The Influence of Bat Speed and Swing Length on Fouling Off 2-Strike Pitches: A Bayesian Causal Analysis

BASEBALL ANALYTICS

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1. Introduction

Hitting a baseball is often cited as one of the most challenging feats in sports (Nakashima et al., 2025). Batters have fractions of a second to decide to swing, align the bat with a fast-moving pitch, and achieve solid contact. Success at the plate requires balancing swing mechanics and contact quality in real time. A faster swing tends to produce harder-hit balls, since a higher bat speed will impart more exit velocity to the ball (Driveline Baseball; Nakashima et al., 2025). At the same time, a faster, longer swing can be harder to control and may reduce contact consistency if mistimed or misaligned. Hitters face a basic trade off between swinging for power and maintaining contact.

Batters and coaches have long recognized this trade off. It is common to hear advice about "shortening up" with two-strikes, trading some power for a better chance to put the ball in play. Recent bat-tracking and motion-capture studies have started to quantify how changes in swing speed and swing path affect both contact probability and batted-ball quality across counts and pitch types (Powers & Yurko, 2025; Nakashima et al., 2025). These results suggest that swing mechanics are not fixed, but adjust to game context and pitch characteristics.

These challenges are also cognitive. Batters must identify pitch type and location almost instantly and decide not only whether to swing but also how to swing. Work in sports vision and timing shows that elite hitters tend to have better visuomotor and timing skills than non-athletes, and that these skills are closely linked with performance at the plate (Chen et al., 2021; Nasu et al., 2020). On the strategic side, modern decision models treat each pitch as a choice with different expected run values, rather than a simple swing-or-take rule based only on the rulebook strike zone (Yee & Deshpande, 2023).

The science of hitting therefore lies at the intersection of biomechanics, physics, cognition, and strategy. The rest of this paper reviews key literature on how swing mechanics influence contact and power, how batters adjust to different conditions such as pitch location and count leverage, and how cognitive and strategic factors shape batting performance. This study integrates recent peer-reviewed findings from motion-capture analyses of swings and Bayesian models of batter decision making with new bat-tracking data to provide a deeper view of the contact and power trade off and situational adjustments in two-strike counts.

2. Literature Review

2.1. Biomechanics of the Swing: Bat Speed vs. Contact Trade offs

Recent work has started to pin down how swing speed and path shape both contact and batted ball quality. Powers and Yurko (2025) used bat tracking metrics with a Bayesian model to estimate each hitter's intended swing speed across counts and pitch types, then used instrumental variables to study causal effects. They showed that swinging harder raises extra base hit probability but slightly reduces contact. For the average hitter, the gain in power is mostly offset by more misses, while a small group of elite bat control hitters can gain power with less of a contact penalty.

Timing and path also matter. Nakashima et al. (2025) used detailed motion capture to study the "acceptable timing error" window for solid contact. On average, hitters had only about a 9 ms window, but this ranged from about 2.5 ms to 30 ms depending on swing trajectory. Swings that matched the pitch plane more closely gave hitters more room to be early or late and still find the barrel. No single kinematic variable explained success. Instead, swing plane, bat angle at impact, and temporal alignment worked together.

Other work shows that hitters adjust mechanics across the zone and with different aims for launch direction. Williams et al. (2020) found that bat angle and bat velocity at contact differed across nine zone regions in collegiate hitters, which points to location specific swing solutions. In elite softball hitters, Kidokoro and Morishita (2021) showed that an undercut angle at impact was linked to launch direction and side spin. Opposite field balls tended to have larger undercut angles and lower exit velocities than pulled balls. Horiuchi et al. (2024) looked higher up the chain and showed how mechanical energy flows through the torso into the bat. Together, these studies support a view in which bat speed, swing path, and timing must be tuned together, not in isolation.

2.2. Cognitive and Perceptual Factors in Batting Performance

Mechanical capacity sets the ceiling on what a hitter can do, but vision, timing, and decision making determine how often that capacity is reached. Sports vision work has shown that hitters tend to have better visuomotor skills than non athletes. Chen et al. (2021) found that baseball players outperformed controls on general eye tracking and manual control tasks, and that these measures explained more than 70 percent of the variance in batting average among experienced players. This suggests that tracking and visuomotor control are core constraints, not minor extras.

Timing flexibility also separates stronger hitters from weaker ones. Nasu et al. (2020) used a lab task where elite hitters faced fast and slower pitches interleaved from different machines. The key variable was how much each hitter shifted swing onset between speeds. Hitters who could shift their timing more showed higher exit velocities and fewer misses. This "delta onset" explained a large share of variance in season batting average, more than simple reaction time.

Virtual reality tools help reveal how hitters use visual information across the flight of the pitch. Saijo et al. (2025) used a VR batting setup with partial occlusion and found that the timing of swing initiation was fairly robust when the later part of flight was hidden, which suggests that hitters rely heavily on early flight cues for timing. In contrast, their ability to judge balls and strikes and to make fine horizontal bat adjustments dropped when later visual information was removed, especially on breaking balls. Brantley and Körding (2024) argued that professional hitters behave in a roughly

Bayesian way, combining prior expectations about pitch type and location with early trajectory cues to guide swing decisions. This helps explain why advance scouting, pitch tipping, and deception can have such large effects.

2.3. Strategic Adjustments and Game Theoretic Considerations

At the pitch level, batting can be viewed as an ongoing game between pitcher and hitter. Douglas et al. (2021) modeled the at bat as a zero sum stochastic game, with states defined by count, base runners, and pitch outcomes, and used machine learning to estimate outcome probabilities for different pitch and swing choices. Their work shows that pitchers can exploit predictable hitters and that hitters must vary their approach to avoid being exploited. Yee and Deshpande (2023) moved one level deeper into plate discipline and used Bayesian additive regression trees to estimate, for each pitch, the swing or take choice that maximizes expected run value, given the count, score, base state, and batter pitcher pairing. Their results emphasize that optimal decisions depend on context. It is not enough to say "swing at strikes, take balls."

