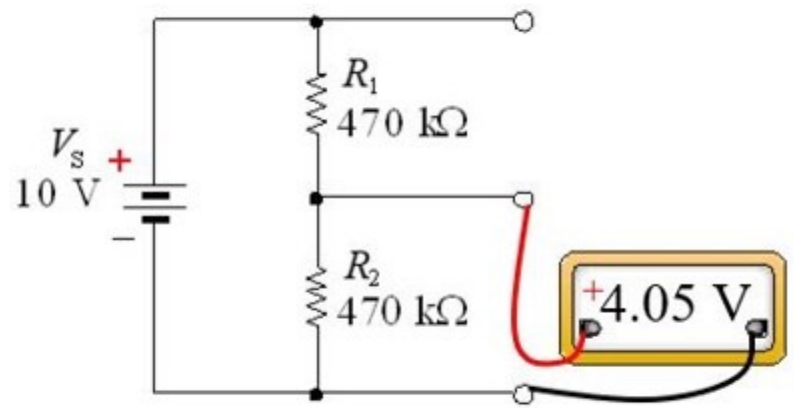
This is 95% of the unloaded voltage, which is 0.5 V_S

Summary (18 of 40)

Loading Effect of a Voltmeter

Assume $V_S = 10\text{ V}$, but the meter reads only 4.05 V when it is across either R_1 or R_2.

Can you explain what is happening?

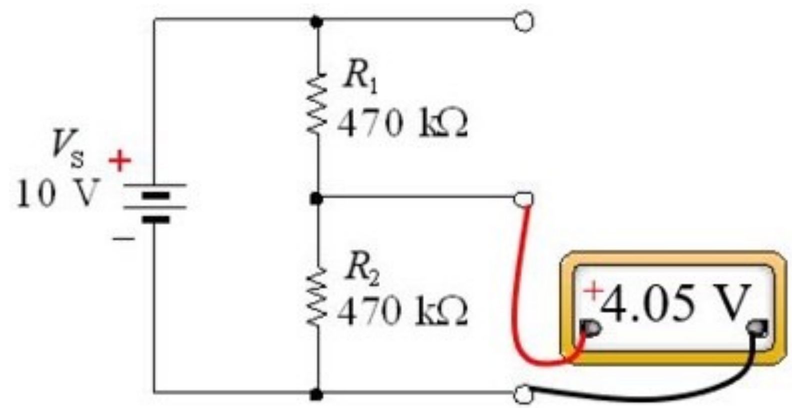


Summary (19 of 40)

Loading Effect of a Voltmeter

Assume $V_S = 10\text{ V}$, but the meter reads only 4.05 V when it is across either R_1 or R_2.

Can you explain what is happening?

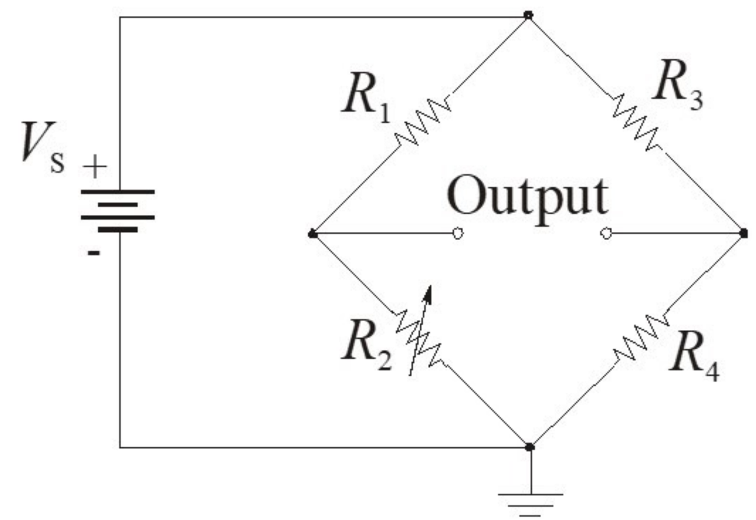


All measurements affect the quantity being measured. A voltmeter has internal resistance, which appears as a load to the circuit under test. In this case, a $1.0\text{ M}\Omega$ internal resistance of the meter accounts for the readings.

Summary (20 of 40)

Wheatstone Bridge

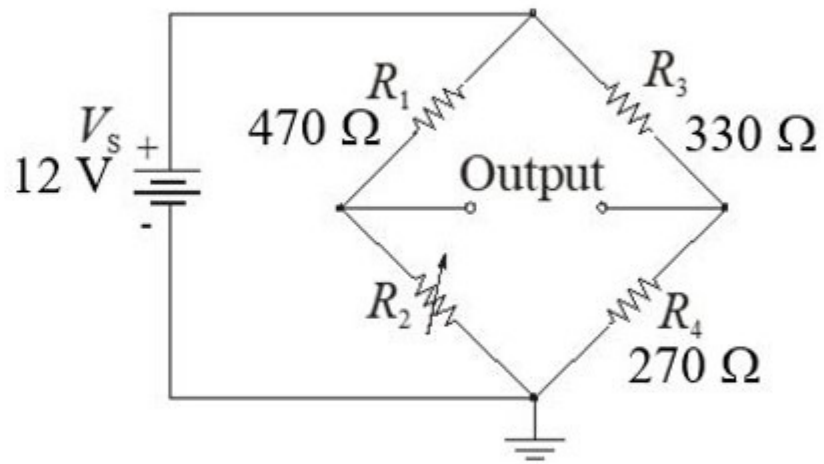
The Wheatstone bridge consists of a dc voltage source and four resistive arms forming two voltage dividers. The output is taken between the dividers. Frequently, one of the bridge resistors is adjustable.



