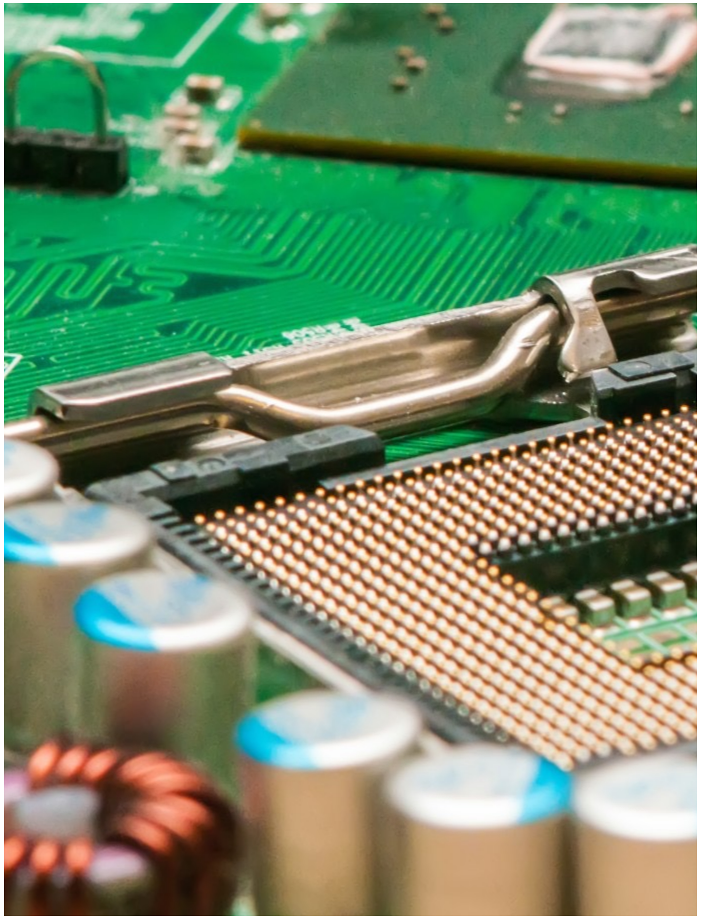
Fundamentals of Electrical & Electronic Engineering

Prof. John G. Breslin, Electrical & Electronic Engineering



Lecture 5

Parallel Circuits

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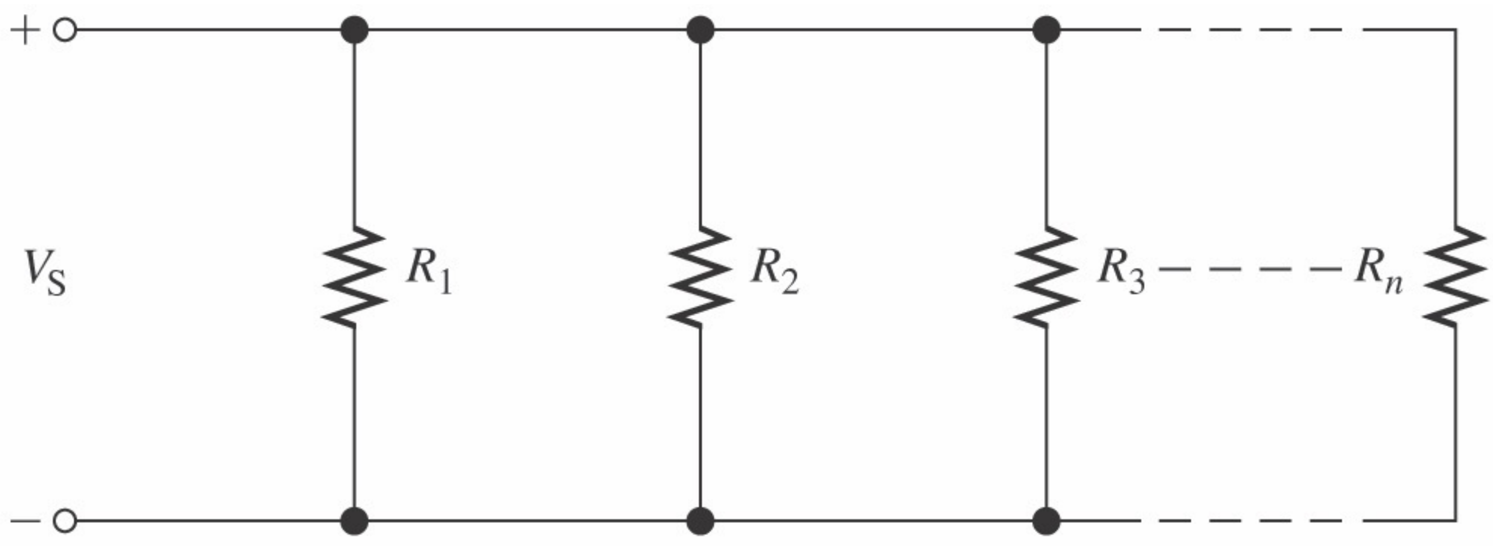
University of Galway