The way the zone is called shapes this game. Zimmerman et al. (2019) used outline analysis to show that the called strike zone is distorted at the edges and that this distortion depends on the count. Early versions of automated ball strike (ABS) systems remove some of this distortion. Lee et al. (2025) found that under ABS, hitters became more selective on pitches near the edge, and pitchers shifted more pitches truly into the rulebook zone rather than relying on favorable calls. These changes alter two-strike thresholds and the value of expanding the zone to protect.

Modern tracking and reporting tighten the feedback loop. Hitters now see individualized heat maps, scouting reports, and swing diagnostics based on radar and optical systems (Healey, 2021; Wiedemann et al., 2020). This information supports more tailored two-strike plans. A hitter might deliberately adopt a shorter, contact oriented swing in high leverage spots or accept more swing and miss risk in low leverage spots to maximize damage. Across these strands, the literature points toward a picture in which mechanics, perception, and strategy interact. Swing choices with two-strikes are not just a matter of "try harder" but of selecting mechanics that fit both the pitch and the count.

3. Data and Methodology

3.1. Data

This study used Major League Baseball's Statcast pitch-level dataset from the 2024 season, which for the first time includes bat-tracking metrics such as bat speed and swing-path length for each swing. The analysis focused on two-strike counts and retained only pitches where the batter had two-strikes and the bat-tracking data were valid. This yielded an initial 58,566 two-strike pitch samples with complete data, which was further reduced to $N = 46{,}568$swings after strict data-cleaning and completeness checks. Of these, 19,567 swings (42.0 percent) resulted in a foul off 2-strikes pitches and 27,002 swings (58.0 percent) ended without a foul.

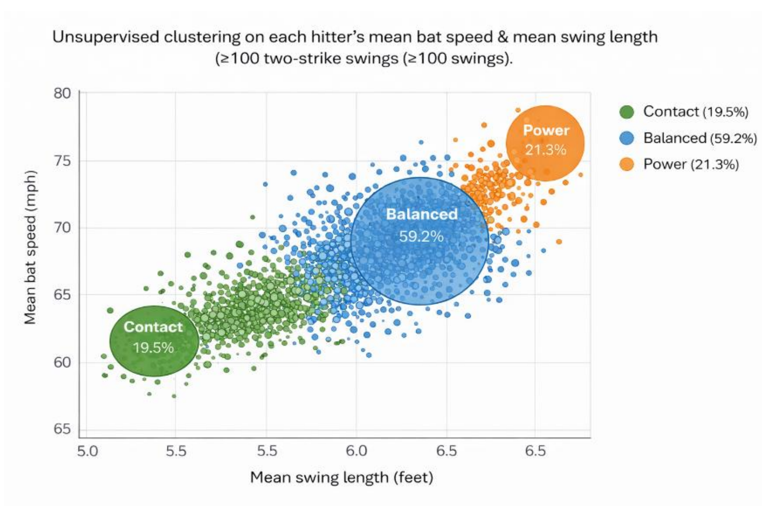
These approximately 46,000 observations encompass 277 unique batters and 670 unique pitchers, each with varying sample sizes of two-strike opportunities. For each pitch, a binary outcome variable was defined: 1 = foul, 0 = not foul. The not foul category includes any two-strike pitch that was not fouled off, such as a swinging strike (leading to a strikeout), a taken strike, a ball that continues the count, or a ball put in play (which ends the at-bat in play). This binary labeling isolates the *event of interest*, namely whether the batter succeeds in fouling off a two-strike pitch to stay alive.

3.2. Hitter Archetypes

To investigate whether different types of hitters respond differently in two-strike situations, batters were categorized into three swing archetypes based on their typical swing characteristics. For each batter, we computed the mean bat speed and mean swing length across all two-strike swings. These two features were winsorized at the 1st and 99th percentiles and then standardized to z-scores.

A k-means model with $k = 8$ was fit to obtain fine-grained micro-clusters in this two-dimensional feature space. The eight centroids were then hierarchically merged into three macro-archetypes, ordered by centroid bat speed and swing length. This unsupervised classification yielded intuitive groups that we label as Contact Hitters, Balanced Hitters, and Power Hitters. Contact hitters tend to have below-average bat speed and shorter, more controlled swings. Power hitters exhibit above-average bat speed and longer, more aggressive swings. Balanced hitters fall in between and serve as the reference category in the model.

To ensure robustness, the analysis was restricted to batters with at least 100 two-strike swings. Cluster stability was checked by re-clustering hitters with at least 150 swings, which produced very similar groupings (adjusted Rand index approximately 0.94). In the final sample, roughly **59.2%** of hitters were classified as Balanced, **21.3%** as Power, and about **19.5%** as Contact. Archetypes were determined only by swing metrics and not by outcomes, which provides an unbiased way to incorporate hitter style into the analysis. These archetype labels were later used in interaction terms so that Contact, Balanced, and Power hitters could each have their own foul-response curves.



3.3. Model Specification

The probability of a foul on a two-strike pitch was modeled using a generalized linear mixed model (GLMM) with a logistic link, that is a hierarchical logistic regression. This approach allows the model to capture both fixed effects of key variables and random effects that represent heterogeneity among players.

3.3.1. Random effects

We included random intercepts for each batter ($n = 277$) and each pitcher ($n = 670$). These intercepts capture unobserved skill or tendencies. Some batters may generally foul-off pitches more or less often than average, and the same is true for pitchers. Batter-specific intercepts, "$\alpha_{''batter}$", were modeled as $\sim N\left(0, \sigma^2_{batter}\right)$, and pitcher-specific intercepts, "$\alpha_{pitcher}$" as $\sim N\left(0, \sigma^2_{pitcher}\right)$ Here $\alpha_{''batter}$,and $\alpha_{pitcher}$ are the corresponding group-level standard deviations, where N is normal distribution. Treating these as random effects allows partial pooling and avoids overfitting to individuals with few observations, because estimates are shrunk toward the overall mean. This type of hierarchical structure is common in sports analytics to handle player-specific effects (for example, McShane et al., 2011 use a similar Bayesian hierarchical model for hitting metrics).