When the bridge is balanced, the output voltage is zero, and the products of resistances in the opposite diagonal arms are equal.

Wheatstone Bridge

Example: What is the value of R_2 if the bridge is balanced?

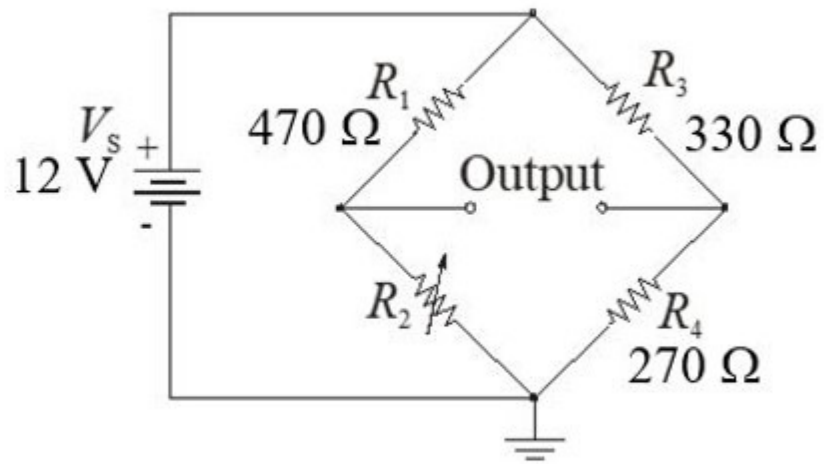


Summary (22 of 40)

Wheatstone Bridge

Example: What is the value of R_2 if the bridge is balanced?

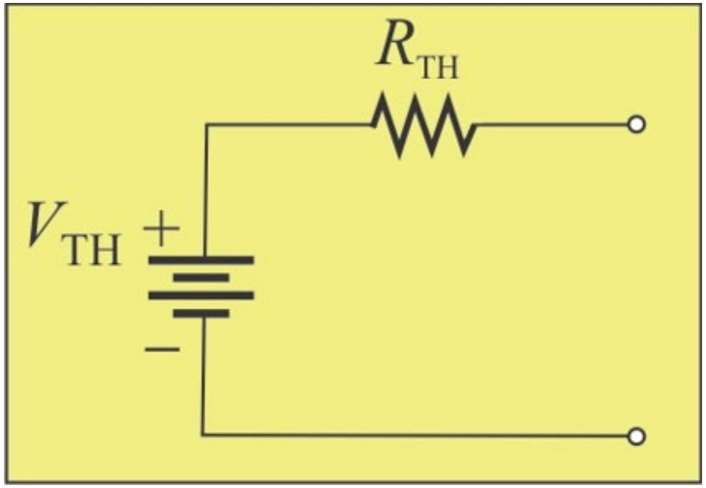
384 Ω



Summary (23 of 40)

Thevenin's Theorem

Thevenin's theorem states that any two-terminal, resistive circuit can be replaced with a simple equivalent circuit when viewed from two output terminals. The equivalent circuit is:

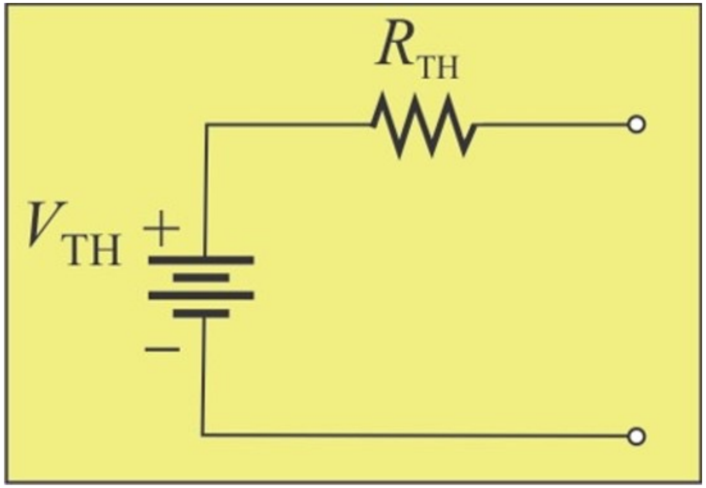


Summary (24 of 40)

Thevenin's Theorem

V_{TH} is defined as the open circuit voltage between the two output terminals of a circuit.

R_{TH} is defined as the total resistance appearing between the two output terminals when all sources have been replaced by their internal resistances.

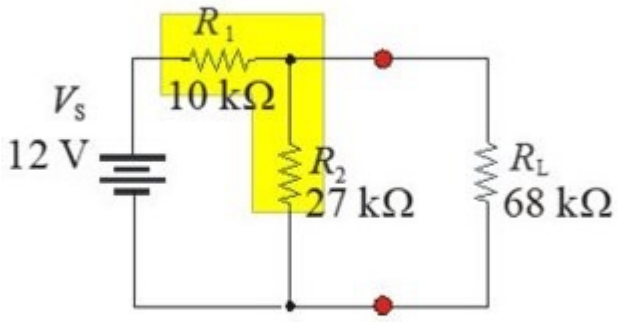


Summary (25 of 40)

Thevenin's Theorem

Example:

Find the Thevenin voltage and resistance for the circuit. To find V_{TH}, apply the voltage divider rule to R_1 and R_2.