Summary (1 of 20)

Resistors in Parallel

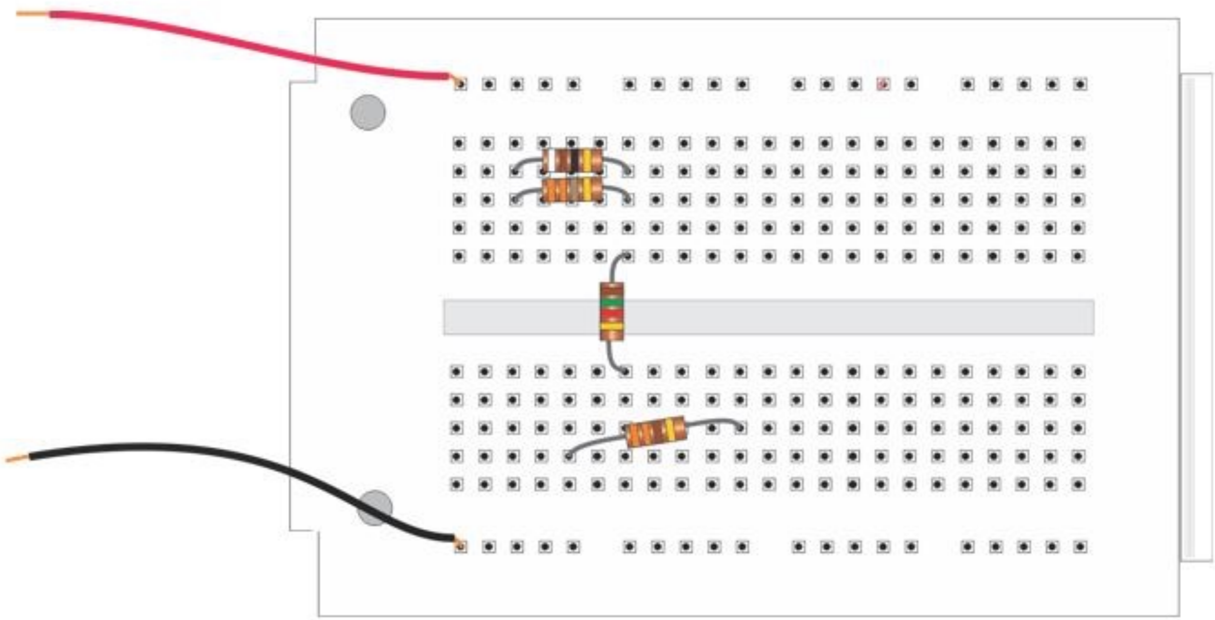
Resistors that are connected between the same two points are said to be in **parallel**.



Summary (2 of 20)

Resistors in Parallel

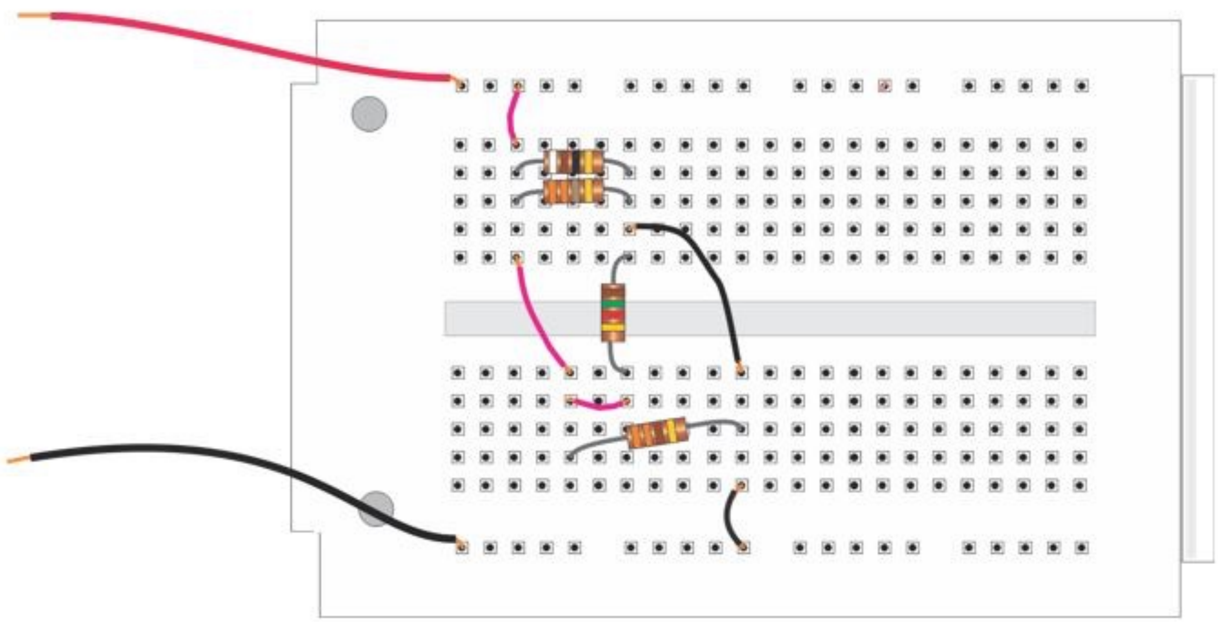
Example Show how to connect the resistors on the protoboard in parallel.



