Beyond Security and Efficiency: On-Demand Ratcheting with Security Awareness

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Abstract. Ratcheting communication strengthens privacy, specifically in the presence of internal state exposures or random coin corruptions. This is called post-compromise security. There have been several such secure protocols proposed in the last few years. The strongest level of security comes with a high cost, because of the need for hierarchical identity-based encryption (HIBE) or at least public-key cryptography.
In this paper, we propose two generic constructions with favorable properties. We start with formal definitions of *security awareness*. Concretely, our first construction allows users to clearly see which of their messages are safe during the communication and to be able to detect active attacks, including trivial attacks which follow a state exposure. The communication between honest participants aborts in the case of an active attack. This is because an active attack indicates that the adversary has synchronized with one of the honest participants. Abortion is guaranteed by our RECOVER security.
In our second construction, we define a hybrid system formed by combining two protocols: typically, a weakly secure "light" protocol and a strongly secure "heavy" protocol. The design goals of our hybrid construction are, first, to let the sender decide which one of to use in order to obtain an efficient protocol with *ratchet on demand*; and second, to restore the communication between honest participants in the case of a message loss or an active attack.
We can apply our generic constructions on *any* existing protocol. For this work, we chose to hybridize the Durak-Vaudenay protocol with a light design liteARCAD of ours that uses only symmetric-key primitives. Hence, there is no post-compromise security until a sender demands to ratchet. Last but not least, we find it quite appealing and intuitive to use the security-aware and ratchet-on-demand protocols together in order to strengthen each other by building one design on top of the other.

1 Introduction

In recent messaging applications, protocols are secured with end-to-end encryption to enable secure communication services for their users. Besides security, there are many other characteristics of communication systems. The nature of two-party protocols is that it is *asynchronous*: the messages should be transmitted regardless of the counterpart being online. Moreover, the protocols do not have any control over the time that participants send messages, and, by the same token, the participants change their roles as a *sender* or a *receiver* arbitrarily.

Many deployed systems are built with some sort of security guarantees. However, they often struggle with security vulnerabilities due to the internal state compromises that occur through *exposures* of participants. In order to prevent the attacker from decrypting past communication after an exposure, a state update procedure is applied. Ideally, such updates are done through one-way functions which delete the old states and generate new ones. This guarantees *forward secrecy*. Additionally, to further prevent the attacker from decrypting future communication, *ratcheting* is used. This adds some source of randomness in every state update to obtain what is called *future secrecy*, or *backward secrecy*, or *post-compromise security*, or even *self-healing*.

Even though forward secrecy or post-compromise security have been integrated for a while, there have been no formal definitions and protocols provably secure under such notions until recently. After the first formal definitions of ratcheting security given by Bellare et al. [2], many

subsequent studies about secure protocols have followed with different security levels and primitives [1, 6–9]. Some of these results are about key-exchange while others study secure messaging. Since secure ratcheted messaging can reduce to secure key exchange, we consider these works as equivalent.

Previous work. Early ratcheting protocols were suggested in Off-the-Record (OTR) and then Signal [3, 10]. The security of Signal was studied by Cohn-Gordon et al. [4]. Unger et al. [11] surveyed many ratcheting techniques. Alwen et al. [1] formalized the concept of "double ratcheting" from Signal.

Cohn-Gordon et al. [5] proposed a ratcheted protocol at CSF 2016 but requiring synchronized roles. Bellare et al. [2] proposed another protocol at CRYPTO 2017, but unidirectional and without forward secrecy. Poettering and Rösler (PR) [9] designed a protocol at CRYPTO 2018 with "*optimal*" security (in the sense that we know no better security so far), but using a random oracle, and heavy algorithms such as hierarchical identity-based encryption (HIBE). Yet, their protocol does not guarantee security against compromised random coins. Jaeger and Stepanovs (JS) [7] proposed a similar protocol with security against compromised random coins: with random coin leakage *before* usage. Their protocol also requires HIBE and a random oracle.

Durak and Vaudenay (DV) [6] proposed a protocol with slightly lower security[3] but relying on neither HIBE nor random oracle. They rely on a public-key cryptosystem, a digital signature scheme, a one-time symmetric encryption scheme, and a collision-resistant hash function. They further show that a unidirectional scheme with post-compromise security implies public-key cryptography, which obviates any hope of having a fully secure protocol solely based on symmetric cryptography. At EUROCRYPT 2019, Jost, Maurer, and Mularczyk (JMM) [8] proposed concurrently and independently a protocol with security between optimal security and the security of the DV protocol.[4] They achieve it even with random coin leakage *after* usage. Contrarily to other protocols achieving security with corrupted random coins, in their protocol, random coin leakage does not necessarily imply revealing part of the state of the participant. In the same conference, Alwen, Coretti, and Dodis [1] proposed two other ratcheting protocols denoted as ACD and ACD-PK with security against adversarially *chosen* random coins and *immediate decryption*. Namely, messages can be decrypted even though some previous messages have not been received yet. The ACD-PK protocol offers a good level of security, although having immediate decryption may lower it a bit as it will be discussed shortly. On the other hand, during a phase when the direction of communication does not change, the ACD protocol is fully based on symmetric cryptography, hence has lower security (in particular, no post-compromise security in this period). However, it is much more efficient. We consider Signal and ACD as equivalent.

We summarize these results in Table 1. The first four rows are based on DV [6, Table 1]. The other rows of the table will be discussed shortly.

We are mostly interested in the DV model [6]. It gives a simple description of the KIND security and FORGE security. The former deals with key indistinguishability where the generated keys are indistinguishable from random strings and the latter states that update messages for ratcheted key exchange are unforgeable. Additionally, they present the notion of RECOVER-security which guarantees that a participant can no longer accept messages from his counterpart after he receives a forged message. Actually, even though FORGE security avoids non-trivial forgeries, there are still (unavoidable) trivial forgeries. They occur when the state of a participant is exposed and the adversary decides to impersonate him. With RECOVER security, when an adversary impersonates someone (say Bob), the impersonated participant is out and can no longer communicate with the counterpart (say Alice). It does not mean to bother participants but rather work for their benefit. Indeed, this security notion guarantees that the attack is eventually detected by Bob if he is still alive. If the protocol has a way to resume secure communication based on an explicit action form the users, this property is particularly appealing.

[3] More precisely, the security is called "*sub-optimal*" as detailed later.
[4] They call this security level "*near-optimal*".

What makes the DV model simple is that all technicalities are hidden in a *cleanness* notion which eliminates trivial attack strategies. The adversary can only win when the attack scenario trace is "clean". This model makes it easy to consider several cleanness notions, specifically for hybrid protocols. The difficulty is perhaps to provide an exhaustive list of criteria for attacks to be clean.

Our objectives. In this paper, we study various security notions for the asynchronous ratcheted communication with additional data which we call ARCAD in short. Experience showed that when we want the protocols to be highly secure, we have to give up the efficiency of the protocol and rely on heavy tools. For instance, DV [6] (which we consider as ARCAD with slight changes) showed that post-compromise secure communication implies public-key cryptography, hence a complexity overhead. Equivalently, when we want protocols to perform fast, then the security should be lowered to a reasonable level. For instance, we know that symmetric cryptography can handle forward secrecy at a very low cost. In real-world applications, the developers do not want to over-complicate or under-perform. At the same time, users seek usability and strong privacy. Therefore, we believe that the confidentiality level of sending messages should be set on demand by the sender or could be tuned by the application itself based on time intervals. For instance, if the users are exchanging hundreds of messages per day, there may not be any real need for ratcheting all the time with strongly secure protocols. Instead, a lighter version of the protocol only with forward secrecy (e.g. symmetric-key ratcheting) should be enough for security. Alternatively, the sender could ask for healing only when an exposure is likely (e.g. because his device was taken by a third party, remained unattended for a while etc.) or just once a day. Healing may actually scarcely occur in intensive communication. Therefore, we construct a protocol called hybridARCAD that runs a healing ratchet *on demand*.

We also define a security notion by improving RECOVER-security from DV [6]. This security implies that when a participant receives a forgery, he should not be able to receive genuine messages any longer. What is also needed is that the participant who has received a forgery should not be able to *send* messages to his counterpart either. This makes sure that forgeries are eventually detected.

Another interesting notion is given in Alwen et al. [1] as *immediate decryption*. It allows receiving messages even though some previous ones were not received. Concretely, it is done by keeping all keys in the state of the receiver to decrypt messages until they are needed. Obviously, it has some consequences with regards to security. Namely, when an adversary prevents a message from being delivered, the key remains in the receiver state and this key may be stolen in the future. Hence, even though communication can continue, the participants have no guarantee about the safety status of this message until it is received. For instance, we can imagine that the adversary may collect a few sensitive messages (e.g. all the large ones as they may contain media content) and decrypt them all after exposure of the receiver state. Immediate decryption is nice when the communication network is not reliable and messages may come in a different order at random. However, we believe that this problem can be solved by independent techniques and need not to be addressed by the cryptographic protocol. More precisely, messages can be encapsulated in containers which makes sure that if a message is missing, it can be requested for a second delivery and the received messages can be held until the sequence is reconstructed with no loss. Adding reliability on the communication channel can indeed be solved by a lower-level protocol. Hence, we *do not* provide immediate-decryption security in our constructions. Instead, we focus on a very important aspect of secure messaging protocol which is described as security awareness. To defeat communication interruption due to a message loss or a forgery, we will propose a way for participants to repair it.

Our contributions. We start with formally and explicitly defining a notion of *security awareness* in which the users detect active attacks by realizing they can no longer communicate; users can be confident that nothing in the protocol can compromise the confidentiality of an acknowledged message if it did not leak before; and users can deduce from an incoming message which of the messages they sent have been delivered when the incoming message was formed.

More concretely, we elaborate on the RECOVER security to offer optimal security awareness. We start by defining a new notion called s-RECOVER. We make sure that not only is a receiver of a forgery no longer able to receive genuine messages via r-RECOVER-security but he can no longer send a message to his counterpart either via s-RECOVER-security. The r-RECOVER security is equal to RECOVER security of the DV protocol. Both r-RECOVER and s-RECOVER notions imply that reception of a genuine message offers a strong guarantee of having no forgery in the past: after an active attack ended, participants realize they can no longer communicate. Our security-awareness notion makes also explicit that the receiver of a message can deduce (in absence of a forgery) which of his messages have been seen by his counterpart (which we call an *acknowledgment extractor*). Hence, each sent message implicitly carries an acknowledgment for all received messages. Finally, what we want from the history of receive/send messages and exposures of a participant is the ability to deduce which message remains private (or "clean"). We call it a *cleanness extractor*.

Then, we tune our direction to give another *generic* construction to compose "any" two protocols with different security levels to allow a sender to select which security level to use. By composing a strongly secure protocol (such as PR, JS, JMM, DV) with a lighter and weakly secure one (such as liteARCAD, which we construct based solely on symmetric cryptography), we obtain the notion of *ratchet on-demand*. When the ratcheting becomes infrequent, we obtain the excellent software performances of liteARCAD as we will show in our implementation results. Hybrid constructions already exist, like Signal/ACD. However, the user has no control on the choice of the protocol to be used. Instead, they ratchet if (and only if) the direction of communication alternates.

Another interesting outcome of our hybrid system is that we can form our hybrid system with *two identical* protocols: an upper one and a lower one. The lower protocol is used to communicate the messages and the upper protocol is used to control the lower protocol: to setup or to reset it. With this hybrid structure with identical protocols, we can repair broken communication in the case of a message loss or active attacks. As far as we observe, the complexity of the hybrid system is the same as the complexity of the underlying protocol. Since our security-aware property breaks communication in the case of an active attack, this repairing construction is a nice additional tool.

Another major contribution of the present paper is the implementation of the existing protocols: PR, JS, DV, JMM, ACD, ACD-PK, together with liteARCAD. We observe that liteARCAD is the fastest one. This is not surprising for all protocols which heavily use public-key cryptography, but it is surprising for ACD. Our goal is to offer a high level of security with the performances of liteARCAD. We reach it with on-demand ratcheting when the participant demands healing scarcely.

Finally, we conclude that security awareness can be added on top of an existing protocol (even a hybrid one) in a generic way to strengthen security. We propose this generic strengthening (called chain) of protocol to obtain r-RECOVER and s-RECOVER security on the top of any protocol. As an example, we apply it on the ratchet-on-demand hybrid protocol composed with DV and liteARCAD and obtain our final protocol.

We provide a comparison of all the protocols with r-RECOVER-security, s-RECOVER-security, acknowledgment extractor and cleanness extractor in Table 1. Note that this table is made to help both the authors and the readers to have a fair understanding of what specified properties each protocol has or not. We stress that "any" protocol could form a hybrid system to provide ratchet-on-demand and repairing a broken communication in the case of message loss or active attacks. The protocol which is shown in the last column is the case where we chose to use DV and liteARCAD to construct our hybrid system.

To summarize, our contributions are:

- we formally define the notion of security awareness, construct a generic protocol strengthening called chain, and prove its security;
- we design liteARCAD with provable (forward) security which turns out to be faster than its symmetric-key primitive peer ACD;
- we define the notion of on-demand ratcheting, construct a generic hybrid protocol called hybrid, define and prove its security;
- we implement PR, JS, DV, JMM, ACD, ACD-PK, and liteARCAD protocols in order to clearly compare their performances.

Table 1: Comparison of Several Protocols with our protocol $\mathsf{chain}(\mathsf{hybrid}(\mathsf{ARCAD}_{\mathsf{DV}}, \mathsf{liteARCAD}))$ from Cor. 27 in Section 3.3: security level; worst case complexity for exchanging $\mathfrak{n}$ messages; types of coin-leakage security; plain model (i.e. no random oracle); PKC or less (i.e. no HIBE). DV and $\mathsf{ARCAD}_{\mathsf{DV}}$ have identical characteristics. The terms "optimal", "near-optimal", and "sub-optimal" from Durak-Vaudenay [6] are defined on p.2. "Pragmatic" degrades a bit security to offer on-demand ratcheting. "id-optimal" is optimal among protocols with immediate decryption.

	PR [9]	JS [7]	JMM [8]	DV [6]	ACD-PK [1]	ours
Security	optimal	optimal	near-optimal	sub-optimal	id-optimal	pragmatic
Worst case complexity	$\mathcal{O}(\mathfrak{n}^2)$	$\mathcal{O}(\mathfrak{n}^2)$	$\mathcal{O}(\mathfrak{n}^2)$	$\mathcal{O}(\mathfrak{n})$	$\mathcal{O}(\mathfrak{n})$	$\mathcal{O}(\mathfrak{n})$
Coins leakage resilience	no	pre-send	post-send	no	chosen coins	no
Plain model (no ROM)	no	no	no	yes	yes	yes
PKC or less	no	no	yes	yes	yes	yes
Immediate decryption	no	no	no	no	yes	no
r-RECOVER security	no	yes	no	yes	no	yes
s-RECOVER security	no	yes	no	no	no	yes
ack. extractor	yes	yes	yes	yes	no	yes
cleanness extractor	yes	yes	yes	yes	yes	yes
category	BARK	ARCAD	ARCAD	BARK	ARCAD	ARCAD

Notations. We have two participants named Alice (A) and Bob (B). Whenever we talk about either one of the participants, we represent it as P, then $\overline{\mathsf{P}}$ refers to P's counterpart. We have two roles send and rec for sender and receiver respectively. We define $\overline{\mathsf{send}} = \mathsf{rec}$ and $\overline{\mathsf{rec}} = \mathsf{send}$. When the communication is unidirectional, the participants are called the *sender* S and the *receiver* R.

Structure of the paper. In Section 2, we revisit the preliminary notions from Durak-Vaudenay [6] and Alwen-Coretti-Dodis [1]. They all are essential to be able to follow our results. In Section 5, we present our implementation results with the figures comparing various protocols. In Section 3, we define a new notion named security awareness and build a protocol with regard to the notion. Finally, in Section 4, we define a new protocol called on-demand ratcheting with better performance than state-of the-art.

2 Preliminaries

2.1 ARCAD Definition and Security

In this section, we recall the DV model [6] and we slightly adapt to define asynchronous ratcheted communication with additional data denoted as ARCAD. That is, we consider message encryption instead of key agreement (BARK: bidirectional asynchronous ratcheted key agreement). The difference between BARK and ARCAD is the same as the difference between KEM and cryptosystems: pt is input to the Send instead of output. Additionally, we treat associated data ad to authenticate. Like DV [6][5], we adopt asymptotic security rather than exact security, for more readability. Adversaries and algorithms are probabilistic polynomially bounded (PPT) in terms of a parameter λ.

As we slightly change our direction from key exchange to encryption, we feel that it is essential to redefine the set of definitions from BARK for ARCAD. In this section, some of the definitions are marked with the reference [6]. It means that these definitions are unchanged except for possible necessary notation changes. The other definitions are straightforward adaptations to fit ARCAD.

[5] Proceedings version.

We try not to overload this section by redefining already existing terminology, hence, we let less essential definitions in Appendix A.2 for completeness.

Definition 1 (ARCAD). *An* asynchronous ratcheted communication with additional data (ARCAD) *consists of the following PPT algorithms:*

- $\mathsf{Setup}(1^\lambda) \xrightarrow{\$} \mathsf{pp}$*: This defines the common public parameters* pp.
- $\mathsf{Gen}(1^\lambda, \mathsf{pp}) \xrightarrow{\$} (\mathsf{sk}, \mathsf{pk})$*: This generates the secret key* sk *and the public key* pk *of a participant.*
- $\mathsf{Init}(1^\lambda, \mathsf{pp}, \mathsf{sk}_\mathsf{P}, \mathsf{pk}_{\overline{\mathsf{P}}}, \mathsf{P}) \to \mathsf{st}_\mathsf{P}$*: This sets up the initial state* st_P *of* P *given his secret key, and the public key of his counterpart.*
- $\mathsf{Send}(\mathsf{st}_\mathsf{P}, \mathsf{ad}, \mathsf{pt}) \xrightarrow{\$} (\mathsf{st}'_\mathsf{P}, \mathsf{ct})$*: it takes as input a plaintext* pt *and some associated data* ad *and produces a ciphertext* ct *along with an updated state* st'_P.
- $\mathsf{Receive}(\mathsf{st}_\mathsf{P}, \mathsf{ad}, \mathsf{ct}) \to (\mathsf{acc}, \mathsf{st}'_\mathsf{P}, \mathsf{pt})$*: it takes as input a ciphertext* ct *and some associated data* ad *and produces a plaintext* pt *with an updated state* st'_P *together with a flag* acc.[6]

An additional $\mathsf{Initall}(1^\lambda, \mathsf{pp}) \to (\mathsf{st}_\mathrm{A}, \mathsf{st}_\mathrm{B}, \mathsf{z})$ *algorithm, which returns the initial states of* A *and* B *as well as public information* z*, is defined as follows:*

$\mathsf{Initall}(1^\lambda, \mathsf{pp})$:
1: $\mathsf{Gen}(1^\lambda, \mathsf{pp}) \to (\mathsf{sk}_\mathrm{A}, \mathsf{pk}_\mathrm{A})$
2: $\mathsf{Gen}(1^\lambda, \mathsf{pp}) \to (\mathsf{sk}_\mathrm{B}, \mathsf{pk}_\mathrm{B})$
3: $\mathsf{st}_\mathrm{A} \leftarrow \mathsf{Init}(1^\lambda, \mathsf{pp}, \mathsf{sk}_\mathrm{A}, \mathsf{pk}_\mathrm{B}, \mathrm{A})$
4: $\mathsf{st}_\mathrm{B} \leftarrow \mathsf{Init}(1^\lambda, \mathsf{pp}, \mathsf{sk}_\mathrm{B}, \mathsf{pk}_\mathrm{A}, \mathrm{B})$
5: $z \leftarrow (\mathsf{pp}, \mathsf{pk}_\mathrm{A}, \mathsf{pk}_\mathrm{B})$
6: ***return*** $(\mathsf{st}_\mathrm{A}, \mathsf{st}_\mathrm{B}, z)$

Initall is defined for convenience as an initialization procedure for all games. None of our security games actually cares about how Initall is made from Gen and Init. This is nice because there is little to change to define a notion of "symmetric-cryptography-based ARCAD": we only need to define Initall. We will do so with liteARCAD and prove that it is a "secure ARCAD" by slight abuse of definition.

In what follows we only consider an ARCAD protocol.

For all global variables ν in the game such as $\mathsf{received}^\mathsf{P}_\mathsf{ct}$, st_P, or ct_P (which appear in Fig. 1 and Fig. 16, for instance), we denote the value of ν at time t by $\nu(\mathsf{t})$. The notion of *time* is participant-specific. It refers to the number of elementary operations he has done. We assume neither synchronization nor central clock. Time for two different participants can only be compared when they are run non-concurrently by an adversary in a game.

In addition to the RATCH oracle (in Fig. 16) which is used to ratchet (either to send or to receive), we define several other oracles (in Fig. 1): EXP_st to obtain the state of a participant; EXP_pt to obtain the last received message pt; CHALLENGE to send either the plaintext or a random string. We give a brief description of the DV security notions [6] as follows.

FORGE**-security:** It makes sure that there is no forgery, except trivial ones.

r-RECOVER**-security**[7]**:** If an adversary manages to forge (trivially or not) a message to one of the participants, then this participant can no longer accept genuine messages from his counterpart.

PREDICT**-security:** The adversary cannot guess the value $(\mathsf{ad}, \mathsf{ct})$ which will be output from the Send algorithm.

KIND**-security:** We omit this security notion which is specific to key exchange. Instead, we consider IND-CCA**-security** in a real-or-random style.

We define the ratcheting security with IND-CCA notion. Before defining it, we like to introduce a predicate called $\mathsf{C}_\mathsf{clean}$ as IND-CCA is relative to this predicate. $\mathsf{C}_\mathsf{clean}$ is defined with a logical combination of sub-predicates. For these helper predicates, we consider several of them as defined in the DV model [6]:

[6] In our work, we assume that $\mathsf{acc} = \mathsf{false}$ implies that $\mathsf{st}'_\mathsf{P} = \mathsf{st}_\mathsf{P}$ and $\mathsf{pt} = \perp$, i.e. the state is not updated when the reception fails. Other authors assume that $\mathsf{st}'_\mathsf{P} = \mathsf{pt} = \perp$, i.e. no further reception can be done.

[7] It is called RECOVER-security in DV [6]. We call it r-RECOVER because we will enrich it with an s-RECOVER notion in Section 3.1.

$\mathsf{C}_{\mathsf{leak}}$: $\mathsf{pt}_{\mathsf{test}}(\mathsf{t}_{\mathsf{test}})$ has no direct or indirect leakage (following Def. 34–35).
$\mathsf{C}^{\mathsf{S}}_{\mathsf{tforge}}$: (with $\mathsf{t} = \mathsf{trivial}$ or t void, and $\mathsf{S} = \{\mathsf{P}_{\mathsf{test}}\}$ or $\mathsf{S} = \{\mathsf{A}, \mathsf{B}\}$) no $\mathsf{P} \in \mathsf{S}$ received any t-forgery until having seen $(\mathsf{ad}, \mathsf{ct})_{\mathsf{test}}$ (following Def. 32–33).
$\mathsf{C}_{\mathsf{ratchet}}$: $(\mathsf{ad}, \mathsf{ct})_{\mathsf{test}}$ was sent by a participant $\mathsf{P}_{\mathsf{test}}$, received and accepted by $\overline{\mathsf{P}}_{\mathsf{test}}$, then some $(\mathsf{ad}', \mathsf{ct}')$ were sent by $\overline{\mathsf{P}}_{\mathsf{test}}$, received and accepted by $\mathsf{P}_{\mathsf{test}}$.