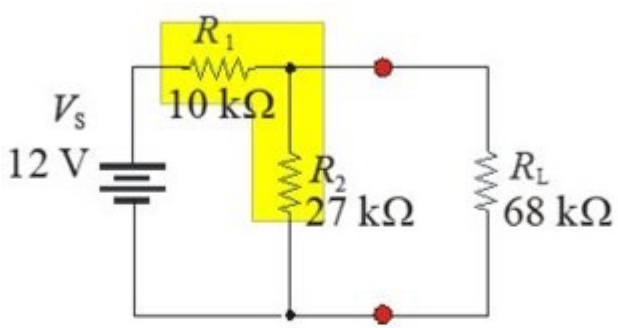
Thevenin's Theorem

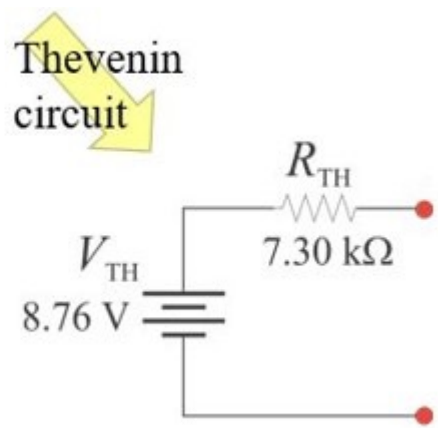
Example:

Find the Thevenin voltage and resistance for the circuit. To find V_{TH}, apply the voltage divider rule to R_1 and R_2.

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_S = \left(\frac{27\ \text{k}\Omega}{10\ \text{k}\Omega + 27\ \text{k}\Omega}\right)12\ \text{V} = 8.76\ \text{V}$$

$$R_{TH} = 10\ \text{k}\Omega \parallel 37\ \text{k}\Omega = 7.30\ \text{k}\Omega$$



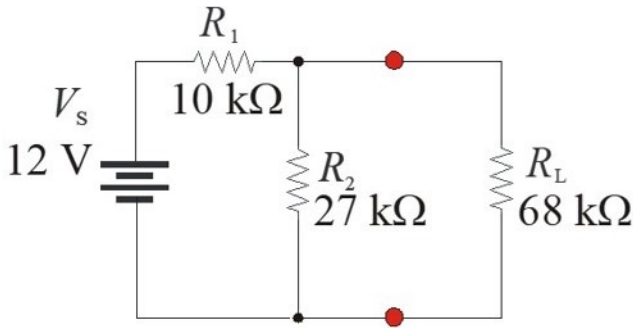


Summary (27 of 40)

Thevenin's Theorem

Follow up:

What is the voltage across R_L?

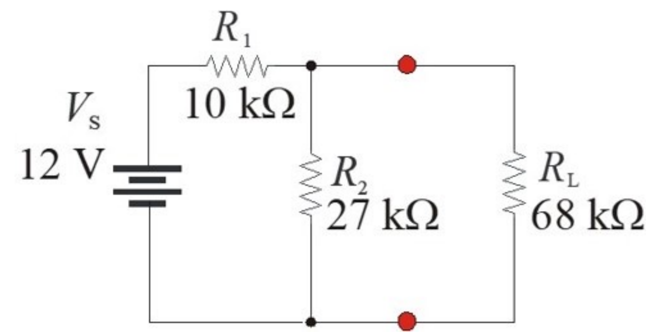


Summary (28 of 40)

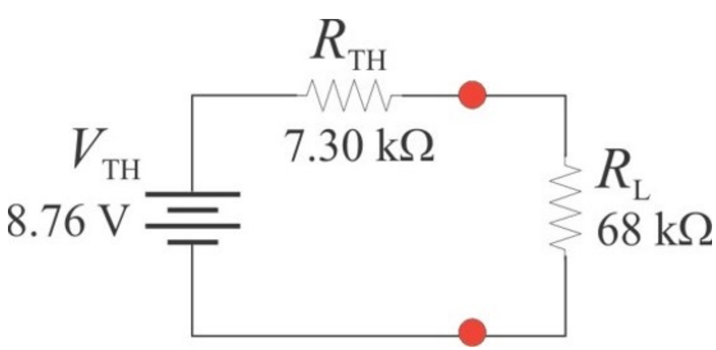
Thevenin's Theorem

Follow up:

What is the voltage across R_L?



Since we know the Thevenin circuit, the easiest way to answer the question is to use it and apply the voltage divider theorem.



$$V_L = \left(\frac{R_L}{R_{TH} + R_L}\right) V_{TH}$$

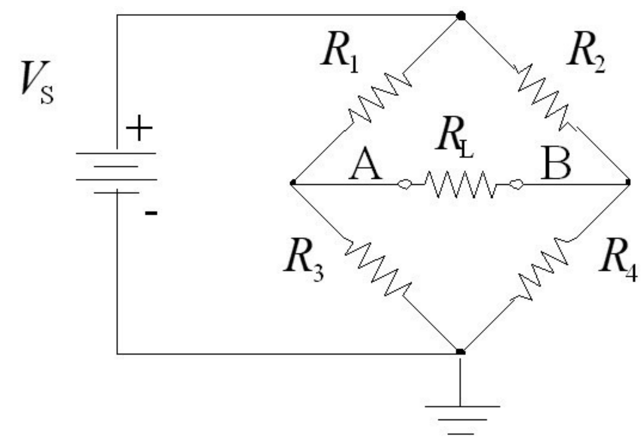
$$= \left(\frac{68\ \text{k}\Omega}{7.3\ \text{k}\Omega + 68\ \text{k}\Omega}\right) 8.76\ \text{V} = 7.91\ \text{V}$$

Summary (29 of 40)

Thevenin's Theorem

Thevenin's theorem is useful for solving the Wheatstone bridge. One way to Thevenize the bridge is to create two Thevenin circuits – from A to ground and from B to ground. This method retains the circuit ground in the Thevenin equivalent circuit.