Summary (3 of 20)

Resistors in Parallel

Solution: This is one way. Notice that one node is colored in red; the other is **black** and *all* resistors are between these two nodes.



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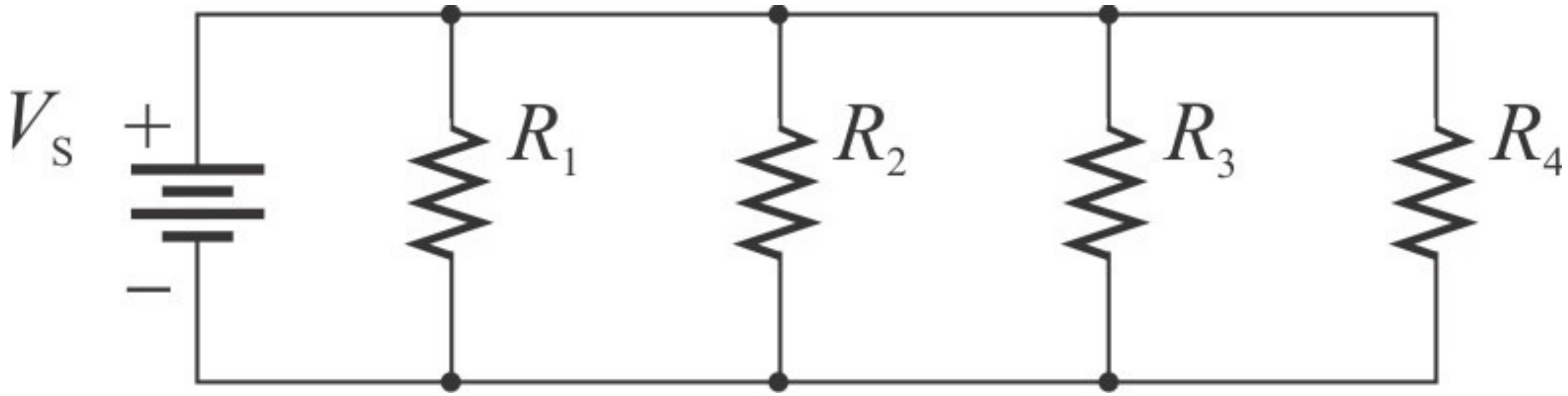
Parallel Circuits

A ***parallel circuit*** is identified by the fact that it has more than one current path (branch) connected to a common voltage source.



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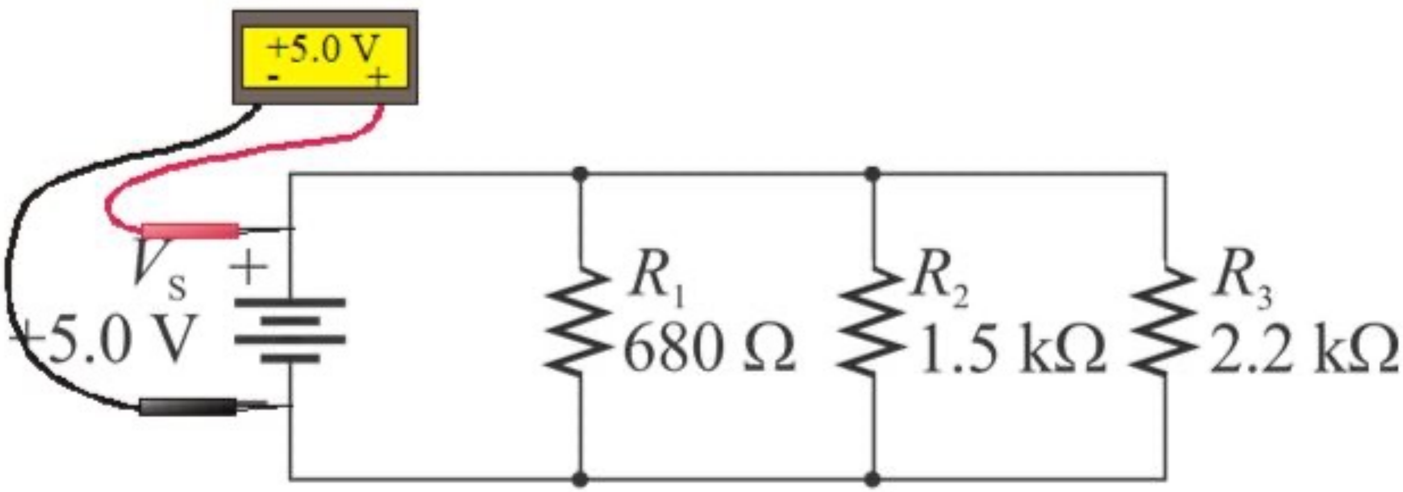
Summary (5 of 20)

Parallel Circuit Rule for Voltage

Because all components are connected across the same voltage source, the voltage across each is the same.