In Table 1, "*optimal*" security refers to $\mathsf{C}_{\mathsf{clean}} = \mathsf{C}_{\mathsf{leak}} \wedge \mathsf{C}^{\mathsf{P}_{\mathsf{test}}}_{\mathsf{trivial\ forge}}$ and "*sub-optimal*" security refers to $\mathsf{C}_{\mathsf{clean}} = \mathsf{C}_{\mathsf{leak}} \wedge \mathsf{C}^{\mathsf{A},\mathsf{B}}_{\mathsf{trivial\ forge}}$.

Game $\mathsf{IND\text{-}CCA}^{\mathcal{A}}_{\mathsf{b},\mathsf{C}_{\mathsf{clean}}}(1^\lambda)$
1: $\mathsf{Setup}(1^\lambda) \xrightarrow{\$} \mathsf{pp}$
2: $\mathsf{Initall}(1^\lambda, \mathsf{pp}) \xrightarrow{\$} (\mathsf{st}_A, \mathsf{st}_B, \mathsf{z})$
3: set all sent^*_* and $\mathsf{received}^*_*$ variables to $\emptyset$
4: set $\mathsf{t}_{\mathsf{test}}$ to $\perp$
5: $\mathsf{b}' \leftarrow \mathcal{A}^{\mathsf{RATCH},\mathsf{EXP}_{\mathsf{st}},\mathsf{EXP}_{\mathsf{pt}},\mathsf{CHALLENGE}}(\mathsf{z})$
6: **if** $\neg \mathsf{C}_{\mathsf{clean}}$ **then return** $\perp$
7: **return** b'

Oracle $\mathsf{EXP}_{\mathsf{st}}(\mathsf{P})$
1: **return** st_{P}

Oracle $\mathsf{CHALLENGE}(\mathsf{P}, \mathsf{ad}, \mathsf{pt})$
1: **if** $\mathsf{t}_{\mathsf{test}} \neq \perp$ **then return** $\perp$
2: **if** $\mathsf{b} = 0$ **then**
3: replace pt by a random string of same length
4: **end if**
5: $\mathsf{ct} \leftarrow \mathsf{RATCH}(\mathsf{P}, \text{“send”}, \mathsf{ad}, \mathsf{pt})$
6: $(\mathsf{t}, \mathsf{P}, \mathsf{ad}, \mathsf{pt}, \mathsf{ct})_{\mathsf{test}} \leftarrow (\mathsf{time}, \mathsf{P}, \mathsf{ad}, \mathsf{pt}, \mathsf{ct})$
7: **return** ct

Oracle $\mathsf{EXP}_{\mathsf{pt}}(\mathsf{P})$
1: **return** pt_{P}

Fig. 1: IND-CCA Game.
(Oracle RATCH is defined in Fig. 16)

Definition 2 ($\mathsf{C}_{\mathsf{clean}}$-IND-CCA **security).** *Let* $\mathsf{C}_{\mathsf{clean}}$ *be a cleanness predicate. We consider the* $\mathsf{IND\text{-}CCA}^{\mathcal{A}}_{\mathsf{b},\mathsf{C}_{\mathsf{clean}}}$ *game of Fig. 1. We say that the* ARCAD *is* $\mathsf{C}_{\mathsf{clean}}$-IND-CCA-secure *if for any PPT adversary, the advantage*

$$\mathsf{Adv}(\mathcal{A}) = \left|\Pr\left[\mathsf{IND\text{-}CCA}^{\mathcal{A}}_{0,\mathsf{C}_{\mathsf{clean}}}(1^\lambda) \to 1\right] - \Pr\left[\mathsf{IND\text{-}CCA}^{\mathcal{A}}_{1,\mathsf{C}_{\mathsf{clean}}}(1^\lambda) \to 1\right]\right|$$

of $\mathcal{A}$ *in* $\mathsf{IND\text{-}CCA}^{\mathcal{A}}_{\mathsf{b},\mathsf{C}_{\mathsf{clean}}}$ *security game is negligible.*

Definition 3 ($\mathsf{C}_{\mathsf{clean}}$-FORGE **security).** *Given a cleanness predicate* $\mathsf{C}_{\mathsf{clean}}$*, consider* $\mathsf{FORGE}^{\mathcal{A}}_{\mathsf{C}_{\mathsf{clean}}}$ *game in Fig. 2 associated to the adversary* $\mathcal{A}$*. Let the advantage of* $\mathcal{A}$ *be the probability that the game outputs 1. We say that* ARCAD *is* $\mathsf{C}_{\mathsf{clean}}$-FORGE-secure *if, for any PPT adversary, the advantage is negligible.*

In this definition, we added the notion of cleanness which determines if an attack is trivial or not. The original notion of FORGE security is equivalent to using the following $\mathsf{C}_{\mathsf{trivial}}$ predicate $\mathsf{C}_{\mathsf{clean}}$:

$\mathsf{C}_{\mathsf{trivial}}$: the last $(\mathsf{ad}, \mathsf{ct})$ message is not a trivial forgery (following Def. 33).

The purpose of this update in the definition is to allow us to easily define a weaker form of FORGE-security in Th. 19 and in Section 3.3.

Definition 4 (r-RECOVER **security [6]).** *Consider the* r-RECOVER$^{\mathcal{A}}$ *game in Fig. 2 associated to the adversary* $\mathcal{A}$*. Let the advantage of* $\mathcal{A}$ *in succeeding the game be* $\Pr(\mathsf{win} = 1)$*. We say that the* ARCAD *is* r-RECOVER-secure*, if for any PPT adversary, the advantage is negligible.*

Definition 5 (PREDICT **security [6]).** *Consider* $\mathsf{PREDICT}^{\mathcal{A}}(1^\lambda)$ *game in Fig. 2 associated to the adversary* $\mathcal{A}$*. Let the advantage of* $\mathcal{A}$ *in succeeding the game be the probability that* 1 *is returned. We say that the* ARCAD *is* PREDICT-secure*, if for any adversary limited to a polynomial number of queries, the advantage is negligible.*

Game $\mathsf{FORGE}^{\mathcal{A}}_{\mathsf{C}_{\mathsf{clean}}}(1^\lambda)$
1: $\mathsf{Setup}(1^\lambda) \xrightarrow{\$} \mathsf{pp}$
2: $\mathsf{Initall}(1^\lambda, \mathsf{pp}) \xrightarrow{\$} (\mathsf{st}_\mathrm{A}, \mathsf{st}_\mathrm{B}, z)$
3: $(\mathsf{P}, \mathsf{ad}, \mathsf{ct}) \leftarrow \mathcal{A}^{\mathsf{RATCH},\mathsf{EXP}_{\mathsf{st}},\mathsf{EXP}_{\mathsf{pt}}}(z)$
4: $\mathsf{RATCH}(\mathsf{P}, \text{“rec”}, \mathsf{ad}, \mathsf{ct}) \to \mathsf{acc}$
5: **if** $\mathsf{acc} = \mathsf{false}$ **then return** 0
6: **if** $\neg \mathsf{C}_{\mathsf{clean}}$ **then return** 0
7: **if** $(\mathsf{ad}, \mathsf{ct})$ is not a forgery (Def. 32) for P **then return** 0
8: **return** 1

Game $\mathsf{r\text{-}RECOVER}^{\mathcal{A}}(1^\lambda)$
1: $\mathsf{win} \leftarrow 0$
2: $\mathsf{Setup}(1^\lambda) \xrightarrow{\$} \mathsf{pp}$
3: $\mathsf{Initall}(1^\lambda, \mathsf{pp}) \xrightarrow{\$} (\mathsf{st}_\mathrm{A}, \mathsf{st}_\mathrm{B}, z)$
4: set all sent^*_* and $\mathsf{received}^*_*$ variables to $\emptyset$
5: $\mathsf{P} \leftarrow \mathcal{A}^{\mathsf{RATCH},\mathsf{EXP}_{\mathsf{st}},\mathsf{EXP}_{\mathsf{pt}}}(z)$
6: **if** we can parse $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}} = (\mathsf{seq}_1, (\mathsf{ad}, \mathsf{ct}), \mathsf{seq}_2)$ and $\overline{\mathsf{sent}^{\mathsf{P}}_{\mathsf{ct}}} = (\mathsf{seq}_3, (\mathsf{ad}, \mathsf{ct}), \mathsf{seq}_4)$ with $\mathsf{seq}_1 \neq \mathsf{seq}_3$ (where $(\mathsf{ad}, \mathsf{ct})$ is a single message and all seq_i are finite sequences of single messages) **then** $\mathsf{win} \leftarrow 1$
7: **return** win

Game $\mathsf{PREDICT}^{\mathcal{A}}(1^\lambda)$
1: $\mathsf{Setup}(1^\lambda) \xrightarrow{\$} \mathsf{pp}$
2: $\mathsf{Initall}(1^\lambda, \mathsf{pp}) \xrightarrow{\$} (\mathsf{st}_\mathrm{A}, \mathsf{st}_\mathrm{B}, z)$
3: $(\mathsf{P}, \mathsf{ad}, \mathsf{pt}) \leftarrow \mathcal{A}^{\mathsf{RATCH},\mathsf{EXP}_{\mathsf{st}},\mathsf{EXP}_{\mathsf{pt}}}(z)$
4: $\mathsf{RATCH}(\mathsf{P}, \text{“send”}, \mathsf{ad}, \mathsf{pt}) \to \mathsf{ct}$
5: **if** $(\mathsf{ad}, \mathsf{ct}) \in \overline{\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}}$ **then return** 1
6: **return** 0

Fig. 2: FORGE, r-RECOVER, and PREDICT Games.
(Oracles RATCH, $\mathsf{EXP}_{\mathsf{st}}$, $\mathsf{EXP}_{\mathsf{pt}}$ are defined in Fig. 1 and Fig. 16.)

An instantiation of ARCAD*:* $\mathsf{ARCAD}_{\mathsf{DV}}$. With slight modifications, we transform the DV protocol [6] into an ARCAD that we call $\mathsf{ARCAD}_{\mathsf{DV}}$. $\mathsf{ARCAD}_{\mathsf{DV}}$ is presented in Appendix A.2.

2.2 The Epoch Notion in Secure Communication

We define the epochs according to the work done by Alwen et al. [1] but in a different way.[8] Epochs are useful to designate the sequence of messages, as both participants may not see exactly the same. We will use epoch numbers in the design of our hybrid scheme for on-demand ratcheting in Section 4.2.

Epochs are a set of consecutive messages going in the same direction. An epoch is identified by an integer counter e. Each message is assigned one epoch counter e_m. Hence, the epochs are non-intersecting. For convenience, each participant P keeps the epoch value $\mathsf{e}^{\mathsf{P}}_{\mathsf{send}}$ of the last sent message and the epoch value $\mathsf{e}^{\mathsf{P}}_{\mathsf{rec}}$ of the last received message. They are used to assign an epoch to a message to be sent.

Definition 6 (Epoch). *Epochs are non-intersecting sets of messages which are defined by an integer. Let* $\mathsf{e}^{\mathsf{P}}_{\mathsf{rec}}$ *(resp.* $\mathsf{e}^{\mathsf{P}}_{\mathsf{send}}$*) be the epoch of the last received (resp. sent) message by* P*. At the very beginning of the protocol, there is no last message. Therefore we define* $\mathsf{e}^{\mathsf{P}}_{\mathsf{send}}$ *and* $\mathsf{e}^{\mathsf{P}}_{\mathsf{rec}}$ *differently. For the participant* A*,* $\mathsf{e}^{\mathsf{A}}_{\mathsf{rec}} = -1$ *and* $\mathsf{e}^{\mathsf{A}}_{\mathsf{send}} = 0$*. For the participant* B*,* $\mathsf{e}^{\mathsf{B}}_{\mathsf{send}} = -1$ *and* $\mathsf{e}^{\mathsf{B}}_{\mathsf{rec}} = 0$*. The procedure to assign an epoch* e_m *to a new sent message follows the rule described next:*
If $\mathsf{e}^{\mathsf{P}}_{\mathsf{rec}} < \mathsf{e}^{\mathsf{P}}_{\mathsf{send}}$*, then the message is put in the epoch* $\mathsf{e}_\mathsf{m} = \mathsf{e}^{\mathsf{P}}_{\mathsf{send}}$*. Otherwise, it is put in epoch* $\mathsf{e}_\mathsf{m} = \mathsf{e}^{\mathsf{P}}_{\mathsf{rec}} + 1$.

Let $\mathsf{e}_\mathsf{P} = \max\{\mathsf{e}^{\mathsf{P}}_{\mathsf{rec}}, \mathsf{e}^{\mathsf{P}}_{\mathsf{send}}\}$. Let $\mathsf{b}_\mathsf{A} = 0$ and $\mathsf{b}_\mathsf{B} = 1$. We have

$$\mathsf{e}^{\mathsf{P}}_{\mathsf{send}} = \begin{cases} \mathsf{e}_\mathsf{P} & \text{if } \mathsf{e}_\mathsf{P} \bmod 2 = \mathsf{b}_\mathsf{P} \\ \mathsf{e}_\mathsf{P} - 1 & \text{otherwise} \end{cases} \qquad \mathsf{e}^{\mathsf{P}}_{\mathsf{rec}} = \begin{cases} \mathsf{e}_\mathsf{P} & \text{if } \mathsf{e}_\mathsf{P} \bmod 2 \neq \mathsf{b}_\mathsf{P} \\ \mathsf{e}_\mathsf{P} - 1 & \text{otherwise} \end{cases}$$

Therefore, it is equivalent to maintain $(\mathsf{e}^{\mathsf{P}}_{\mathsf{rec}}, \mathsf{e}^{\mathsf{P}}_{\mathsf{send}})$ or e_P. The procedure to manage e_P and e_m is described by Alwen et al. [1].

[8] The notion of epoch appeared in Poettering-Rösler [9] before.

We depict a sample of a bidirectional communication in Fig. 3. The figure shows the epoch number assignments based on our definitions.

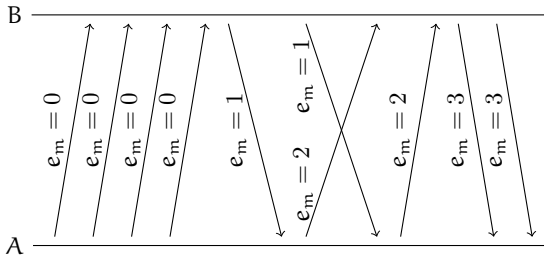

Fig. 3: Bidirectional Exchanges between A and B with Epoch Numbers.

Property 7. From the epoch definition, we have the following properties.

1. At all times, $|\mathsf{e}^{\mathsf{P}}_{\mathsf{send}} - \mathsf{e}^{\mathsf{P}}_{\mathsf{rec}}| \leqslant 1$.
2. The epoch numbers for a unidirectional stream of messages are even if the sender is the participant A and it is odd if the sender is B.
3. A new epoch for a participant P always starts with a $\mathsf{RATCH}(\mathsf{P}, \text{“}\mathsf{send}\text{”})$ calls and ends with $\mathsf{RATCH}(\mathsf{P}, \text{“}\mathsf{rec}\text{”})$ calls.
4. If a participant P accepts a message corresponding to an epoch number e_m, then $\mathsf{e}^{\mathsf{P}}_{\mathsf{send}} \geqslant \mathsf{e}_\mathsf{m} + 1$.

We will use a counter c for each epoch e. We will use the order on (e, c) pairs defined by

$$(\mathsf{e}, \mathsf{c}) < (\mathsf{e}', \mathsf{c}') \iff (\mathsf{e} < \mathsf{e}' \vee (\mathsf{e} = \mathsf{e}' \wedge \mathsf{c} < \mathsf{c}'))$$

3 Security Awareness

3.1 s-RECOVER Security

We gave the DV r-RECOVER security definition [6] in Def. 4. It is an important notion to capture that P cannot accept a genuine ct from $\overline{\mathsf{P}}$ after P receives a forgery. However, r-RECOVER-security does not capture the fact that when it is $\overline{\mathsf{P}}$ who receives a forgery, P could still accept messages which come from $\overline{\mathsf{P}}$. We strengthen r-RECOVER security with another definition called s-RECOVER.

Definition 8 (s-RECOVER security). *In the* $\mathsf{s\text{-}RECOVER}^{\mathcal{A}}$ *game in Fig. 4 with the adversary* $\mathcal{A}$*, we let the advantage of* $\mathcal{A}$ *in succeeding the game be* $\Pr(\mathsf{win} = 1)$*. We say that the* ARCAD *is* s-RECOVER-secure*, if for any PPT adversary, the advantage is negligible.*

Ideally, what we want from the protocol is that participants can detect forgeries by realizing that they are no longer able to communicate to each other. We cannot prevent impersonation to happen after a state exposure but we want to make sure that the normal exchange between the participants is cut. Hence, if a participant eventually receives a genuine message, he should feel safe that no forgeries happened. Contrarily, detecting a communication cut requires an action from the participants, such as restoring communication using a super hybrid structure, as we will suggest in Section 4.2.

We directly obtain the following useful result:

Lemma 9. *If an* ARCAD *is* r-RECOVER, s-RECOVER*, and* PREDICT *secure, whenever* P *receives a genuine message from* $\overline{\mathsf{P}}$ *(i.e., an* $(\mathsf{ad}, \mathsf{ct})$ *pair sent by* $\overline{\mathsf{P}}$ *is accepted by* P*),* P *is in a matching status (following Def. 30), except with negligible probability.*

Game $\mathsf{s\text{-}RECOVER}^{\mathcal{A}}(1^\lambda)$
1: $\mathsf{win} \leftarrow 0$
2: $\mathsf{Setup}(1^\lambda) \xrightarrow{\$} \mathsf{pp}$
3: $\mathsf{Initall}(1^\lambda, \mathsf{pp}) \xrightarrow{\$} (\mathsf{st}_A, \mathsf{st}_B, z)$
4: set all sent^*_* and $\mathsf{received}^*_*$ variables to $\emptyset$
5: $\mathsf{P} \leftarrow \mathcal{A}^{\mathsf{RATCH},\mathsf{EXP}_{\mathsf{st}},\mathsf{EXP}_{\mathsf{pt}}}(z)$
6: **if** $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}$ is a prefix of $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$ **then**
7: set $\bar{t}$ to the time when $\overline{\mathsf{P}}$ sent the last message in $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}$
8: **if** $\mathsf{received}^{\overline{\mathsf{P}}}_{\mathsf{ct}}(\bar{t})$ is not a prefix of $\mathsf{sent}^{\mathsf{P}}_{\mathsf{ct}}$ **then** $\mathsf{win} \leftarrow 1$
9: **end if**
10: **return** win

Fig. 4: s-RECOVER Security Game.
(RATCH and EXP oracles are defined in Fig. 16 and Fig. 1.)

Proof. Let Γ be a game. Let $(\mathsf{ad}, \mathsf{ct})$ be a message which was sent by $\overline{\mathsf{P}}$ then received and accepted by P.

We consider an r-RECOVER adversary $\mathcal{A}$ which simulates Γ until P receives $(\mathsf{ad}, \mathsf{ct})$, and output P. We can parse $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}} = (\mathsf{seq}_1, (\mathsf{ad}, \mathsf{ct}), \mathsf{seq}_2)$ with seq_2 empty and $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}} = (\mathsf{seq}_3, (\mathsf{ad}, \mathsf{ct}), \mathsf{seq}_4)$. Due to r-RECOVER security, we have $\mathsf{seq}_1 = \mathsf{seq}_3$, but with negligible cases. Hence, $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}$ is prefix of $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$, except with negligible probability.

We now let the same $\mathcal{A}$ play the s-RECOVER security game. Due to s-RECOVER security, since $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}$ is prefix of $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$, then $\mathsf{received}^{\overline{\mathsf{P}}}_{\mathsf{ct}}(\bar{t})$ is a prefix of $\mathsf{sent}^{\mathsf{P}}_{\mathsf{ct}}$, but with negligible probability. Due to PREDICT-security, no message arrives before it is sent. Hence, P is in a matching status, except with negligible probability. □

Our notion of RECOVER-security and forgery is quite strong in the sense that it focuses on the ciphertext. Some protocols such as JMM [8] focus on the plaintext. In JMM, ct includes some encrypted data and some signature but only the encrypted data is hashed. Hence, an adversary can replace the signature by another signature after exposure of the signing key. It can be seen as not so important because it must sign the same content. However, the signature has a key update and the adversary can make the receiver update to any verifying key to desynchronize, then re-synchronize at will. Consequently, the JMM protocol does not offer RECOVER security as we defined it. Contrarily, PR [9] hashes $(\mathsf{ad}, \mathsf{ct})$ but does not use it in the next ad or to compute the next ct. Thus, PR has no RECOVER security, either.[9] We can fix those protocols by transforming them with our chain construction in Section 3.3, thanks to Lemma 15.

3.2 Security Awareness

To have a security-awareness notion, we want r-RECOVER, s-RECOVER, and PREDICT security[10], we want to have an acknowledgment extractor (to be aware of message delivery), and we want to have a cleanness extractor (to be aware of the cleanness of every message, if not subject to trivial exposure). The last two notions are defined below. This means that on the one hand, impersonations are eventually discovered, and on the other hand, by assuming that no impersonation occurs and assuming that exposures are known, a participant P knows exactly which messages are safe, at least after one round-trip occurred.

[9] More precisely, in PR, if A is exposed then issues a message ct, the adversary can actually forge a ciphertext ct′ transporting the same pk and vfk and deliver it to B in a way which makes B accept. If A issues a new message ct″, delivering ct″ to B will pass the signature verification. The decryption following-up may fail, except if the kuKEM encryption scheme taking care of encryption does not check consistency, which is the case in the proposed one [9, Fig. 3, eprint version]. Therefore, ct″ may be accepted by B so PR is not r-RECOVER secure. The same holds for s-RECOVER security.

[10] We want those security notions to be able to apply Lemma 9 and be aware of matching status.

Definition 10 (Security-awareness). *A protocol is* $\mathsf{C_{clean}}$-security-aware *if*

- *it is* r-RECOVER, s-RECOVER, *and* PREDICT-*secure;*
- *there is an acknowledgment extractor (Def. 12);*
- *there is a cleanness extractor for* $\mathsf{C_{clean}}$ *(Def. 13).*

To make participants aware of the security status of any (challenge) message, they need to know the history of exposures, they need to be able to reconstruct the history of RATCH calls from their own view, and they need to be able to evaluate the $\mathsf{C_{clean}}$ predicate. Thankfully, the $\mathsf{C_{clean}}$ predicates that we consider only depend on these histories. We first formally define the notion of transcript.

Definition 11 (Transcript). *In a game, for a participant* P*, we define the* transcript *of* P *as the chronological sequence* $\mathsf{T_P}$ *of all* $(\mathsf{oracle}, \mathsf{extra})$ *pairs involving* P *where each pair represents an oracle call to* oracle *with* P *as input (i.e. either* $\mathsf{RATCH}(\mathsf{P}, \text{“rec”}, ., .)$*,* $\mathsf{RATCH}(\mathsf{P}, \text{“send”}, ., .)$*,* $\mathsf{EXP_{pt}}(\mathsf{P})$*,* $\mathsf{EXP_{st}}(\mathsf{P})$*, or* $\mathsf{CHALLENGE}(\mathsf{P})$*), except the unsuccessful* RATCH *calls which are omitted. For each pair with a* RATCH *or* CHALLENGE *oracle,* extra *specifies the role (*“send” *or* “rec”*) and the message* $(\mathsf{ad}, \mathsf{ct})$ *of the oracle call. For other pairs,* $\mathsf{extra} = \bot$*.*

The partial transcript *of* P *up to time* t *is the prefix* $\mathsf{T_P}(\mathsf{t})$ *of* $\mathsf{T_P}$ *of all oracle calls until time* t*. The* RATCH-transcript *of* P *is the list* $\mathsf{T_P^{RATCH}}$ *of all* extra *elements in* $\mathsf{T_P}$ *which are not* $\bot$ *(i.e. it only includes* RATCH*/*CHALLENGE *calls). Similarly, the* partial RATCH-transcript *of* P *up to time* t *is the list* $\mathsf{T_P^{RATCH}}(\mathsf{t})$ *of* extra *elements in* $\mathsf{T_P}(\mathsf{t})$ *which are not* $\bot$*.*

Next, we formalize that a participant can be aware of which of his messages were received by his counterpart.

