Transmuted New Weibull-Pareto Distribution

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4/16/2022

Contents

Libraries

```
library(maxLik)
options(scipen = 999, digits = 3)
```

Density Plot

```
## PDF #============================================

x <- seq(0, 3, by = 0.0001)
beta <- 1.5
delta <- 0.5
theta <- 0.5
lambda <- 0.0

tnwp.pdf <- function(x, beta=1.5, delta=0.5,
                     theta=0.5, lambda=0.0){
  (beta*delta/theta) * ((x/theta)^(beta-1))*
    exp(-(delta*(x/theta)^beta)) *(1-lambda + 2*lambda*exp(-(delta*(x/theta)^ beta)))
}


 par(mfrow = c(2,2), cex.axis = 0.5, cex.lab = 0.5,
     mar = c(3.5, 3.5, 3.5, 3.5) - 2.5)
```

```
plot(x, tnwp.pdf(x,1.5,0.5,0.5,0.0), type = "l", xlab = "x",
     ylab = "density", ylim = c(0, 1.7),
     main = "", cex.axis = 0.5, cex.lab = 0.5, ann = TRUE,
     family  = "mono")

lines(x, tnwp.pdf(x,1.5,0.5,0.5,0.2), col = 2)
lines(x, tnwp.pdf(x,1.5,0.5,0.5,0.4), col = 3)
lines(x, tnwp.pdf(x,1.5,0.5,0.5,0.6), col = 4)
lines(x, tnwp.pdf(x,1.5,0.5,0.5,0.8), col = 5)
lines(x, tnwp.pdf(x,1.5,0.5,0.5,1.0), col = 6)
legend("topright", cex=0.7,
       c(expression(paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=0.0"),
                    paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=0.2"),
                    paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=0.4"),
                    paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=0.6"),
                    paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=0.8"),
                    paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=1.0"))),
       col = c(1,2,3,4,5,6),
       lty = c(1,1,1,1,1,1),
       lwd = c(2,2,2,2,2,2),
       horiz = FALSE,
       bg = "grey96")


####========== 2nd plot


plot(x, tnwp.pdf(x,1.5,0.5,0.5,0.0), type = "l", xlab = "x",
     ylab = "density", ylim = c(0, 1.5))
lines(x, tnwp.pdf(x,1.5,0.5,0.5,-0.2), col = 2)
lines(x, tnwp.pdf(x,1.5,0.5,0.5,-0.4), col = 3)
lines(x, tnwp.pdf(x,1.5,0.5,0.5,-0.6), col = 4)
lines(x, tnwp.pdf(x,1.5,0.5,0.5,-0.8), col = 5)
lines(x, tnwp.pdf(x,1.5,0.5,0.5,-1.0), col = 6)
legend("topright", cex=0.7,
```

```
       c(expression(paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=0.0"),
                    paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=-0.2"),
                    paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=-0.4"),
                    paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=-0.6"),
                    paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=-0.8"),
                    paste(beta, "=1.5, ",
                          delta, "=0.5, ",
                          theta, "=0.5, ",
                          lambda, "=-1.0"))),
       col = c(1,2,3,4,5,6),
       lty = c(1,1,1,1,1,1),
       lwd = c(2,2,2,2,2,2),
       horiz = F,
       bg = "grey96")


#####========== 3rd plot


plot(x, tnwp.pdf(x,3.0,5.0,2.0,0.0), type = "l", xlab = "x",
