Exercises - Graphs and Random Graphs - Network Science

Tutorial April 29, 2020

1. **Random graph:** Consider a random graph with $n = 3000$ and $p = 10^{-3}$.
   (a) What is the expected number of edges?
   (b) In which regime is this network?
   (c) Given this n, what probability would you need to choose such that the network is at its critical point?
   (d) Given this $p = 10^{-3}$, how big would n need to be such that it is almost surely connected?

2. **Clustering coefficient I:** Consider $G(n, p)$.
   (a) Show that for large n, the expected number of triangles is roughly $\langle k \rangle^3/6$.
   (b) Show that for large n, the expected number of triplets is roughly $n\langle k \rangle^2/2$.
   (c) What is the global clustering coefficient?

3. **Closeness centrality:** Consider a cycle of length $n + 1$ for even n (so it has odd length). Each node has the same closeness centrality. What is it?

4. **Clustering coefficient II:**
   (a) How would you compute the (local) clustering coefficient of a vertex? Give an efficient algorithm and analyze its running time.
   (b) How would you compute the global clustering coefficient of a graph? Give an efficient algorithm and analyze its running time.
   (c) Consider the following graph on n vertices numbered $0, \ldots, n-1$: node i and j are adjacent if and only if i and j differ by at most k (here we treat indices modulo n). Calculate the clustering coefficient of a node and the global clustering coefficient as a function of k. You may assume $n > 5k$.

5. **Programming exercise:**
   (a) Download the `tut1.py` file from Blackboard. Modify line 21 so that it points to the correct path. Run it.
   (b) Create a table that lists, for each of the networks in the dataset, the number of vertices, number of edges, average degree, maximum degree, diameter, average shortest path length, and global clustering coefficient.
   (c) Now create random graphs for each of the average degrees, and see how the maximum degree, diameter, average shortest path length, and global clustering coefficient compare.