In order to reflect these differences in attribute values, we introduce a modified version of the Damerau-Levenshstein distance, that not only reflects the difference between two process instances, but also the attribute values. We achieve this by introducing a cost function $cost_{a_i,b_j}$, which applies to a normed vector-space¹. Concretely, we formulate the modified Damerau-Levenshstein distance as shown in Equation 1. For the remainder, we will denote this edit-distance as Semi-strucured Damerau-Levenshtein distance (SSDLD).

$$d_{a,b}(i,j) = \min \begin{cases} d_{a,b}(i-1,j) + cost(\mathbf{0},b_j) & \text{if } i > 0 \\ d_{a,b}(i,j-1) + cost(a_i,\mathbf{0}) & \text{if } j > 0 \\ d_{a,b}(i-1,j-1) + cost(a_i,b_j) & \text{if } i,j > 0 \\ & & & \overline{a}_i = \overline{b}_j \\ d_{a,b}(i-1,j-1) + cost(a_i,\mathbf{0}) + cost(\mathbf{0},b_j) & \text{if } i,j > 0 \\ & & & & \overline{a}_i \neq \overline{b}_j \\ d_{a,b}(i-2,j-2) + cost(a_i,b_{j-1}) + cost(a_{i-1},b_j) & \text{if } i,j > 1 \\ & & & & \overline{a}_i = \overline{b}_{j-1} \\ & & & & & \overline{a}_{i-1} = \overline{b}_j \\ 0 & & & & & & & \\ 0 & & & & & & \\ \end{cases}$$

Here, $d_{a,b}(i,j)$ is the recursive form of the Damerau-Levenshtein-Distance. a and b are sequences and i and j specific elements of the sequence. cost(a,b) is a cost function which takes the attribute values of a and b into account. The first two terms correspond to a deletion and an insertion from a to b. The idea is to compute the maximal cost for that the wrongfully deleted or inserted event. The third term adds the difference between two events with identical activities \overline{a}_i and \overline{b}_j . As mentioned earlier, two events that refer to the same activity can still be different due to event attributes. The distance between the event attributes determines how different these events are. The fourth term handles the substitution of two events. Here, we compute the substitution cost as the sum of an insertion and a deletion. The fifth term computes the cost after transposing both events. This cost is similar to term 3 only that we now consider the differences between both events after they were aligned. The last term relates to the stopping criterion of the recursive formulation of the Damerau-Levenshstein distance.

¹A normed vector-space is a vector space, in which all vectors have the same dimensionality. For instance, if all vectors have three dimensions, we can call the vector-space normed.