Definition 12 (Acknowledgment extractor). *We consider a game* Γ *where the transcript* $\mathsf{T_P}$ *is formed for a participant* P*. Given a message* $(\mathsf{ad}, \mathsf{ct})$ *successfully received by* P *at time* t *and which was sent by* $\overline{\mathsf{P}}$ *at time* $\overline{\mathsf{t}}$*, we let* $(\mathsf{ad}', \mathsf{ct}')$ *be the last message successfully received by* $\overline{\mathsf{P}}$ *before time* $\overline{\mathsf{t}}$*. (If there is no such message, we set it to* $\bot$*.)*

An acknowledgment extractor *is an efficient function* f *such that* $\mathsf{f}(\mathsf{T_P^{RATCH}}(\mathsf{t})) = (\mathsf{ad}', \mathsf{ct}')$ *for any time* t *when* P *is in a matching status (Def. 30).*

Given this extractor, P can iteratively reconstruct the entire flow of messages, and which messages crossed each other during transmission.

We formalize awareness of a participant for the safety of each message.

Definition 13 (Cleanness extractor). *We consider a game* Γ *where the transcript* $\mathsf{T_P}$ *is formed for a participant* P*. Let* t *be a time for* P *and* $\overline{\mathsf{t}}$ *be a time for* $\overline{\mathsf{P}}$*. Let* $\mathsf{T_P}(\mathsf{t})$ *and* $\mathsf{T}_{\overline{\mathsf{P}}}(\overline{\mathsf{t}})$ *be the partial transcripts at those time. We say that there is a* cleanness extractor for $\mathsf{C_{clean}}$ *if there is an efficient function* g *such that* $\mathsf{g}(\mathsf{T_P}(\mathsf{t}), \mathsf{T}_{\overline{\mathsf{P}}}(\overline{\mathsf{t}}))$ *has the following properties: if there is one* CHALLENGE *in the* $\mathsf{T_P}(\mathsf{t})$ *transcript and, either* P *received* $(\mathsf{ad_{test}}, \mathsf{ct_{test}})$ *or there is a round trip* $\mathsf{P} \to \overline{\mathsf{P}} \to \mathsf{P}$ *starting with* P *sending* $(\mathsf{ad_{test}}, \mathsf{ct_{test}})$ *to* $\overline{\mathsf{P}}$*, then* $\mathsf{g}(\mathsf{T_P}(\mathsf{t}), \mathsf{T}_{\overline{\mathsf{P}}}(\overline{\mathsf{t}})) = \mathsf{C_{clean}}(\Gamma)$*. Otherwise,* $\mathsf{g}(\mathsf{T_P}(\mathsf{t}), \mathsf{T}_{\overline{\mathsf{P}}}(\overline{\mathsf{t}})) = \bot$*.*

The function g is able to predict whether the game is “clean” for any challenge message. The case with an incomplete round trip $\mathsf{P} \to \overline{\mathsf{P}} \to \mathsf{P}$ starting with P sending $(\mathsf{ad_{test}}, \mathsf{ct_{test}})$ to $\overline{\mathsf{P}}$ is when the tested message was sent but somehow never acknowledged for the reception. If the message never arrived, we cannot say for sure if the game is clean because the counterpart may later either receive it and make the game clean or have a state exposure and make the game not clean. In other cases, the cleanness can be determined for sure.

3.3 Strongly Secure ARCAD with Security Awareness

In this section, we take a secure ARCAD (it could be $\mathsf{ARCAD_{DV}}$ (in Appendix A.2) or the hybrid one defined in Section 4) which we denote by ARCAD_0 and we transform it into another secure ARCAD which we denote by $\mathsf{ARCAD}_1 = \mathsf{chain}(\mathsf{ARCAD}_0)$, that is *security aware*. We achieve security awareness by keeping some hashes in the states of participants. The intuitive way to build it is

to make chains of hash of ciphertexts (like a blockchain) which will be sent and received and to associate each message to the digest of the chain. This enables a participant P to acknowledge its counterpart about received messages whenever P sends a new message.

We define a tuple $(\mathsf{Hsent}, \mathsf{Hreceived}, \mathsf{Asent}, \mathsf{Areceived})$ and store it in the state of a participant. Hsent is the hash of all sent ciphertexts. It is computed by the sender and delivered to the counterpart along with ct. It is updated with hashing key hk and the old Hsent every time a new Send operation is called. Likewise, $\mathsf{Hreceived}$ is the hash of all received ciphertexts. It is computed with hk and the last stored $\mathsf{Hreceived}$ by the receiver upon receiving a message. It is updated every time a new $\mathsf{Receive}$ operation is run.

$\mathsf{Areceived}$ is a counter of received messages which need to be reported when the next Send operation is run. For each Send operation, the protocol attaches to ct the last $\mathsf{Hreceived}$ to acknowledge for received messages and reset $\mathsf{Areceived}$ to 0. $\mathsf{Areceived}$ is incremented by each $\mathsf{Receive}$.

Asent is a *list of the hash* of sent ciphertexts which are waiting for an acknowledgment. Basically, it is initialized to an empty array in the beginning and whenever a new Hsent is computed, it is accumulated in this array. The purpose of such a list is to keep track of the sent messages for which the sender expects an acknowledgment. More precisely, when the participant P keeps its list of sent ciphertexts in Asent, the counterpart $\overline{\mathsf{P}}$ keeps a counter $\mathsf{Areceived}$ telling that an acknowledgment is needed. Remember that $\overline{\mathsf{P}}$ sends $\mathsf{Hreceived}$ back to the participant P to acknowledge him about received messages. As soon as $\overline{\mathsf{P}}$ acknowledges, P deletes the hash of the acknowledged ciphertexts from Asent.

The principle of our construction is that if an adversary starts to impersonate a participant after exposure, there is a fork in the list of message chains which is viewed by both participants and those chains can never merge again without making a collision.

We give our security aware protocol on Fig. 5. The security of the protocol is proved with the following lemmas.

$\mathsf{ARCAD.Setup}(1^\lambda)$
1: $\mathsf{ARCAD}_0.\mathsf{Setup}(1^\lambda) \xrightarrow{\$} \mathsf{pp}_0$
2: $\mathsf{H.Gen}(1^\lambda) \xrightarrow{\$} \mathsf{hk}$
3: $\mathsf{pp} \leftarrow (\mathsf{hk}, \mathsf{pp}_0)$
4: **return** pp

$\mathsf{ARCAD.Gen} = \mathsf{ARCAD}_0.\mathsf{Gen}$

$\mathsf{ARCAD.Init}(1^\lambda, \mathsf{pp}, \mathsf{sk}_\mathsf{P}, \mathsf{pk}_{\overline{\mathsf{P}}}, \mathsf{P})$
1: parse $\mathsf{pp} = (\mathsf{hk}, \mathsf{pp}_0)$
2: $\mathsf{ARCAD}_0.\mathsf{Init}(1^\lambda, \mathsf{pp}_0, \mathsf{sk}_\mathsf{P}, \mathsf{pk}_{\overline{\mathsf{P}}}, \mathsf{P}) \xrightarrow{\$} \mathsf{st}'_\mathsf{P}$
3: $\mathsf{Hsent}, \mathsf{Hreceived} \leftarrow \bot$
4: $\mathsf{Asent} \leftarrow []$, $\mathsf{Areceived} \leftarrow 0$
5: $\mathsf{st}_\mathsf{P} \leftarrow (\mathsf{st}'_\mathsf{P}, \mathsf{hk}, \mathsf{Hsent}, \mathsf{Hreceived}, \mathsf{Asent}, \mathsf{Areceived})$
6: **return** st_P

$\mathsf{ARCAD.Send}(\mathsf{st}_\mathsf{P}, \mathsf{ad}, \mathsf{pt})$
1: parse st_P as $(\mathsf{st}'_\mathsf{P}, \mathsf{hk}, \mathsf{Hsent}, \mathsf{Hreceived}, \mathsf{Asent}, \mathsf{Areceived})$
2: **if** $\mathsf{Areceived} = 0$ **then** $\mathsf{ack} \leftarrow \bot$ **else** $\mathsf{ack} \leftarrow \mathsf{Hreceived}$
3: $\mathsf{ad}' \leftarrow (\mathsf{ad}, \mathsf{Hsent}, \mathsf{ack})$
4: $\mathsf{ARCAD}_0.\mathsf{Send}(\mathsf{st}'_\mathsf{P}, \mathsf{ad}', \mathsf{pt}) \xrightarrow{\$} (\mathsf{st}'_\mathsf{P}, \mathsf{ct}')$
5: $\mathsf{ct} \leftarrow (\mathsf{ct}', \mathsf{Hsent}, \mathsf{ack})$
6: $\mathsf{Areceived} \leftarrow 0$
7: $\mathsf{Hsent} \leftarrow \mathsf{H.Eval}(\mathsf{hk}, \mathsf{Hsent}, \mathsf{ad}, \mathsf{ct})$
8: $\mathsf{Asent} \leftarrow (\mathsf{Asent}, \mathsf{Hsent})$
9: $\mathsf{st}_\mathsf{P} \leftarrow (\mathsf{st}'_\mathsf{P}, \mathsf{hk}, \mathsf{Hsent}, \mathsf{Hreceived}, \mathsf{Asent}, \mathsf{Areceived})$
10: **return** $(\mathsf{st}_\mathsf{P}, \mathsf{ct})$

$\mathsf{ARCAD.Receive}(\mathsf{st}_\mathsf{P}, \mathsf{ad}, \mathsf{ct})$
1: parse st_P as $(\mathsf{st}'_\mathsf{P}, \mathsf{hk}, \mathsf{Hsent}, \mathsf{Hreceived}, \mathsf{Asent}, \mathsf{Areceived})$
2: parse ct as $(\mathsf{ct}', \mathsf{h}, \mathsf{ack})$
3: **if** $\mathsf{h} \neq \mathsf{Hreceived}$ or $\mathsf{ack} \notin \{\bot\} \cup \mathsf{Asent}$ **then**
4: **return** $(\mathsf{false}, \mathsf{st}_\mathsf{P}, \bot)$
5: **end if**
6: $\mathsf{ad}' \leftarrow (\mathsf{ad}, \mathsf{h}, \mathsf{ack})$
7: $\mathsf{ARCAD}_0.\mathsf{Receive}(\mathsf{st}'_\mathsf{P}, \mathsf{ad}', \mathsf{ct}') \rightarrow (\mathsf{acc}, \mathsf{st}'_\mathsf{P}, \mathsf{pt}')$
8: **if** acc **then**
9: $\mathsf{Hreceived} \leftarrow \mathsf{H.Eval}(\mathsf{hk}, \mathsf{Hreceived}, \mathsf{ad}, \mathsf{ct})$
10: $\mathsf{Areceived} \leftarrow \mathsf{Areceived} + 1$
11: **if** $\mathsf{ack} \neq \bot$ **then** remove in Asent all elements of Asent until ack (included)
12: $\mathsf{st}_\mathsf{P} \leftarrow (\mathsf{st}'_\mathsf{P}, \mathsf{hk}, \mathsf{Hsent}, \mathsf{Hreceived}, \mathsf{Asent}, \mathsf{Areceived})$
13: **end if**
14: **return** $(\mathsf{acc}, \mathsf{st}_\mathsf{P}, \mathsf{pt}')$

Fig. 5: Our Security-Aware $\mathsf{ARCAD}_1 = \mathsf{chain}(\mathsf{ARCAD}_0)$ Protocol.

Theorem 14. *If* ARCAD_0 *is correct, then* $\mathsf{chain}(\mathsf{ARCAD}_0)$ *is correct.*

The proof is straightforward.

Lemma 15. *If* H *is collision-resistant,* $\mathsf{chain}(\mathsf{ARCAD}_0)$ *is* $\mathsf{s\text{-}RECOVER}$, $\mathsf{r\text{-}RECOVER}$*-secure.*

Proof. All $(\mathsf{ad}, \mathsf{ct})$ messages seen by one participant P in one direction (send or receive) are chained by hashing. Hence, if $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}} = (\mathsf{seq}_1, (\mathsf{ad}, \mathsf{ct}), \mathsf{seq}_2)$, the $(\mathsf{ad}, \mathsf{ct})$ message includes (in the second field of ct) the hash h of seq_1. If $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}} = (\mathsf{seq}_3, (\mathsf{ad}, \mathsf{ct}), \mathsf{seq}_4)$, the $(\mathsf{ad}, \mathsf{ct})$ message includes the hash h of seq_3. If H is collision-resistant, then $\mathsf{seq}_1 \neq \mathsf{seq}_3$ with negligible probability. Hence, we have $\mathsf{r\text{-}RECOVER}$ security.

Additionally, all genuine $(\mathsf{ad}, \mathsf{ct})$ messages include (in the third field of ct) the hash ack of messages which are received by the counterpart. This list must be approved by P, thus it must match the list of hash of messages that P sent. Hence, if $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}$ is prefix of $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$ and $\bar{\mathsf{t}}$ is the time when $\overline{\mathsf{P}}$ sent the last message in $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}$, then this message includes the hash of $\mathsf{received}^{\overline{\mathsf{P}}}_{\mathsf{ct}}(\bar{\mathsf{t}})$ which must be a hash of a prefix of $\mathsf{sent}^{\mathsf{P}}_{\mathsf{ct}}$. Thus, unless there is a collision in the hash function, $\mathsf{received}^{\overline{\mathsf{P}}}_{\mathsf{ct}}(\bar{\mathsf{t}})$ is a prefix of $\mathsf{sent}^{\mathsf{P}}_{\mathsf{ct}}$ and we have $\mathsf{s\text{-}RECOVER}$ security. □

Lemma 16. $\mathsf{chain}(\mathsf{ARCAD}_0)$ *has an acknowledgment extractor.*

Proof. Let $(\mathsf{ad}, \mathsf{ct})$ be a message sent by $\overline{\mathsf{P}}$ to P in a matching status. Let $(\mathsf{ad}', \mathsf{ct}')$ be the last message received by $\overline{\mathsf{P}}$ before sending $(\mathsf{ad}, \mathsf{ct})$. Due to the protocol, ct includes the value of $\mathsf{Hreceived}$ after receiving $(\mathsf{ad}', \mathsf{ct}')$. Since this message is from P, P recognizes this hash $\mathsf{Hreceived} = \mathsf{Hsent}$ from Asent. Both $(\mathsf{ad}', \mathsf{ct}')$ and this hash can be computed from $\mathsf{T}^{\mathsf{RATCH}}_{\mathsf{P}}(\mathsf{t})$. Hence, $\mathsf{chain}(\mathsf{ARCAD}_0)$ has an extractor. □

Lemma 17. $\mathsf{chain}(\mathsf{ARCAD}_0)$ *has a cleanness extractor for* $\mathsf{C}_{\mathsf{leak}}$, $\mathsf{C}^{\mathsf{S}}_{\mathsf{tforge}}$ *(*$\mathsf{t} = \mathsf{trivial}$ *or void,* $\mathsf{S} = \mathsf{P}_{\mathsf{test}}$ *or* $\mathsf{S} = \{\mathsf{A}, \mathsf{B}\}$*),* $\mathsf{C}_{\mathsf{ratchet}}$, *and* $\mathsf{C}_{\mathsf{noexp}}$.

Hence, there is an extractor for all cleanness predicates which we considered.

Proof. For $\mathsf{C}_{\mathsf{noexp}}$ and $\mathsf{C}_{\mathsf{ratchet}}$, we just look at the appropriate oracle calls. For $\mathsf{C}^{\mathsf{S}}_{\mathsf{tforge}}$, we can directly see from transcripts where the forgeries are and we can determine if they are trivial or not. For $\mathsf{C}_{\mathsf{leak}}$, we can easily inspect all cases of direct and indirect leakage and see that they can be deduced from the available transcripts. □

The following result is trivial.

Lemma 18. *If* ARCAD_0 *is* $\mathsf{PREDICT}$*-secure, then* $\mathsf{chain}(\mathsf{ARCAD}_0)$ *is* $\mathsf{PREDICT}$*-secure.*

Consequently, if ARCAD_0 is $\mathsf{PREDICT}$-secure, $\mathsf{chain}(\mathsf{ARCAD}_0)$ is security-aware.

In Section 4.5, we will extend these results for our hybrid ratchet-on demand construction.

4 On-Demand Ratcheting

In this section, we define a bidirectional secure communication messaging protocol with *hybrid on-demand* ratcheting. The aim is to design such a protocol to integrate two ratcheting protocols with different security levels: a strongly secure protocol using public-key cryptography and a weaker but much more efficient protocol with symmetric key primitive. The core of the protocol is to use the weak protocol with frequent exchanges and to use the strong one on demand by the sending participant. Hence, we build a more efficient protocol with on-demand ratcheting. Yet, it comes with a security drawback. Even though the security for the former is to provide the post-compromise security, we secure part of the communication only with the forward secure protocol.

The sender uses a flag to tell which level of security the communication will have and apply ratcheting with public-key cryptography or the lighter primitives such as the $\mathsf{liteARCAD}$ protocol given in Section 4.1. The flag is set in the ad input and it is denoted as $\mathsf{ad.flag}$. We call the strong protocol as $\mathsf{ARCAD}_{\mathsf{main}}$ and the weak one as $\mathsf{ARCAD}_{\mathsf{sub}}$. Ideally, the time to set the flag

for specific security can be decided during the deployment of the application using the protocol. This choice may also be left to the users who can decide based on the confidentiality-level of their communication. The more often the protocol turns the flag on, the more secure is the hybrid on-demand protocol. If we do it for every message exchange, then we obtain $\mathsf{ARCAD}_{\mathsf{main}}$ without $\mathsf{ARCAD}_{\mathsf{sub}}$. If we do it for no message exchange, then we obtain $\mathsf{ARCAD}_{\mathsf{sub}}$. The evolution of states is depicted on Fig. 6. The details are explained shortly in the following sections.

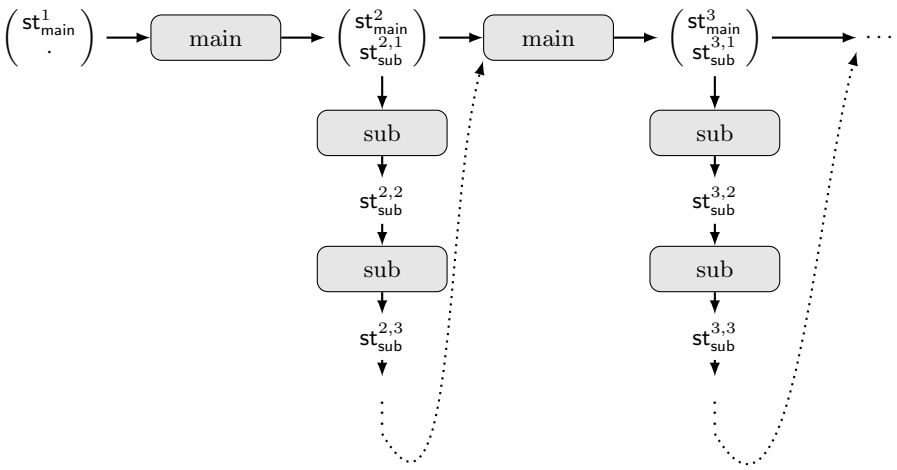

Fig. 6: Evolution of States in the Hybrid Protocol

We start by designing liteARCAD that will be used as $\mathsf{ARCAD}_{\mathsf{sub}}$.

4.1 liteARCAD: a Light Protocol without Post-Compromise Security

On Fig. 8, we adapt $\mathsf{ARCAD}_{\mathsf{DV}}$ of Fig. 15 by replacing the signcryption SC[11] by a symmetric one-time authenticated encryption (OTAE) scheme.[12] We obtain a lightweight ARCAD which achieves most of the security properties except post-compromise security. In fact, it is known that a secure and a correct unidirectional ARCAD implies public-key encryption [6]. Therefore, we do not expect full security from this symmetric-only protocol.

When there is a state exposure, it allows simulating every subsequent reception of messages. Additionally, it also allows to decrypt what is sent and to simulate a new state exposure. Therefore, there is no possible healing after a state exposure. To formalize our IND-CCA-security, we prune out post-compromise security but leave forward secrecy by using the following cleanness predicate.

$\mathsf{C}_{\mathsf{noexp}}$**:** neither A nor B had an $\mathsf{EXP}_{\mathsf{st}}$ before seeing $(\mathsf{ad}, \mathsf{ct})_{\mathsf{test}}$.

When $\mathsf{C}_{\mathsf{noexp}}$ holds, the notion of direct and indirect leakage (Def. 34–35) boils down to the cases based on $\mathsf{EXP}_{\mathsf{pt}}$ leakages. Hence, $\mathsf{C}_{\mathsf{leak}} \wedge \mathsf{C}_{\mathsf{noexp}} = \mathsf{C}_{\mathsf{sym}}$ can be defined as follows:

[11] SC is a public-key primitive that combines encryption and signature; see Appendix A.1.
[12] OTAE consists of a key space $\mathsf{OTAE}.\mathcal{K}_\lambda$ and the OTAE.Enc and OTAE.Dec algorithms.

$\mathsf{C_{sym}}$: the following conditions are all satisfied (see Fig. 7)

- (no direct $\mathsf{EXP_{pt}}$ leakage) there is no $\mathsf{EXP_{pt}}(\mathsf{P_{test}})$ after time $\mathsf{t_{test}}$ until there is a $\mathsf{RATCH}(\mathsf{P_{test}}, .)$;
- (no indirect $\mathsf{EXP_{pt}}$ leakage) if $\mathsf{P_{test}}$ is in a matching status (Def. 30) at time $\mathsf{t_{test}}$ and its $\mathsf{CHALLENGE}$ call corresponds (Def. 31) to some $\mathsf{RATCH}(\overline{\mathsf{P}}_{\mathsf{test}}, , \text{“rec”}, .)$ at some time $\overline{\mathsf{t}}$ in a matching status, then there is no $\mathsf{EXP_{pt}}(\overline{\mathsf{P}}_{\mathsf{test}})$ after time $\overline{\mathsf{t}}$ until there is another $\mathsf{RATCH}(\overline{\mathsf{P}}_{\mathsf{test}}, .)$ call;
- (no $\mathsf{EXP_{st}}$ leakage) neither A nor B had an $\mathsf{EXP_{st}}$ before seeing $(\mathsf{ad}, \mathsf{ct})_{\mathsf{test}}$.

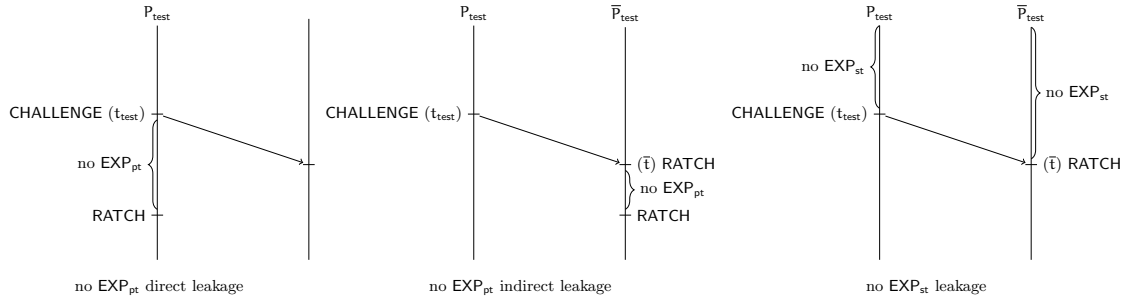

Fig. 7: $\mathsf{C_{sym}}$ Cleanness

Similarly, the notion of trivial forgery changes as the exposure of the state of P now allows to forge for P as well, due to the symmetric key. (Before, it was only allowing to forge for $\overline{\mathsf{P}}$ as keys were asymmetric.) Thus, a forgery becomes trivial when an $\mathsf{EXP_{st}}$ occurs. Hence, the FORGE game cannot allow any state exposure at all. We formalize the security by using the $\mathsf{C_{noexp}}$ cleanness predicate in FORGE-security. There is no $(\mathsf{ad}, \mathsf{ct})_{\mathsf{test}}$ message in FORGE-security, thus $\mathsf{C_{noexp}}$ means no $\mathsf{EXP_{st}}$ at all.