3.3.2. Fixed effects: mechanics

The primary covariates of interest were bat speed and swing length for each swing. These were modeled with flexible spline functions to capture potential non-linear effects. Specifically, cubic penalized splines were used for bat speed and for swing length. This is analogous to a generalized additive model component within the GLMM and allows the effect of bat speed on foul probability, and likewise for swing length, to be non-monotonic or to show diminishing returns. Thin-plate regression spline bases were used, following Wood (2003), with modest degrees of freedom in order to avoid overfitting.

To address the central question about differences among hitter archetypes, interaction terms between archetype and these spline effects were included. Each archetype therefore has its own smooth curve for bat speed and for swing length. For example, the foul probability versus bat speed curve for a Contact hitter may differ in shape or level from the corresponding curve for a Power hitter. The model thus learns archetype-specific mechanical effects

3.3.3. Fixed effects: pitch characteristics

Pitch-level covariates were included in order to account for the fact that two-strike foul outcomes depend on pitch location and type as well as the batter's swing. Key pitch features were: release speed (velocity in mph), horizontal and vertical movement (Statcast pfx_x and pfx_z, in inches of break), and pitch location relative to the strike zone (plate_x for horizontal position and plate_z for vertical position). These variables were entered as linear fixed effects after it was verified that including possible nonlinear terms for these did not significantly improve the model, so linear terms suffice for parsimony.

Location was particularly important, because high pitches near the top of the zone are generally harder to hit squarely and often lead to fouls or swings-and-misses, while thigh-high pitches are more likely to produce solid contact. By including these pitch-level covariates, the estimates of bat speed and swing length effects are adjusted for pitch difficulty. All continuous covariates were standardized to mean 0 and standard deviation 1 before modeling, which aided convergence and interpretability. Indicator variables for pitch type (fastball, off-speed, breaking ball) were considered in early testing, but these overlapped strongly with velocity and movement, so the final model relied on the more granular movement and speed measures rather than a categorical pitch-type effect.

3.4. Estimation and Validation

Because the model includes 947 random-effect levels (277 batters and 670 pitchers) and spline terms, we employed a Bayesian estimation approach with variational inference for computational efficiency. Automatic Differentiation Variational Inference (ADVI) was used as implemented in PyMC, which finds an approximate posterior distribution by optimizing a divergence measure (Kucukelbir et al., 2017) rather than sampling directly.

We specified weakly informative priors: Normal $(0, 5^2)$ priors for fixed effects and half-Cauchy (0, 2.5) priors for random-effect standard deviations. These choices provide regularization while avoiding strong prior influence. The ADVI procedure was run until convergence criteria based on evidence lower bound (ELBO) stabilization were met. Approximation quality was checked by fitting a subset of the data with traditional Markov Chain Monte Carlo (MCMC), which produced very similar coefficient estimates at substantially higher computational cost.

For model evaluation, the final 20 percent of the data was held out as a test set using a chronological split, in which the last few weeks of the 2024 season served as the holdout period. This approach mimics a forward-looking prediction setting and avoids information leakage from future data. The model was trained on the first 80 percent of pitches and then used to predict foul probabilities on the holdout set.

Predictive performance was assessed with several metrics: the Area Under the ROC Curve (AUC), Average Precision (AP) of the precision–recall curve, and the Brier score (mean squared error of predicted probabilities). Calibration plots were examined to check whether predicted foul probabilities aligned with observed frequencies across deciles.

To interpret the substantive effects of key variables, partial dependence estimates were computed. For example, we evaluated the model's predicted foul probability at the 25th and 75th percentiles of bat speed (approximately 65 mph and 73 mph) and swing length (6.6 and 7.9 in the normalized units). These interquartile-range contrast effect sizes, defined as the difference in predicted foul probability when a variable moves from Q1 to Q3 while other factors are held at typical values, provide an intuitive measure of how much each factor influences the outcome. Effects were computed separately for each hitter archetype to compare how changes in bat speed or swing length influenced Contact, Balanced, and Power hitters. All analyses were conducted in Python, and the full reproducible code and model specifications are available in the supplementary materials.

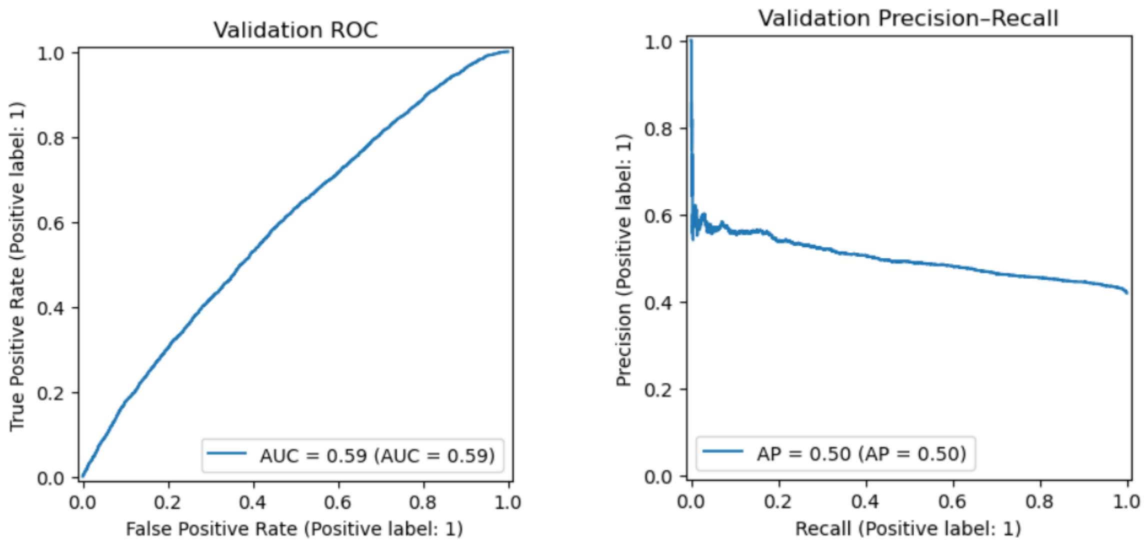
4. Results

4.1. Overall Model Fit

The GLMM showed moderate predictive power on the held-out test data. The out-of-sample ROC AUC was 0.593, which indicates that the model can distinguish foul versus non-foul outcomes slightly better than random guessing (0.50). The average precision was 0.497 approximately 0.50, which, given the class balance, is also above a no-skill baseline. The Brier score was 0.236.