The resistance between point A and ground is $R_1 \| R_3$ and the resistance from B to ground is $R_2 \| R_4$. The voltage on each side of the bridge is found using the voltage divider rule.

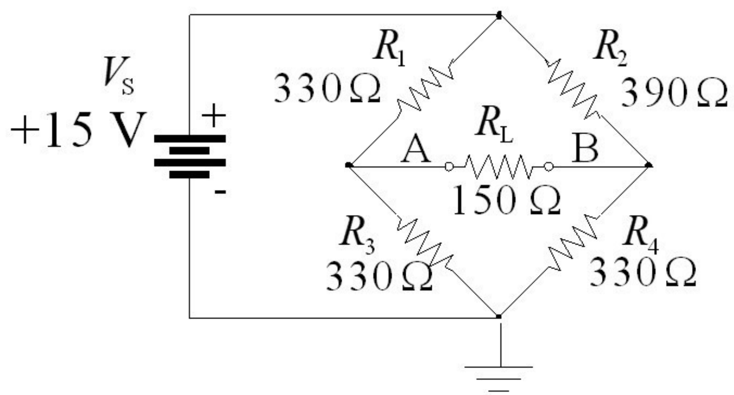


Summary (30 of 40)

Thevenin's Theorem

Example:

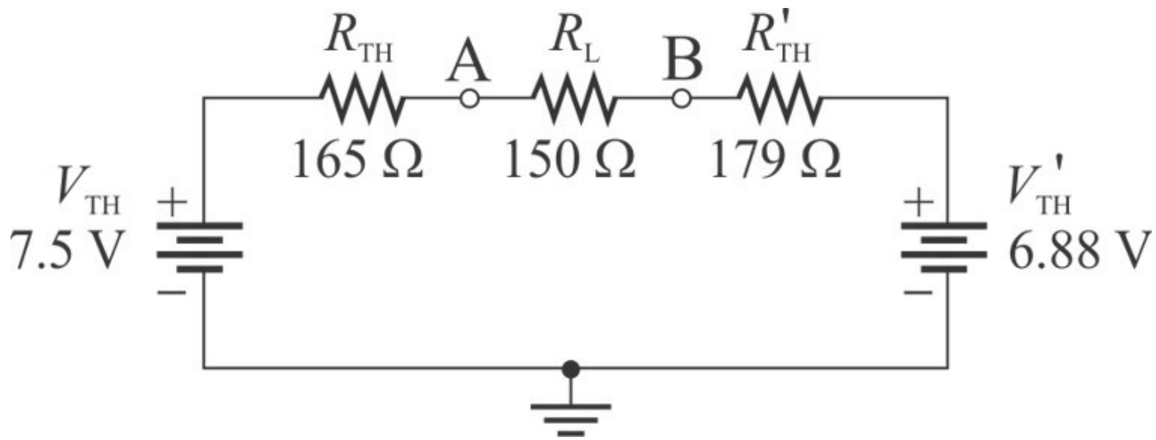
For the bridge shown, $R_1 \| R_3 = 165\ \Omega$ and $R_2 \| R_4 = 179\ \Omega$. The voltage from A to ground (with no load) is 7.5 V and from B to ground (with no load) is 6.88 V.



The Thevenin circuits for each of the bridge are shown on the following slide.

Summary (31 of 40)

Thevenin's Theorem



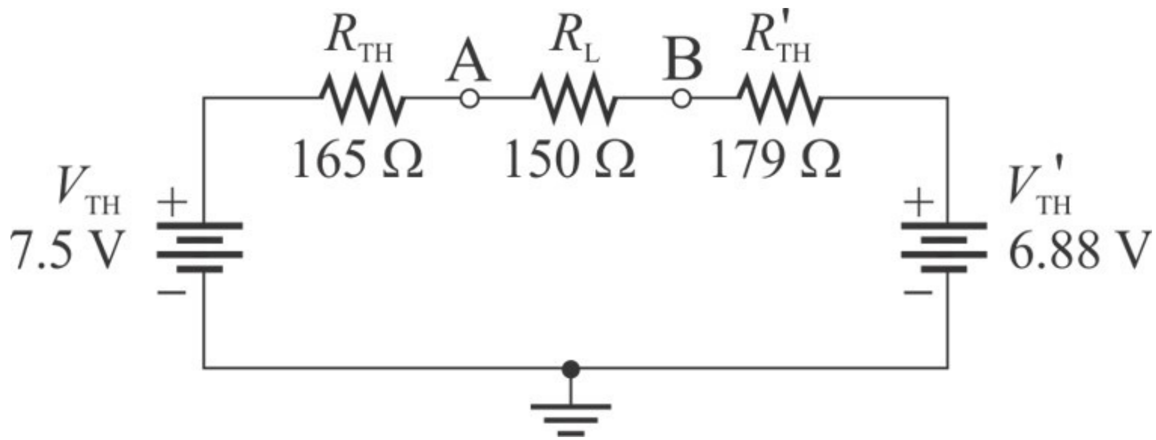
Add the load on the Thevenin circuits. Algebraically add the voltage sources and solve for the current to calculate the load current.

The load current is:

Prof. John G. Breslin, University of Galway

Summary (32 of 40)

Thevenin's Theorem



Add the load on the Thevenin circuits. Algebraically add the voltage sources and solve for the current to calculate the load current.