     ylab = "density", ylim = c(0, 1.4))
lines(x, tnwp.pdf(x,3.0,5.0,2.0,0.2), col = 2)
lines(x, tnwp.pdf(x,3.0,5.0,2.0,0.4), col = 3)
lines(x, tnwp.pdf(x,3.0,5.0,2.0,0.6), col = 4)
lines(x, tnwp.pdf(x,3.0,5.0,2.0,0.8), col = 5)
lines(x, tnwp.pdf(x,3.0,5.0,2.0,1.0), col = 6)
legend("topright",cex=0.7,
       c(expression(paste(beta, "=3.0, ",
                          delta, "=5.0, ",
                          theta, "=2.0, ",
                          lambda, "=0.0"),
                    paste(beta, "=3.0, ",
                          delta, "=5.0, ",
                          theta, "=2.0, ",
                          lambda, "=0.2"),
                    paste(beta, "=3.0, ",
                          delta, "=5.0, ",
                          theta, "=2.0, ",
```

```
                        lambda, "=0.4"),
                  paste(beta, "=3.0, ",
                        delta, "=5.0, ",
                        theta, "=2.0, ",
                        lambda, "=0.6"),
                  paste(beta, "=3.0, ",
                        delta, "=5.0, ",
                        theta, "=2.0, ",
                        lambda, "=0.8"),
                  paste(beta, "=3.0, ",
                        delta, "=5.0, ",
                        theta, "=2.0, ",
                        lambda, "=1.0"))),
       col = c(1,2,3,4,5,6),
       lty = c(1,1,1,1,1,1),
       lwd = c(2,2,2,2,2,2),
       horiz = F,
       bg = "grey96")

#########========== 4th plot


plot(x, tnwp.pdf(x,0.4,0.5,0.0,-1.0), type = "l",
     xlab = "x",
     ylab = "density",
     ylim = c(0, 1.8))
lines(x, tnwp.pdf(x,0.9,1.2,1.1,0.6), col = 2)
lines(x, tnwp.pdf(x,4.6,2.9,2.8,-0.5), col = 3)
lines(x, tnwp.pdf(x,4.0,1.9,1.5,0.2), col = 4)
lines(x, tnwp.pdf(x,5.5,5.0,6.2,1.0), col = 5)
lines(x, tnwp.pdf(x,5.9,6.2,7.5,1.0), col = 6)
legend("topright",cex=0.7,
       c(expression(paste(beta, "=0.4, ",
                        delta, "=0.5, ",
                        theta, "=0.0, ",
                        lambda, "=-1.0"),
                  paste(beta, "=0.9, ",
                        delta, "=1.2, ",
                        theta, "=1.1, ",
                        lambda, "=0.6"),
                  paste(beta, "=4.6, ",
                        delta, "=2.9, ",
                        theta, "=2.8, ",
                        lambda, "=-0.5"),
                  paste(beta, "=4.0, ",
                        delta, "=1.9, ",
                        theta, "=1.5, ",
                        lambda, "=0.2"),
                  paste(beta, "=5.5, ",
                        delta, "=5.0, ",
                        theta, "=6.2, ",
                        lambda, "=1.0"),
                  paste(beta, "=5.9, ",
```

```
                        delta, "=6.2, ",
                        theta, "=7.5, ",
                        lambda, "=1.0"))),
       col = c(1,2,3,4,5,6),
       lty = c(1,1,1,1,1,1),
       lwd = c(2,2,2,2,2,2),
       horiz = F,
       bg = "grey96")
```