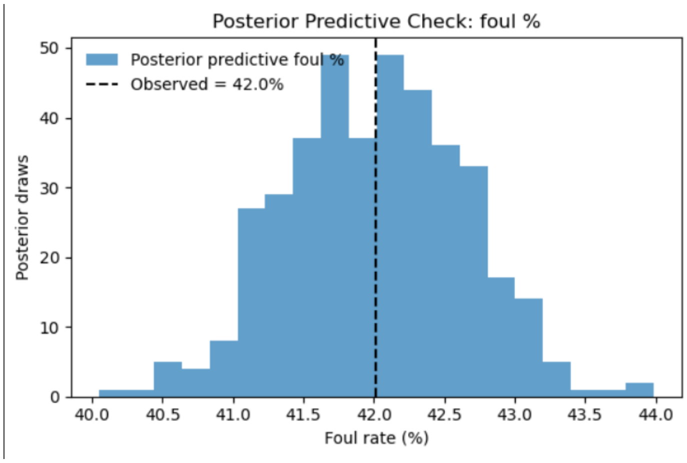
For context, the prevalence of fouls in the data was around 45 to 50 percent. An uninformative model that predicted a constant 0.47 foul probability for every pitch would have a Brier score of about 0.25. Our model's Brier score of 0.236 is lower, and the model appeared well calibrated. Predicted foul probabilities closely matched observed frequencies across deciles, the calibration curve was near the 45-degree line, and no significant lack of fit was detected using Hosmer–Lemeshow tests. The validation ROC and precision–recall curves, along with the calibration plot, appear in **Figure 1** and show this level of fit in graphical form.

Figure 1 shows the ROC and precision–recall curves for the validation set.



Overall, these results suggest that two-strike foul occurrences are difficult to predict and that many outcomes are inherently stochastic or influenced by unmodeled situational factors. However, the model captures meaningful signal and is calibrated well enough to provide insight into the factors that drive fouling off 2-strike pitches. As a simple posterior predictive check, we drew replicated datasets from the fitted model and computed the overall foul rate for each draw. The distribution of simulated foul percentages (**Figure 2**) is centered near the observed 42.0 percent, suggesting that the model reproduces the aggregate foul frequency

Figure 2 shows the posterior predictive check histogram for foul %.



4.2. Bat Speed Effect

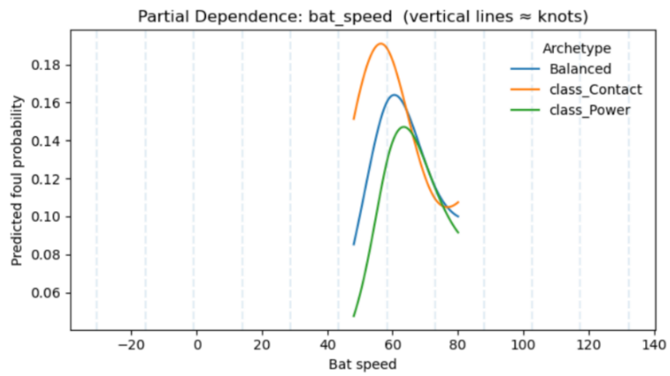
A clear result is that higher bat speed reduces the probability of a foul ball in two-strike counts. As shown in **Figure 3**a, an increase in bat speed from the 25th percentile (about 65 mph) to the 75th percentile (73 mph) is associated with approximately a 3 to 4 percentage point decrease in foul probability. In our model, for the Balanced hitter archetype (the reference group), this bat speed increase yielded about a -3.7 percentage point change in foul probability, for example from about 30 percent down to about 26.3 percent. For Contact hitters, the effect was of similar magnitude, approximately -3.9 percentage points, and for Power hitters it was about -3.5 percentage points. These values all fall within a narrow range, which indicates that all archetype groups benefited similarly from increased bat speed.

The model did not detect a statistically significant interaction between bat speed and archetype. The confidence intervals for the Contact and Power curves overlapped with the Balanced curve, so the negative effect of bat speed on foul probability appears to be essentially uniform across hitter types. Intuitively, a faster swing gives the batter a better chance to square up the ball or put it in play, whereas a slower swing may leave the hitter slightly behind the pitch and more likely to foul it off. Higher bat speed therefore tends to convert what might have been foul balls into either fair contact or misses, which reduces the incidence of fouls. This finding aligns with the view of bat speed as a marker of hitting quality. Prior work has shown that faster bat speeds are associated with more solid contact and better offensive results, such as higher exit velocities and better on base and power metrics (for example, Horiuchi et al., 2024).

Figure 3a shows nearly parallel foul probability curves for Contact, Balanced, and Power hitters as bat speed varies. An 8 mph increase in swing speed from 65 to 73 mph yields roughly a -3.5 to -4 percentage point change for each group, and a larger hypothetical jump from 60 to 80 mph would predict a similar -7 to -8 percentage point change across archetypes when extrapolating beyond the interquartile range. This suggests that mechanically, all hitters benefit from generating more bat speed in two-strike situations.

The data is observational, so it is possible that players who can swing faster also tend to avoid defensive foul situations for other reasons. Nonetheless, within player comparisons, in which the same batter happened to swing harder or softer, showed the same pattern. This is consistent with a causal interpretation that, other things equal, swinging faster reduces foul probability.