Theorem 19 (Security of liteARCAD). *Let* liteARCAD *be the* ARCAD *scheme on Fig. 8. It is correct. If* $\mathsf{Sym.kl} = \Omega(\lambda)$, liteARCAD *is* PREDICT*-secure. If* OTAE *is* SEF-OTCMA *and* IND-OTCCA*-secure,* Sym *is* IND-OTCCA*-secure, and* H *is collision-resistant, then* liteARCAD *is* $\mathsf{C_{noexp}}$-FORGE*-secure and* $\mathsf{C_{sym}}$-IND-CCA*-secure.*

Proof. We start from an initial game Γ which has a “special message” $(\mathsf{ad}, \mathsf{ct})$. The notions of game Γ and of special message will differ for the proof of FORGE security (where the special message is the final forgery) and IND-CCA security (where the special message is the challenge). It will be made precise later in the proof. We denote by Q the participant who plays the sender role for the special message. In the game Γ, we define the event E that no participant P has an $\mathsf{EXP_{st}}(\mathsf{P})$ query before having seen the special message. We assume that the game Γ has the property that whenever E does not occur, then Γ never returns 1. Typically, this will be the case because $\neg\mathsf{E}$ implies a non-clean game, as it will be made more precise later. We define below for every tuple $(\mathsf{Q}, \mathsf{m_{send}}, \mathsf{m_{rec}})$, the hybrids $\Gamma_{\mathsf{Q},\mathsf{m_{send}},\mathsf{m_{rec}}}$ and $\Gamma'_{\mathsf{Q},\mathsf{m_{send}},\mathsf{m_{rec}}}$ which essentially guess Q and how many messages are sent and received by Q when sending the special message.

First of all, we extend the data structure of a stored OTAE key sk by adding a boolean object sk.flag. By default, sk.flag is down. When we raise the flag, we say that the key sk is marked. Our aim is to mark keys which should not leak. The defined hybrids $\Gamma_{\mathsf{Q},\mathsf{m_{send}},\mathsf{m_{rec}}}$ and $\Gamma'_{\mathsf{Q},\mathsf{m_{send}},\mathsf{m_{rec}}}$ essentially mark the keys in the $\mathsf{m_{send}}$ first messages by Q and the $\mathsf{m_{rec}}$ first messages by $\overline{\mathsf{Q}}$ but abort if any marked key is exposed. Note that sk is encrypted without the extension sk.flag.

We denote by ν_{P} the length of $\mathsf{st}_{\mathsf{P}}^{\mathsf{rec}}$, which is also one plus the number of sent messages by P. Similarly, we denote by μ_{P} the length of $\mathsf{st}_{\mathsf{P}}^{\mathsf{send}}$, which is also one plus the number of received messages by P.

$\mathsf{liteARCAD.Setup} = \mathsf{H.Gen}$

$\mathsf{liteARCAD.Initall}(1^\lambda, \mathsf{hk})$
1: pick $\mathsf{sk}_1, \mathsf{sk}_2$ in $\mathsf{OTAE}.\mathcal{K}_\lambda$
2: $\mathsf{st}_A^{\mathsf{send}} \leftarrow (\lambda, \mathsf{hk}, (\mathsf{sk}_1), (\mathsf{sk}_2))$
3: $\mathsf{st}_B^{\mathsf{send}} \leftarrow (\lambda, \mathsf{hk}, (\mathsf{sk}_2), (\mathsf{sk}_1))$
4: **return** $(\mathsf{st}_A, \mathsf{st}_B, \perp)$

$\mathsf{onion.Send}(1^\lambda, \mathsf{hk}, \mathsf{st}_S^1, \ldots, \mathsf{st}_S^n, \mathsf{ad}, \mathsf{pt})$
1: pick $\mathsf{k}_1, \ldots, \mathsf{k}_n$ in $\{0,1\}^{\mathsf{Sym.kl}(\lambda)}$
2: $\mathsf{k} \leftarrow \mathsf{k}_1 \oplus \cdots \oplus \mathsf{k}_n$
3: $\mathsf{ct}_{n+1} \leftarrow \mathsf{Sym.Enc}(\mathsf{k}, \mathsf{pt})$
4: $\mathsf{ad}_{n+1} \leftarrow \mathsf{ad}$
5: **for** $i = n$ down to 1 **do**
6: $\quad \mathsf{ad}_i \leftarrow \mathsf{H.Eval}(\mathsf{hk}, \mathsf{ad}_{i+1}, n, \mathsf{ct}_{i+1})$
7: $\quad \mathsf{ct}_i \leftarrow \mathsf{OTAE.Enc}(\mathsf{st}_S^i, \mathsf{ad}_i, \mathsf{k}_i)$
8: **end for**
9: **return** $(\mathsf{ct}_1, \ldots, \mathsf{ct}_{n+1})$

$\mathsf{onion.Receive}(\mathsf{hk}, \mathsf{st}_R^1, \ldots, \mathsf{st}_R^n, \mathsf{ad}, \vec{\mathsf{ct}})$
1: parse $\vec{\mathsf{ct}} = (\mathsf{ct}_1, \ldots, \mathsf{ct}_{n+1})$
2: $\mathsf{ad}_{n+1} \leftarrow \mathsf{ad}$
3: **for** $i = n$ down to 1 **do**
4: $\quad \mathsf{ad}_i \leftarrow \mathsf{H.Eval}(\mathsf{hk}, \mathsf{ad}_{i+1}, n, \mathsf{ct}_{i+1})$
5: $\quad \mathsf{OTAE.Dec}(\mathsf{st}_R^i, \mathsf{ad}_i, \mathsf{ct}_i) \to \mathsf{k}_i$
6: $\quad$ **if** $\mathsf{k}_i = \perp$ **then return** $\perp$
7: **end for**
8: $\mathsf{k} \leftarrow \mathsf{k}_1 \oplus \cdots \oplus \mathsf{k}_n$
9: $\mathsf{pt} \leftarrow \mathsf{Sym.Dec}(\mathsf{k}, \mathsf{ct}_0)$
10: **return** pt

$\mathsf{uni.Init}(1^\lambda)$
1: pick sk in $\mathsf{OTAE}.\mathcal{K}_\lambda$
2: $\mathsf{st}_S \leftarrow \mathsf{sk}$
3: $\mathsf{st}_R \leftarrow \mathsf{sk}$
4: **return** $(\mathsf{st}_S, \mathsf{st}_R)$

$\mathsf{uni.Send}(1^\lambda, \mathsf{hk}, \vec{\mathsf{st}}_S, \mathsf{ad}, \mathsf{pt})$
1: pick sk in $\mathsf{OTAE}.\mathcal{K}_\lambda$
2: $\mathsf{pt}' \leftarrow (\mathsf{sk}, \mathsf{pt})$
3: $\mathsf{onion.Enc}(1^\lambda, \mathsf{hk}, \vec{\mathsf{st}}_S, \mathsf{ad}, \mathsf{pt}') \to \vec{\mathsf{ct}}$
4: **return** $(\mathsf{sk}, \vec{\mathsf{ct}})$

$\mathsf{uni.Receive}(\mathsf{hk}, \vec{\mathsf{st}}_R, \mathsf{ad}, \vec{\mathsf{ct}})$
1: $\mathsf{onion.Dec}(\mathsf{hk}, \vec{\mathsf{st}}_R, \mathsf{ad}, \vec{\mathsf{ct}}) \to \mathsf{pt}'$
2: **if** $\mathsf{pt}' = \perp$ **then**
3: $\quad$ **return** $(\mathsf{false}, \perp, \perp)$
4: **end if**
5: parse $\mathsf{pt}' = (\mathsf{sk}, \mathsf{pt})$
6: **return** $(\mathsf{true}, \mathsf{sk}, \mathsf{pt})$

$\mathsf{liteARCAD.Send}(\mathsf{st}_P, \mathsf{ad}, \mathsf{pt})$
1: parse $\mathsf{st}_P = (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},\nu}))$
2: $\mathsf{uni.Init}(1^\lambda) \xrightarrow{\$} (\mathsf{st}_{S\mathsf{new}}, \mathsf{st}_P^{\mathsf{rec},\nu+1})$ ▷ append a new receive state to the $\mathsf{st}_P^{\mathsf{rec}}$ list
3: $\mathsf{pt}' \leftarrow (\mathsf{st}_{S\mathsf{new}}, \mathsf{pt})$ ▷ then, $\mathsf{st}_{S\mathsf{new}}$ is erased to avoid leaking
4: take the smallest i s.t. $\mathsf{st}_P^{\mathsf{send},i} \neq \perp$ ▷ $i = u - n$ if we had n Receive since the last Send
5: $\mathsf{uni.Send}(1^\lambda, \mathsf{hk}, \mathsf{st}_P^{\mathsf{send},i}, \ldots, \mathsf{st}_P^{\mathsf{send},u}, \mathsf{ad}, \mathsf{pt}') \xrightarrow{\$} (\mathsf{st}_P^{\mathsf{send},u}, \mathsf{ct})$ ▷ update $\mathsf{st}_P^{\mathsf{send},u}$
6: $\mathsf{st}_P^{\mathsf{send},i}, \ldots, \mathsf{st}_P^{\mathsf{send},u-1} \leftarrow \perp$ ▷ flush the send state list: only $\mathsf{st}_P^{\mathsf{send},u}$ remains
7: $\mathsf{st}'_P \leftarrow (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},\nu+1}))$
8: **return** $(\mathsf{st}'_P, \mathsf{ct})$

$\mathsf{liteARCAD.Receive}(\mathsf{st}_P, \mathsf{ad}, \mathsf{ct})$
9: parse $\mathsf{st}_P = (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},\nu}))$
10: set $n+1$ to the number of components in ct ▷ the onion has n layers
11: set i to the smallest index such that $\mathsf{st}_P^{\mathsf{rec},i} \neq \perp$
12: **if** $i + n - 1 > \nu$ **then return** $(\mathsf{false}, \mathsf{st}_P, \perp)$
13: $\mathsf{uni.Receive}(\mathsf{hk}, \mathsf{st}_P^{\mathsf{rec},i}, \ldots, \mathsf{st}_P^{\mathsf{rec},i+n-1}, \mathsf{ad}, \mathsf{ct}) \to (\mathsf{acc}, \mathsf{st}'^{\mathsf{rec},i+n-1}_P, \mathsf{pt}')$
14: **if** $\mathsf{acc} = \mathsf{false}$ **then return** $(\mathsf{false}, \mathsf{st}_P, \perp)$
15: parse $\mathsf{pt}' = (\mathsf{st}_P^{\mathsf{send},u+1}, \mathsf{pt})$ ▷ a new send state is added in the list
16: $\mathsf{st}_P^{\mathsf{rec},i}, \ldots, \mathsf{st}_P^{\mathsf{rec},i+n-2} \leftarrow \perp$ ▷ update stage 1: $n-1$ entries of $\mathsf{st}_P^{\mathsf{rec}}$ were erased
17: $\mathsf{st}_P^{\mathsf{rec},i+n-1} \leftarrow \mathsf{st}'^{\mathsf{rec},i+n-1}_P$ ▷ update stage 2: update $\mathsf{st}_P^{\mathsf{rec},i+n-1}$
18: $\mathsf{st}'_P \leftarrow (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u+1}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},\nu}))$
19: **return** $(\mathsf{acc}, \mathsf{st}'_P, \mathsf{pt})$

Fig. 8: liteARCAD Protocol (Adapted from $\mathsf{ARCAD}_{\mathsf{DV}}$ in Fig. 15).

We show on Fig. 9 the modifications of the game to define the hybrids.

The Initall code is modified by marking the initial keys sk_1 and sk_2. If an $\mathsf{EXP}_{\mathsf{st}}$ reveals a marked key, the game aborts. This is enforced by the following change in $\mathsf{EXP}_{\mathsf{st}}$:

Oracle $\mathsf{EXP}_{\mathsf{st}}(\mathsf{P})$
1: **if** $\mathsf{st}_P^{\mathsf{send}}$ or $\mathsf{st}_P^{\mathsf{rec}}$ in st_P contain any sk with $\mathsf{sk.flag}$ up **then**
2: $\quad$ abort the game
3: **end if**
4: **return** st_P

When the special message is identified as coming from P, the following verification is made by the game:

1: **if** $P \neq Q$ or $\nu_Q + 1 \neq m_{\mathsf{send}}$ or $u_Q + 1 \neq m_{\mathsf{receive}}$ **then**
2: $\quad$ abort the game
3: **end if**

$\mathsf{liteARCAD.Setup} = \mathsf{H.Gen}$

$\mathsf{liteARCAD.Initall}(1^\lambda, \mathsf{hk})$
1: pick $\mathsf{sk}_1, \mathsf{sk}_2$ in $\mathsf{OTAE}.\mathcal{K}_\lambda$
2: raise sk_1.flag and sk_2.flag
3: $\mathsf{st}_A^{\mathsf{send}} \leftarrow (\lambda, \mathsf{hk}, (\mathsf{sk}_1), (\mathsf{sk}_2))$
4: $\mathsf{st}_B^{\mathsf{send}} \leftarrow (\lambda, \mathsf{hk}, (\mathsf{sk}_2), (\mathsf{sk}_1))$
5: **return** $(\mathsf{st}_A, \mathsf{st}_B, \perp)$

$\mathsf{onion.Send}(1^\lambda, \mathsf{hk}, \mathsf{st}_S^1, \ldots, \mathsf{st}_S^n, \mathsf{ad}, \mathsf{pt})$
1: get P, u from higher calls
2: pick $k_1, \ldots, k_n$ in $\{0,1\}^{\mathsf{Sym.kl}(\lambda)}$
3: $k \leftarrow k_1 \oplus \cdots \oplus k_n$
4: **if** game is Γ' and $\exists i\ \mathsf{st}_S^i.\mathsf{flag}$ **then**
5: pick ρ of same size as pt
6: $\mathsf{ct}_{n+1} \leftarrow \mathsf{Sym.Enc}(k, \rho)$
7: $S[P, u, n, \mathsf{ct}_{n+1}] \leftarrow \mathsf{pt}$
8: **else**
9: $\mathsf{ct}_{n+1} \leftarrow \mathsf{Sym.Enc}(k, \mathsf{pt})$
10: **end if**
11: $\mathsf{ad}_{n+1} \leftarrow \mathsf{ad}$
12: **for** $i = n$ down to 1 **do**
13: $\mathsf{ad}_i \leftarrow \mathsf{H.Eval}(\mathsf{hk}, \mathsf{ad}_{i+1}, n, \mathsf{ct}_{i+1})$
14: **if** game is Γ' and $\mathsf{st}_S^i.\mathsf{flag}$ **then**
15: pick $r \in \{0,1\}^{\mathsf{Sym.kl}(\lambda)}$
16: $\mathsf{ct}_i \leftarrow \mathsf{OTAE.Enc}(\mathsf{st}_S^i, \mathsf{ad}_i, r)$
17: $S[P, u, n, i, \mathsf{ad}_i, \mathsf{ct}_i] \leftarrow k_i$
18: **else**
19: $\mathsf{ct}_i \leftarrow \mathsf{OTAE.Enc}(\mathsf{st}_S^i, \mathsf{ad}_i, k_i)$
20: **end if**
21: **end for**
22: **return** $(\mathsf{ct}_1, \ldots, \mathsf{ct}_{n+1})$

$\mathsf{onion.Receive}(\mathsf{hk}, \mathsf{st}_R^1, \ldots, \mathsf{st}_R^n, \mathsf{ad}, \vec{\mathsf{ct}})$
1: get P, i from higher calls
2: $u \leftarrow i + n - 1$
3: parse $\vec{\mathsf{ct}} = (\mathsf{ct}_1, \ldots, \mathsf{ct}_{n+1})$
4: $\mathsf{ad}_{n+1} \leftarrow \mathsf{ad}$
5: **for** $i = n$ down to 1 **do**
6: $\mathsf{ad}_i \leftarrow \mathsf{H.Eval}(\mathsf{hk}, \mathsf{ad}_{i+1}, n, \mathsf{ct}_{i+1})$
7: **if** game is Γ' and $\mathsf{st}_S^i.\mathsf{flag}$ and $S[P, u, n, i, \mathsf{ad}_i, \mathsf{ct}_i]$ defined **then**
8: $k_i \leftarrow S[P, u, n, i, \mathsf{ad}_i, \mathsf{ct}_i]$
9: **else**
10: $\mathsf{OTAE.Dec}(\mathsf{st}_R^i, \mathsf{ad}_i, \mathsf{ct}_i) \to k_i$
11: **end if**
12: **if** $k_i = \perp$ **then return** $\perp$
13: **end for**
14: $k \leftarrow k_1 \oplus \cdots \oplus k_n$
15: **if** game is Γ' and $S[Q, u, n, \mathsf{ct}_{n+1}]$ exists **then**
16: pick ρ of same size as pt
17: $\mathsf{pt} \leftarrow S[Q, u, n, \mathsf{ct}_{n+1}]$
18: **else**
19: $\mathsf{pt} \leftarrow \mathsf{Sym.Dec}(k, \mathsf{ct}_0)$
20: **end if**
21: **return** pt

$\mathsf{uni.Init}(1^\lambda)$
1: pick sk in $\mathsf{OTAE}.\mathcal{K}_\lambda$
2: $\mathsf{st}_S \leftarrow \mathsf{sk}$
3: $\mathsf{st}_R \leftarrow \mathsf{sk}$
4: **return** $(\mathsf{st}_S, \mathsf{st}_R)$

$\mathsf{uni.Send}(1^\lambda, \mathsf{hk}, \vec{\mathsf{st}}_S, \mathsf{ad}, \mathsf{pt})$
1: pick sk in $\mathsf{OTAE}.\mathcal{K}_\lambda$
2: $\mathsf{pt}' \leftarrow (\mathsf{sk}, \mathsf{pt})$
3: $\mathsf{onion.Enc}(1^\lambda, \mathsf{hk}, \vec{\mathsf{st}}_S, \mathsf{ad}, \mathsf{pt}') \to \vec{\mathsf{ct}}$
4: **return** $(\mathsf{sk}, \vec{\mathsf{ct}})$

$\mathsf{uni.Receive}(\mathsf{hk}, \vec{\mathsf{st}}_R, \mathsf{ad}, \vec{\mathsf{ct}})$
1: $\mathsf{onion.Dec}(\mathsf{hk}, \vec{\mathsf{st}}_R, \mathsf{ad}, \vec{\mathsf{ct}}) \to \mathsf{pt}'$
2: **if** $\mathsf{pt}' = \perp$ **then**
3: **return** $(\mathsf{false}, \perp, \perp)$
4: **end if**
5: parse $\mathsf{pt}' = (\mathsf{sk}, \mathsf{pt})$
6: **return** $(\mathsf{true}, \mathsf{sk}, \mathsf{pt})$

$\mathsf{liteARCAD.Send}(\mathsf{st}_P, \mathsf{ad}, \mathsf{pt})$
1: parse $\mathsf{st}_P = (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},v}))$
2: $\mathsf{uni.Init}(1^\lambda) \xrightarrow{\$} (\mathsf{st}_{S\mathsf{new}}, \mathsf{st}_P^{\mathsf{rec},v+1})$
3: $\mathsf{pt}' \leftarrow (\mathsf{st}_{S\mathsf{new}}, \mathsf{pt})$
4: take the smallest i s.t. $\mathsf{st}_P^{\mathsf{send},i} \neq \perp$
5: $\mathsf{uni.Send}(1^\lambda, \mathsf{hk}, \mathsf{st}_P^{\mathsf{send},i}, \ldots, \mathsf{st}_P^{\mathsf{send},u}, \mathsf{ad}, \mathsf{pt}') \xrightarrow{\$} (\mathsf{st}_P^{\mathsf{send},u}, \mathsf{ct})$
6: $\mathsf{st}_P^{\mathsf{send},i}, \ldots, \mathsf{st}_P^{\mathsf{send},u-1} \leftarrow \perp$
7: **if** $(P = Q$ and $v < m_{\mathsf{send}})$ or $(P = \overline{Q}$ and $v < m_{\mathsf{rec}})$ **then**
8: raise $\mathsf{st}_P^{\mathsf{send},u}.\mathsf{flag}$ and $\mathsf{st}_P^{\mathsf{rec},v+1}.\mathsf{flag}$
9: **end if**
10: $\mathsf{st}'_P \leftarrow (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},v+1}))$
11: **return** $(\mathsf{st}'_P, \mathsf{ct})$

$\mathsf{liteARCAD.Receive}(\mathsf{st}_P, \mathsf{ad}, \mathsf{ct})$
12: parse $\mathsf{st}_P = (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},v}))$
13: set $n+1$ to the number of components in ct
14: set i to the smallest index such that $\mathsf{st}_P^{\mathsf{rec},i} \neq \perp$
15: **if** $i + n - 1 > v$ **then return** $(\mathsf{false}, \mathsf{st}_P, \perp)$
16: $\mathsf{uni.Receive}(\mathsf{hk}, \mathsf{st}_P^{\mathsf{rec},i}, \ldots, \mathsf{st}_P^{\mathsf{rec},i+n-1}, \mathsf{ad}, \mathsf{ct}) \to (\mathsf{acc}, \mathsf{st}'^{\,\mathsf{rec},i+n-1}_P, \mathsf{pt}')$
17: **if** $\mathsf{acc} = \mathsf{false}$ **then return** $(\mathsf{false}, \mathsf{st}_P, \perp)$
18: parse $\mathsf{pt}' = (\mathsf{st}_P^{\mathsf{send},u+1}, \mathsf{pt})$
19: $\mathsf{st}_P^{\mathsf{rec},i}, \ldots, \mathsf{st}_P^{\mathsf{rec},i+n-2} \leftarrow \perp$
20: $\mathsf{st}_P^{\mathsf{rec},i+n-1} \leftarrow \mathsf{st}'^{\,\mathsf{rec},i+n-1}_P$
21: **if** $(P = Q$ and $v < m_{\mathsf{send}})$ or $(P = \overline{Q}$ and $v < m_{\mathsf{rec}})$ **then**
22: raise $\mathsf{st}_P^{\mathsf{send},u+1}.\mathsf{flag}$ and $\mathsf{st}_P^{\mathsf{rec},i+n-1}.\mathsf{flag}$
23: **end if**
24: $\mathsf{st}'_P \leftarrow (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u+1}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},v}))$
25: **return** $(\mathsf{acc}, \mathsf{st}'_P, \mathsf{pt})$

Fig. 9: Proof for liteARCAD (modifications are in gray)

In liteARCAD, Send and Receive are also modified in order to mark the keys in the $\mathfrak{m}_{\mathsf{send}}$ first messages by Q and the $\mathfrak{m}_{\mathsf{rec}}$ first messages by $\overline{\mathsf{Q}}$. onion.Send and onion.Receive are also modified but in a way which does not change the result. Those modifications will become useful in $\Gamma'_{\mathsf{Q},\mathfrak{m}_{\mathsf{send}},\mathfrak{m}_{\mathsf{rec}}}$.