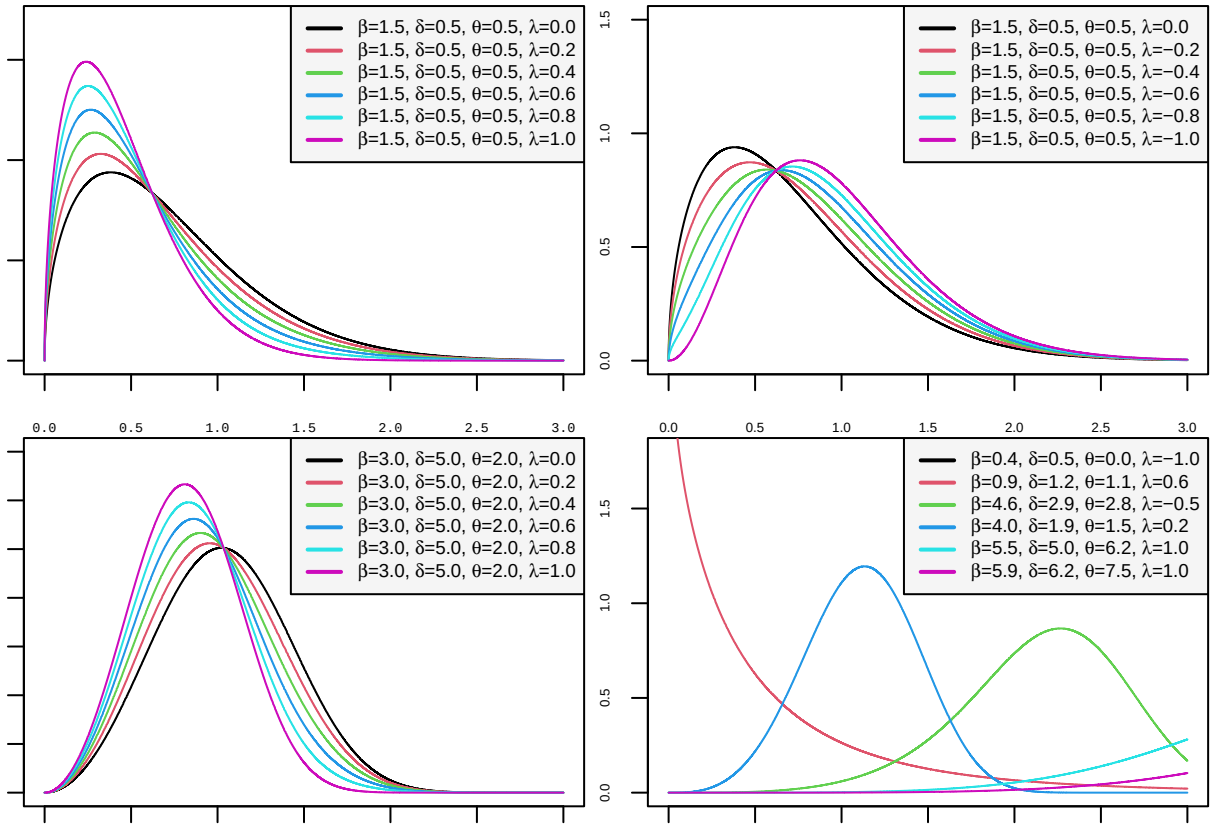


Figure 1: TNWP density plot using different values of the paramters

```
dev.off()
```

```
## null device
##           1
```

CDF Plot

```
## ==== R-code for the CDF plot ==========================

x <- seq(0,10,0.025)

cd <- function(x,beta,delta,theta,lambda){

  (1-exp(-(delta*(x/theta)^beta)))*
```

```
    (1+lambda*(exp(-(delta*(x/theta)^beta))))

  }


c1 <- cd(x,beta=1,delta=2,theta=1,lambda=1)

plot(x,c1, main="CDF of Transmuted Weibull Pareto Distribution",
     type="n", xlab = "x",ylab = "F(x)", cex.main = 0.8, cex.lab = 0.7,
     family  = "mono", cex.axis = 0.7)

lines(x, cd(x,beta=1,delta=2,theta=1,lambda=1),
      col="red",lwd=2,lty=1)

lines(x, cd(x,beta=2,delta=2,theta=1,lambda=0),
      col="blue",lwd=2,lty=1)

lines(x, cd(x,beta=3,delta=2,theta=1.5,lambda=0.5),
      col="green",lwd=2,lty=1)

lines(x, cd(x,beta=4,delta=2,theta=1.5,lambda=1),
      col="black",lwd=2,lty=1)


legend("bottomright",cex=0.7,
       c(expression(paste(beta, "=1, ",
                          delta, "=2, ",
                          theta, "=1, ",
                          lambda, "=1"),
                    paste(beta, "=2, ",
                          delta, "=2, ",
                          theta, "=1, ",
                          lambda, "=0"),
                    paste(beta, "=3, ",
                          delta, "=2, ",
                          theta, "=1.5, ",
                          lambda, "=0.5"),
                    paste(beta, "=4, ",
                          delta, "=2, ",
                          theta, "=1.5, ",
                          lambda, "=1"))),
       horiz=F,
       lty=c(1,1,1,1),
       lwd=c(2,2,2,2),
       col=c("red","blue","green","black"),
       bg="grey96")
```