For example, the source voltage is +5.0 V. What will a voltmeter read if it is placed across each of the resistors?



Summary (6 of 20)

Parallel Circuit Rule for Voltage

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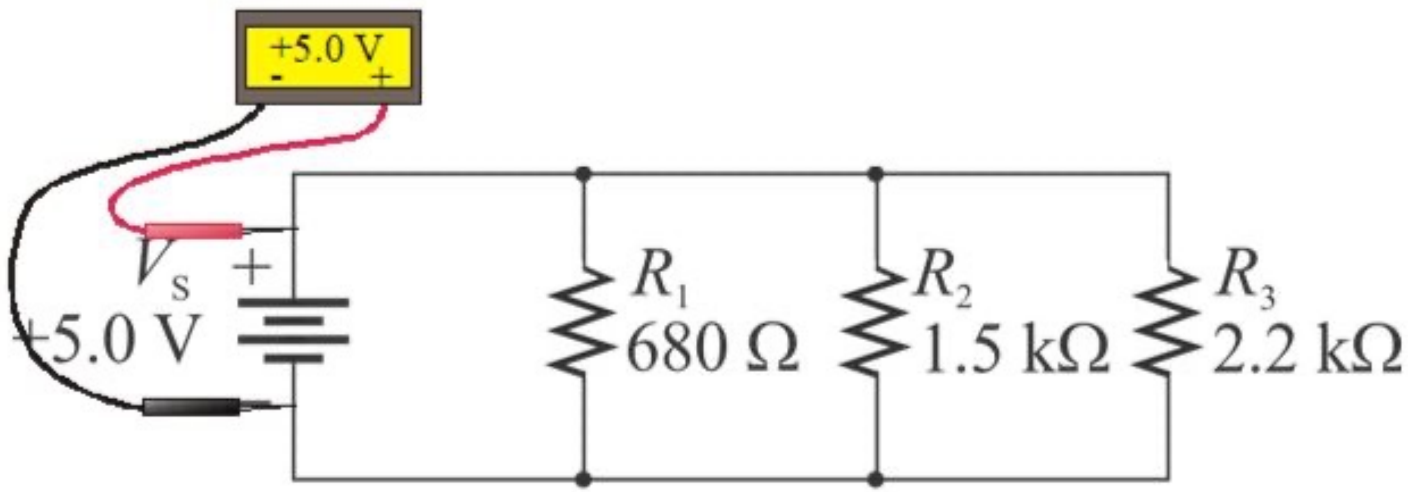
For example, the source voltage is +5.0 V. What will a voltmeter read if it is placed across each of the resistors?

+5.0 V



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Summary (7 of 20)

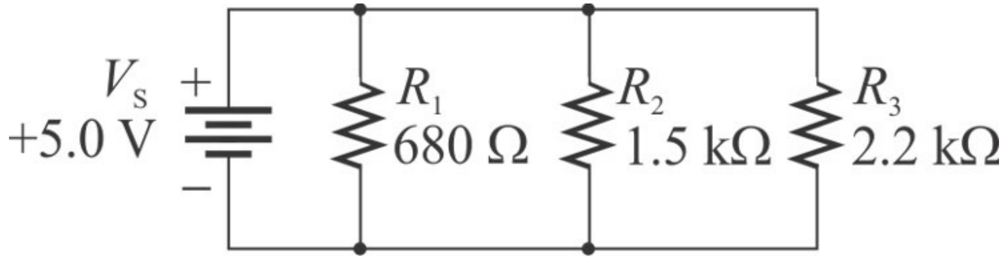
Parallel Resistors

The total resistance of resistors in parallel is the reciprocal of the sum of the sum of the reciprocals of the individual resistors.

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}}$$

Example:

The resistors in a certain parallel circuit are 680 Ω, 1.5 kΩ, and 2.2 kΩ. What is the total resistance?