Clearly, the only behavior difference between Γ and $\Gamma_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}}$ is that $\Gamma_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}}$ may abort if a marked key is requested to be revealed or if $(\mathsf{Q},\mathfrak{m}_{\mathsf{send}},\mathfrak{m}_{\mathsf{rec}})$ is a wrong guess. Because of the property of Γ, we know that the former abort cases imply Γ not returning 1. The latter abort cases actually partition the success cases into several $(\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}})$ parameters. Hence, we have

$$\Pr[\Gamma \to 1] = \sum_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}} \Pr[\Gamma_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}} \to 1]$$

Next, we can focus on the modification in onion.Send and onion.Receive. When onion.Send applies a OTAE.Enc with a marked key, the plaintext k_i is replaced by a random one r and the value of pt is saved in a dictionary S with a key $\mathfrak{u}$ referring to the sender P, an identifier $\mathfrak{i}$ of the key, and the ciphertext $(\mathsf{ad},\mathsf{ct})$. We can easily see that $(\mathfrak{u},\mathfrak{n},\mathfrak{i})$ uniquely identifies which key was used by P to encrypt with OTAE. Hence, we store pt in $[\mathsf{P},\mathfrak{u},\mathfrak{n},\mathfrak{i},\mathsf{ad},\mathsf{ct}]$. The values of $\mathsf{P},\mathfrak{u},\mathfrak{n}$ should be passed by the algorithm Send (which we did not explicitly write for simplicity). This dictionary is to remember that if we decrypt $(\mathsf{ad},\mathsf{ct})$ with a key corresponding to $(\mathsf{P},\mathfrak{u},\nu,\mathfrak{i})$, the result should be pt instead of the actual decryption. This is what onion.Receive is doing. Similarly, there is a dictionary $\mathsf{S}[\mathsf{P},\mathfrak{u},\mathfrak{n},\mathsf{ct}]$ for the encryptions using Sym.

We construct a sequence of hybrids starting by $\Gamma_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}}$ and ending by $\Gamma'_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}}$ in which we treat all keys one after the other. In $\Gamma'_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}}$, none of the marked key is ever used for anything but encryption or decryption. Each key is used to encrypt only one message. The IND-OTCCA game can simulate the difference between two hybrids. Hence, we obtain that $\Pr[\Gamma_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}} \to 1] - \Pr[\Gamma'_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}} \to 1]$ is negligible.

We deduce that the difference between $\Pr[\Gamma \to 1]$ and $\sum_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}} \Pr[\Gamma'_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}} \to 1]$ is negligible.

FORGE-*security.* We continue from the same setting. We take Γ as the $\mathsf{C}_{\mathsf{noexp}}$-FORGE game. The extra $\mathsf{RATCH}(\mathsf{P},\text{“rec”},\mathsf{ad},\mathsf{ct})$ query to P by the FORGE game defines the special message $(\mathsf{ad},\mathsf{ct})$. Namely, we set $\mathsf{Q} = \overline{\mathsf{P}}$ and $\mathfrak{m}_{\mathsf{sent}}$ (resp. $\mathfrak{m}_{\mathsf{rec}}$) to the number of messages sent (resp. received) by $\overline{\mathsf{P}}$ at the end of the game. The property of Γ is satisfied: there is no $\mathsf{EXP}_{\mathsf{st}}$. The game $\Gamma'_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}}$ returns 1 if the special message is a forgery. In this case, the special message is not sent by Q. It is a forgery for the onion scheme. We can apply the collision-resistance of H and the SEF-OTCMA security of OTAE on this game to show that $\Pr[\Gamma'_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}}} \to 1]$ is negligible. Hence, $\Pr[\Gamma \to 1]$ is negligible.

IND-CCA-*security.* In the IND-CCA game, $\Gamma_\mathfrak{b} = \mathsf{IND\text{-}CCA}^{\mathcal{A}}_{\mathfrak{b},\mathsf{C}_{\mathsf{sym}}}$ and the special message is the one of the CHALLENGE query. Again, the property of Γ is satisfied: no participant has a $\mathsf{EXP}_{\mathsf{st}}$ before seeing the special message. We add the subscript $\mathfrak{b}$ in $\Gamma'_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}},\mathfrak{b}}$ game. We can use the IND-OTCCA security again to show that $\Pr[\Gamma'_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}},0} \to 1] - \Pr[\Gamma'_{\mathsf{Q},\mathfrak{m}_{\mathsf{sent}},\mathfrak{m}_{\mathsf{rec}},1} \to 1]$ is negligible. Actually, pt_0 is replaced twice by some random pt_0 for $\mathfrak{b} = 0$ and once for $\mathfrak{b} = 1$. The difference is what is stored in $\mathsf{pt}_{\overline{\mathsf{Q}}}$ after receiving the special message, since it is not revealed due to $\mathsf{C}_{\mathsf{sym}}$ cleanness, there is no difference in the game. Hence $\Pr[\Gamma_0 \to 1] - \Pr[\Gamma_1 \to 1]$ is negligible.

PREDICT-*security.* Like in DV [6], due to the correctness of OTAE, guessing ct before it is produced by RATCH implies guessing the $\mathsf{k}_\mathfrak{n}$ key which RATCH will select on onion.Send. Hence, we obtain PREDICT-security. □

4.2 Our Hybrid On-Demand ARCAD Protocol

We give our on-demand ARCAD protocol on Fig. 10. It uses two sub-protocols called $\mathsf{ARCAD}_{\mathsf{main}}$ and $\mathsf{ARCAD}_{\mathsf{sub}}$. The former is to represent a strong-but-slow protocol such as $\mathsf{ARCAD}_{\mathsf{DV}}$ (Fig. 15). The latter is typically a weaker-but-faster protocol like liteARCAD (Fig. 8). The use of one or

the other is based on a flag that can be turned on and off in ad (it is checked with ad.flag operation in the protocol). To have the flag on lets the protocol run $\mathsf{ARCAD}_{\mathsf{main}}$ while setting the flag off means to run $\mathsf{ARCAD}_{\mathsf{sub}}$. Assuming that $\mathsf{ARCAD}_{\mathsf{main}}$ is ratcheting (i.e. post-compromise secure) and $\mathsf{ARCAD}_{\mathsf{sub}}$ is not, this defines on-demand ratcheting. We denote our hybrid protocol as $\mathsf{hybridARCAD} = \mathsf{hybrid}(\mathsf{ARCAD}_{\mathsf{main}}, \mathsf{ARCAD}_{\mathsf{sub}})$.

We use as a reference the (e, c) number of messages in the $\mathsf{ARCAD}_{\mathsf{main}}$ thread. Every $\mathsf{ARCAD}_{\mathsf{main}}$ message creates a new $\mathsf{ARCAD}_{\mathsf{sub}}$ send/receive state pair. The sending participant keeps the generated send state in a $\mathsf{sub}[e, c]$ register under the (e, c) number of the message and sends the generated receive state together with his message. The very first message which a participant sees (either in sending or receiving) forces the flag to indicate $\mathsf{ARCAD}_{\mathsf{main}}$ as we have no initial $\mathsf{ARCAD}_{\mathsf{sub}}$ state. The (e, c) number if authenticated and also explicitly added in the ciphertext. The receiving participant checks that (e, c) increases and uses the $\mathsf{sub}[e, c]$ register state to receive the message.

Theorem 20. *If the protocols* $\mathsf{ARCAD}_{\mathsf{main}}$ *and* $\mathsf{ARCAD}_{\mathsf{sub}}$ *are both correct, then the protocol* $\mathsf{hybrid}(\mathsf{ARCAD}_{\mathsf{main}}, \mathsf{ARCAD}_{\mathsf{sub}})$ *is correct.*

Proof. We want to prove that the list of sent $(\mathsf{ad}, \mathsf{ct})$ by P matches the list of received $(\mathsf{ad}, \mathsf{ct})$ by $\overline{\mathsf{P}}$, for each P. We first rewrite a list of $(\mathsf{ad}, \mathsf{ct})$ as follows. We note that ct is in the form of (ct', e, c). We define below a bit b for each $(\mathsf{ad}, \mathsf{ct})$ from the list, then $\mathsf{ad}' = (\mathsf{ad}, \mathsf{b}, e, c)$, and we rewrite each $(\mathsf{ad}, \mathsf{ct})$ into $(\mathsf{ad}', \mathsf{ct}')$. Hence, we rewrite $(\mathsf{ad}, (\mathsf{ct}', e, c))$ into $((\mathsf{ad}, \mathsf{b}, e, c), \mathsf{ct}')$. Clearly, the only significant change is to add this bit b.

The bit b indicates which ARCAD protocol this message belongs to. We define b for $(\mathsf{ad}, \mathsf{ct})$ in the list as follows. If one of the two following conditions is satisfied, we define $\mathsf{b} = 1$. Otherwise, we define $\mathsf{b} = 0$.

C1: ad.flag is set.
C2: $(e, c) = (0, 0)$ and there is no prior message in the list with $(e, c) = (0, 0)$.

Condition C2 is to identify the very first message for which the sender is forced to use $\mathsf{ARCAD}_{\mathsf{main}}$. We now show that $\mathsf{b} = 1$ if and only if the message belongs to $\mathsf{ARCAD}_{\mathsf{main}}$ and use the correctness of both protocols to conclude.

Clearly, if ad.flag is set, $\mathsf{ARCAD}_{\mathsf{main}}$ is used.

For a sent message by P, the condition $\mathsf{ctr}[\max(e_{\mathsf{send}}, e_{\mathsf{rec}})] = -1$ and Condition C2 are both equivalent to that P received no message and is sending his very first one. Hence, they are equivalent.

For a received message by P, the condition $(e, \mathsf{ctr}[0]) = (0, -1)$ and Condition C2 are similarly equivalent to that $\mathsf{P} = \mathsf{B}$ and this is the very first received message. Hence, they are equivalent.

Therefore, the conditions in Send and Receive defining whether the message belongs to $\mathsf{ARCAD}_{\mathsf{main}}$ are equivalent to $\mathsf{b} = 1$.

Once we have rewritten the messages with (b, e, c), we can clearly identify which protocol they belong to. If $\mathsf{b} = 1$, this is an $\mathsf{ARCAD}_{\mathsf{main}}$ message. If $\mathsf{b} = 0$, this is an $\mathsf{ARCAD}_{\mathsf{sub}}$ message in a session state indexed by (e, c). We can extract from the lists all messages with $\mathsf{b} = 1$ and use the correctness of $\mathsf{ARCAD}_{\mathsf{main}}$ to show that they match. Similarly, for each (e, c), we can extract from the lists all messages with $\mathsf{b} = 0$ and this (e, c) index and use the correctness of $\mathsf{ARCAD}_{\mathsf{sub}}$ to show that they match. Next, we observe that (e, c) is non-decreasing in each list and that the first message with an index (e, c) must have $\mathsf{b} = 1$. Hence, there cannot be any order mismatch. The list of sent $(\mathsf{ad}', \mathsf{ct}')$ by P matches the list of received $(\mathsf{ad}', \mathsf{ct}')$ by $\overline{\mathsf{P}}$, for each P. So is the list of sent $(\mathsf{ad}, \mathsf{ct})$ and the list of received $(\mathsf{ad}, \mathsf{ct})$. □

4.3 Application: Super-Scheme to (Re)set a Protocol

Our hybrid construction finds another application than on-demand ratcheting: defense against message loss or active attacks. Indeed, by using $\mathsf{ARCAD}_{\mathsf{main}} = \mathsf{ARCAD}_{\mathsf{sub}}$, we can set ad.flag to

$\mathsf{hybridARCAD.Setup}(1^\lambda)$
1: $\mathsf{pp}_{\mathsf{main}} \leftarrow \mathsf{ARCAD}_{\mathsf{main}}.\mathsf{Setup}(1^\lambda)$
2: $\mathsf{pp}_{\mathsf{sub}} \leftarrow \mathsf{ARCAD}_{\mathsf{sub}}.\mathsf{Setup}(1^\lambda)$
3: **return** $(\mathsf{pp}_{\mathsf{main}}, \mathsf{pp}_{\mathsf{sub}})$

$\mathsf{hybridARCAD.Gen}(1^\lambda, \mathsf{pp}_{\mathsf{main}}, \mathsf{pp}_{\mathsf{sub}})$
4: **return** $\mathsf{ARCAD}_{\mathsf{main}}.\mathsf{Gen}(1^\lambda, \mathsf{pp}_{\mathsf{main}})$

$\mathsf{hybridARCAD.Init}(1^\lambda, (\mathsf{pp}_{\mathsf{main}}, \mathsf{pp}_{\mathsf{sub}}), \mathsf{sk}_{\mathrm{P}}, \mathsf{pk}_{\overline{\mathrm{P}}}, \mathrm{P})$
1: $\mathsf{ARCAD}_{\mathsf{main}}.\mathsf{Init}(1^\lambda, \mathsf{pp}_{\mathsf{main}}, \mathsf{sk}_{\mathrm{P}}, \mathsf{pk}_{\overline{\mathrm{P}}}, \mathrm{P}) \rightarrow \mathsf{st}_{\mathsf{main}}$
2: initialize array $\mathsf{st}_{\mathsf{sub}}[]$ to empty
3: **if** $\mathrm{P} = \mathrm{A}$ **then** $(\mathsf{e}_{\mathsf{send}}, \mathsf{e}_{\mathsf{rec}}) \leftarrow (0, -1)$
4: **else** $(\mathsf{e}_{\mathsf{send}}, \mathsf{e}_{\mathsf{rec}}) \leftarrow (-1, 0)$
5: **end if**
6: initialize array ctr with $\mathsf{ctr}[0] = -1$
7: $\mathsf{st}_{\mathrm{P}} \leftarrow (\lambda, \mathsf{pp}_{\mathsf{sub}}, \mathsf{st}_{\mathsf{main}}, \mathsf{st}_{\mathsf{sub}}[], \mathsf{e}_{\mathsf{send}}, \mathsf{e}_{\mathsf{rec}}, \mathsf{ctr}[])$
8: **return** st_{P}

$\mathsf{hybridARCAD.Send}(\mathsf{st}_{\mathrm{P}}, \mathsf{ad}, \mathsf{pt})$
1: parse st_{P} as $(\lambda, \mathsf{pp}_{\mathsf{sub}}, \mathsf{st}_{\mathsf{main}}, \mathsf{st}_{\mathsf{sub}}[], \mathsf{e}_{\mathsf{send}}, \mathsf{e}_{\mathsf{rec}}, \mathsf{ctr}[])$
2: $e \leftarrow \max(\mathsf{e}_{\mathsf{send}}, \mathsf{e}_{\mathsf{rec}})$; $\mathsf{c} \leftarrow \mathsf{ctr}[e]$ ▷ current epoch
3: **if** $\mathsf{ad.flag}$ or $\mathsf{c} = -1$ **then**
4: **if** $\mathsf{e}_{\mathsf{send}} < \mathsf{e}_{\mathsf{rec}}$ **then** $e \leftarrow \mathsf{e}_{\mathsf{rec}} + 1$; $\mathsf{c} \leftarrow 0$
5: **else** $e \leftarrow \mathsf{e}_{\mathsf{send}}$; $\mathsf{c} \leftarrow \mathsf{ctr}[e] + 1$
6: **end if**
7: $\mathsf{ARCAD}_{\mathsf{sub}}.\mathsf{Initall}(1^\lambda, \mathsf{pp}_{\mathsf{sub}}) \xrightarrow{\$} (\mathsf{st}_{\mathrm{S}}, \mathsf{st}_{\mathrm{R}}, z)$ ▷ create a new sub-state.
8: $\mathsf{st}_{\mathsf{sub}}[e, \mathsf{c}] \leftarrow \mathsf{st}_{\mathrm{S}}$
9: $\mathsf{pt}' \leftarrow (\mathsf{st}_{\mathrm{R}}, \mathsf{pt})$; $\mathsf{ad}' \leftarrow (\mathsf{ad}, 1, e, \mathsf{c})$
10: $\mathsf{ARCAD}_{\mathsf{main}}.\mathsf{Send}(\mathsf{st}_{\mathsf{main}}, \mathsf{ad}', \mathsf{pt}') \xrightarrow{\$} (\mathsf{st}_{\mathsf{main}}, \mathsf{ct}')$ ▷ send using the main state.
11: $\mathsf{ct} \leftarrow (\mathsf{ct}', e, \mathsf{c})$
12: $\mathsf{e}_{\mathsf{send}} \leftarrow e$; $\mathsf{ctr}[\mathsf{e}_{\mathsf{send}}] \leftarrow \mathsf{c}$
13: **else**
14: $\mathsf{ad}' \leftarrow (\mathsf{ad}, 0, e, \mathsf{c})$
15: $\mathsf{ARCAD}_{\mathsf{sub}}.\mathsf{Send}(\mathsf{st}_{\mathsf{sub}}[e, \mathsf{c}], \mathsf{ad}', \mathsf{pt}) \xrightarrow{\$} (\mathsf{st}_{\mathsf{sub}}[e, \mathsf{c}], \mathsf{ct}')$ ▷ send using the sub-state.
16: $\mathsf{ct} \leftarrow (\mathsf{ct}', e, \mathsf{c})$
17: **end if**
18: clean-up: erase $\mathsf{st}_{\mathsf{sub}}[e, \mathsf{c}]$ for all (e, c) such that $(e, \mathsf{c}) < (\mathsf{e}_{\mathsf{send}}, \mathsf{ctr}[\mathsf{e}_{\mathsf{send}}])$ and $(e, \mathsf{c}) < (\mathsf{e}_{\mathsf{rec}}, \mathsf{ctr}[\mathsf{e}_{\mathsf{rec}}])$
19: clean-up: erase $\mathsf{ctr}[e]$ for all e such that $e < \mathsf{e}_{\mathsf{send}}$ and $e < \mathsf{e}_{\mathsf{rec}}$
20: $\mathsf{st}_{\mathrm{P}} \leftarrow (\lambda, \mathsf{pp}_{\mathsf{sub}}, \mathsf{st}_{\mathsf{main}}, \mathsf{st}_{\mathsf{sub}}[], \mathsf{e}_{\mathsf{send}}, \mathsf{e}_{\mathsf{rec}}, \mathsf{ctr}[])$
21: **return** $(\mathsf{st}_{\mathrm{P}}, \mathsf{ct})$

$\mathsf{hybridARCAD.Receive}(\mathsf{st}_{\mathrm{P}}, \mathsf{ad}, \mathsf{ct})$
22: parse st_{P} as $(\lambda, \mathsf{pp}_{\mathsf{sub}}, \mathsf{st}_{\mathsf{main}}, \mathsf{st}_{\mathsf{sub}}[], \mathsf{e}_{\mathsf{send}}, \mathsf{e}_{\mathsf{rec}}, \mathsf{ctr}[])$
23: parse ct as $(\mathsf{ct}', e, \mathsf{c})$
24: **if** $(e, \mathsf{c}) < (\mathsf{e}_{\mathsf{rec}}, \mathsf{ctr}[\mathsf{e}_{\mathsf{rec}}])$ **then return** $(\mathsf{false}, \mathsf{st}_{\mathrm{P}}, \perp)$ ▷ (e, c) must increase
25: **if** $\mathsf{ad.flag}$ or $(\mathsf{e} = 0$ and $\mathsf{ctr}[0] = -1)$ **then**
26: $\mathsf{ad}' \leftarrow (\mathsf{ad}, 1, e, \mathsf{c})$
27: $\mathsf{ARCAD}_{\mathsf{main}}.\mathsf{Receive}(\mathsf{st}_{\mathsf{main}}, \mathsf{ad}', \mathsf{ct}') \rightarrow (\mathsf{acc}, \mathsf{st}_{\mathsf{main}}, \mathsf{pt}')$
28: parse pt' as $(\mathsf{st}_{\mathrm{R}}, \mathsf{pt})$
29: **if** acc **then**
30: $\mathsf{st}_{\mathsf{sub}}[e, \mathsf{c}] \leftarrow \mathsf{st}_{\mathrm{R}}$
31: $\mathsf{e}_{\mathsf{rec}} \leftarrow e$; $\mathsf{ctr}[e] \leftarrow \mathsf{c}$
32: **end if**
33: **else**
34: $\mathsf{ad}' \leftarrow (\mathsf{ad}, 0, e, \mathsf{c})$
35: **if** $\mathsf{st}_{\mathsf{sub}}[e, \mathsf{c}]$ undefined **then return** $(\mathsf{false}, \mathsf{st}_{\mathrm{P}}, \perp)$
36: $\mathsf{ARCAD}_{\mathsf{sub}}.\mathsf{Receive}(\mathsf{st}_{\mathsf{sub}}[e, \mathsf{c}], \mathsf{ad}', \mathsf{ct}') \rightarrow (\mathsf{acc}, \mathsf{st}_{\mathsf{sub}}[e, \mathsf{c}], \mathsf{pt})$
37: **end if**
38: clean-up: erase $\mathsf{st}_{\mathsf{sub}}[e, \mathsf{c}]$ for all (e, c) such that $(e, \mathsf{c}) < (\mathsf{e}_{\mathsf{send}}, \mathsf{ctr}[\mathsf{e}_{\mathsf{send}}])$ and $(e, \mathsf{c}) < (\mathsf{e}_{\mathsf{rec}}, \mathsf{ctr}[\mathsf{e}_{\mathsf{rec}}])$
39: clean-up: erase $\mathsf{ctr}[e]$ for all e such that $e < \mathsf{e}_{\mathsf{send}}$ and $e < \mathsf{e}_{\mathsf{rec}}$
40: $\mathsf{st}_{\mathrm{P}} \leftarrow (\lambda, \mathsf{pp}_{\mathsf{sub}}, \mathsf{st}_{\mathsf{main}}, \mathsf{st}_{\mathsf{sub}}[], \mathsf{e}_{\mathsf{send}}, \mathsf{e}_{\mathsf{rec}}, \mathsf{ctr}[])$
41: **return** $(\mathsf{acc}, \mathsf{st}_{\mathrm{P}}, \mathsf{pt})$

Fig. 10: On-Demand $\mathsf{hybridARCAD} = \mathsf{hybrid}(\mathsf{ARCAD}_{\mathsf{main}}, \mathsf{ARCAD}_{\mathsf{sub}})$ Protocol.

restore an $\mathsf{ARCAD}_{\mathsf{sub}}$ communication which was broken due to a message loss. Normal communication works in the $\mathsf{ARCAD}_{\mathsf{sub}}$ session, hence with a flag down. However, we may use $\mathsf{ARCAD}_{\mathsf{main}}$ to start a new $\mathsf{ARCAD}_{\mathsf{sub}}$ session. If $\mathsf{ARCAD}_{\mathsf{sub}}$ gets broken due to a message loss or an active attack on it, $\mathsf{ARCAD}_{\mathsf{main}}$ can be used to restart a new $\mathsf{ARCAD}_{\mathsf{sub}}$ session. We cannot resume if the $\mathsf{ARCAD}_{\mathsf{main}}$ session is broken. However, we can also make nested hybrid protocols with more than two levels of protocols inside for safety. It may increase the state sizes but the performance should be nearly the same. Then, only persistent message drop attacks would succeed to make a denial of service.

4.4 Security Definitions

We modify the predicates and the notion of FORGE-security from Section 2. In our hybrid protocol, each message $(\mathsf{ad}, \mathsf{ct})$ has a clearly defined (e, c) pair. A ct which is input or output from RATCH comes with an ad which has a clearly defined $\mathsf{ad.flag}$ bit.

Sub-games. Given a game Γ for the $\mathsf{hybridARCAD}$ scheme with an adversary $\mathcal{A}$, we define a game $\mathsf{main}(\Gamma)$ for $\mathsf{ARCAD}_{\mathsf{main}}$ with an adversary $\mathcal{A}'$ which simulates everything but the $\mathsf{ARCAD}_{\mathsf{main}}$ calls in Γ. Namely, $\mathcal{A}'$ simulates the enrichment of the states and all $\mathsf{ARCAD}_{\mathsf{sub}}$ management together with $\mathcal{A}$.