Survival Plot

```
## ====R-code for the Survival Plot ======

x <- seq(0,10,0.025)
```

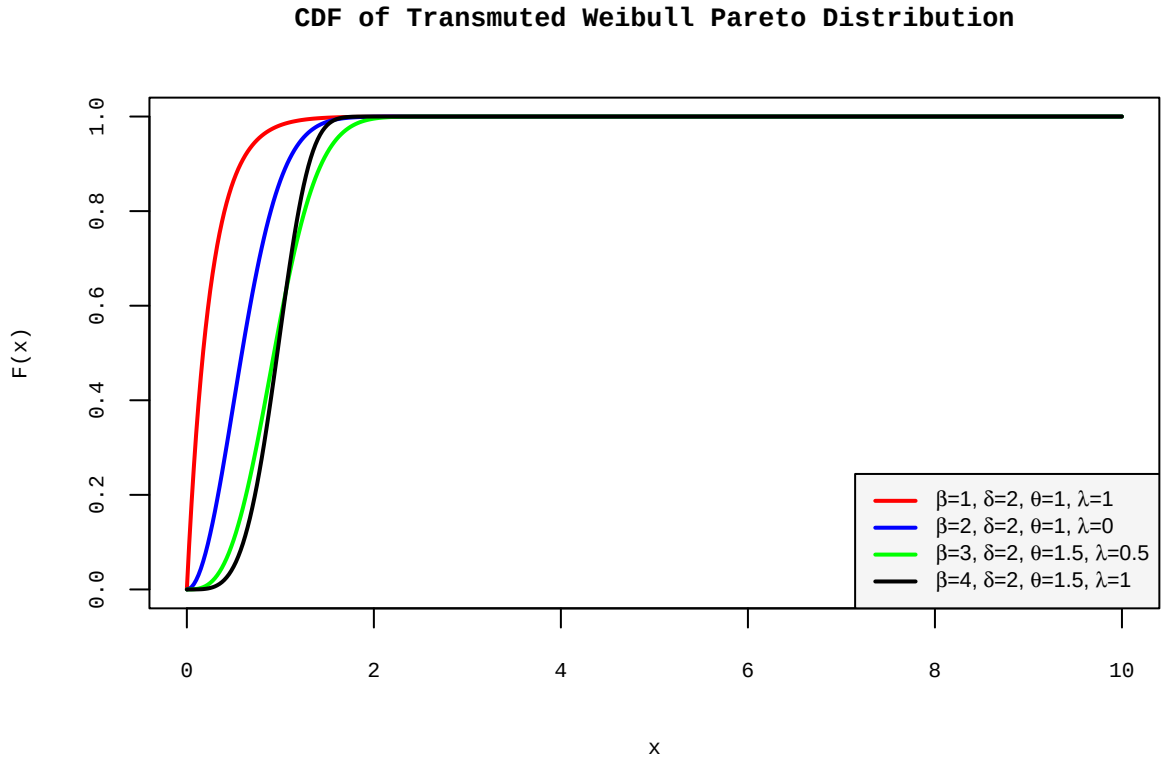


Figure 2: CDF of Transmuted Weibull Pareto Distribution

```r
h <- function(x, beta, delta, theta, lambda){

  (1-((1-exp(-(delta*(x/theta)^beta)))*
        (1+(lambda*(exp(-(delta*(x/theta)^beta)))))))

}

x1 <- h(x,beta=1,delta=2,theta=1,lambda=1)

x2 <- h(x,beta=2,delta=2,theta=1,lambda=0)

x3 <- h(x,beta=3,delta=2,theta=1.5,lambda=0.5)

x4 <- h(x,beta=4,delta=2,theta=1.5,lambda=1)

plot(x, x1,
     main = "Survival Plot of Transmuted Weibull Pareto Dist.",
     ylim = c(0,1),type = "n", xlab = "x",ylab = "S(x)",
     cex.main = 0.8, cex.lab = 0.7,
     family  = "mono", cex.axis = 0.7)

lines(x,x1,col="red",lwd=2,lty=1)

lines(x,x2,col="blue",lwd=2,lty=1)

lines(x,x3,col="black",lwd=2,lty=1)

lines(x,x4,col="green",lwd=2,lty=1)


legend("topright",cex=0.7,
       c(expression(paste(beta, "=1, ",
                          delta, "=2, ",
                          theta, "=1, ",
                          lambda, "=1"),
                    paste(beta, "=2, ",
                          delta, "=2, ",
                          theta, "=1, ",
                          lambda, "=0"),
                    paste(beta, "=3, ",
                          delta, "=2, ",
                          theta, "=1.5, ",
                          lambda, "=0.5"),
                    paste(beta, "=4, ",
                          delta, "=2, ",
                          theta, "=1.5, ",
                          lambda, "=1"))),
       horiz=F,
       lty=c(1,1,1,1),
       lwd=c(2,2,2,2),
       col=c("red","blue","black","green"),
       bg="grey96")
```