Summary (8 of 20)

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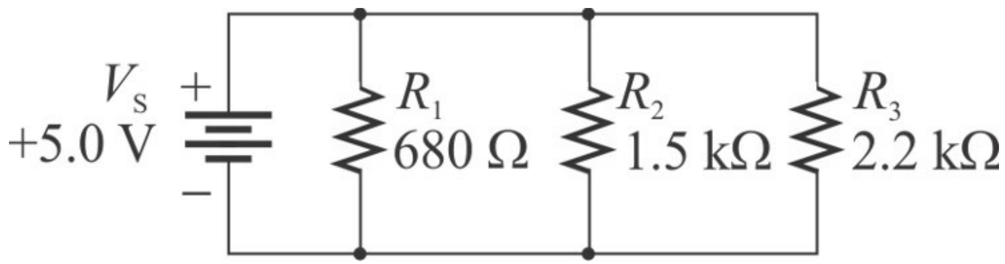
The resistors in a certain parallel circuit are 680 Ω, 1.5 kΩ, and 2.2 kΩ. What is the total resistance? 386 Ω



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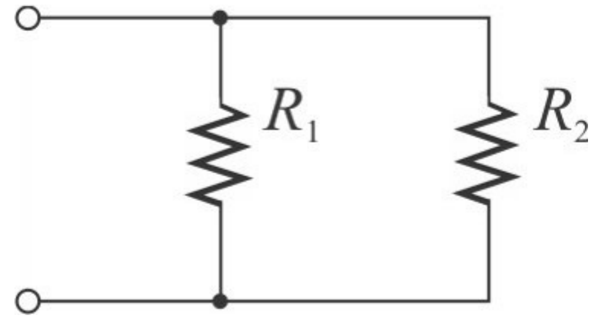
Two Parallel Resistors



The special case of resistance of two parallel resistors can be found by:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

which can be reduced to

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

This is called the product-over-sum rule.

Example: Calculate the total resistance if $R_1 = 27\ \text{k}\Omega$ and $R_2 = 56\ \text{k}\Omega$.

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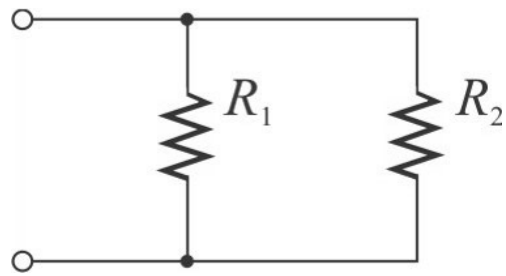
Two Parallel Resistors



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This is called the product-over-sum rule.

Example: Calculate the total resistance if $R_1 = 27\ \text{k}\Omega$ and $R_2 = 56\ \text{k}\Omega$.

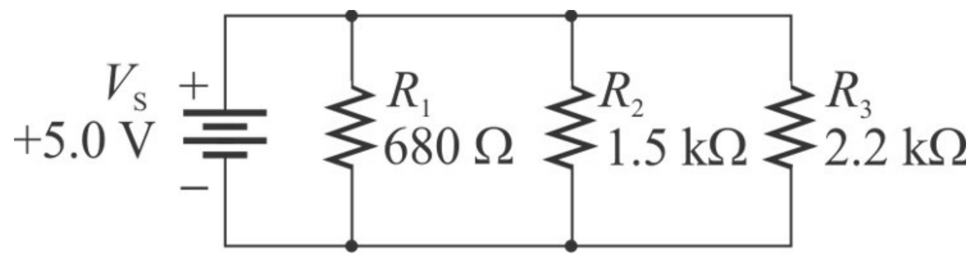
18.2 kΩ

With a calculator, you can enter the values in kΩ and append kΩ to the result. The key sequence varies by calculator; for the TI-84, you can enter the following:

2 7 x^{-1} + 5 6 x^{-1} enter x^{-1} enter

Summary (11 of 20)

Parallel Circuits



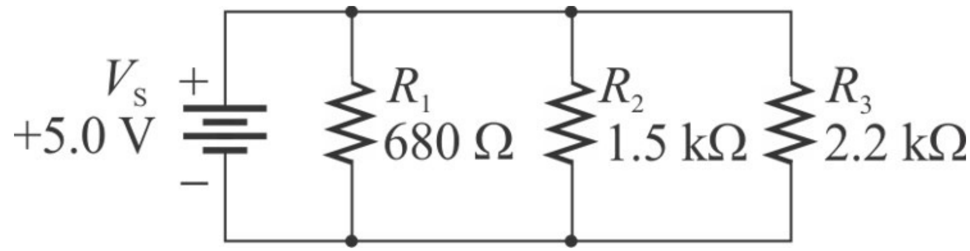
Tabulating current, resistance, voltage, and power is a useful way to summarize parameters in a parallel circuit.

Continuing with the previous example, complete the parameters listed in the Table.

I_1 =	R_1 = 0.68 kΩ	V_1 =	P_1 =
I_2 =	R_2 = 1.50 kΩ	V_2 =	P_2 =
I_3 =	R_3 = 2.20 kΩ	V_3 =	P_3 =
I_T =	R_T = 386 Ω	V_S = 5.0 V	P_T =

Summary (12 of 20)

Parallel Circuits



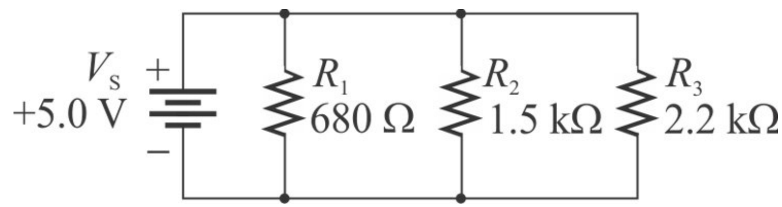
Tabulating current, resistance, voltage, and power is a useful way to summarize parameters in a parallel circuit.