The load current is: 1.27 mA

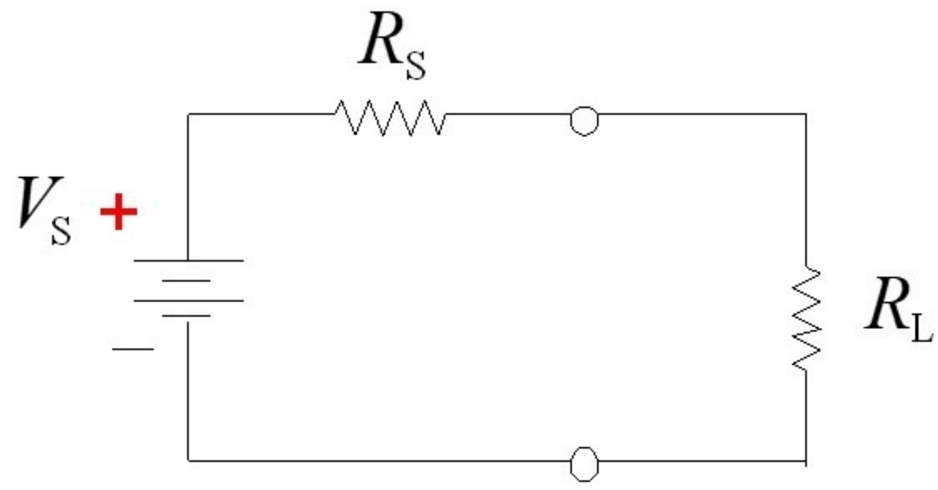
The dual Thevenin circuits used in this analysis have the advantage of retaining the ground from the original circuit.

Prof. John G. Breslin, University of Galway

Summary (33 of 40)

Maximum Power Transfer

The maximum power is transferred from a source to a load when the load resistance is equal to the internal source resistance.



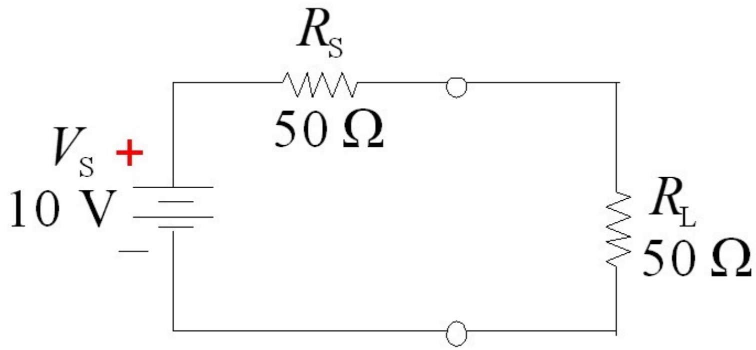
The maximum power transfer theorem assumes the source voltage and resistance are fixed.

Summary (34 of 40)

Maximum Power Transfer

Example:

What is the power delivered to the matching load?



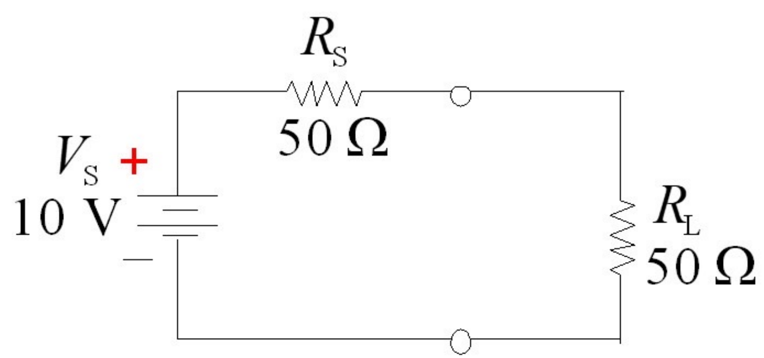
Maximum Power Transfer

Example:

What is the power delivered to the matching load?

Solution:

The voltage to the load is 5.0 V.
The power delivered is



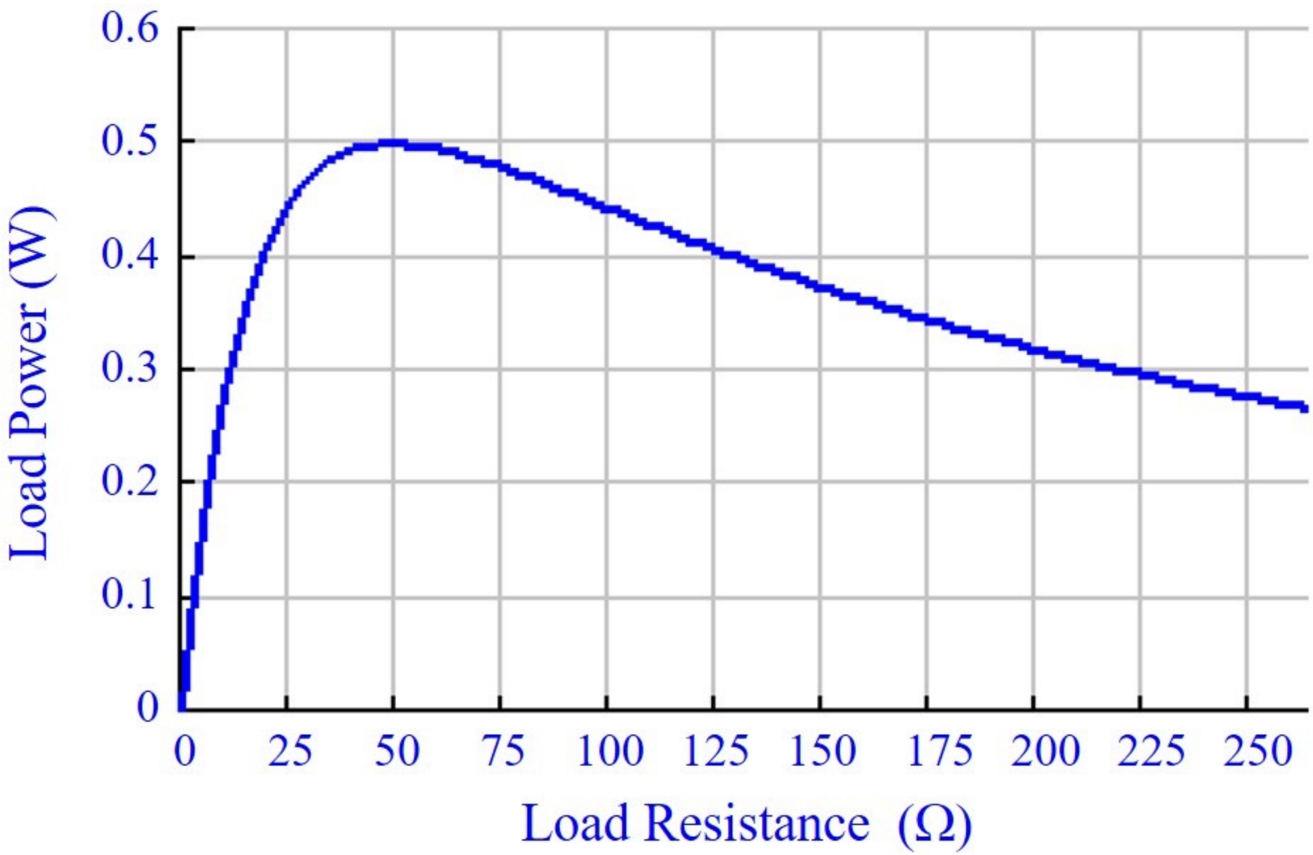
$$P_L = \frac{V^2}{R_L} = \frac{(5.0\ \text{V})^2}{50\ \Omega} = 0.5\ \text{W}$$

The next slide shows a plot of load power as a function of load resistance (plotted using a graphing calculator).

Prof. John G. Breslin, University of Galway

Summary (36 of 40)

Maximum Power Transfer

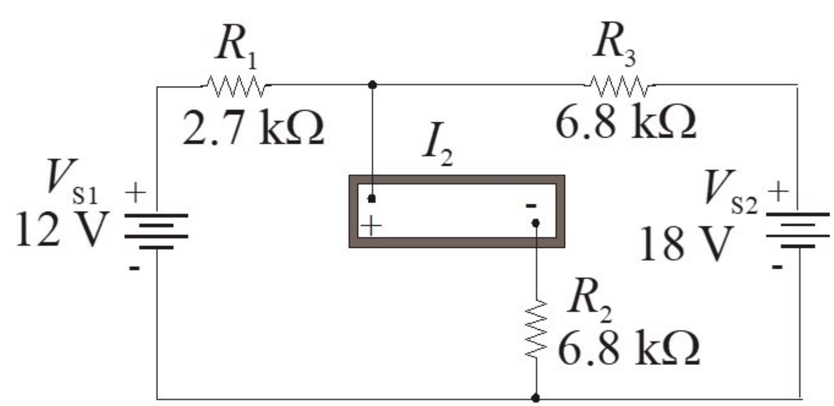


Summary (37 of 40)

Superposition theorem

The **superposition theorem** is a way to determine currents and voltages in a linear circuit that has multiple sources by taking one source at a time and algebraically summing the results.

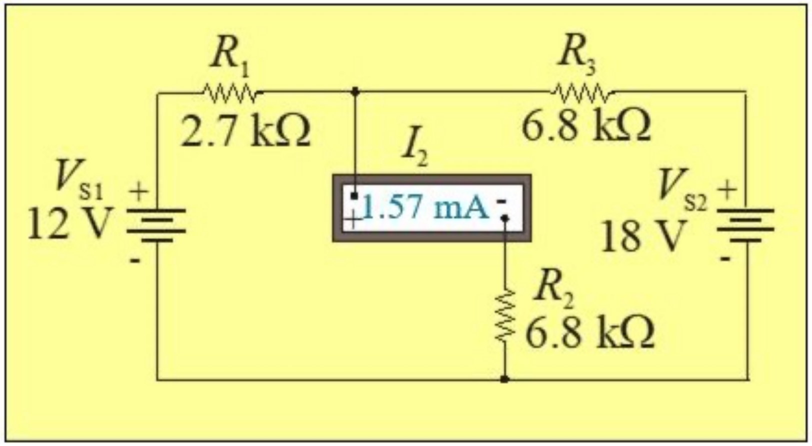
Example: What does the ammeter read? (See next slide for the solution.)



Summary (38 of 40)

What does the ammeter read?

Set up a table of pertinent information and solve for each quantity listed:



Source 1:	$R_{T(S1)}$= 6.10 kΩ	I_1= 1.97 mA	I_2= 0.98 mA
Source 2:	$R_{T(S2)}$= 8.73 kΩ	I_3= 2.06 mA	I_2= 0.59 mA
Both sources			1.57 mA

The total current is the algebraic sum.

Summary (39 of 40)

Troubleshooting

The effective troubleshooter must think logically about circuit operation.

Analysis: Understand normal circuit operation and find out the symptoms of the failure.

Planning: Decide on a logical set of steps to find the fault.

Measurement: Following the plan, make measurements to isolate the problem. Modify the plan if necessary.