Given a game Γ_{main} for $\mathsf{ARCAD}_{\mathsf{main}}$ using no CHALLENGE oracle and an (e, c) pair, we denote by $\mathsf{main}_{\mathsf{e},\mathsf{c}}(\Gamma_{\mathsf{main}})$ the variant of Γ_{main} in which the RATCH Send call making the message $(\mathsf{ad}, \mathsf{ct})$ with pair (e, c) is replaced by a CHALLENGE query with $\mathsf{b} = 1$. This perfectly simulates Γ_{main} and produces the same value, and we can evaluate a predicate $\mathsf{C}_{\mathsf{clean}}$ relative to this challenge message. We define $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{clean}}(\Gamma_{\mathsf{main}}) = \mathsf{C}_{\mathsf{clean}}(\mathsf{main}_{\mathsf{e},\mathsf{c}}(\Gamma_{\mathsf{main}}))$. Intuitively, $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{clean}}(\Gamma_{\mathsf{main}})$ means that the message of pair (e, c) was safely encrypted and should be considered as private because no trivial attack leaks it.

We also define $\mathsf{sub}_{\mathsf{e},\mathsf{c}}(\Gamma)$ and $\mathsf{sub}'_{\mathsf{e},\mathsf{c}}(\Gamma)$. We let P be the sending participant of the $\mathsf{ARCAD}_{\mathsf{main}}$ message of pair (e, c). In $\mathsf{sub}'_{\mathsf{e},\mathsf{c}}(\Gamma)$, the adversary $\mathcal{A}'$ simulates everything but the $\mathsf{ARCAD}_{\mathsf{sub}}$ calls involving messages with pair (e, c). The initial states of P and $\overline{\mathsf{P}}$ are also set by the game $\mathsf{sub}'_{\mathsf{e},\mathsf{c}}(\Gamma)$. However, it makes an $\mathsf{EXP}_{\mathsf{st}}(\overline{\mathsf{P}})$ call at the beginning of the protocol to get the initial state st_{R} for $\mathsf{ARCAD}_{\mathsf{sub}}$. With this state, $\mathcal{A}'$ can simulate the encryption of st_{R} with $\mathsf{ARCAD}_{\mathsf{main}}$ and all the rest. Clearly, the simulation is perfect but it adds an initial $\mathsf{EXP}_{\mathsf{st}}(\overline{\mathsf{P}})$ call.

The $\mathsf{sub}_{\mathsf{e},\mathsf{c}}(\Gamma)$ game is a variant of $\mathsf{sub}'_{\mathsf{e},\mathsf{c}}(\Gamma)$ without the additional $\mathsf{EXP}_{\mathsf{st}}(\overline{\mathsf{P}})$. To simulate the encryption of st_{R}, $\mathcal{A}'$ encrypts a random string instead. When it comes to decrypt the obtained ciphertext, the random plaintext is ignored and the RATCH calls with st_{R} are simulated with the RATCH calls for the $\mathsf{ARCAD}_{\mathsf{sub}}$ game. The simulation is no longer perfect but it does not add an $\mathsf{EXP}_{\mathsf{st}}(\overline{\mathsf{P}})$ call.

Hybrid cleanness. We assume two cleanness predicates $\mathsf{C}_{\mathsf{clean}}$ and $\mathsf{C}_{\mathsf{main}}$ (which could be the same) for $\mathsf{ARCAD}_{\mathsf{main}}$ and one cleanness predicate $\mathsf{C}_{\mathsf{sub}}$ for $\mathsf{ARCAD}_{\mathsf{sub}}$. We define a hybrid predicate $\mathsf{C}^{\mathsf{C}_{\mathsf{clean}}}_{\mathsf{C}_{\mathsf{main}},\mathsf{C}_{\mathsf{sub}}}$ as follows. By abuse of notation, we write $\mathsf{C}^{\mathsf{clean}}_{\mathsf{main,sub}}$ instead, for more readability. Let Γ be a game played by an adversary $\mathcal{A}$ against $\mathsf{hybridARCAD}$.

We let $(\mathsf{ad}, \mathsf{ct})$ be the challenge message $(\mathsf{ad}_{\mathsf{test}}, \mathsf{ct}_{\mathsf{test}})$ if it exists. Otherwise, $(\mathsf{ad}, \mathsf{ct})$ is the last message in Γ. We let (e, c) be the number of $(\mathsf{ad}, \mathsf{ct})$. We let

$$\mathsf{C}^{\mathsf{clean}}_{\mathsf{main,sub}}(\Gamma) = \begin{cases} \text{if } (\mathsf{ad},\mathsf{ct}) \text{ belongs to } \mathsf{ARCAD}_{\mathsf{main}}: & \mathsf{C}_{\mathsf{main}}(\mathsf{main}(\Gamma)) \\ \text{else}: & \begin{cases} \text{if } \mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{clean}}(\mathsf{main}(\Gamma)): & \mathsf{C}_{\mathsf{sub}}(\mathsf{sub}_{\mathsf{e},\mathsf{c}}(\Gamma)) \\ \text{else}: & \mathsf{C}_{\mathsf{sub}}(\mathsf{sub}'_{\mathsf{e},\mathsf{c}}(\Gamma)) \end{cases} \end{cases}$$

This means that if the challenge holds on an $\mathsf{ARCAD}_{\mathsf{main}}$ message, we only care for $\mathsf{main}(\Gamma)$ to be $\mathsf{C}_{\mathsf{main}}$-clean. Otherwise, either the $\mathsf{ARCAD}_{\mathsf{main}}$ message initiating the relevant $\mathsf{ARCAD}_{\mathsf{sub}}$ session is $\mathsf{C}_{\mathsf{clean}}$ or not. If it is clean, we can replace it and consider $\mathsf{C}_{\mathsf{sub}}$-cleanness for $\mathsf{sub}_{\mathsf{e},\mathsf{c}}(\Gamma)$. Otherwise, the initial $\mathsf{ARCAD}_{\mathsf{sub}}$ state st_{R} trivially leaked (or was exposed, equivalently) and we consider

$\mathsf{C}_{\mathsf{sub}}$-cleanness for $\mathsf{sub}'_{\mathsf{e},\mathsf{c}}(\Gamma)$. The role of $\mathsf{C}_{\mathsf{clean}}$ is to control which of the two games to use. $\mathsf{C}_{\mathsf{clean}}$ must be a privacy cleanness notion for main. Contrarily, $\mathsf{C}_{\mathsf{main}}$ and $\mathsf{C}_{\mathsf{sub}}$ could be either privacy or authenticity notions.

Note that $\mathsf{C}_{\mathsf{sub}}(\mathsf{sub}'_{\mathsf{e},\mathsf{c}}(\Gamma)) = \mathsf{false}$ for $\mathsf{C}_{\mathsf{sub}} = \mathsf{C}_{\mathsf{noexp}}$, due to the $\mathsf{EXP}_{\mathsf{st}}$ call.

We easily obtain the following result.

Lemma 21. *If* $\mathsf{ARCAD}_{\mathsf{main}}$ *is* $\mathsf{C}_{\mathsf{main}}$-IND-CCA-*secure and* $\mathsf{ARCAD}_{\mathsf{sub}}$ *is* $\mathsf{C}_{\mathsf{sub}}$-IND-CCA-*secure, then* $\mathsf{hybridARCAD}$ *is* $\mathsf{C}_{\mathsf{clean}}$-IND-CCA *with* $\mathsf{C}_{\mathsf{clean}} = \mathsf{C}^{\mathsf{main}}_{\mathsf{main},\mathsf{sub}}$.

Proof. Let Γ be an IND-CCA game for $\mathsf{hybridARCAD}$. Let us assume that Γ is clean with our new cleanness notion $\mathsf{C}_{\mathsf{clean}} = \mathsf{C}^{\mathsf{main}}_{\mathsf{main},\mathsf{sub}}$.

Let $(\mathsf{ad},\mathsf{ct})$ be the challenge message. If there is no challenge message in Γ, we let $(\mathsf{ad},\mathsf{ct})$ be the last message sent by any participant in Γ. The $(\mathsf{ad},\mathsf{ct})$ message belongs to either $\mathsf{ARCAD}_{\mathsf{main}}$ or $\mathsf{ARCAD}_{\mathsf{sub}}$. It depends on $\mathsf{ad.flag}$ and on whether this is the very first message of the participant or not (because we force to use $\mathsf{ARCAD}_{\mathsf{main}}$ in this case).

We define the following non-overlapping events/cases:

- $\mathsf{C}_{\mathsf{main}}$: $(\mathsf{ad},\mathsf{ct})$ belongs to $\mathsf{ARCAD}_{\mathsf{main}}$;
- $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{true}}$: $(\mathsf{ad},\mathsf{ct})$ belongs to $\mathsf{ARCAD}_{\mathsf{sub}}$, has number (e,c), and $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{main}}(\mathsf{main}(\Gamma))$ is true;
- $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{false}}$: $(\mathsf{ad},\mathsf{ct})$ belongs to $\mathsf{ARCAD}_{\mathsf{sub}}$, has number (e,c), and $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{main}}(\mathsf{main}(\Gamma))$ is false.

We know that Γ is clean following $\mathsf{C}_{\mathsf{clean}}$. In the $\mathsf{C}_{\mathsf{main}}$ case ($(\mathsf{ad},\mathsf{ct})$ belongs to $\mathsf{ARCAD}_{\mathsf{main}}$), by definition of $\mathsf{C}_{\mathsf{clean}}$, we deduce that $\mathsf{main}(\Gamma)$ is $\mathsf{C}_{\mathsf{main}}$-clean. The outcome of $\mathsf{main}(\Gamma)$ and Γ is obviously the same. So is the advantage. Due to the $\mathsf{C}_{\mathsf{main}}$-IND-CCA security of $\mathsf{ARCAD}_{\mathsf{main}}$, the advantage in Γ conditioned to $\mathsf{C}_{\mathsf{main}}$ is negligible.

In what follows, we consider that $(\mathsf{ad},\mathsf{ct})$ belongs to $\mathsf{ARCAD}_{\mathsf{sub}}$.

$\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{main}}$ indicates if the $\mathsf{ARCAD}_{\mathsf{main}}$ message of pair (e,c) can be replaced by the encryption of something random to produce the same result, except with negligible probability: If $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{C}_{\mathsf{main}}}$ is true, $\mathsf{sub}_{\mathsf{e},\mathsf{c}}(\Gamma)$ produces the same outcome as Γ. So, the advantages of Γ and $\mathsf{sub}_{\mathsf{e},\mathsf{c}}(\Gamma)$ have a negligible difference when $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{true}}$ holds. By definition of $\mathsf{C}_{\mathsf{clean}}$, $\mathsf{sub}_{\mathsf{e},\mathsf{c}}(\Gamma)$ must be $\mathsf{C}_{\mathsf{sub}}$-clean. Due to the $\mathsf{C}_{\mathsf{sub}}$-IND-CCA security of $\mathsf{ARCAD}_{\mathsf{sub}}$, the advantage in $\mathsf{sub}_{\mathsf{e},\mathsf{c}}(\Gamma)$ is negligible. Hence, the advantage in Γ conditioned to $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{true}}$ is negligible.

Similarly, if $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{clean}}(\Gamma)$ does not hold, $\mathsf{C}_{\mathsf{clean}}$ implies that $\mathsf{sub}'_{\mathsf{e},\mathsf{c}}(\Gamma)$ is clean. This game produces exactly the same outcome as Γ when $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{false}}$ holds. So is the advantage. Due to the $\mathsf{C}_{\mathsf{sub}}$-IND-CCA security of $\mathsf{ARCAD}_{\mathsf{sub}}$, the advantage in Γ conditioned to $\mathsf{C}^{\mathsf{e},\mathsf{c}}_{\mathsf{false}}$ is negligible.

In all cases, the advantage in Γ is negligible. As the number of cases is polynomially bounded, the advantage in Γ is negligible. □

In the FORGE game, we replace the $\mathsf{C}_{\mathsf{trivial}}$ predicate. Typically, by taking $\mathsf{C}_{\mathsf{main}}$ as the predicate that tests if the last $(\mathsf{ad},\mathsf{ct})$ message is a trivial forgery and by taking $\mathsf{C}_{\mathsf{sub}}$ as the predicate that additionally tests if no $\mathsf{EXP}_{\mathsf{st}}$ occurred, the $\mathsf{C}^{\mathsf{clean}}_{\mathsf{main},\mathsf{sub}}$ predicate defines a new FORGE notion for $\mathsf{hybrid}(\mathsf{ARCAD}_{\mathsf{DV}},\mathsf{liteARCAD})$. More generally, if $\mathsf{ARCAD}_{\mathsf{main}}$ is $\mathsf{C}_{\mathsf{main}}$-FORGE-secure and $\mathsf{ARCAD}_{\mathsf{sub}}$ is $\mathsf{C}_{\mathsf{sub}}$-FORGE-secure, we would like to have $\mathsf{C}^{\mathsf{C}_{\mathsf{clean}}}_{\mathsf{C}_{\mathsf{main}},\mathsf{C}_{\mathsf{sub}}}$-FORGE-security.

We almost have the reduction but there is something missing. Namely, a forgery for $\mathsf{hybridARCAD}$ in Γ may not be a forgery for neither $\mathsf{ARCAD}_{\mathsf{main}}$ in $\mathsf{main}(\Gamma)$ nor $\mathsf{ARCAD}_{\mathsf{sub}}$ in $\mathsf{sub}_{\mathsf{e},\mathsf{c}}(\Gamma)$. This happens if the adversary in Γ drops the delivery of the last messages in a sub scheme. We relax FORGE-security using the FORGE^* game in Fig. 11. Only Steps 4 and 8 are new. Our chain strengthening in Section 3 can later make the protocols fully FORGE-secure. We easily prove the following result.

Lemma 22. *If* $\mathsf{ARCAD}_{\mathsf{main}}$ *is* $\mathsf{C}_{\mathsf{clean}}$-IND-CCA-*secure and* $\mathsf{C}_{\mathsf{main}}$-$\mathsf{FORGE}^*$-*secure and if* $\mathsf{ARCAD}_{\mathsf{sub}}$ *is* $\mathsf{C}_{\mathsf{sub}}$-$\mathsf{FORGE}^*$-*secure, then* $\mathsf{hybridARCAD}$ *is* $\mathsf{C}_{\mathsf{hybrid}}$-$\mathsf{FORGE}^*$*, where* $\mathsf{C}_{\mathsf{hybrid}} = \mathsf{C}^{\mathsf{clean}}_{\mathsf{main},\mathsf{sub}}$.

Proof. We proceed like in the proof of Lemma 21. Let Γ be a FORGE^* game for $\mathsf{hybridARCAD}$. Let $(\mathsf{P},\mathsf{ad},\mathsf{ct})$ be the output of the adversary. The $(\mathsf{ad},\mathsf{ct})$ message belongs to either $\mathsf{ARCAD}_{\mathsf{main}}$ or $\mathsf{ARCAD}_{\mathsf{sub}}$. We show below that

$$\mathsf{Adv}(\Gamma) \leqslant \mathsf{Adv}(\mathsf{main}(\Gamma)) + \sum_{\mathsf{e},\mathsf{c}} \mathsf{Adv}(\mathsf{sub}_{\mathsf{e},\mathsf{c}}(\Gamma)) + \sum_{\mathsf{e},\mathsf{c}} \mathsf{Adv}(\mathsf{sub}'_{\mathsf{e},\mathsf{c}}(\Gamma)) + \mathsf{negl}$$

Game $\mathsf{FORGE}^{*,\mathcal{A}}_{\mathsf{C}_{\mathsf{clean}}}(1^\lambda)$

1: $\mathsf{Setup}(1^\lambda) \xrightarrow{\$} \mathsf{pp}$
2: $\mathsf{Initall}(1^\lambda, \mathsf{pp}) \xrightarrow{\$} (\mathsf{st}_\mathrm{A}, \mathsf{st}_\mathrm{B}, z)$
3: $(\mathsf{P}, \mathsf{ad}, \mathsf{ct}) \leftarrow \mathcal{A}^{\mathsf{RATCH},\mathsf{EXP}_{\mathsf{st}},\mathsf{EXP}_{\mathsf{pt}}}(z)$
4: **if** one participant (or both) is NOT in a matching status **then return** 0
5: $\mathsf{RATCH}(\mathsf{P}, \text{“rec”}, \mathsf{ad}, \mathsf{ct}) \to \mathsf{acc}$
6: **if** $\mathsf{acc} = \mathsf{false}$ **then return** 0
7: **if** $\neg \mathsf{C}_{\mathsf{clean}}$ **then return** 0
8: **if** we can parse $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}} = (\mathsf{seq}_1, (\mathsf{ad}, \mathsf{ct}))$ and $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}} = (\mathsf{seq}_1, \mathsf{seq}_2, (\mathsf{ad}, \mathsf{ct}), \mathsf{seq}_3)$ **then return** 0
9: **return** 1

Fig. 11: Relaxed FORGE Security.

Applying FORGE^* security for the three terms, $\mathsf{Adv}(\Gamma)$ is negligible. To prove the above inequality, we show that when Γ returns 1, then at least one of the three other games return 1, with negligible exceptions.

We first assume that $(\mathsf{ad}, \mathsf{ct})$ belongs to $\mathsf{ARCAD}_{\mathsf{main}}$ and $\Gamma = \mathsf{FORGE}^*$ succeeds to return 1. Since Γ returns 1, both participants are in a matching status before we deliver the forgery to P. Hence, both participants are in a matching status in $\mathsf{main}(\Gamma)$ too. Similarly, since $(\mathsf{ad}, \mathsf{ct})$ is accepted by $\mathsf{RATCH}(\mathsf{P}, .)$ in Γ and it belongs to $\mathsf{main}(\Gamma)$, it is accepted by $\mathsf{RATCH}(\mathsf{P}, .)$ in $\mathsf{main}(\Gamma)$ too. Let seq_1 be the value of $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}$ in Γ before receiving $(\mathsf{ad}, \mathsf{ct})$. Since both participants were in a matching status, we know that $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$ starts with seq_1 in Γ. As Γ returns 1, we know that $(\mathsf{ad}, \mathsf{ct})$ does not appear anywhere in $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$ after seq_1. In $\mathsf{main}(\Gamma)$, the values of $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}$ and $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$ are sub-sequences of the values in Γ. By the same reasoning, we have $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}} = (\mathsf{seq}'_1, (\mathsf{ad}, \mathsf{ct}))$ in $\mathsf{main}(\Gamma)$ and $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$ starts with seq'_1. But seq'_1 must be a sub-sequence of seq_1 so $(\mathsf{ad}, \mathsf{ct})$ cannot appear after it in $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$. Finally, since $\mathsf{C}_{\mathsf{hybrid}}$ holds and $(\mathsf{ad}, \mathsf{ct})$ belongs to $\mathsf{ARCAD}_{\mathsf{main}}$, $\mathsf{C}_{\mathsf{main}}(\mathsf{main}(\Gamma))$ holds, by definition of $\mathsf{C}_{\mathsf{hybrid}}$. This means that $\mathsf{main}(\Gamma)$ is $\mathsf{C}_{\mathsf{main}}$-clean. We deduce that $\mathsf{main}(\Gamma)$ succeeds to return 1 as well.

Similarly, if $(\mathsf{ad}, \mathsf{ct})$ belongs to $\mathsf{ARCAD}_{\mathsf{sub}}$ and Γ returns 1, we treat two cases depending on whether $\mathsf{C}^{e,\mathsf{c}}_{\mathsf{clean}}(\Gamma)$ holds or not. Let Γ' be the game in which ct is replaced by the encryption of a random string. If $\mathsf{C}^{e,\mathsf{c}}_{\mathsf{clean}}(\Gamma)$ is true, thanks to $\mathsf{C}_{\mathsf{clean}}$-IND-CCA security, Γ and Γ' produce the same output, but with negligible probability. Hence, Γ' outputs 1, except in negligible cases. Like in the previous case, we deduce that $\mathsf{sub}_{e,\mathsf{c}}(\Gamma)$ outputs 1:

- RATCH accepts in Γ' implies that RATCH accepts in $\mathsf{sub}_{e,\mathsf{c}}(\Gamma)$;
- $(\mathsf{ad}, \mathsf{ct})$ appears in $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$ in neither Γ' nor $\mathsf{sub}_{e,\mathsf{c}}(\Gamma)$);
- $\mathsf{sub}_{e,\mathsf{c}}(\Gamma)$ is $\mathsf{C}_{\mathsf{sub}}$-clean because $(\mathsf{ad}, \mathsf{ct})$ belongs to $\mathsf{ARCAD}_{\mathsf{sub}}$ and $\mathsf{C}^{e,\mathsf{c}}_{\mathsf{clean}}(\Gamma)$ is true.

Finally, if $\mathsf{C}^{e,\mathsf{c}}_{\mathsf{clean}}(\Gamma)$ is false, we apply the same reasoning with $\mathsf{sub}'_{e,\mathsf{c}}(\Gamma)$. □

What FORGE^* security does not guarantee is that some forgeries in a sub-scheme may occur in the far future, due to state exposure. Fortunately, our protocol mitigates this problem by making sure that old sub-protocols become obsolete. Indeed, our protocol makes sure that sent messages always have an increasing sequence of (e, c) pairs, and the same for received messages. Hence, we cannot have a forgery with an old (e, c) pair. Another problem which is explicit in Step 8 of the game is that the adversary may prevent P from receiving a sequence seq_2 sent from $\overline{\mathsf{P}}$ (namely in a sub-protocol). In Section 3, making the protocol r-RECOVER-secure fixes both problems. (See Lemma 24.) Hence, we will obtain FORGE-security.

4.5 Security-aware Hybrid Construction

In this section, we apply our results from Section 3.3 to our hybrid constructions.