Continuing with the previous example, complete the parameters listed in the Table.

I_1 = 7.4 mA	R_1 = 0.68 kΩ	V_1 = 5.0 V	P_1 = 36.8 mW
I_2 = 3.3 mA	R_2 = 1.50 kΩ	V_2 = 5.0 V	P_2 = 16.7 mW
I_3 = 2.3 mA	R_3 = 2.20 kΩ	V_3 = 5.0 V	P_3 = 11.4 mW
I_T = 13.0 mA	R_T = 386 Ω	V_S = 5.0 V	P_T = 64.8 mW

Summary (13 of 20)

Kirchhoff's Current Law



Kirchhoff's current law (KCL) is generally stated as:

The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Notice in the previous example that the current from the source is equal to the sum of the branch currents.

I_1 = 7.4 mA	R_1 = 0.68 kΩ	V_1 = 5.0 V	P_1 = 36.8 mW
I_2 = 3.3 mA	R_2 = 1.50 kΩ	V_2 = 5.0 V	P_2 = 16.7 mW
I_3 = 2.3 mA	R_3 = 2.20 kΩ	V_3 = 5.0 V	P_3 = 11.4 mW
I_T = 13.0 mA	R_T = 386 Ω	V_S = 5.0 V	P_T = 64.8 mW

Current Divider

When current enters a node (junction) it divides inversely proportional to each branch resistance.

The most widely used formula for the current divider is the two-resistor equation. For resistors R_1 and R_2,

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T \quad \text{and} \quad I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T$$

Notice the subscripts. The resistor in the numerator is not the same as the one for which current is found.

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Current Divider

Example Assume that R_1 is a $2.2\ \text{k}\Omega$ resistor that is in parallel with R_2, which is $4.7\ \text{k}\Omega$. If the total current into the resistors is 8.0 mA, what is the current in each resistor?

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Current Divider

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Solution

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T = \left(\frac{4.7\ \text{k}\Omega}{6.9\ \text{k}\Omega}\right) 8.0\ \text{mA} = 5.45\ \text{mA}$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T = \left(\frac{2.2\ \text{k}\Omega}{6.9\ \text{k}\Omega}\right) 8.0\ \text{mA} = 2.55\ \text{mA}$$

Notice that the *larger* resistor has the *smaller* current.

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Power in Parallel Circuits

Power in parallel resistors can be calculated with any of the standard power formulas. Most of the time, the voltage and resistance are known,

so $P_T = \frac{V^2}{R}$ is the most convenient one to use.

The total power is the sum of the powers dissipated in each resistor.

Example: Assume 10 V is applied to the parallel combination of $R_1 = 270\ \Omega$ and $R_2 = 150\ \Omega$. Calculate the total power.

Solution:

$$P_1 = \frac{V^2}{R} = \frac{(10\text{ V})^2}{270\ \Omega} = 0.370\text{ W};\quad P_1 = \frac{V^2}{R} = \frac{(10\text{ V})^2}{150\ \Omega} = 0.667\text{ W}$$

Alternatively,

$$R_T = \left(\frac{R_1 R_2}{R_1 + R_2}\right) = \frac{(270\ \Omega)(150\ \Omega)}{270\ \Omega + 150\ \Omega} = 96.4\ \Omega \quad P_T = \frac{V^2}{R} = \frac{(10\text{ V})^2}{96.4\ \Omega}$$

Summary (18 of 20)

Power in Parallel Circuits

Power in parallel resistors can be calculated with any of the standard power formulas. Most of the time, the voltage and resistance are known,

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Summary (19 of 20)

Power in Parallel Circuits

Question: Assume there are 8 wires having equal resistance that form a rear window defroster for an automobile. The wires are in parallel.

(a) If the defroster dissipates 90 W when connected to a 12.6 V source, what power is dissipated by each resistive wire?

(b) What is the total resistance of the defroster?

Answer: (a) Each of the 8 wires will dissipate 1/8 of the total power or

$$\frac{90\ \text{W}}{8\ \text{wires}}$$

(b) The total resistance is

$$R = \frac{V^2}{P} = \frac{(12.6\ \text{V})^2}{90\ \text{W}}$$

Follow up: What is the resistance of each wire? $1.76\ \Omega \times 8$

Summary (20 of 20)

Power in Parallel Circuits

Question: Assume there are 8 wires having equal resistance that form a rear window defroster for an automobile. The wires are in parallel.