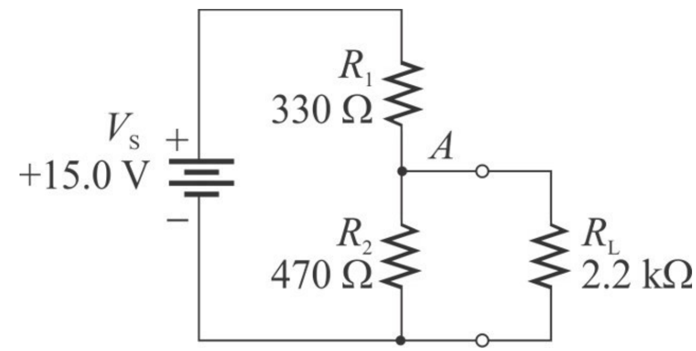
Prof. John G. Breslin, University of Galway

Summary (40 of 40)

Troubleshooting

Example:

The output of the voltage-divider is 6.0 V. Describe how you would use analysis and planning in finding the fault.



Analysis: From an earlier calculation, V_L should equal **8.10 V**. A low voltage is most likely caused by a low source voltage or incorrect resistors (possibly R_1 and R_2 reversed). If the circuit is new, incorrect components are possible.

Planning: Decide on a logical set of steps to locate the fault. You could decide to 1) check the source voltage, 2) disconnect the load and check the output voltage, and if it is correct, 3) check the load resistance. If R_L is correct, check the other resistors.

Selected Key Terms (1 of 2)

Loading	The effect on the output of a circuit when an element that draws current from the circuit is connected across the output terminals.
Load current	The output current supplied to a load.
Bleeder current	The current left after the load current is subtracted from the total current into the circuit.
Wheatstone bridge	A four-legged type of bridge circuit with which an unknown resistance can be accurately measured using the balanced state. Deviations in resistance can be measured using the unbalanced state.

Selected Key Terms (2 of 2)

Thevenin's theorem	A circuit theorem that provides for reducing any two-terminal resistive circuit to a single equivalent voltage source in series with an equivalent resistance.
Maximum power transfer	The current left after the load current is subtracted from the total current into the circuit.
Superposition	A method for analyzing circuits with two or more sources by examining the effects of each source by itself and then combining the effects.

Quiz (1 of 11)

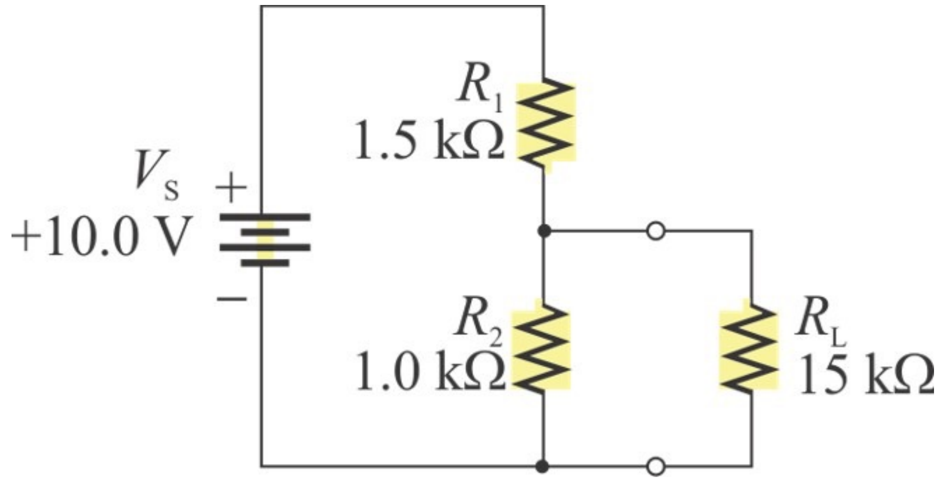
1. The load voltage is

   a. 3.85 V

   b. 5.00 V

   c. 6.15 V

   d. 10.0 V



Quiz (2 of 11)

2. If a series equivalent circuit is drawn for a complex circuit, the equivalent circuit can be analyzed with

   a. the voltage divider theorem.

   b. Kirchhoff’s voltage law.

   c. both of the above.

   d. none of the above.

Quiz (3 of 11)

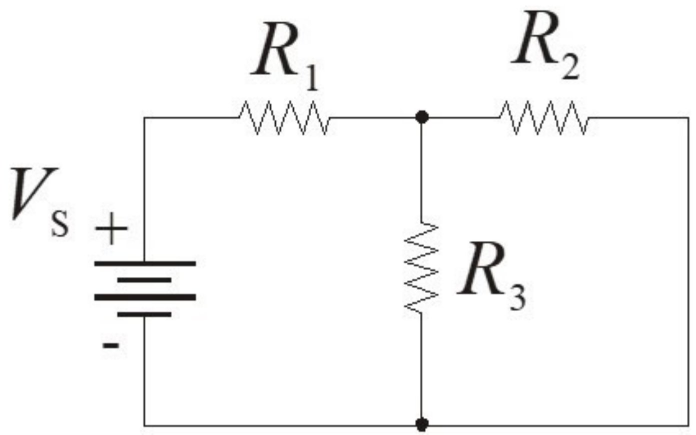
3. For the circuit shown,

   a. R_1 is in series with R_2.

   b. R_1 is in parallel with R_2.

   c. R_2 is in series with R_3.

   d. R_2 is in parallel with R_3.