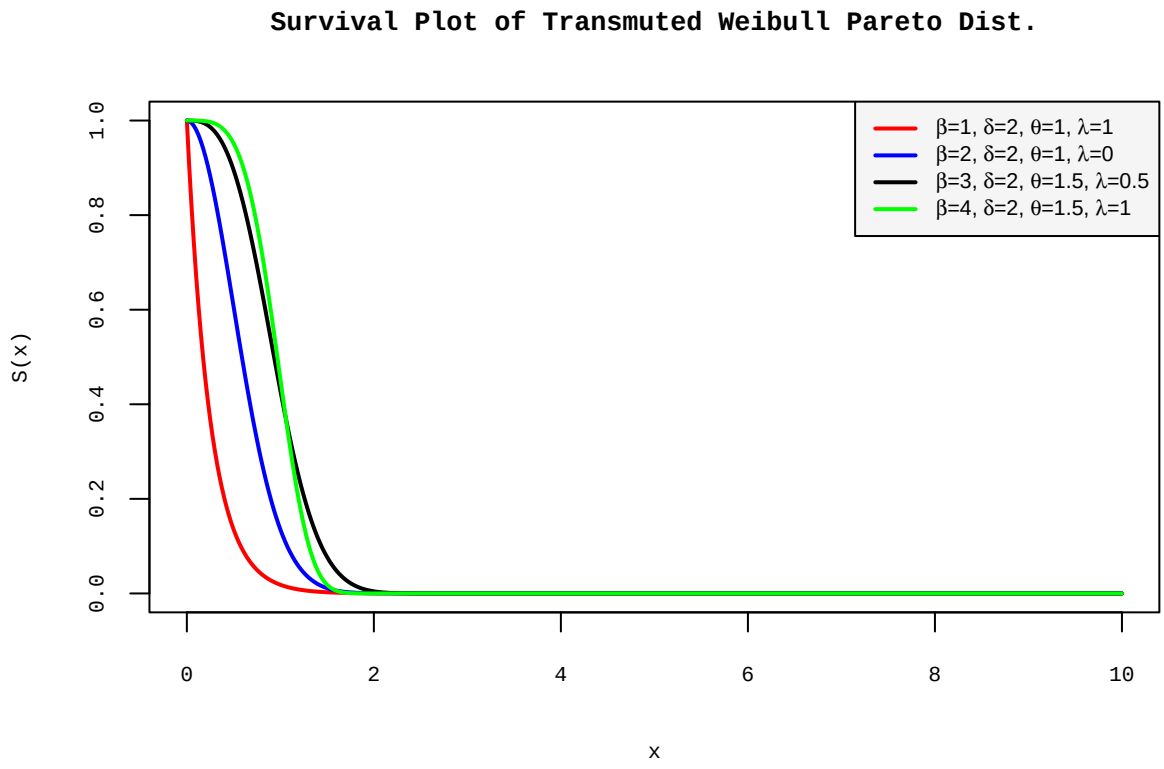