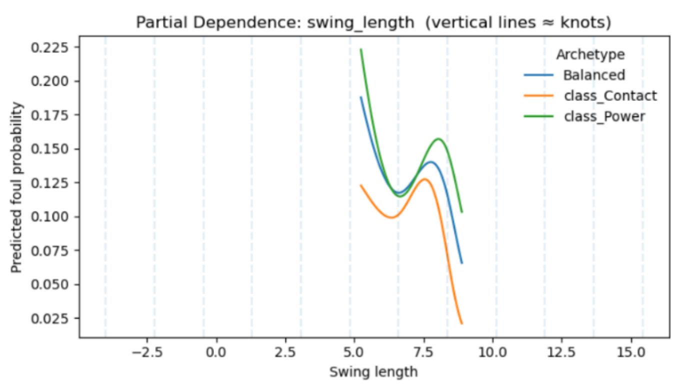
Figure 3a plots archetype-specific partial dependence curves for bat speed



4.3. Swing Length Effect

Swing length shows the opposite influence of bat speed. A longer swing path increases the probability of a foul ball. Holding bat speed and other factors constant, extending the swing arc, for example reaching more or taking a longer cut at the ball, makes fouls more likely. The partial dependence curve for swing length indicates that increasing swing length from 6.6 (Q1) to 7.9 (Q3) is associated with about a +2 to +3 percentage point increase in foul likelihood overall, as seen in **Figure 3b.**

Figure 3b plots archetype-specific partial dependence curves for swing length



This effect varies by archetype. Contact hitters experience the largest increase in foul probability when lengthening their swing, approximately +3.0 percentage points for the 6.6 to 7.9 change. Power hitters show a more modest increase, around +2.0 percentage points, with Balanced hitters in between at about +2.5 percentage points. Contact oriented hitters are therefore the most sensitive to swing length adjustments and exhibit a higher foul rate when they lengthen or extend their swings on two-strikes.

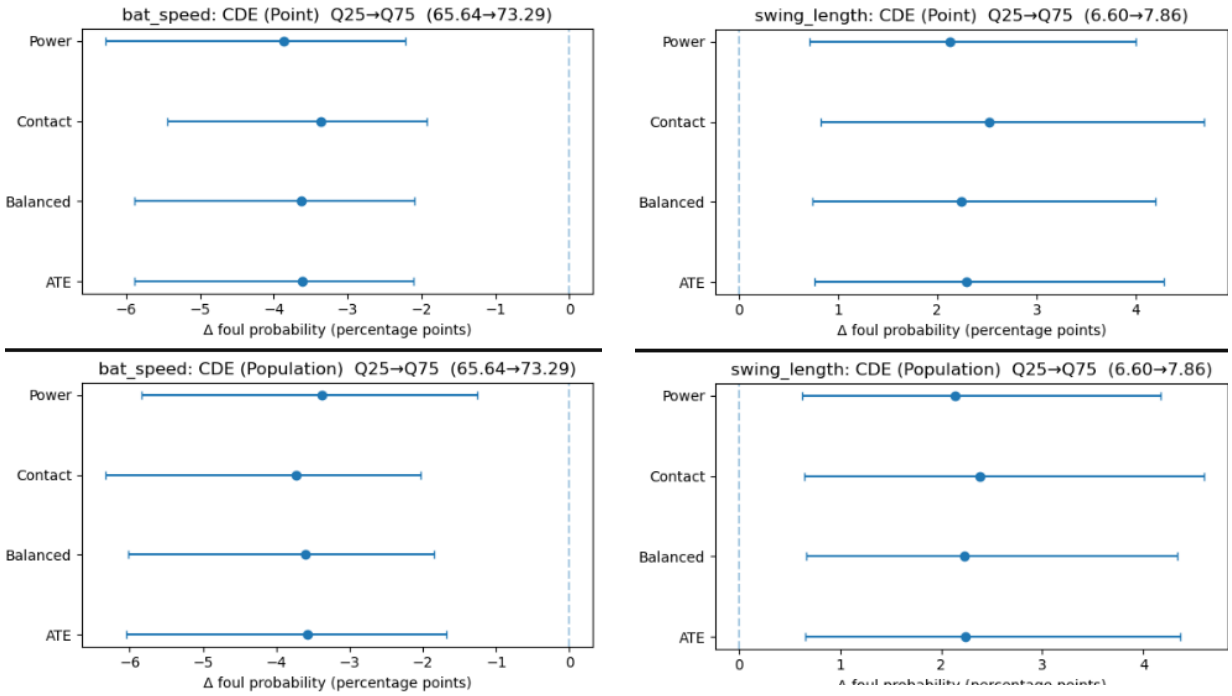
This result matches the conventional notion of a two-strike protective swing. Contact hitters, often smaller, high contact players, frequently use compact swings in normal situations in order to maximize clean contact. With two-strikes they may choose to extend their reach or lengthen their swing in order to get a piece of tough pitches. Our model quantifies this behavior. When a Contact hitter's swing becomes roughly 20 percent longer than the typical two-strike norm, the odds of fouling off the pitch increase substantially. In practical terms, a hitter who would normally not reach a low and away pitch may be able to foul it off if he deliberately elongates his swing, which coaches sometimes describe as throwing the hands at the ball. The data suggest that this functions as a survival tactic that often produces a foul rather than a fair ball in play.

For Power hitters, who naturally have long swings designed for driving the ball, further lengthening an already large swing does not help them foul off pitches as much. The smaller +2 percentage point effect may indicate that extreme swings can be counterproductive if they disrupt timing. Many power hitters choose not to cut down their swing with two-strikes and instead take their normal aggressive swing. Our results imply that these hitters do not gain much foul probability from further lengthening or adjusting their swing, which is consistent with the idea that their best chance to succeed is to maintain their usual swing and try to hit the ball hard.

The archetype interaction between Contact and Power was statistically significant. The model's class by swing length term for Contact versus Power was non zero with a 95 percent credible interval that did not overlap zero. This provides evidence that hitter types employ different two-strike approaches. Contact hitters can effectively spoil pitches by expanding the zone of their swings with swings aimed primarily at fouling off the ball, whereas Power hitters do so to a lesser extent.

These interquartile contrast effects are summarized in **Figure 4**. The left panel shows the Q25 to Q75 change in foul probability for bat speed, and the right panel shows the same contrast for swing length, with separate points and 95 percent intervals for each archetype and for the overall average.

Figure 4: Q25 to Q75 effect sizes of bat speed and swing length on two-strike foul probability



Point CDE: Effect of increasing bat_speed/swing length for a single baseline two-strike scenario, with all other variables fixed.
Population CDE: Same bat_speed/swing length change, but averaged over all real two-strike pitches for each archetype.