Lemma 23. *Let* $\mathsf{C_{clean}} \in \{\mathsf{C_{trivial}}, \mathsf{C_{noexp}}\}$ *and* $\mathsf{ARCAD}_1 = \mathsf{chain}(\mathsf{ARCAD}_0)$*. If* ARCAD_0 *is* $\mathsf{C_{clean}}$-FORGE-*secure (resp.* $\mathsf{C_{clean}}$-FORGE^*-*secure), then* ARCAD_1 *is* $\mathsf{C_{clean}}$-FORGE-*secure (resp.* $\mathsf{C_{clean}}$-FORGE^*-*secure).*

Proof. We reduce an adversary playing the FORGE game with ARCAD_1 to an adversary playing the FORGE game with ARCAD_0 by simulating the hashings. ARCAD_1 is an extension of ARCAD_0 such that an ARCAD_1 message $(\mathsf{ad}, (\mathsf{ct}', \mathsf{h}, \mathsf{ack}))$ is equivalent to an ARCAD_0 message $((\mathsf{ad}, \mathsf{h}, \mathsf{ack}), \mathsf{ct}')$. It is just reordering $(\mathsf{ad}, \mathsf{ct})$. Hence, a forgery for ARCAD_1 must be a forgery for ARCAD_0. FORGE^*-security works the same. □

Lemma 24. *Given* $\mathsf{ARCAD_{main}}$ *and* $\mathsf{ARCAD_{sub}}$*, let*

$$\mathsf{ARCAD}_0 = \mathsf{hybrid}(\mathsf{ARCAD_{main}}, \mathsf{ARCAD_{sub}}) \ , \ \mathsf{ARCAD}_1 = \mathsf{chain}(\mathsf{ARCAD}_0)$$

If $\mathsf{ARCAD_{main}}$ *is* $\mathsf{C_{clean}}$-IND-CCA-*secure and* $\mathsf{C_{main}}$-FORGE^*-*secure and* $\mathsf{ARCAD_{sub}}$ *is* $\mathsf{C_{sub}}$-FORGE^*-*secure, then* ARCAD_1 *is* $\mathsf{C}^{\mathsf{clean}}_{\mathsf{main,sub}}$-$\mathsf{FORGE}^*$-*secure. If* H *is additionally collision-resistant, then* ARCAD_1 *is* $\mathsf{C}^{\mathsf{clean}}_{\mathsf{main,sub}}$-FORGE-*secure.*

Proof. Due to Lemma 22, $\mathsf{C}^{\mathsf{clean}}_{\mathsf{main,sub}}$-$\mathsf{FORGE}^*$-security works like in the previous result. To extend to $\mathsf{C}^{\mathsf{clean}}_{\mathsf{main,sub}}$-FORGE-security, we just observe that ARCAD_1 is r-RECOVER-secure due to Lemma 15. We thus deduce $\mathsf{seq}_2 = \bot$ from having $\mathsf{receive}^{\mathsf{P}}_{\mathsf{ct}} = (\mathsf{seq}_1, (\mathsf{ad}, \mathsf{ct}))$ and $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}} = (\mathsf{seq}_1, \mathsf{seq}_2, (\mathsf{ad}, \mathsf{ct}), \mathsf{seq}_3)$. Hence, we have a full forgery, except with negligible probability. □

Lemma 25. *Let* $\mathsf{C_{clean}} = \mathsf{C_{leak}}$*,* $\mathsf{C_{ratchet}}$*,* $\mathsf{C_{noexp}}$*, or* $\mathsf{C}^{\mathsf{S}}_{\mathsf{tforge}}$ *(*$\mathsf{t} = \mathsf{trivial}$ *or* $\bot$*},* $\mathsf{S} = \mathsf{P_{test}}$ *or* $\{\mathrm{A}, \mathrm{B}\}$*), If* ARCAD_0 *is* $\mathsf{C_{clean}}$-IND-CCA-*secure, then* ARCAD_1 *is* $\mathsf{C_{clean}}$-IND-CCA-*secure.*

Proof. We reduce an adversary playing the IND-CCA game with ARCAD_1 to an adversary playing the IND-CCA game with ARCAD_0 by simulating the hashings. We easily see that the cleanness is the same and that the simulation is perfect. □

We easily extend this result to hybrid constructions. We conclude with our final result.

Theorem 26. *Given* $\mathsf{ARCAD_{main}}$ *and* $\mathsf{ARCAD_{sub}}$*, let*

$$\mathsf{ARCAD}_0 = \mathsf{hybrid}(\mathsf{ARCAD_{main}}, \mathsf{ARCAD_{sub}}) \ , \ \mathsf{ARCAD}_1 = \mathsf{chain}(\mathsf{ARCAD}_0)$$

We assume that 1. H *is collision-resistant; 2.* $\mathsf{ARCAD_{main}}$ *is* $\mathsf{C_{clean}}$-IND-CCA-*secure and* $\mathsf{C_{main}}$-FORGE^*-*secure; 3.* $\mathsf{ARCAD_{sub}}$ *is* $\mathsf{C_{sub}}$-FORGE^*-*secure and* $\mathsf{C'_{clean}}$-IND-CCA-*secure. Then,* ARCAD_1 *is 1.* r-RECOVER-*secure, 2.* s-RECOVER-*secure, 3.* $\mathsf{C}^{\mathsf{clean}}_{\mathsf{main,sub}}$-FORGE-*secure, 4.* $\mathsf{C}^{\mathsf{clean}}_{\mathsf{clean,clean'}}$-IND-CCA-*secure, 5. with acknowledgement extractor.*

Corollary 27. *Let* $\mathsf{ARCAD}_1 = \mathsf{chain}(\mathsf{hybrid}(\mathsf{ARCAD_{DV}}, \mathsf{liteARCAD}))$ *and let* $\mathsf{C_{clean}} = \mathsf{C_{leak}} \wedge \mathsf{C}^{\mathrm{A},\mathrm{B}}_{\mathsf{forge}}$*. With the assumptions from Th. 28 and Th. 19, if* H *is collision-resistant,* ARCAD_1 *is* $\mathsf{C}^{\mathsf{clean}}_{\mathsf{trivial,noexp}}$-FORGE-*secure,* $\mathsf{C}^{\mathsf{clean}}_{\mathsf{clean,sym}}$-IND-CCA-*secure, and with security-awareness.*

In particular, when a sender deduces an acknowledgment for his message $\mathfrak{m}$ from a received message $\mathfrak{m}'$, if he can make sure that $\mathfrak{m}'$ is genuine and that no trivial exposure for $\mathfrak{m}$ happened, then he can be sure that his message $\mathfrak{m}$ is private, no matter what happened before or what will happen next.

5 Implementations/Comparisons with Existing Protocols

We compare the performances of $\mathsf{ARCAD_{DV}}$ and liteARCAD to other ratcheted messaging and key agreement protocols that have surfaced since 2018. In particular, we implemented five other schemes from the literature. They are the bidirectional asynchronous key-agreement protocol

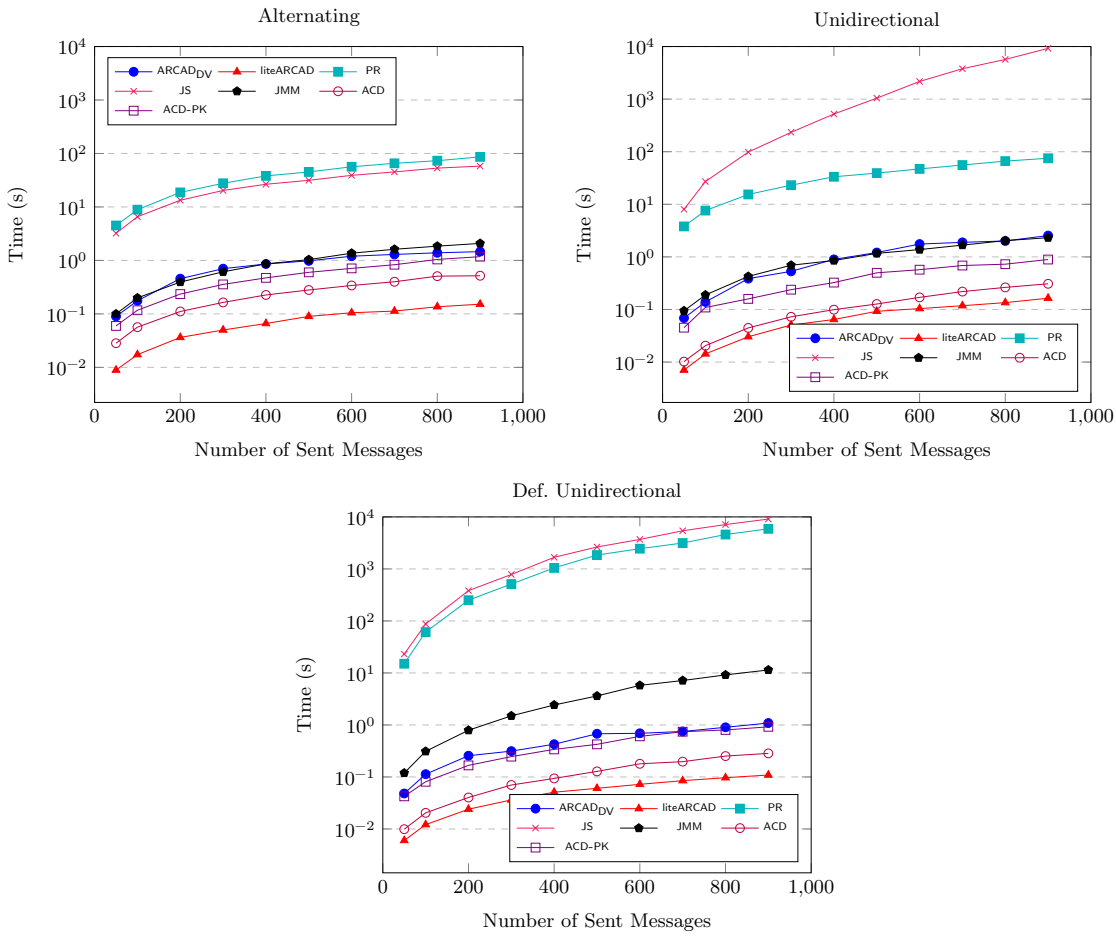

Fig. 12: Runtime Benchmarks
The protocol in [9] is represented with PR; [7] with JS; [8] with JMM; and [1] with ACD and ACD-PK. ACD-PK is the public-key version with stronger security.

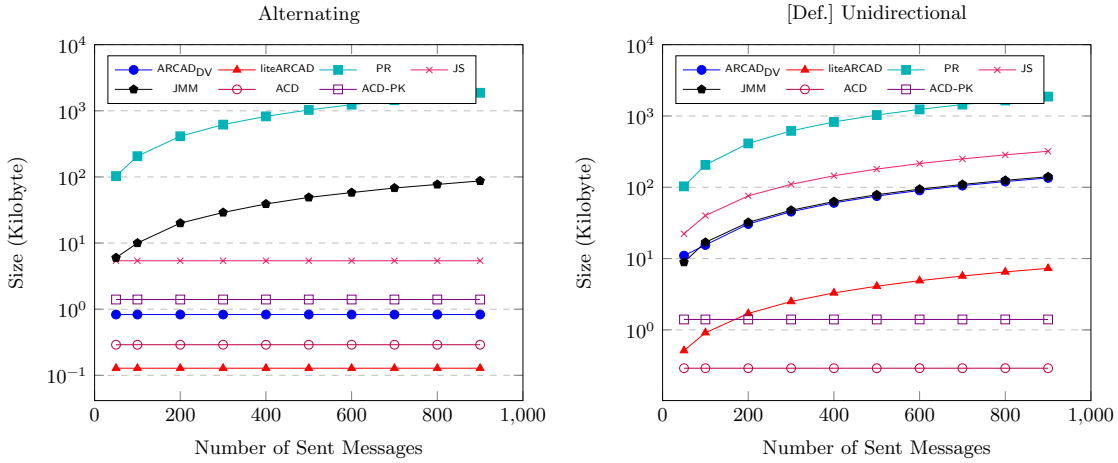

Fig. 13: State Size Benchmarks
Due to the equivalent state sizes in unidirectional and deferred unidirectional traffic, one figure is omitted

BRKE by PR [9], the similar secure messaging protocol by JS [7], the secure messaging protocol by JMM [8] and a modularized version of two protocols by ACD [1]. In ACD [1], the given protocols are both with symmetric key cryptography ACD and public-key cryptography ACD-PK. We did not implement the DV protocol [6], as $\mathsf{ARCAD}_{\mathsf{DV}}$ is a slightly modified version of DV, hence has identical performances.

All the protocols were implemented in Go [13] and measured with its built-in benchmarking suite [14] on a regular fifth generation Intel Core i5 processor. In order to mitigate potential overheads garbage collection has been disabled for all runs. Go is comparable in speed to C/C++ though further performance gains are within reach when the protocols are re-implemented in the latter two. Additionally, some protocols deploy primitives for which no standard implementations exists, which is for example the case for the HIBE constructions used in the PR and JS protocols, making custom implementations necessary that can certainly be improved upon. For the deployed primitives, when we needed an AEAD scheme, we used AES-GCM. For public key cryptosystem, we used elliptic curve version of ElGamal (ECIES); for the signature scheme, we used ECDSA. And, finally for the PRF-PRNG in [1] protocol, we used HKDF with SHA-256. Lastly, the protocols themselves may offer some room for performance tweaks.

The benchmarks can be categorized into two types as depicted in Fig. 12–13.

(a) Runtime designates the total required time to exchange $\mathfrak{n}$ messages, ignoring potential latency that normally occurs in a network.
(b) State size shows the maximal size of a user state throughout the exchange of $\mathfrak{n}$ messages.

A state is all the data that is kept in memory by a user. Each type itself is run on three canonical ways traffic can be shaped when two participants are communicating. In alternating traffic the parties are synchronized, i.e. take turns sending messages. In unidirectional traffic one participant first sends $\frac{\mathfrak{n}}{2}$ messages which are received by the partner who then sends the other half. Finally, in deferred unidirectional traffic both participants send $\frac{\mathfrak{n}}{2}$ messages before they start receiving. ACD-PK adds some public-key primitives to the double ratchet by ACD [1] to plug some post-compromise security gaps. These two variations serve as baselines to see how the metrics of a protocol can change when some of its internals are replaced or extended. Also note that due to the equivalent state sizes in unidirectional and deferred unidirectional traffic one figure is omitted.

[13] `https://golang.org/`
[14] `https://golang.org/pkg/testing/`

As we can see, overall, the fastest protocol is liteARCAD, followed by the two ACD protocols, then $\mathsf{ARCAD}_{\mathsf{DV}}$, then the JMM protocol, and lastly the strongest protocols PR and JS. $\mathsf{ARCAD}_{\mathsf{DV}}$ and JMM may be comparable except for deferred unidirectional communication.

The smallest state size is obtained with liteARCAD. $\mathsf{ARCAD}_{\mathsf{DV}}$ performs well in terms of state size.

Clearly, $\mathsf{hybrid}(\mathsf{ARCAD}_{\mathsf{DV}}, \mathsf{liteARCAD})$ has performances which are weighted averages of the ones of $\mathsf{ARCAD}_{\mathsf{DV}}$ and liteARCAD, depending on the frequency of on-demand ratcheting.

6 Conclusion

We revisited the DV security model. We proposed an additional lite protocol liteARCAD. We compared the performance of existing protocols with liteARCAD. Based on the good results of liteARCAD, we proposed an hybrid construction which would mostly use liteARCAD and occasionally a stronger protocol, upon the choice of the sender, thus achieving on-demand ratcheting. Finally, we proposed the notion of security awareness to enable participants to have a better idea on the safety of their communication. We achieved what we think is the optimal awareness. Concretely, a participant is aware of which of his messages arrived to his counterpart when he sent the last received one. We make sure that any forgery (possibly due to exposure) would fork the chain of messages which is seen by both participants and result in making them unable to continue communication. We also make sure that assuming that the exposure history is known, participants can deduce which messages leaked.

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A Appendix

A.1 $\mathsf{ARCAD_{DV}}$ Formal Protocol

$\mathsf{ARCAD_{DV}}$ is based on a hash function H [15], a one-time symmetric cipher Sym[16], a digital signature scheme DSS[17], and a public-key cryptosystem PKC[18].

$\mathsf{ARCAD_{DV}}$, just as DV, consists of many modules which are built on top of each other. The "smallest" module is a "naive" signcryption scheme SC which can be of the form

$$\mathsf{SC.Enc}(\overbrace{\mathsf{sk_S}, \mathsf{pk_R}}^{\mathsf{st_S}}, \mathsf{ad}, \mathsf{pt}) = \mathsf{PKC.Enc}(\mathsf{pk_R}, (\mathsf{pt}, \mathsf{DSS.Sign}(\mathsf{sk_S}, (\mathsf{ad}, \mathsf{pt}))))$$
$$\mathsf{SC.Dec}(\underbrace{\mathsf{sk_R}, \mathsf{pk_S}}_{\mathsf{st_R}}, \mathsf{ad}, \mathsf{ct}) = \begin{bmatrix} (\mathsf{pt}, \sigma) \leftarrow \mathsf{PKC.Dec}(\mathsf{sk_R}, \mathsf{ct}) \ ; \\ \mathsf{DSS.Verify}(\mathsf{pk_S}, (\mathsf{ad}, \mathsf{pt}), \sigma) \ ? \ \mathsf{pt} \ : \ \perp \end{bmatrix}$$

SC extends to a multiple-state (and multiple-key) encryption called onion. It handles the the case where the states get accumulated during a sequential send or receive operation during the communication. It generates a secret key to encrypt a plaintext. This secret key is, then, secret shared and encrypted under different states so that if a state is exposed, its shares would still remain confidential. onion leads to a unidirectional scheme called uni where participants have fixed roles as either senders or receivers. The underlying idea of unidirectional communication is to let the sender generate the next send/receive states for the future exchange during the current send operation and transmit the next receive state to the receiver. These future states are shown as $\mathsf{st'_S}$ and $\mathsf{st'_R}$ in the second row of Fig. 14. After each $\mathsf{uni.Send}$ and $\mathsf{uni.Rec}$ operations, the states are completely flushed to ensure security.

Finally, unidirectional communication allow us to construct the bidirectional $\mathsf{ARCAD_{DV}}$ as shown in the last row of Fig.14. Since the communication become bidirectional, the participant P also keeps states for receiving. More specifically, the sender generates a pair of fresh states and transmits the send state to the counterpart so that s/he can use it to send a reply to back to the sender with this states.

$\mathsf{ARCAD_{DV}}$ is depicted on Fig. 15.

Note that we removed some parts of the protocol which ensure $\mathsf{r\text{-}RECOVER}$ security. This is because the generic transformation in Section 3 which we apply on $\mathsf{ARCAD_{DV}}$ will restore it in a stronger and generic way.

Theorem 28 (Security of $\mathsf{ARCAD_{DV}}$ [6]). $\mathsf{ARCAD_{DV}}$ *is correct. If* $\mathsf{Sym.kl}(\lambda) = \Omega(\lambda)$, H *is collision-resistant,* DSS *is* $\mathsf{SEF\text{-}OTCMA}$, PKC *is* $\mathsf{IND\text{-}CCA}$*-secure, and* Sym *is* $\mathsf{IND\text{-}OTCCA}$*-secure, then* $\mathsf{ARCAD_{DV}}$ *is* $\mathsf{C_{trivial}}$-FORGE*-secure,* $(\mathsf{C_{leak}} \wedge \mathsf{C}^{\mathrm{A,B}}_{\mathsf{forge}})$-$\mathsf{IND\text{-}CCA}$*-secure and* $\mathsf{PREDICT}$*-secure.*[19,20]

[15] H uses a common key hk generated by $\mathsf{H.Gen}$ and an algorithm $\mathsf{H.Eval}$.

[16] Sym uses a key of length $\mathsf{Sym.kl}$, encrypts over the domain $\mathsf{Sym}.\mathcal{D}$ with algorithm $\mathsf{Sym.Enc}$ and decrypts with $\mathsf{Sym.Dec}$.

[17] DSS uses a key generation $\mathsf{DSS.Gen}$, a signing algorithm $\mathsf{DSS.Sign}$, and a verification algorithm $\mathsf{DSS.Verify}$.

[18] PKC uses a key generation $\mathsf{PKC.Gen}$, an encryption algorithm $\mathsf{PKC.Enc}$, and a decryption algorithm $\mathsf{PKC.Dec}$.

[19] $\mathsf{SEF\text{-}OTCMA}$ is the strong existential one-time chosen message attack. $\mathsf{IND\text{-}OTCCA}$ is the real-or-random indistinguishability under one-time chosen plaintext and chosen ciphertext attack. Their definitions are given in [6].

[20] Following Durak-Vaudenay [6], for a $\mathsf{C_{trivial}}$-FORGE-secure scheme, $(\mathsf{C_{leak}} \wedge \mathsf{C}^{\mathrm{A,B}}_{\mathsf{forge}})$-$\mathsf{IND\text{-}CCA}$ security is equivalent to $(\mathsf{C_{leak}} \wedge \mathsf{C}^{\mathrm{A,B}}_{\mathsf{trivial\ forge}})$-$\mathsf{IND\text{-}CCA}$ security, which corresponds to the "sub-optimal" security in Table 1.

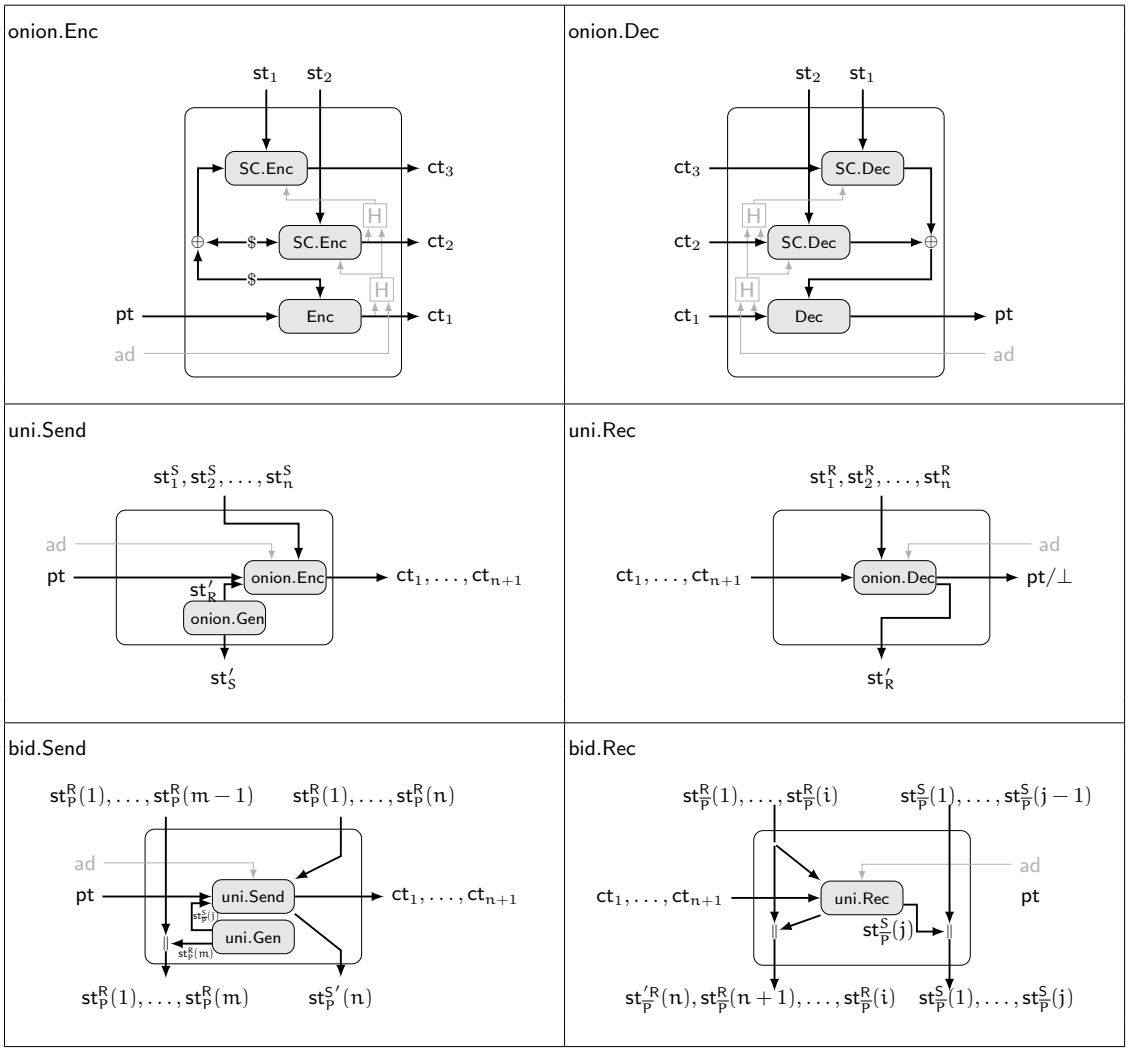

Fig. 14: $\mathsf{ARCAD}_{\mathsf{DV}}$ Protocol Adapted from DV [6] without RECOVER-Security.

A.2 ARCAD Definitions Adapted from the DV Model

We give some necessary definitions to borrow from DV and adapt to ARCAD. They are as follows.