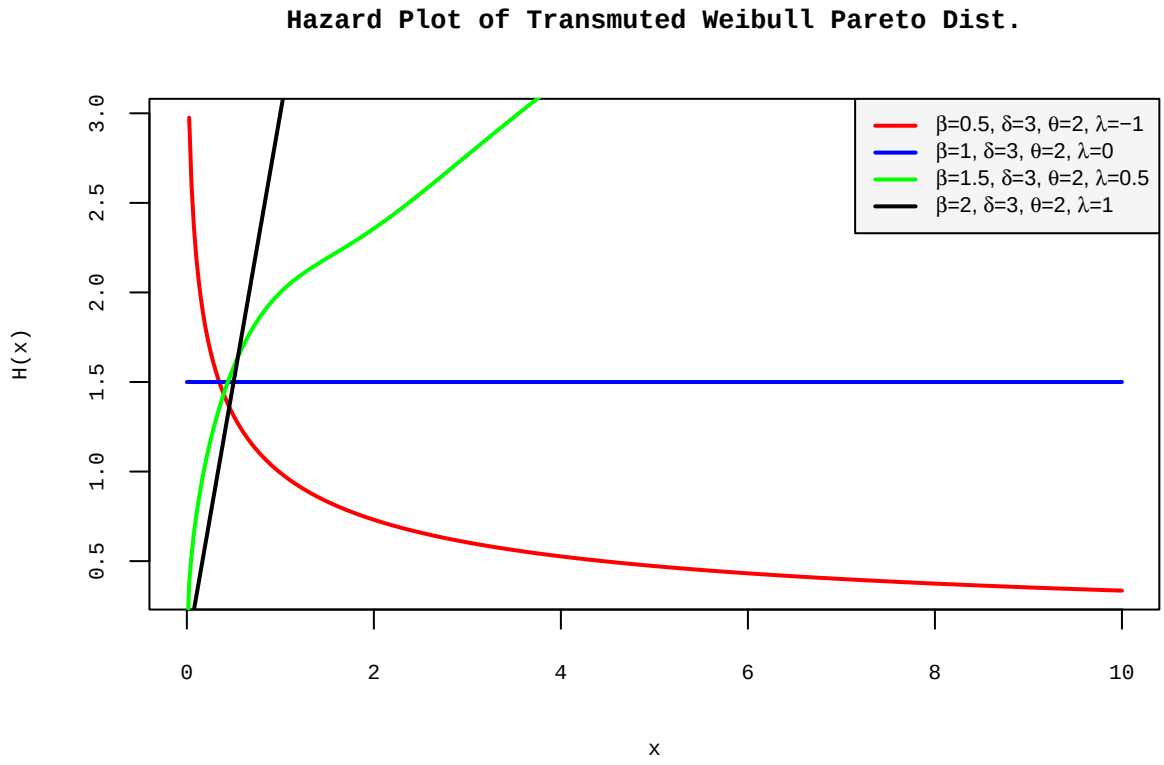