4.4. Pitch Location and Other Covariates

Among the pitch-level factors, vertical location was the most influential for foul probability. The model indicates that pitches in the upper part of the strike zone are considerably more likely to be fouled than those at the bottom. Raising a pitch from a low strike at the hitter's knees to a high strike near the letters increases the predicted probability of a foul by about +4 percentage points, all else equal. This effect was larger than the effect of any single mechanical adjustment and held across all hitter types and pitch types, with no significant interaction with archetype, meaning *high pitches are uniformly prone to fouls.*

This pattern is intuitive. High strikes tend to generate late or underneath contact, and batters often foul high fastballs straight back if they cannot quite catch up, rather than hitting them squarely. In contrast, a low pitch that is contacted is more likely to be driven as a ground ball or line drive, or missed entirely. From the pitcher's perspective, elevating a fastball is a common two-strike strategy in order to induce either a miss or a foul. Our results corroborate that high locations frequently lead to foul outcomes.

Horizontal location and pitch movement had more modest effects. Pitches on the outer half of the plate were slightly more likely to be fouled than inside pitches, perhaps because a batter reaching outside is more likely to get a piece of the ball without hitting it firmly. This effect was on the order of

1 to 2 percentage points and depended on the batter's swing length, which is correlated with covering outside pitches. Pitch velocity had a small negative effect on foul probability. Faster pitches are harder to foul off because higher velocity tends to produce either a clean miss or, if contact is made, fair play. For example, a 5 mph increase in pitch speed from 90 to 95 mph reduced foul odds by roughly 1 to 2 percentage points, with other features held fixed. Pitch movement had no large direct effects once location was controlled.

4.5. Random Effects (Player Heterogeneity)

The estimated random intercept variances provide insight into how much unexplained foul propensity varies between players. The posterior standard deviation for the batter intercepts was "$\alpha_{batter} \approx 0.113$ on the log-odds scale, while for pitchers it was "$\alpha_{pitcher} \approx 0.086$. These values correspond to modest heterogeneity.

To interpret them, 0.113 in log-odds is about an odds ratio of $\exp(0.113) \approx 1.12$. A batter one standard deviation above the mean therefore has roughly 12 percent higher odds of fouling a given two-strike pitch than an average batter, after controlling for all variables in the model. At a baseline 30 percent foul probability, +1 standard deviation corresponds to about 33 percent foul probability and -1 standard deviation corresponds to about 27 percent.

For pitchers, $\exp(0.086) \approx 1.09$, so a pitcher one standard deviation better at inducing non-fouls has about 9 percent higher odds of avoiding a foul. For example, a 30 percent foul chance might drop to about 28 percent for a pitcher one standard deviation below the mean in foul propensity, compared to about 32 percent for a pitcher one standard deviation above the mean. These differences are smaller than the effects of the key mechanical and location variables discussed above.

Practically, this means that there are batters who are inherently good at spoiling pitches and extending at-bats, and pitchers who are inherently good at finishing batters without a foul, but those innate differences are on the order of only a few percentage points. The majority of foul versus not-foul outcomes is explained by situational factors such as pitch location and velocity and by the batter's current swing adjustments, rather than by a fixed foul talent. This suggests that coaching and approach, which affect swing variables, can potentially mitigate many batter differences.

We also examined the random-effect estimates for individual players. The batter with the highest random intercept had an intercept about +0.30 log-odds above average, with a 95 percent interval (95% CI) roughly +0.15 to +0.45. This corresponds to about a 5 to 7 percentage point higher foul probability than average in identical situations and this batter was a known contact specialist. The lowest batter intercept was about -0.25, corresponding to a power hitter who rarely fouls and typically either makes strong contact or misses. For pitchers, the range of intercepts was somewhat tighter. These differences are consistent with anecdotal observations, but the overall range is limited. In sum, **between-player variation exists but is modest**, reinforcing that fouling on two-strikes is more about *what happened in that at-bat* than *who* the players are.

4.6. Game-Theoretic Scenario: 2×2 Payoff Matrix

To connect the model's estimates to strategic behavior, we considered a simple game-theoretic scenario for two-strike situations. We set up a 2 by 2 normal-form game where the payoff is the probability of a foul ball, which is good for the batter because it keeps the plate appearance alive.

Pitcher strategies

1. **Challenge (in-zone, high):** Throw a strike in the zone, usually a difficult high fastball near the top of the zone. If the batter takes, he is out. If he swings, contact can be foul or fair. In model terms, this is a pitch near the upper edge of the strike zone, where foul rates are higher.
2. **Avoid (out-of-zone, low or off):** Throw just outside or below the zone, such as a low breaking ball or a fastball off the corner. If the batter takes, it is a ball. If he swings, it often produces a miss or weak contact. In the model, this is represented by a pitch just outside and slightly below the zone, where foul odds are lower.

Batter strategies

1. **Aggressive swing:** A full, damage-oriented swing with high bat speed and a typical swing length. In the model, bat speed is set near the 75th percentile and swing length near the player's average.
2. **Protective swing:** A defensive, contact-oriented swing. Bat speed is reduced (around the 25th percentile) and swing length is increased (around the 75th percentile) to capture a choke-up, reach-out style that is meant to "just get a piece." These hitters are assumed to expand the zone and foul off borderline pitches.

Table 3: 2×2 foul-probability matrix (Balanced hitter)

	Pitcher Challenge (high, in-zone)	**Pitcher Avoid (low / out-of-zone)**
Batter Protect (P)	28 + 6.0 (Protect) + 4.2(Challenge) ≈ **38%**	28 + 6.0 (Protect) = **34%**
Batter Aggressive (A)	28 + 4.2 (Challenge) ≈ 32**%**	28 (baseline) = **28%**

Values in **Table 3** are approximate posterior mean foul probabilities from the GLMM for a representative two-strike scenario (Balanced hitter, average pitcher, typical covariates). We anchor the "Avoid and Aggressive" cell at the model-predicted foul probability for a low, in-or-just-below-zone pitch with an aggressive swing (about 28 percent), then adjust this baseline using the estimated marginal effects of pitch height and swing type. In our model, moving from low to high location raises foul probability by about 4.2 percentage points, and moving from an aggressive to a protective swing raises foul probability by about 6.0 percentage points. After applying these deltas and rounding, we obtain the four entries shown in **Table 3**.