Quiz (4 of 11)

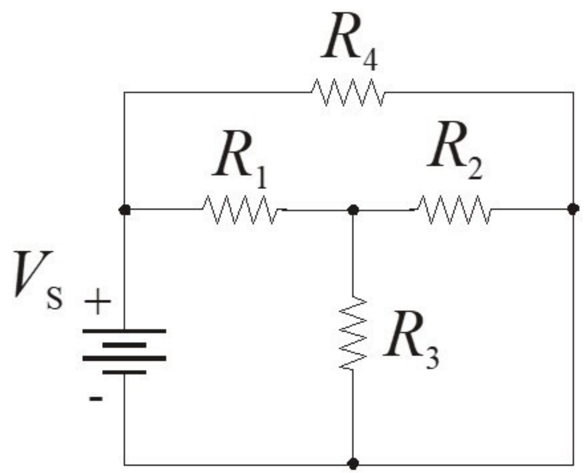
4. For the circuit shown,

   a. R_1 is in series with R_2.

   b. R_4 is in parallel with R_1.

   c. R_2 is in parallel with R_3.

   d. none of the above.



Quiz (5 of 11)

5. A signal generator has an output voltage of 2.0 V with no load. When a 600 Ω load is connected to it, the output drops to 1.0 V. The Thevenin resistance of the generator is

   a. 300 Ω.
   b. 600 Ω.
   c. 900 Ω.
   d. 1200 Ω.

Prof. John G. Breslin, University of Galway

Quiz (6 of 11)

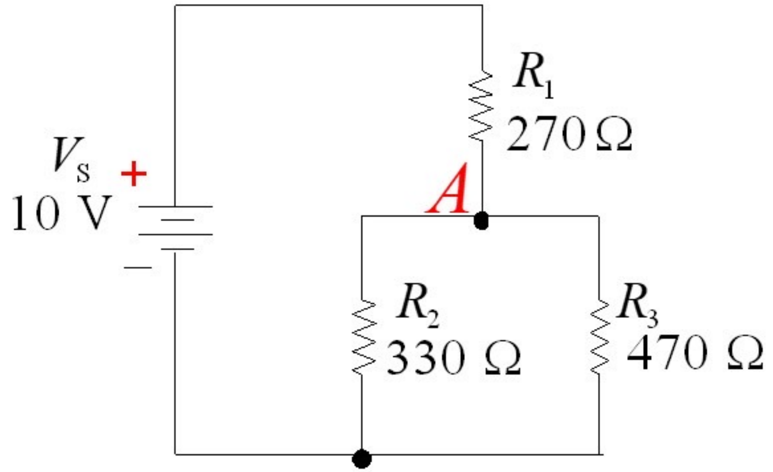
6. For the circuit shown, Kirchhoff's voltage law

   a. applies only to the outside loop.

   b. applies only to the *A* junction.

   c. can be applied to any closed path.

   d. does not apply.



Quiz (7 of 11)

7. The effect of changing a measured quantity due to connecting an instrument to a circuit is called

   a. loading.

   b. clipping.

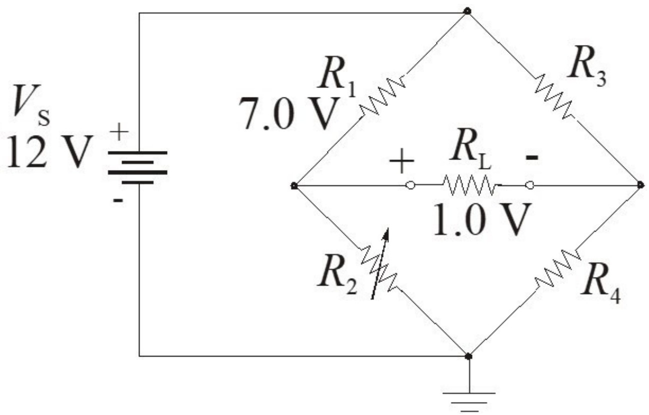
   c. distortion.

   d. loss of precision.

Quiz (8 of 11)

8. An unbalanced Wheatstone bridge has the voltages shown. The voltage across R_4 is

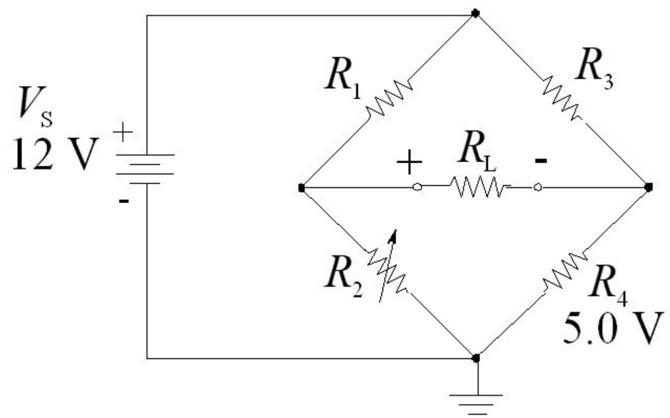
   a. 4.0 V.
   b. 5.0 V.
   c. 6.0 V.
   d. 7.0 V.



Quiz (9 of 11)

9. Assume R_2 is adjusted until the Wheatstone bridge is balanced. At this point, the voltage across R_4 is measured and found to be 5.0 V. The voltage across R_1 will be

a. 4.0 V.
b. 5.0 V.
c. 6.0 V.
d. 7.0 V.



Quiz (10 of 11)

10. Maximum power is transferred from a fixed source when

    a. the load resistance is ½ the source resistance.

    b. the load resistance is equal to the source resistance.

    c. the load resistance is twice the source resistance.

    d. none of the above.

Prof. John G. Breslin, University of Galway

Quiz (11 of 11)

Answers:

1. a
2. c
3. d
4. d
5. b
6. c
7. a
8. a
9. d
10. b

Prof. John G. Breslin, University of Galway