(a) If the defroster dissipates 90 W when connected to a 12.6 V source, what power is dissipated by each resistive wire?

(b) What is the total resistance of the defroster?

Answer: (a) Each of the 8 wires will dissipate 1/8 of the total power or

$$\frac{90\ \text{W}}{8\ \text{wires}} = 11.25\ \text{W}$$

(b) The total resistance is

$$R = \frac{V^2}{P} = \frac{(12.6\ \text{V})^2}{90\ \text{W}} = 1.76\ \Omega$$

Follow up: What is the resistance of each wire? $1.76\ \Omega \times 8 = 14.1\ \Omega$

Key Terms

Parallel The relationship in electric circuits in which two or more current paths are connected between two separate points (nodes).

Branch One current path in a parallel circuit.

Kirchhoff's current law A law stating the total current into a node equals the total current out of the node.

Node A point or junction in a circuit at which two or more components are connected.

Current divider A parallel combination of resistances in which the input current divides in inverse proportion to the branch resistances.

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Quiz (1 of 11)

1. The total resistance of parallel resistors is equal to

   a. the sum of the resistances.

   b. the sum of the reciprocals of the resistances.

   c. the sum of the conductances.

   d. none of the above.

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Quiz (2 of 11)

2. The number of nodes in a parallel circuit is

   a. one.

   b. two.

   c. three.

   d. any number.

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Quiz (3 of 11)

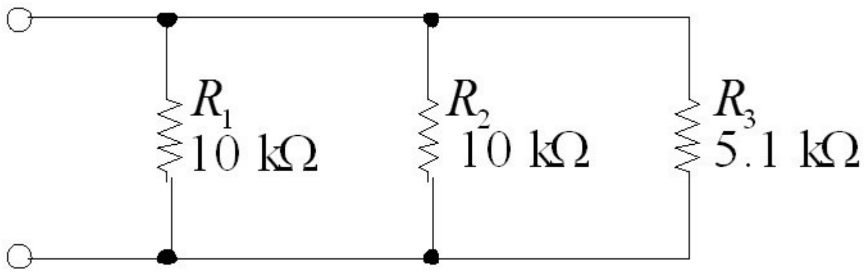
3. The total resistance of the parallel resistors is

   a. 2.52 kΩ

   b. 3.35 kΩ

   c. 5.1 kΩ

   d. 25.1 kΩ



Quiz (4 of 11)

4. If three equal resistors are in parallel, the total resistance is

   a. one third the value of one resistor.

   b. the same as one resistor.

   c. three times the value of one resistor.

   d. the product of the three resistors.

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Quiz (5 of 11)

5. In any circuit the total current entering a node is

   a. less than the total current leaving the node.
   b. equal to the total current leaving the node.
   c. greater than the total current leaving the node.
   d. can be any of the above, depending on the circuit.

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Quiz (6 of 11)

6. The current divider formula to find I_1 for the special case of two resistors is

   a. $I_1 = \left(\dfrac{R_1}{R_T}\right) I_T$

   b. $I_1 = \left(\dfrac{R_2}{R_T}\right) I_T$

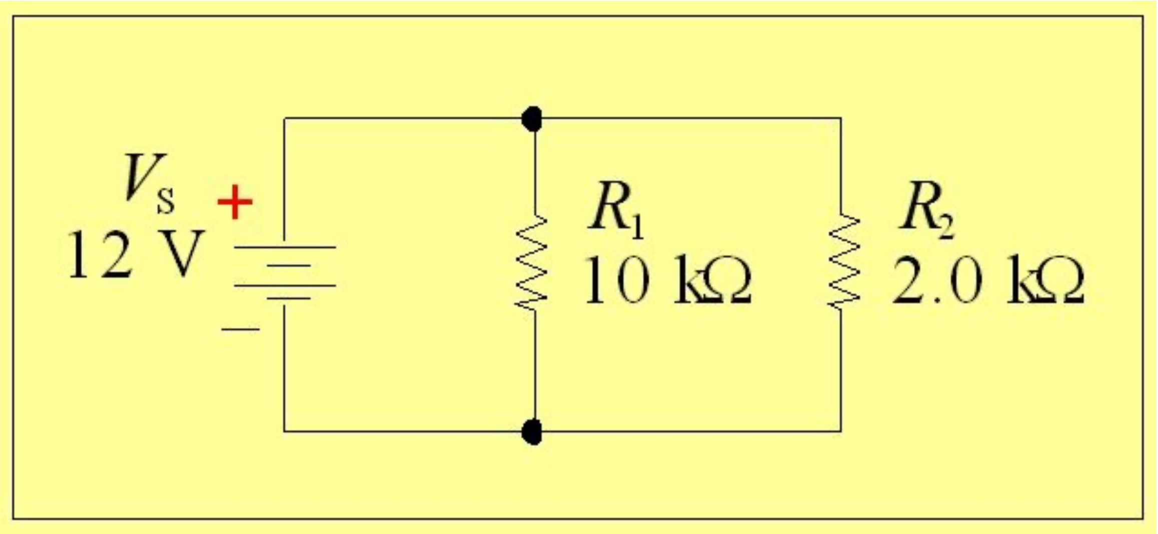
   c. $I_1 = \left(\dfrac{R_2}{R_1 + R_2}\right) I_T$

   d. $I_1 = \left(\dfrac{R_1}{R_1 + R_2}\right) I_T$

Quiz (7 of 11)