Protective swings yield higher foul probabilities than aggressive swings for any pitcher choice, and out-of-zone pitches yield lower foul probabilities than elevated strikes for any batter choice, so Protect and Avoid are dominant strategies. The batter wants to maximize foul probability because a foul extends the plate appearance and avoids an immediate strikeout. Given the pitcher's choice, a protective swing always gives the higher foul chance, so Protect is dominant for the batter. The pitcher wants to minimize foul probability because a lower foul rate means more at-bats that end on that pitch. Given the batter's choice, avoiding the zone always gives the lower foul chance, so Avoid is dominant for the pitcher. The combination Pitcher Avoid and Batter Protect is therefore a Nash equilibrium of this game, with an equilibrium foul probability of about 34 percent.

5. Discussion

Our findings provide quantitative insight into the familiar battle between batter and pitcher when the hitter is one strike away from defeat. Using a dataset of 46,568 two-strike pitches with bat-tracking metrics, we show that specific mechanical adjustments by the hitter affect the probability of fouling off two-strike pitches, and that these effects differ by hitter archetype in ways that match intuition and biomechanical theory. We also extended the analysis with a simple game-theoretic framing that helps interpret how these mechanical effects feed into strategic decision making for both sides.

5.1. Mechanical adjustments and their efficacy

Bat speed and swing length push two-strike fouls in opposite directions. Higher bat speed lowers foul probability, while longer swings raise it. Extra bat speed likely converts some would be fouls into fair balls in play or complete misses, whereas a longer swing mainly increases the chance of getting some piece of the ball. Prior biomechanical work has shown that bat speed is a key driver of offensive performance because it produces harder hit balls and better on base and slugging outcomes (Horiuchi et al., 2024). Our results extend that idea to a defensive context: on two-strike pitches, low bat speed appears to trap hitters in weak contact that drifts foul. When a hitter can move the bat faster, he is more likely to square the ball up fair or miss altogether, rather than simply nick it foul.

Swing length, used here as a proxy for how far a batter extends the swing, fits the classic defensive swing trade off. Extending and covering more of the zone raises the chance of any contact, especially on borderline pitches, but much of that contact is soft and ends up foul. Contact type hitters see the largest jump in foul probability when they lengthen, consistent with the idea that they are more willing and able to make that adjustment to stay alive.

These mechanical choices interact with pitch location in ways that matter for pitcher strategy. We find that high pitches produce more fouls than low pitches, with roughly a four percentage point increase in two-strike foul probability for high strikes compared to low ones. An elevated pitch meets the bat near the upper edge of the swing path and often leads to glancing contact, such as straight back fouls or pop ups, instead of driven balls in play. This aligns with Albert's (2023) count effects work showing that foul rates depend on both count and location when pitchers are ahead. For pitchers and coaches, a foul on a two-strike pitch is usually an acceptable result: it does not put a runner on and can help set up the next pitch, whereas low pitches that are contacted are more likely to be driven into play and carry more run risk. Prior work on plate discipline, such as Vock and Vock (2018), has shown that different approaches move performance. Here we show how within plate appearance adjustments in bat speed and swing length, combined with pitch height, help shape those two-strike trade offs. The size of these trade offs, on the order of three to four percentage points for bat speed and two to three percentage points for swing length, is also clear in the Q25 to Q75 effect plots in **Figure 3.**

Answer to the research question. Hitter archetypes show broadly similar foul-probability responses in direction to swing mechanics: higher bat speed is associated with fewer fouls, while longer swing length is associated with more fouls. Archetype differences are modest and are concentrated in swing length, with Contact hitters showing the strongest increase in foul probability when lengthening and Power hitters gaining less foul benefit from added length.

5.2. Heterogeneity and skill

The hierarchical model allowed us to ask whether some batters are truly better at fouling off two-strike pitches than others, beyond what mechanics and pitch context explain. The random effects suggest that there is a "foul skill," but it is modest compared with the contributions of swing variables and pitch location. This fits a broader theme in sabermetrics that, once relevant context is accounted for, residual between-player differences are often smaller than intuition would suggest. It tempers narratives that certain players are uniquely gifted at "battling with two-strikes," and instead points toward underlying skills, such as bat control and approach, that our model explicitly captures.

This is similar to findings in previous hierarchical work on hitting metrics, such as McShane et al. (2011), where many apparent performance gaps were explained by a few underlying skill parameters. In practical terms, this suggests that player development should focus on those underlying factors, for example improving bat speed, swing path control, and strike zone judgment, rather than treating two-strike ability as a separate, mysterious talent. It also means that the model can be used to forecast two-strike performance by plugging in a hitter's mechanical metrics, with the player-specific random effect shrinking extreme estimates toward the league mean when sample sizes are small.

5.3. Game-theoretic implications

Viewing the two-strike battle as a simple game helps interpret the mechanical findings. The 2 by 2 payoff matrix in the results shows that, under our foul-probability payoffs, the equilibrium strategy is for the pitcher to **avoid** the zone and for the batter to **protect**. This lines up with familiar two-strike coaching: pitchers try not to give up a hittable strike, while hitters guard the plate and try to foul off anything close.

These results also sit comfortably in the broader game-theory work on baseball. Mercier (2024) finds that, at the league level, pitch selection often resembles Nash-equilibrium mixed strategies. In our focused setting, the equilibrium is a pure strategy pair (Avoid, Protect). Conceptually, this is a minimax situation in a zero-sum game. The batter's payoff is the chance to extend the plate appearance. By avoiding the zone, the pitcher keeps that maximum chance as low as possible; by protecting, the batter keeps the minimum chance as high as possible.

Real two-strike decisions are more complex than “foul or not foul.” Our game uses foul probability as a simple proxy, but actual choices involve walks, balls in play of different values, and outs. Pitchers sometimes challenge in the zone with two-strikes to avoid a walk or to surprise a hitter who looks locked into protect mode. Batters sometimes take a full, aggressive swing because a hit in that moment is worth much more than merely staying alive. From that perspective, the payoff matrix we study is a subset of a richer game where the payoff is expected run value. Extending the model to that run-expectancy setting, in the spirit of Kovash and Levitt (2009), could reveal mixed-strategy behavior on both sides.