Definition 29 (Correctness of ARCAD). *Consider the correctness game given on Fig. 16.*[21] *We say that an* ARCAD *protocol is* correct *if for all sequence* sched *of tuples of the form* $(\mathsf{P}, \text{“send”}, \mathsf{ad}, \mathsf{pt})$ *or* $(\mathsf{P}, \text{“rec”})$*, the game never returns 1. Namely,*

- *at each stage, for each* P*,* $\mathsf{received}^{\mathsf{P}}_{\mathsf{pt}}$ *is prefix of* $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{pt}}$[22]*;*
- *each* $\mathsf{RATCH}(\mathsf{P}, \text{“rec”})$ *call returns* $\mathsf{acc} = \mathsf{true}$*.*

Definition 30 (Matching status [6]). *We say that* P *is in a* matching status *at time* t *for* P *if*

1. *at any moment of the game before time* t *for* P*,* $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}$ *is a prefix of* $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{ct}}$ *— this defines the time* $\overline{\mathsf{t}}$ *for* $\overline{\mathsf{P}}$ *when* $\overline{\mathsf{P}}$ *sent the last message in* $\mathsf{received}^{\mathsf{P}}_{\mathsf{ct}}(\mathsf{t})$*;*

[21] We use the programming technique of “function overloading” to define the RATCH oracle: there are two definitions depending on whether the second input is “rec” or “send”.

[22] By saying that $\mathsf{received}^{\mathsf{P}}_{\mathsf{pt}}$ is prefix of $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{pt}}$, we mean that $\mathsf{sent}^{\overline{\mathsf{P}}}_{\mathsf{pt}}$ is the concatenation of $\mathsf{received}^{\mathsf{P}}_{\mathsf{pt}}$ with a (possible empty) list of $(\mathsf{ad}, \mathsf{pt})$ pairs.

$\mathsf{onion.Enc}(1^\lambda, \mathsf{hk}, \mathsf{st}_S^1, \ldots, \mathsf{st}_S^n, \mathsf{ad}, \mathsf{pt})$
1: pick $\mathsf{k}_1, \ldots, \mathsf{k}_n$ in $\{0,1\}^{\mathsf{Sym.kl}(\lambda)}$
2: $\mathsf{k} \leftarrow \mathsf{k}_1 \oplus \cdots \oplus \mathsf{k}_n$
3: $\mathsf{ct}_{n+1} \leftarrow \mathsf{Sym.Enc}(\mathsf{k}, \mathsf{pt})$
4: $\mathsf{ad}_{n+1} \leftarrow \mathsf{ad}$
5: **for** $i = n$ down to 1 **do**
6: $\quad \mathsf{ad}_i \leftarrow \mathsf{H.Eval}(\mathsf{hk}, \mathsf{ad}_{i+1}, n, \mathsf{ct}_{i+1})$
7: $\quad \mathsf{ct}_i \leftarrow \mathsf{SC.Enc}(\mathsf{st}_S^i, \mathsf{ad}_i, \mathsf{k}_i)$
8: **end for**
9: **return** $(\mathsf{ct}_1, \ldots, \mathsf{ct}_{n+1})$

$\mathsf{onion.Dec}(\mathsf{hk}, \mathsf{st}_R^1, \ldots, \mathsf{st}_R^n, \mathsf{ad}, \vec{\mathsf{ct}})$
1: **if** $|\vec{\mathsf{ct}}| \neq n+1$ **then return** $\perp$
2: parse $\vec{\mathsf{ct}} = (\mathsf{ct}_1, \ldots, \mathsf{ct}_{n+1})$
3: $\mathsf{ad}_{n+1} \leftarrow \mathsf{ad}$
4: **for** $i = n$ down to 1 **do**
5: $\quad \mathsf{ad}_i \leftarrow \mathsf{H.Eval}(\mathsf{hk}, \mathsf{ad}_{i+1}, n, \mathsf{ct}_{i+1})$
6: $\quad \mathsf{SC.Dec}(\mathsf{st}_R^i, \mathsf{ad}_i, \mathsf{ct}_i) \to \mathsf{k}_i$
7: $\quad$ **if** $\mathsf{k}_i = \perp$ **then return** $\perp$
8: **end for**
9: $\mathsf{k} \leftarrow \mathsf{k}_1 \oplus \cdots \oplus \mathsf{k}_n$
10: $\mathsf{pt} \leftarrow \mathsf{Sym.Dec}(\mathsf{k}, \mathsf{ct}_{n+1})$
11: **return** pt

$\mathsf{uni.Init}(1^\lambda)$
1: $\mathsf{SC.Gen}_S(1^\lambda) \xrightarrow{\$} (\mathsf{sk}_S, \mathsf{pk}_S)$
2: $\mathsf{SC.Gen}_R(1^\lambda) \xrightarrow{\$} (\mathsf{sk}_R, \mathsf{pk}_R)$
3: $\mathsf{st}_S \leftarrow (\mathsf{sk}_S, \mathsf{pk}_R)$
4: $\mathsf{st}_R \leftarrow (\mathsf{sk}_R, \mathsf{pk}_S)$
5: **return** $(\mathsf{st}_S, \mathsf{st}_R)$

$\mathsf{uni.Send}(1^\lambda, \mathsf{hk}, \vec{\mathsf{st}}_S, \mathsf{ad}, \mathsf{pt})$
1: $\mathsf{SC.Gen}_S(1^\lambda) \xrightarrow{\$} (\mathsf{sk}'_S, \mathsf{pk}'_S)$
2: $\mathsf{SC.Gen}_R(1^\lambda) \xrightarrow{\$} (\mathsf{sk}'_R, \mathsf{pk}'_R)$
3: $\mathsf{st}'_S \leftarrow (\mathsf{sk}'_S, \mathsf{pk}'_R)$
4: $\mathsf{st}'_R \leftarrow (\mathsf{sk}'_R, \mathsf{pk}'_S)$
5: $\mathsf{pt}' \leftarrow (\mathsf{st}'_R, \mathsf{pt})$
6: $\mathsf{onion.Enc}(1^\lambda, \mathsf{hk}, \vec{\mathsf{st}}_S, \mathsf{ad}, \mathsf{pt}') \to \vec{\mathsf{ct}}$
7: **return** $(\mathsf{st}'_S, \vec{\mathsf{ct}})$

$\mathsf{uni.Receive}(\mathsf{hk}, \vec{\mathsf{st}}_R, \mathsf{ad}, \vec{\mathsf{ct}})$
1: $\mathsf{onion.Dec}(\mathsf{hk}, \vec{\mathsf{st}}_R, \mathsf{ad}, \vec{\mathsf{ct}}) \to \mathsf{pt}'$
2: **if** $\mathsf{pt}' = \perp$ **then**
3: $\quad$ **return** $(\mathsf{false}, \perp, \perp)$
4: **end if**
5: parse $\mathsf{pt}' = (\mathsf{st}'_R, \mathsf{pt})$
6: **return** $(\mathsf{true}, \mathsf{st}'_R, \mathsf{pt})$

$\mathsf{ARCAD}_{\mathsf{DV}}.\mathsf{Setup}(1^\lambda)$
1: $\mathsf{H.Gen}(1^\lambda) \xrightarrow{\$} \mathsf{hk}$
2: **return** hk

$\mathsf{ARCAD}_{\mathsf{DV}}.\mathsf{Gen}(1^\lambda, \mathsf{hk})$
1: $\mathsf{SC.Gen}_S(1^\lambda) \xrightarrow{\$} (\mathsf{sk}_S, \mathsf{pk}_S)$
2: $\mathsf{SC.Gen}_R(1^\lambda) \xrightarrow{\$} (\mathsf{sk}_R, \mathsf{pk}_R)$
3: $\mathsf{sk} \leftarrow (\mathsf{sk}_S, \mathsf{sk}_R)$
4: $\mathsf{pk} \leftarrow (\mathsf{pk}_S, \mathsf{pk}_R)$
5: **return** $(\mathsf{sk}, \mathsf{pk})$

$\mathsf{ARCAD}_{\mathsf{DV}}.\mathsf{Init}(1^\lambda, \mathsf{pp}, \mathsf{sk}_P, \mathsf{pk}_{\overline{P}}, P)$
1: parse $\mathsf{sk}_P = (\mathsf{sk}_S, \mathsf{sk}_R)$
2: parse $\mathsf{pk}_{\overline{P}} = (\mathsf{pk}_S, \mathsf{pk}_R)$
3: $\mathsf{st}_P^{\mathsf{send}} \leftarrow (\mathsf{sk}_S, \mathsf{pk}_R)$
4: $\mathsf{st}_P^{\mathsf{rec}} \leftarrow (\mathsf{sk}_R, \mathsf{pk}_S)$
5: $\mathsf{st}_P \leftarrow (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send}}), (\mathsf{st}_P^{\mathsf{rec}}))$
6: **return** st_P

$\mathsf{ARCAD}_{\mathsf{DV}}.\mathsf{Send}(\mathsf{st}_P, \mathsf{ad}, \mathsf{pt})$
1: parse $\mathsf{st}_P = (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},v}))$
2: $\mathsf{uni.Init}(1^\lambda) \xrightarrow{\$} (\mathsf{st}_{S\mathsf{new}}, \mathsf{st}_P^{\mathsf{rec},v+1})$ ▷ append a new receive state to the $\mathsf{st}_P^{\mathsf{rec}}$ list
3: $\mathsf{pt}' \leftarrow (\mathsf{st}_{S\mathsf{new}}, \mathsf{pt})$ ▷ then, $\mathsf{st}_{S\mathsf{new}}$ is erased to avoid leaking
4: take the smallest i s.t. $\mathsf{st}_P^{\mathsf{send},i} \neq \perp$ ▷ $i = u - n$ if we had n Receive since the last Send
5: $\mathsf{uni.Send}(1^\lambda, \mathsf{hk}, \mathsf{st}_P^{\mathsf{send},i}, \ldots, \mathsf{st}_P^{\mathsf{send},u}, \mathsf{ad}, \mathsf{pt}') \xrightarrow{\$} (\mathsf{st}_P^{\mathsf{send},u}, \mathsf{ct})$ ▷ update $\mathsf{st}_P^{\mathsf{send},u}$
6: $\mathsf{st}_P^{\mathsf{send},i}, \ldots, \mathsf{st}_P^{\mathsf{send},u-1} \leftarrow \perp$ ▷ flush the send state list: only $\mathsf{st}_P^{\mathsf{send},u}$ remains
7: $\mathsf{st}'_P \leftarrow (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},v+1}))$
8: **return** $(\mathsf{st}'_P, \mathsf{ct})$

$\mathsf{ARCAD}_{\mathsf{DV}}.\mathsf{Receive}(\mathsf{st}_P, \mathsf{ad}, \mathsf{ct})$
9: parse $\mathsf{st}_P = (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},v}))$
10: set $n+1$ to the number of components in ct ▷ the onion has n layers
11: set i to the smallest index such that $\mathsf{st}_P^{\mathsf{rec},i} \neq \perp$
12: **if** $i + n - 1 > v$ **then return** $(\mathsf{false}, \mathsf{st}_P, \perp)$
13: $\mathsf{uni.Receive}(\mathsf{hk}, \mathsf{st}_P^{\mathsf{rec},i}, \ldots, \mathsf{st}_P^{\mathsf{rec},i+n-1}, \mathsf{ad}, \mathsf{ct}) \to (\mathsf{acc}, \mathsf{st}'^{\,\mathsf{rec},i+n-1}_P, \mathsf{pt}')$
14: **if** $\mathsf{acc} = \mathsf{false}$ **then return** $(\mathsf{false}, \mathsf{st}_P, \perp)$
15: parse $\mathsf{pt}' = (\mathsf{st}_P^{\mathsf{send},u+1}, \mathsf{pt})$ ▷ a new send state is added in the list
16: $\mathsf{st}_P^{\mathsf{rec},i}, \ldots, \mathsf{st}_P^{\mathsf{rec},i+n-2} \leftarrow \perp$ ▷ update stage 1: $n-1$ entries of $\mathsf{st}_P^{\mathsf{rec}}$ were erased
17: $\mathsf{st}_P^{\mathsf{rec},i+n-1} \leftarrow \mathsf{st}'^{\,\mathsf{rec},i+n-1}_P$ ▷ update stage 2: update $\mathsf{st}_P^{\mathsf{rec},i+n-1}$
18: $\mathsf{st}'_P \leftarrow (\lambda, \mathsf{hk}, (\mathsf{st}_P^{\mathsf{send},1}, \ldots, \mathsf{st}_P^{\mathsf{send},u+1}), (\mathsf{st}_P^{\mathsf{rec},1}, \ldots, \mathsf{st}_P^{\mathsf{rec},v}))$
19: **return** $(\mathsf{acc}, \mathsf{st}'_P, \mathsf{pt})$

Fig. 15: $\mathsf{ARCAD}_{\mathsf{DV}}$ Protocol Adapted from DV [6] without RECOVER-Security.

2. *at any moment of the game before time* $\overline{t}$ *for* $\overline{P}$, $\mathsf{received}_{\mathsf{ct}}^{\overline{P}}$ *is a prefix of* $\mathsf{sent}_{\mathsf{ct}}^{P}$.

We further say that time t *for* P originates *from time* $\overline{t}$ *for* $\overline{P}$.

Intuitively, P is in a matching status at a given time if his state is not influenced by an active attack (i.e. message injection/modification/erasure/replay). The PREDICT-security will become

Oracle $\mathsf{RATCH}(\mathsf{P}, \text{“rec”}, \mathsf{ad}, \mathsf{ct})$
1: $\mathsf{ct}_\mathsf{P} \leftarrow \mathsf{ct}$
2: $\mathsf{ad}_\mathsf{P} \leftarrow \mathsf{ad}$
3: $(\mathsf{acc}, \mathsf{st}'_\mathsf{P}, \mathsf{pt}_\mathsf{P}) \leftarrow \mathsf{Receive}(\mathsf{st}_\mathsf{P}, \mathsf{ad}_\mathsf{P}, \mathsf{ct}_\mathsf{P})$
4: **if** acc **then**
5: $\quad \mathsf{st}_\mathsf{P} \leftarrow \mathsf{st}'_\mathsf{P}$
6: $\quad$ append $(\mathsf{ad}_\mathsf{P}, \mathsf{pt}_\mathsf{P})$ to $\mathsf{received}^\mathsf{P}_\mathsf{pt}$
7: $\quad$ append $(\mathsf{ad}_\mathsf{P}, \mathsf{ct}_\mathsf{P})$ to $\mathsf{received}^\mathsf{P}_\mathsf{ct}$
8: **end if**
9: **return** acc

Oracle $\mathsf{RATCH}(\mathsf{P}, \text{“send”}, \mathsf{ad}, \mathsf{pt})$
10: $\mathsf{pt}_\mathsf{P} \leftarrow \mathsf{pt}$
11: $\mathsf{ad}_\mathsf{P} \leftarrow \mathsf{ad}$
12: $(\mathsf{st}'_\mathsf{P}, \mathsf{ct}_\mathsf{P}) \leftarrow \mathsf{Send}(\mathsf{st}_\mathsf{P}, \mathsf{ad}_\mathsf{P}, \mathsf{pt}_\mathsf{P})$
13: $\mathsf{st}_\mathsf{P} \leftarrow \mathsf{st}'_\mathsf{P}$
14: append $(\mathsf{ad}_\mathsf{P}, \mathsf{pt}_\mathsf{P})$ to $\mathsf{sent}^\mathsf{P}_\mathsf{pt}$
15: append $(\mathsf{ad}_\mathsf{P}, \mathsf{ct}_\mathsf{P})$ to $\mathsf{sent}^\mathsf{P}_\mathsf{ct}$
16: **return** ct_P

Game Correctness(sched)
1: set all sent^*_* and $\mathsf{received}^*_*$ to $\emptyset$
2: $\mathsf{Setup}(1^\lambda) \xrightarrow{\$} \mathsf{pp}$
3: $\mathsf{Initall}(1^\lambda, \mathsf{pp}) \xrightarrow{\$} (\mathsf{st}_\mathsf{A}, \mathsf{st}_\mathsf{B}, z)$
4: initialize two FIFO lists $\mathsf{incoming}_\mathsf{A}, \mathsf{incoming}_\mathsf{B} \leftarrow \emptyset$
5: $\mathsf{i} \leftarrow 0$
6: **loop**
7: $\quad \mathsf{i} \leftarrow \mathsf{i} + 1$
8: $\quad$ **if** $\mathsf{sched}_\mathsf{i}$ of form (P, “rec”) **then**
9: $\quad\quad$ **if** $\mathsf{incoming}_\mathsf{P}$ is empty **then return** 0
10: $\quad\quad$ pull $(\mathsf{ad}, \mathsf{ct})$ from $\mathsf{incoming}_\mathsf{P}$
11: $\quad\quad$ $\mathsf{acc} \leftarrow \mathsf{RATCH}(\mathsf{P}, \text{“rec”}, \mathsf{ad}, \mathsf{ct})$
12: $\quad\quad$ **if** $\mathsf{acc} = \mathsf{false}$ **then return** 1
13: $\quad$ **else**
14: $\quad\quad$ parse $\mathsf{sched}_\mathsf{i} = (\mathsf{P}, \text{“send”}, \mathsf{ad}, \mathsf{pt})$
15: $\quad\quad$ $\mathsf{ct} \leftarrow \mathsf{RATCH}(\mathsf{P}, \text{“send”}, \mathsf{ad}, \mathsf{pt})$
16: $\quad\quad$ push $(\mathsf{ad}, \mathsf{ct})$ to $\mathsf{incoming}_{\overline{\mathsf{P}}}$
17: $\quad$ **end if**
18: $\quad$ **if** $\mathsf{received}^\mathsf{A}_\mathsf{pt}$ not prefix of $\mathsf{sent}^\mathsf{B}_\mathsf{pt}$ **then return** 1
19: $\quad$ **if** $\mathsf{received}^\mathsf{B}_\mathsf{pt}$ not prefix of $\mathsf{sent}^\mathsf{A}_\mathsf{pt}$ **then return** 1
20: **end loop**

Fig. 16: The Correctness Game of ARCAD Protocol.

useful to reduce this definition to the two conditions that $\mathsf{received}^\mathsf{P}_\mathsf{ct}$ is a prefix of $\mathsf{sent}^{\overline{\mathsf{P}}}_\mathsf{ct}$ at time t for P and $\mathsf{received}^{\overline{\mathsf{P}}}_\mathsf{ct}$ is a prefix of $\mathsf{sent}^\mathsf{P}_\mathsf{ct}$ at time $\overline{\mathsf{t}}$ for $\overline{\mathsf{P}}$.

Definition 31 (Corresponding RATCH calls [6]). *Let* P *be a participant. We consider only the* $\mathsf{RATCH}(\mathsf{P}, \text{“rec”}, ., .)$ *calls by* P *returning* true. *We say that the* i^{th} *call* corresponds *to the* j^{th} $\mathsf{RATCH}(\overline{\mathsf{P}}, \text{“send”}, ., .)$ *call by* $\overline{\mathsf{P}}$ *if* $\mathsf{i} = \mathsf{j}$ *and* P *is in matching status at the time of this* i^{th} *accepting* $\mathsf{RATCH}(\mathsf{P}, \text{“rec”}, ., .)$ *call.*

Definition 32 (Forgery). *Given a participant* P *in a game, we say that* $(\mathsf{ad}, \mathsf{ct}) \in \mathsf{received}^\mathsf{P}_\mathsf{ct}$ *is a forgery if at the moment of the game just before* P *received* $(\mathsf{ad}, \mathsf{ct})$*,* P *was in a matching status, but no longer after receiving* $(\mathsf{ad}, \mathsf{ct})$*.*

Definition 33 (Trivial forgery). *Let* $(\mathsf{ad}, \mathsf{ct})$ *be a forgery received by* P*. At the time* t *just before the* $\mathsf{RATCH}(\mathsf{P}, \text{“rec”}, \mathsf{ad}, \mathsf{ct})$ *call,* P *was in a matching status. We assume that time* t *for* P *originates from time* $\overline{\mathsf{t}}$ *for* $\overline{\mathsf{P}}$*. If there is an* $\mathsf{EXP}_\mathsf{st}(\overline{\mathsf{P}})$ *call between time* $\overline{\mathsf{t}}$ *for* $\overline{\mathsf{P}}$ *and the next* $\mathsf{RATCH}(\overline{\mathsf{P}}, \text{“send”}, ., .)$ *call (or just after time* $\overline{\mathsf{t}}$ *is there is no further* $\mathsf{RATCH}(\overline{\mathsf{P}}, \text{“send”}, ., .)$ *call), we say that* $(\mathsf{ad}, \mathsf{ct})$ *is a* trivial forgery*.*

Definition 34 (Direct leakage). *Let* t *be a time and* P *be a participant. We say that* $\mathsf{pt}_\mathsf{P}(\mathsf{t})$ *has a* direct leakage *if one of the following conditions is satisfied:*

- *The last* RATCH *call before time* t *is a* $\mathsf{RATCH}(\mathsf{P}, \text{“send”}, \mathsf{ad}, \mathsf{pt})$ *call by the adversary defining* $\mathsf{pt}_\mathsf{P}(\mathsf{t}) = \mathsf{pt}$*.*
- *There is an* $\mathsf{EXP}_\mathsf{pt}(\mathsf{P})$ *at a time* t_e *such that the last* RATCH *call which is executed by* P *before time* t *and the last* RATCH *call which is executed by* P *before time* t_e *are the same.*
- P *is in a matching status and there exists* $\mathsf{t}_0 \leqslant \mathsf{t}_\mathsf{e} \leqslant \mathsf{t}_{\mathsf{RATCH}} \leqslant \mathsf{t}$ *and* $\overline{\mathsf{t}}$ *such that time* t *originates from time* $\overline{\mathsf{t}}$*; time* $\overline{\mathsf{t}}$ *originates from time* t_0*; there is one* $\mathsf{EXP}_\mathsf{st}(\mathsf{P})$ *at time* t_e*; there is one* $\mathsf{RATCH}(\mathsf{P}, \text{“rec”}, ., .)$ *at time* $\mathsf{t}_{\mathsf{RATCH}}$*; and there is no* $\mathsf{RATCH}(\mathsf{P}, ., ., .)$ *between time* $\mathsf{t}_{\mathsf{RATCH}}$ *and time* t*.*

The first condition is specific to ARCAD: Obviously, an adversarial RATCH send call counts as an EXP_pt call.

Definition 35 (Indirect leakage [6]). *We consider a time* t *and a participant* P*. Let* $\mathsf{t}_{\mathsf{RATCH}}$ *be the time of the last successful* RATCH *call and* role *be its input role. We say that* $\mathsf{pt}_{\mathsf{P}}(\mathsf{t})$ *has an* indirect leakage *if* P *is in matching status at time* t *and one of the following conditions is satisfied*

- *There exists a* $\mathsf{RATCH}(\overline{\mathsf{P}}, \overline{\mathsf{role}}, ., .)$ *corresponding to that* $\mathsf{RATCH}(\mathsf{P}, \mathsf{role}, ., .)$ *and making a* $\mathsf{pt}_{\overline{\mathsf{P}}}$ *which has a direct leakage for* $\overline{\mathsf{P}}$*.*
- *There exists* $\mathsf{t}' \leqslant \mathsf{t}_{\mathsf{RATCH}} \leqslant \mathsf{t}$ *and* $\overline{\mathsf{t}} \leqslant \overline{\mathsf{t}}_{\mathsf{e}}$ *such that* $\overline{\mathsf{P}}$ *is in a matching status at time* $\overline{\mathsf{t}}_{\mathsf{e}}$*,* t *originates from* $\overline{\mathsf{t}}$*,* $\overline{\mathsf{t}}_{\mathsf{e}}$ *originates from* t'*, there is one* $\mathsf{EXP}_{\mathsf{st}}(\overline{\mathsf{P}})$ *at time* t_{e}*, and* $\mathsf{role} = \text{“}\mathsf{send}\text{”}$*.*

Lemma 36 (Trivial attacks [6]). *Assume that* ARCAD *is correct. For any* t *and* P*, if* $\mathsf{pt}_{\mathsf{P}}(\mathsf{t})$ *has a direct or indirect leakage, the adversary can deduce* $\mathsf{pt}_{\mathsf{P}}(\mathsf{t})$*.*