7. The total current leaving the source is

   a. 1.0 mA
   b. 1.2 mA
   c. 6.0 mA
   d. 7.2 mA



Quiz (8 of 11)

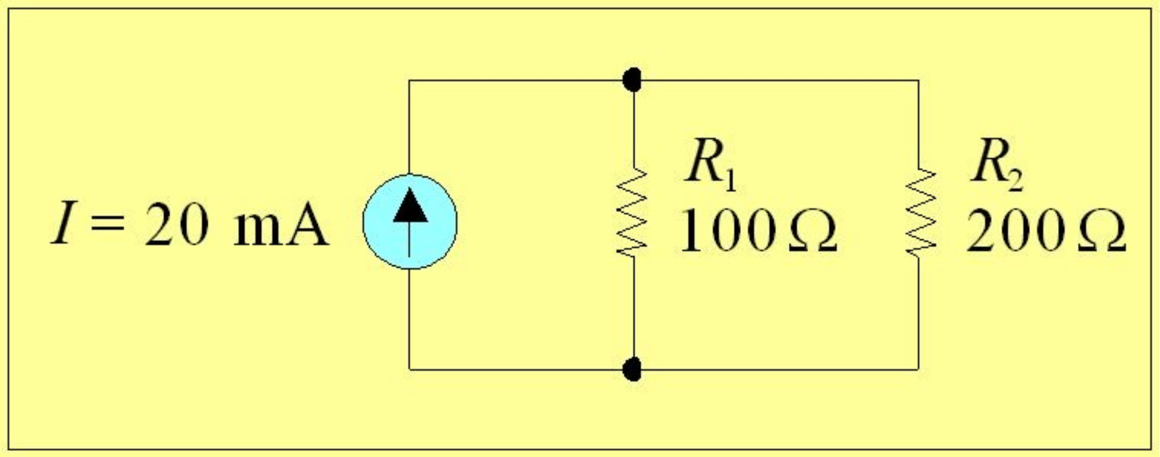
8. The current in R_1 is

a. 6.7 mA

b. 13.3 mA

c. 20 mA

d. 26.7 mA



Quiz (9 of 11)

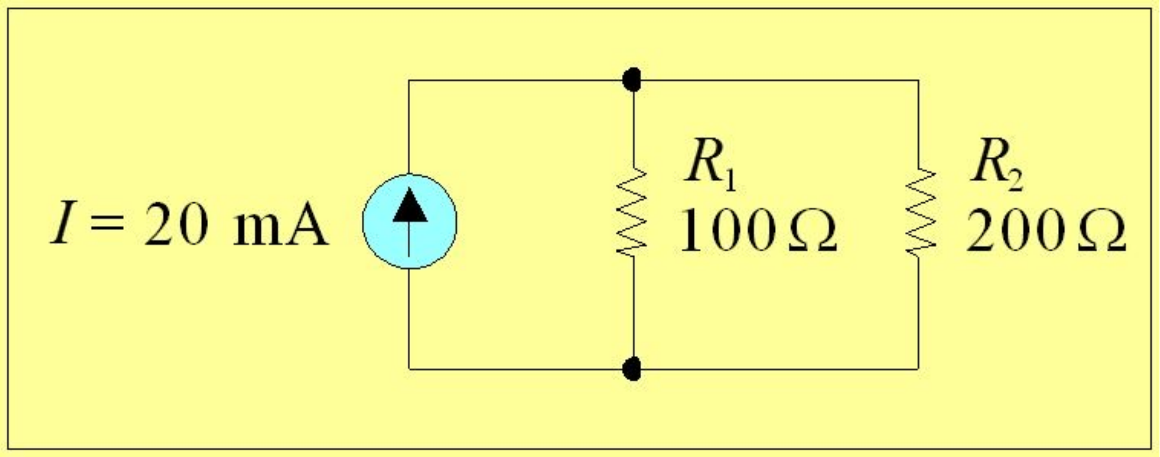
9. The voltage across R_2 is

   a. 0 V

   b. 0.67 V

   c. 1.33 V

   d. 4.0 V



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Quiz (10 of 11)

10. The total power dissipated in a parallel circuit is equal to the

    a. power in the largest resistor.
    b. power in the smallest resistor.
    c. average of the power in all resistors.
    d. sum of the power in all resistors.

Quiz (11 of 11)

Answers:

1. d
2. b
3. a
4. a
5. b
6. c
7. d
8. b
9. c
10. d

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