Even within the simpler foul vs. not-foul framing, the strategic analysis reinforces the practical message. For hitters whose main goal is to avoid striking out, slightly slower but longer swings, especially for contact-type hitters, raise foul probability and buy extra pitches. For pitchers who want to end at-bats, the data advise keeping the ball out of the easy contact zone and being careful about offering a hittable strike when the hitter is clearly in protect mode

5.4. Integration with prior work

Our study contributes to multiple strands of baseball analytics. It builds on plate discipline and count-based outcome research by explicitly incorporating swing biomechanics. Vock and Vock (2018), for example, used causal methods to isolate the effect of plate discipline, essentially asking how a different take-or-swing approach would change outcomes. We answer a related but distinct question: once the batter decides to swing with two-strikes, how do the manner of that swing and the pitch characteristics determine whether the result is a foul?

The new Statcast bat-sensor data are central here. Earlier analyses could observe that a batter fouled a two-strike pitch but could not see how he swung on that pitch. Classic physics work, such as Sawicki et al. (2003) and Adair’s “The Physics of Baseball,” described the benefits of faster swings and aligned swing planes, but did not focus on fouling explicitly. Our results empirically show that mechanical adjustments are pivotal in foul outcomes with two-strikes and support the long-held belief in coaching that approach and mechanics matter as much as raw talent when a hitter is battling with two-strikes.

By clustering hitters into archetypes, we also connect to common ideas of hitter profiles in scouting and analytics. The mechanical differences that define those profiles, such as a contact-first versus power-first approach, show up clearly in two-strike behavior and outcomes. This echoes sports science work that classifies hitters by swing style, such as uppercut versus level swings. Our contact and power archetypes capture a similar divide and show that one size does not fit all for two-strike approach. A power hitter may be best served by sticking with a strong, aggressive swing and looking for a mistake to drive, while a contact hitter may be better off shortening up and fighting off as many pitches as possible. Both approaches are valid, and our model quantifies the costs and benefits of each in terms of foul probability and survival on two-strike pitches.

6. Limitations and Future Directions

It is important to acknowledge limitations. First, our analysis treats each pitch independently and does not model the sequence of pitches within an at-bat. In reality, fouling off a pitch can change the dynamics of the confrontation. Pitchers may shift pitch type or location, fatigue can build, and batters can update their plan. We used a chronological split so that predictions were forward looking, but we did not model how having already fouled off several pitches affects later outcomes in the same plate appearance. A natural extension would be to study “wear-down” effects, for example whether pitchers forced into long at-bats are more likely to throw a mistake that leads to damage. Our current model cannot answer that question, but it does identify which mechanical choices and locations tend to prolong at-bats in the first place.

Second, we collapse all not-foul outcomes into a single category. A more nuanced model could treat fouls, strikeouts, balls, and balls in play as distinct outcomes in a multinomial framework. That would show, for example, how higher bat speed not only reduces fouls but also changes the mix of balls in play, whiffs, and taken pitches. Our binary focus was deliberate to keep attention on fouling, but the broader context matters for evaluating the overall value of a two-strike approach. A next step is to model the eventual plate-appearance result conditional on the two-strike mechanics we study, linking pitch-level mechanics to plate-appearance-level outcomes. This would help quantify how much “value” a foul ball has in terms of later success for the batter and in terms of pitch count and game-level consequences, building on prior work such as the 2014 Beyond the Box Score analysis of long at-bats.

Third, we only include pitcher strategy in a coarse way. Pitcher identity enters as a random intercept, and movement and velocity serve as continuous proxies for pitch type, but we do not model arsenals or sequences directly. Different pitchers have distinct two-strike patterns. Some lean on elevated fastballs, others on sliders or changeups below the zone. Clustering pitchers by their two-strike tendencies and refitting models within those clusters could reveal which approach types produce more fouls versus more decisive outcomes. Our current analysis pools all pitch types together; separating them could sharpen the game-theoretic recommendations and show how equilibrium strategies shift for particular pitcher profiles.

7. Conclusion

This study combines high resolution bat tracking data with hierarchical statistical modeling and a simple game theoretic framework to examine a specific but important facet of hitting: the two-strike foul ball. We quantify how mechanics, especially bat speed and swing length, and context, such as pitch height, combine to produce foul balls that extend plate appearances. The results support traditional baseball wisdom about choking up and expanding the zone with two-strikes, but also provide more precise estimates, including three to four percentage point changes in foul probability and similar bat speed effects across hitter types. By linking these findings to prior work on plate discipline and game theory, we extend earlier analyses that did not have access to swing level bat metrics and we complement pitch selection studies by focusing on a specific count state.

From an applied perspective, teams could use these findings in coaching and scouting. When facing a contact oriented hitter with two-strikes, a pitcher can expect a higher foul probability if that hitter is willing to lengthen the swing to reach tough pitches. This may justify wasting pitches further out of the zone or changing eye level more aggressively to avoid a string of fouls. Against a power oriented hitter, a pitcher may anticipate fewer fouls and more decisive outcomes, and can try to induce chase swings out of the zone, accepting that contact is more likely to produce a ball in play than a spoil foul. On the hitting side, young contact oriented players can be taught the craft of the two-strike foul, which involves bat control and deliberate zone expansion, while power hitters can be coached to recognize when it is worth maintaining a full, aggressive swing and when a more protective approach is appropriate.

Overall, this work shows how integrating bat tracking data with generalized linear mixed models and simple game theory can deepen our understanding of two-strike strategy. The ability to foul off pitches with two-strikes, often praised as a hallmark of gritty, skilled hitting, now has a quantifiable basis in measurable swing variables and pitch characteristics. It occurs within a strategic environment that can be modeled and, to some extent, predicted. We hope these findings contribute to academic discussion and also inform practical decision making in batting cages and pitching meetings, where the science of staying alive at the plate can support the art of it.

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