Figure 3: Survival Plot of Transmuted Weibull Pareto Distribution

Hazard Plot

```
## ==== R-code for the Hazard Plot

x <- seq(0,10,0.025)

ha <- function(x,beta,delta,theta,lambda){

  ((beta*delta/theta)*((x/theta)^(beta-1))*
     (1-lambda+2*lambda*exp(-(delta*(x/theta)^beta))))/
    (lambda*exp(-(delta*(x/theta)^beta))+1-lambda)

  }

x1 <- ha(x,beta=0.5,delta=3,theta=2,lambda=-1)

x2 <- ha(x,beta=1,delta=3,theta=2,lambda=0)

x3 <- ha(x,beta=1.5,delta=3,theta=2,lambda=0.5)

x4 <- ha(x,beta=2,delta=3,theta=2,lambda=1)

plot(x,x1, main="Hazard Plot of Transmuted Weibull Pareto Dist.",type="n",xlab = "x",ylab = "H(x)",cex.m
     cex.lab = 0.7,
     family  = "mono", cex.axis = 0.7)

lines(x,x1,col="red",lwd=2,lty=1)

lines(x,x2,col="blue",lwd=2,lty=1)

lines(x,x3,col="green",lwd=2,lty=1)

lines(x,x4,col="black",lwd=2,lty=1)

legend("topright",cex=0.7,
       c(expression(paste(beta, "=0.5, ",
                          delta, "=3, ",
                          theta, "=2, ",
                          lambda, "=-1"),
                    paste(beta, "=1, ",
                          delta, "=3, ",
                          theta, "=2, ",
                          lambda, "=0"),
                    paste(beta, "=1.5, ",
                          delta, "=3, ",
                          theta, "=2, ",
                          lambda, "=0.5"),
                    paste(beta, "=2, ",
                          delta, "=3, ",
                          theta, "=2, ",
                          lambda, "=1"))),
       horiz=F,
       lty=c(1,1,1,1),
       lwd=c(2,2,2,2),
```

```
       col=c("red","blue","green","black"),
       bg="grey96")
```



Maximum Likelihood Estimation

```
## ===R-code for the Maximum likelihood estimation =====

x <- c(1.7,2.2,14.4,1.1,0.4,20.6, 5.3 ,0.7 ,
       1.4 ,18.7, 8.5, 25.5, 11.6, 14.1, 22.1,
       1.1, 0.6, 2.2, 39.0, 0.3, 15.0, 11.0,
       7.3, 22.9, 0.9, 1.7, 7.0, 20.1, 0.4,
       2.8, 14.1, 9.9, 5.6, 30.8, 13.3, 4.2,
       25.5, 3.4, 11.9, 21.5, 1.5 ,2.5 ,27.4,
       1.0, 27.1, 20.2, 16.8, 5.3 ,1.9 ,10.4,
       13.0, 10.7, 12.0, 30.0, 9.3, 3.6, 2.5,
       27.6, 14.4, 36.4, 1.7, 2.7, 37.6, 64.0,
       1.7, 9.7, 0.1, 27.5, 1.1, 2.5, 0.6, 27.0)


loglik <- function(a) {

  beta <- a[1]
  delta <- a[2]
  theta <- a[3]
```

```
  lambda <- a[4]

log_lik <-   n*log(a[1])+n*log(a[2])-n*log(a[3])-(a[2])*
  sum(x/a[3])^a[1]+(a[1]-1)*sum(log(x/a[3]))+
  sum(log(1 - a[4] + 2 * a[4]*exp(-(a[2]*(x/a[3])^a[1]))))

  }

n <- length(x)

mle_estimate <- maxLik(loglik, start = c(3.421, 8.6231, 2.25, -0.4608))

summary(mle_estimate)
```

```
## --------------------------------------------
## Maximum Likelihood estimation
## Newton-Raphson maximisation, 11 iterations
## Return code 3: Last step could not find a value above the current.
## Boundary of parameter space?
## Consider switching to a more robust optimisation method temporarily.
## Log-Likelihood: 649
## 4  free parameters
## Estimates:
##           Estimate    Std. error   t value             Pr(> t)
## [1,]      0.032089      0.003653      8.78 <0.0000000000000002 ***
## [2,]     23.412154      2.446398      9.57 <0.0000000000000002 ***
## [3,]      0.000673           NaN       NaN                  NaN
## [4,] -79754.920145      2.427643 -32852.82 <0.0000000000000002 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## --------------------------------------------
```

Citation

- R Core Team (2021). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/.
- Arne Henningsen and Ott Toomet (2011). maxLik: A package for maximum likelihood estimation in R. Computational Statistics 26(3), 443-458. DOI 10.1007/s00180-010-0217-1.